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THE TWELFTH STREET TRAFFICWAY VIADUCT, KANSAS CITY, MISSOURI*

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With Discussion by Messrs. F. W. Green, L. J. Mensch, Howard W. Holmes, M. M. Upson, E. A. Slettum, L. R. Ash, John Lyle Harrington, and E. E. Howard.

Synopsis.

This paper describes the planning, the design, the equipment for, and methods of, construction, of a double-deck, reinforced concrete, trafficway viaduct, 2 300 ft. long, 60 ft. wide, and 120 ft. high, recently opened for traffic in Kansas City, Mo. Some special applications of general theories involved in the calculations for design are given, and summarized and classified costs are included. The features deemed of particular interest are the design of a large arch forming a portion of the structure, and the arrangement of the contractor's plant and equipment.

For convenience the paper is divided into the following parts:

1.—Introductory and historical.
2.—General description of structure.
3.—Loadings, specifications for, and methods of, design.
A. Investigation of columns.
B. Design of arch.
4.—Specifications for materials and workmanship.
5.—Equipment for, and methods of, construction.
6.—Quantities and costs.
7.—Personnel.

* Presented at the meeting of September 1st, 1915.

The description of features and details which may be grasped readily from an inspection of the accompanying illustrations is omitted.

1.—Introductory and Historical.

The Twelfth Street Trafficway Viaduct forms the principal part of the Twelfth Street Trafficway, a general municipal improvement designed to facilitate usual street traffic between sections of Kansas City, Mo., which are separated topographically. The total cost of the whole improvement, including land damages, was about $1 000 000, the viaduct structure costing about $650 000. The funds were provided by a portion of a general bond issue of $475 000, by assessments against a benefit district of special tax bills of $325 000, and by a contribution of $200 000 from the local street-car company in return for operating privileges over the structure.

The rather flat, level valley of the Kaw River, in Kansas City, locally known as the "West Bottoms", is about 1 mile wide, and is occupied by railroad yards, freight stations, warehouses, packing-houses, wholesale and manufacturing concerns, and the large stock yards interests, including a considerable percentage of the city's business. It lies at an elevation some 150 ft. below the principal residential, retail, and office building districts of the city, the break between the two elevations being rather precipitous bluffs and hills from 100 to 200 ft. high. Twelfth Street is the principal east-and-west thoroughfare of the city, and may be said to be a central axis, both with respect to the up-town part of the city, and the industries in the Bottoms. There have been only three routes for vehicular travel between these levels: the northern routes, requiring a diversion of about ½ mile north of Twelfth Street, and a southern route, requiring a similar diversion of more than 1 mile south. About thirty years ago, a steel viaduct for cable cars was built on Twelfth Street with a grade of about 13%, and car service was maintained until the present construction was commenced.

A roadway suitable for vehicular, as well as street-car, traffic on a viaduct to the top of the hill, on a viaduct to the bluffs, and in a tunnel emerging at some convenient point, and other such arrangements, have been advocated for many years, and have been made a feature of more than one political campaign, and of franchise negotiations with street-railway interests. The improvement finally realized as the Trafficway

extends from Liberty Street to Broadway, a distance of ¾ mile, of which the viaduct occupies about 2 300 ft. The remaining portion consists of earth embankments and cuts, with the usual street improvements on Twelfth Street, and the regrading of certain side streets to intersect its new grade. A street 60 ft. wide is thus provided on a continuous grade of about 5.5% for a distance of 3 500 ft., substantially from end to end of the improvement. It was physically impossible, without involving prohibitory expense and damage to existing structures, to extend the end of the grade in either direction, so the gradient practically fixed itself. To accommodate traffic desiring a less steep grade, and willing, for such advantage, to travel by a less direct route, a roadway on a lower deck is provided, with a grade of about 2.5 per cent. This lower roadway begins one block east of the upper one and extends to the bluff, a distance of 1 800 ft.; thus enabling all classes of traffic to avoid the grade crossings over the heavy service railway tracks on the street below. Ascending roadways or streets alongside the bluff from the end of the lower deck will terminate in streets at the summit of the hill, giving easy grades, although by indirect routes for team traffic bound up town. These connecting streets are not yet built.

Preliminary studies and estimates were made for viaducts of steel and of reinforced concrete; in the latter case, structures of arch spans, and also of girder spans, were considered. The city officials were favorable to the engineers' recommendations of concrete, and the girder type of structure was selected, the final general plan being as shown on Plate XVI. The suspended lower deck over Santa Fé Street, of the final plan, was developed instead of short spans with intermediate columns, to meet the desire of the railways for a clear span the full width of the street.

2.—General Description of Structure.

The Twelfth Street Trafficway Viaduct is a reinforced concrete structure consisting of girder spans supported by columns founded in part on rock, in part on soil, and in the main part on concrete piles. Over eight railway tracks in Santa Fé Street a long span is provided by the construction of an arch. The structure is about 2 300 ft. long, 60 ft. wide, and 120 ft. high at its highest point. The columns are in pairs transversely, and are placed so that the upper deck, which has a total width of 60 ft., cantilevers beyond the columns on each side, and

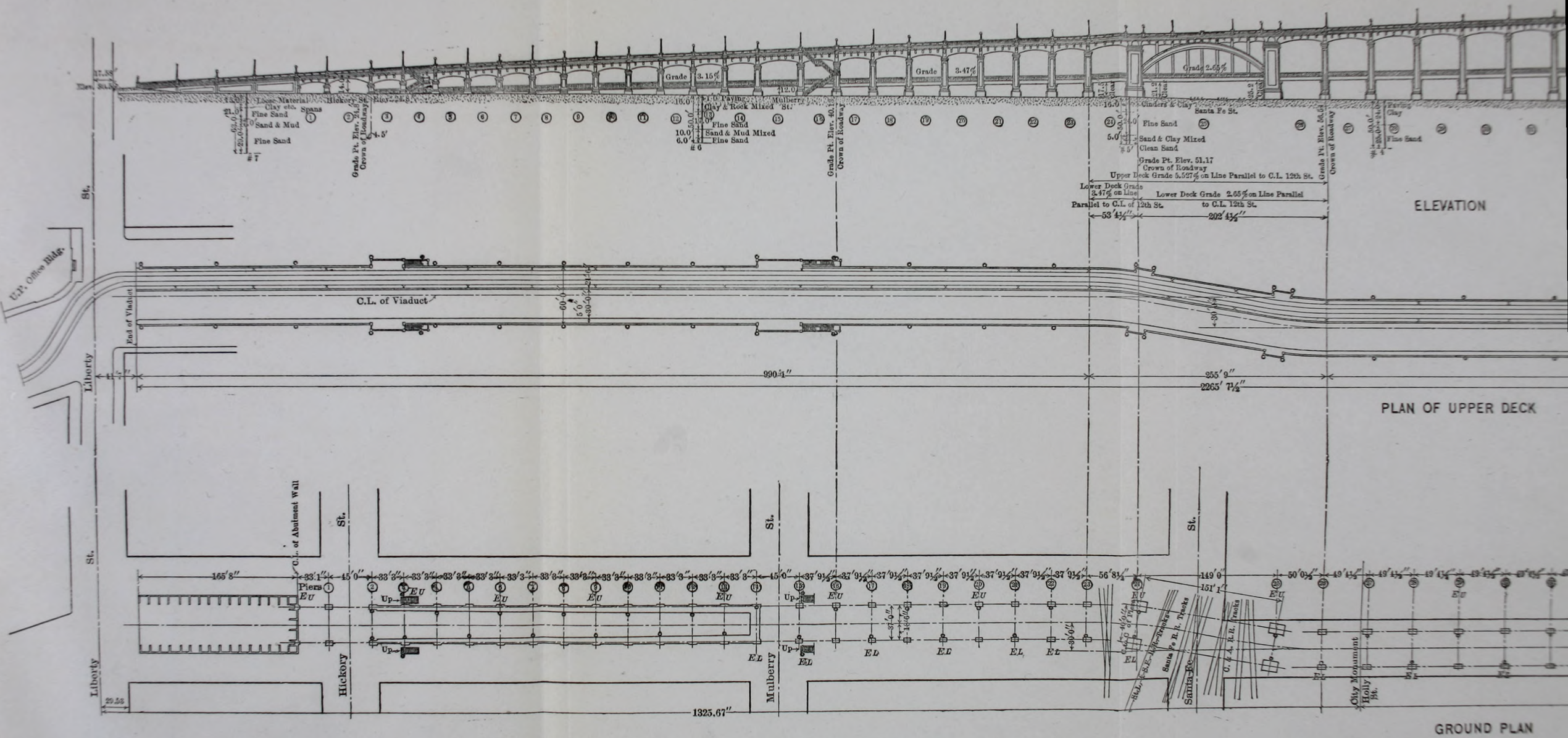
ELEVATION
PLAN OF UPPER DECK
GROUND PLAN
Grade 3.15%
Grade 3.47%
Grade 2.65%
Loose Material
Clay etc.
Spans
Fine Sand
Sand & Mud
Fine Sand
Hickory St.
Mulberry St.
Paving
Clay & Rock Mixed
Fine Sand
Sand & Mud Mixed
Fine Sand
Cinders & Clay
Santa Fe St.
Fine Sand
Sand & Clay Mixed
Clean Sand
Existing Clay
Fine Sand
Grade Pt. Elev. 51.17
Crown of Roadway
Upper Deck Grade 5.527% on Line Parallel to C.L. 12th St.
Lower Deck Grade 2.47% on Line Parallel to C.L. of 12th St.
Lower Deck Grade 2.65% on Line Parallel to C.L. 12th St.
Grade Pt. Elev. 40.0
Crown of Roadway
Grade Pt. Elev. 36.03
Crown of Roadway
U.P. Office Bldg.
C.L. of Viaduct
End of Viaduct
Liberty St.
990½″
255′9″
2265′7½″
C.L. of Abutment Wall
Hickory St.
Mulberry St.
Santa Fe St.
Piers
Up
EU
EL
ED
Santa Fe R.R. Tracks
O. & A. R.R. Tracks
City Monument
Holly St.
1325.67″

PLATE XVI.
TRANS. AM. SOC. CIV. ENGRS.
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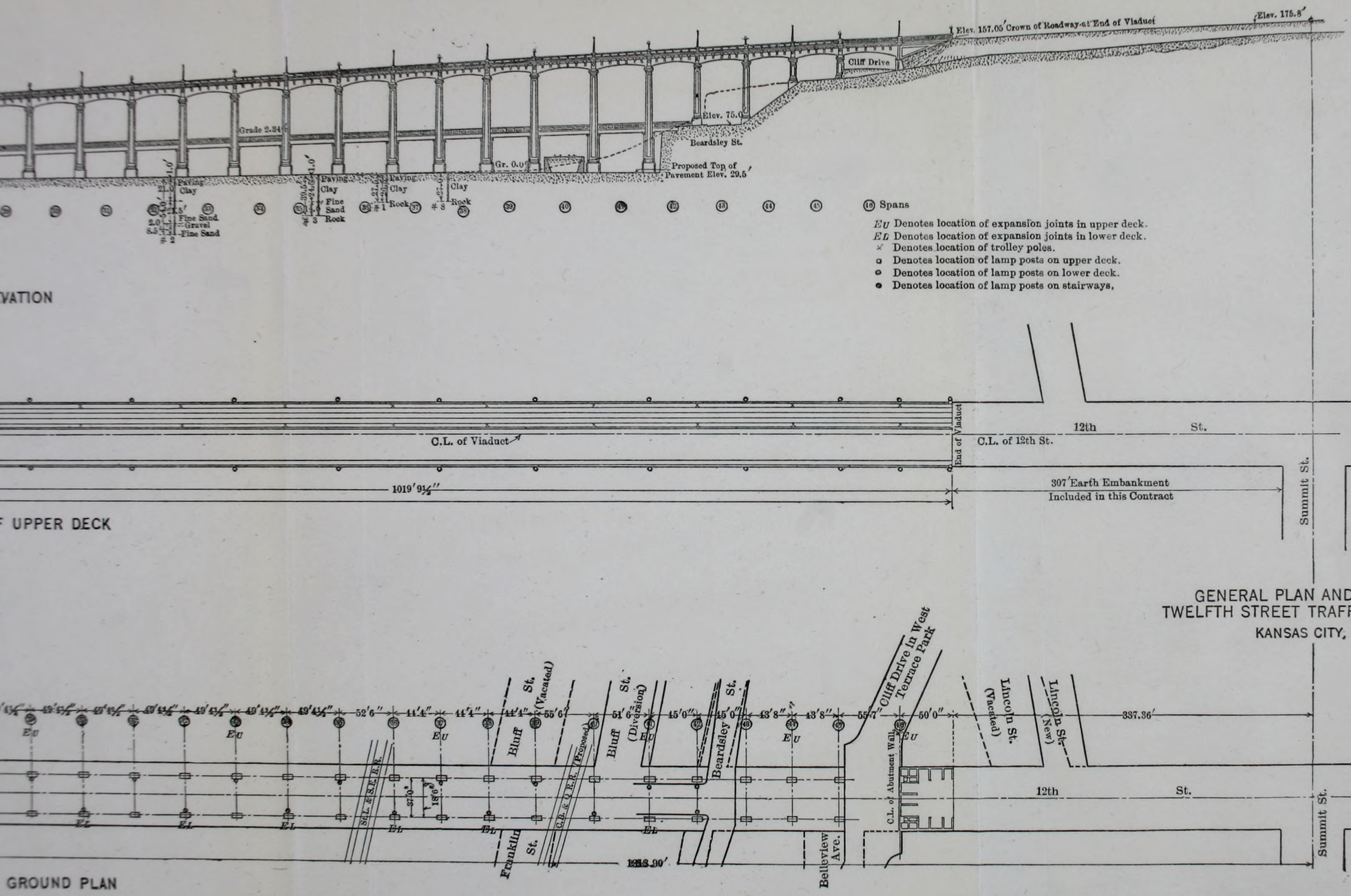
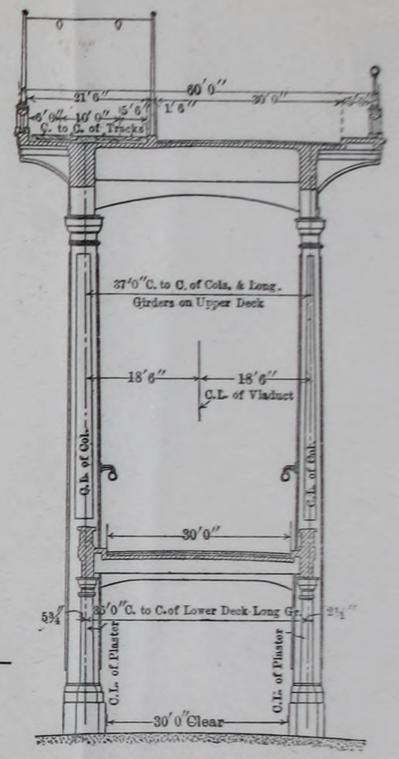
HOWARD ON
TWELFTH ST. TRAFFICWAY VIADUCT,
KANSAS CITY, MO.





TYPICAL CROSS-SECTION

GENERAL PLAN AND ELEVATION
TWELFTH STREET TRAFFICWAY VIADUCT,
KANSAS CITY, MO.

the lower deck roadway passes between them. The upper deck provides a roadway 30 ft. wide paved with creosoted blocks, a sidewalk 5 ft. wide, and a street-car space 22 ft. wide. The roadway is separated from the street-car space by a concrete curb, and on each side of the structure there is a concrete hand-rail. The two street-car tracks are built with the usual wooden cross-ties set in ballast, so that no other traffic can use the area which they occupy. This arrangement, somewhat uneconomical from a highway standpoint, was specified by the city authorities. Iron trolley poles on each side of the street-car space support the usual overhead trolleys. Stairways are provided at Hickory and Mulberry Streets to give access for pedestrians to the roadway from the ground surface. The lower deck, beginning at Hickory Street and ending at Beardsley Street, provides a single roadway, 30 ft. wide, paved with creosoted blocks. The longitudinal girders supporting this deck extend far enough above the roadway to form side barriers.

The upper deck comprises forty-five deck-girder spans of two girders each, varying in length from 33 to about 56 ft., the arch span, and two earth-filled approaches. The lower deck comprises twenty-seven through-girder spans of two girders each, supported on the same columns that carry the upper deck, the suspended deck of the arch span, and the earth-filled approaches. The floor-slabs are supported on cross-girders and cantilever beams. For the crossings of both decks over the streets, near the west end, very shallow floors are necessary, and concrete-encased steel beams in the floors, and concrete-covered steel girders, are used instead of the ordinary reinforcement. Both upper and lower roadways are lighted with incandescent electric lights, placed above the hand-rail for the upper deck, and on brackets on columns for the lower deck.

Below the structure Twelfth Street is about level from Santa Fé Street to the hillside, and gives access to Bluff Street leading northward along the bottom of the bluff. The space transversely between columns at the street level is 30 ft. Near the bluff, passing under the lower deck, the Kansas City Terminal Railway Company has two tracks carried by a concrete girder span above the ground surface of Twelfth Street, making four traffic levels at this point. Higher up on the hillside, just in front of the east abutment, the Kersey Coates Drive of the Park System passes below the upper-deck level.

Twelfth Street from Broadway to Santa Fé Street is 60 ft. wide, but for the three blocks, from Santa Fé Street to Liberty Street, it is only 30 ft. wide. Eleventh Street for these three blocks is 60 ft. wide, and the blocks between Twelfth Street and Eleventh Street are only 40 ft. wide. By securing these three very narrow blocks of property, a single street, 130 ft. wide for a distance of three blocks, was obtained, and the viaduct was placed about midway between, affording an excellent approach and retaining both Twelfth Street and Eleventh Street. This caused a shift in the alignment of about 30 ft., made by two easy curves at Santa Fé Street.

Careful attention was given to the architectural treatment in an effort to secure something more than a plain series of posts and beams. The treatment developed considers the columns as columns with plinths and capitals, and not merely as posts. The bottoms of the upper-deck girders are curved to give an arched or high cambered appearance. The lower-deck girders are straight and in effect are supported by secondary pilasters set out from the main columns. Certain limiting conditions of the railway and street locations crossing under the structure practically determined certain column locations, but the span lengths were made smaller near the lower end of the structure, affording uniformity of appearance and preserving the unity of effect, in spite of the great variation in height of the different columns.

The large arch at Santa Fé Street provides a central feature for the structure and was thus developed. The piers of the arch are carried up to the upper roadway and there support, on cantilevers, bays projecting beyond the usual hand-rail line. The capitals, plinths, mouldings, the projecting panels of the columns, and such features, are bold and heavy, without excess of fine lines, in keeping with the general dimensions of the structure. The architectural appearance has been favorably commented on, and it is believed effects a satisfactory solution of a rather difficult problem of a straight-line structure to be built with all possible economy and without any extra funds for adornment.

3.—Loadings, Specifications for, and Methods of, Design.

Assumed Live Loads.—The following loadings were assumed for design:

1.—For Slabs and Floor Systems:

On street-car tracks, Class 25, consisting of two coupled cars, each 52 ft. long and each weighing 50 tons, with impact allowance loads consisting of percentages of the preceding static loads, determined by the formula,

$$I = \frac{200}{L + 270},$$

where L is the length of track loaded;

On roadways, Class A, with an added impact allowance percentage determined by the formula,

$$I = \frac{100}{L + 150},$$

which, in total, consists of a load varying from 147 lb. per sq. ft., for a loaded length of 100 ft., to 200 lb. per sq. ft.;

On the sidewalk, Class B and impact, a corresponding loading varying from 123 to 167 lb. per sq. ft.

2.—For Girders and Arch Ribs:

On street-car tracks, Class 20, consisting of two coupled 40-ton cars, with impact percentage added;

On the roadways, Class B with impact;

On the sidewalk, Class C with impact, consisting of a load varying from 98 to 134 lb. per sq. ft.

The roadways were also considered to carry a 15-ton road roller, but without impact.

The wind loads figured were 30 lb. per sq. ft. of exposed surface for the unloaded structure, and 15 lb. per sq. ft. of exposed surface for the loaded structure. When considering maximum combined conditions of wind and temperature stresses, the allowable stresses were increased 30%, or as described later.

Allowed Unit Stresses.—For these assumed loadings, the following unit stresses were allowed in the design:

"For steel in tension, 15 000 lb. per sq. in.; for concrete in bending compression, 600 lb. per sq. in., tension zero; for concrete in direct compression, 450 lb. per sq. in., with not more than 400 lb. per sq. in. under expansion plates; for concrete in direct shear, 150 lb. per sq. in.; for concrete in diagonal tension, 35 lb. per sq. in.; for concrete in diagonal tension with part of steel bent up, 50 lb. per sq. in.; for concrete in diagonal tension with full shear reinforcement, 100 lb. per sq. in.; adhesion between concrete and deformed bar, 100 lb. per sq. in."

Designing Rules.—Some of the general rules for proportioning were:

"Column reinforcement shall have hoops spaced not farther apart than fifteen diameters of the vertical bars. The vertical steel shall be not less than 0.8%, nor more than 2.0%, of the minimum section of the column. The ratio of moduli of elasticity shall be taken as 15. The span length shall be taken as the distance from center to center of supports, but not to exceed the clear openings plus the depth of the slab or beam. The width of T-beams shall be considered to be not more than one-fourth of the span. Tension bars shall be spaced apart not less than three diameters of the bar, nor more than the thickness of the slab. If bond stress exceeds 100 lb. per sq. in., the ends of the bars shall be bent into a hook. The distance of the center line of bars to the surface of the concrete shall be not less than 1.3 *d* plus 0.17 in.; or, say, $\frac{3}{4}$ in. for $\frac{3}{8}$-in. bars, 1 in. for $\frac{1}{2}$ and $\frac{5}{8}$-in. bars, $1\frac{1}{2}$ in. for $\frac{7}{8}$ and 1-in. bars, and $1\frac{3}{4}$ in. for $1\frac{1}{8}$ and $1\frac{1}{4}$-in. bars. When stirrups are bent around the bars, allow an extra distance equal to the thickness of the stirrup, when the above distance is 1 in. or less, and equal to one-half the thickness when the above distance is more than 1 inch."

Controlling Conditions and General Methods.—The necessities of the surface street under the structure, from Santa Fé Street east to the bluff, required a transverse spacing of columns which placed one column almost midway under the street-car tracks and the other under the roadway. Thus, the columns, girders, and other parts of one side of the viaduct are much larger and heavier than those of the other side.

All parts of the structure, which could be so constructed, are designed for continuous action. The upper girders were figured as continuous beams for the various combinations of span lengths, assuming a uniform moment of inertia, a condition substantially existing; and the various sections of the girders were proportioned for the figured moments. The lower-deck girders were considered as beams restrained at their ends; the end sections were proportioned for the moment of truly fixed beams, and the center sections for a moment 10% in excess of the actual figured moment for the same condition.

The cross-girders and cantilevers of the upper deck were designed for the usual continuous conditions existing, and the lower-deck cross-girders as simple beams between the supporting girders. Considering longitudinal girders alone, it is easily apparent that the great number of spans of different length, and the varying combinations and num-

bers of spans between expansion points, required rather voluminous calculations for design. It may be of interest to state that the calculations for this structure cover about 300 closely-written, 10 by 16-in. sheets.

Two features of special interest, developed in the design, are the investigation of the columns and the calculation of the arch span at Santa Fé Street.

A.—Investigation of Columns.

The great height of part of the structure and the omission of all diagonal bracing in vertical planes, either transversely or longitudinally, made necessary a thorough investigation of the stresses due not only to every combination of live loads, to wind loads, and to changes in temperature, but especially to the secondary stresses from these loads, due to the rigid connections between the columns and the upper-deck and lower-deck cross-beams and longitudinal girders. It was clearly unnecessary to go to the extreme of attempting to design every separate column by calculating its every possible stress; so that a typical series of columns fairly representing the average was selected, and calculations, as nearly complete as might be, were made for them, in order to ascertain the maximum unit stresses that could be allowed for the direct loads as a simplified basis for column design, which would give results satisfactorily accurate. The essential features of this development are here presented in abbreviated form.

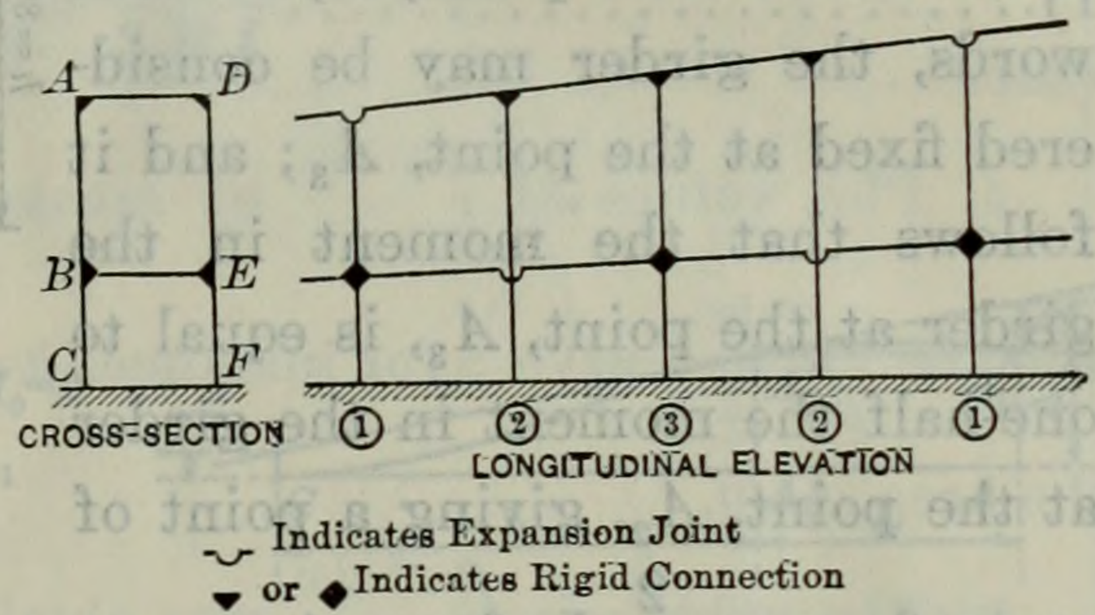


FIG. 1.

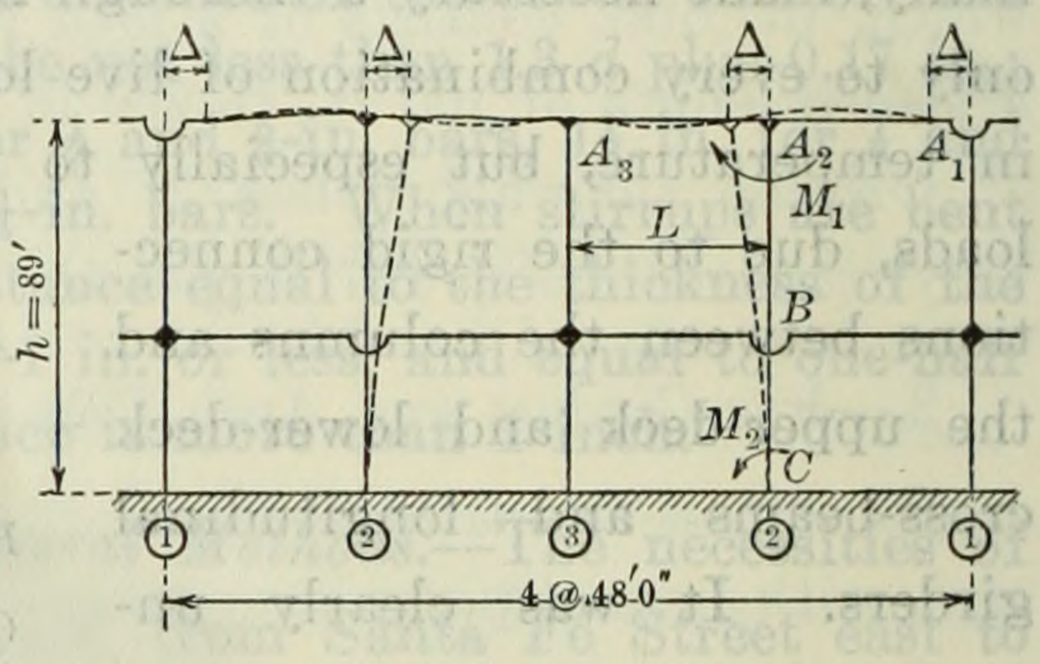
Fig. 1 shows a typical longitudinal section and cross-section of the viaduct. It will be noted that expansion joints are placed at every other column for the lower deck, thus eliminating all temperature stresses in columns, due to the shortening or lengthening of the lower deck. The upper deck has expansion joints generally at every fourth column, these joints staggering with those of the lower deck; all expansion joints for both decks are what may be called "double" expansion joints, the girders on both sides of the column being on sliding plates. The columns numbered (2) are, therefore, the only ones which have

stresses due to temperature changes. A complete investigation of this bent, therefore, was carried through.

To simplify the calculations, it will be assumed that all bents are of equal height and that the dimensions are as shown on Fig. 2.

Under the influence of a fall in temperature, the tops of Columns 2 will move toward the top of Column 3 equal distances, which are called Δ. Assuming that the lower-deck girders rest on frictionless rollers at the points, B, Columns 2 will deflect as if the lower deck were omitted.

Consider the upper girder between Columns 2 and 3; the deflection of Column 2 produces a moment, which will be called M_1, at the point, A_2. As the structure is symmetrical about the point, A_3, the tangent to the deflection curve will remain horizontal at that point, or, in other words, the girder may be considered fixed at the point, A_3; and it follows that the moment in the girder at the point, A_3, is equal to one-half the moment in the girder at the point, A_2, giving a point of contraflexure, $\frac{2}{3} L$, from A_2.



Indicates Expansion Joint
or Indicates Rigid Connection

FIG. 2.

It will simplify matters to substitute for Girder A_3 A_2, a girder which is rigidly connected to the column at A_2 and is freely supported on an expansion bearing at the other end; the length of girder must be such as to give the same moment, M_1, at Column 2. This length will be $\frac{3}{4} L$. It is somewhat more simple to work with a girder length equal to L, so it will be assumed that the girder has a length, L, and that its moment of inertia, I, is increased to $\frac{4}{3} I$.

Evidently, the moments in the column will be unchanged if, for the two girders connecting to the column at A_2, one girder is substituted having a moment of inertia equal to their sum; the moment of inertia of this one girder will be taken, therefore, as $\frac{4}{3} I + I = \frac{7}{3} I$, where I is the moment of inertia of the actual girder.

As the columns are battered from top to bottom on all faces, there is a considerable variation in moment of inertia throughout their

lengths. Any formulas that are applied for the solution of the desired moments, M_1 and M_2, must take into account this variation in moment of inertia of the column. Consider the column and attached girder as a beam on three supports having moments, M_0, M_1, and M_2, at the three supports and having a straight-line moment variation, as indicated in Fig. 3; the beam is assumed to be fixed at C.

Fig. 4 shows the beam as deflected by the moments.

Bearing in mind that the deflection of any point, as A_1, with respect to a tangent at A_2, is equal to the moment about A_1 of the area under the $\frac{M}{E\,I}$ curve, we can write the following equations:

$$\delta_1 = \int \frac{M\,x\,d\,x}{E\,I}.$$

But, $M = \frac{M_1\,x}{L}$, and I is constant for this span, therefore, we have

$$\delta_1 = \frac{M_1\,L^2}{3\,E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

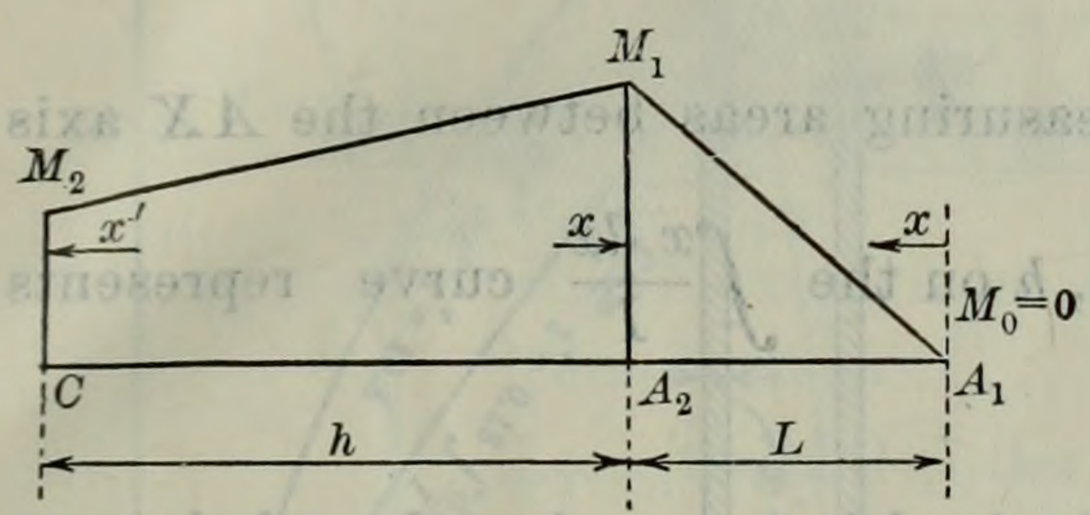


FIG. 3.

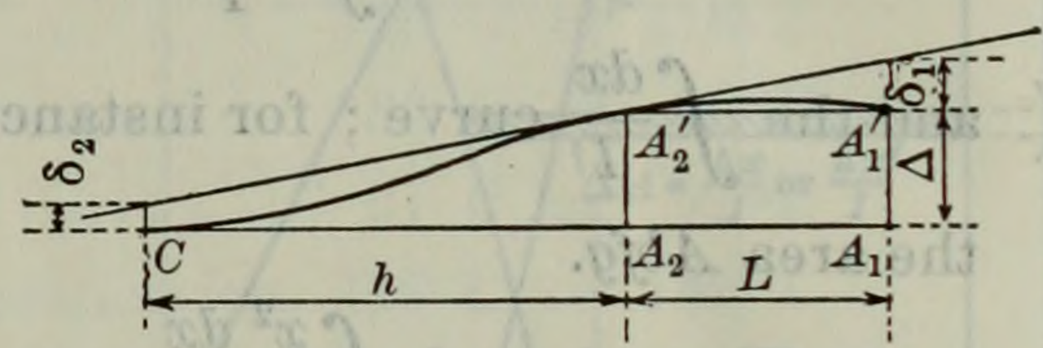


FIG. 4.

similarly, $$\delta_2 = \frac{M_2}{E}\int\frac{x'\,dx'}{I} + \frac{M_1}{E\,h}\int\frac{x'^2\,dx'}{I} - \frac{M^2}{E\,h}\int\frac{x'^2_1\,dx'}{I} \quad \ldots (2)$$

We have also the relation, $\frac{\delta_1}{L} = \frac{\Delta - \delta_2}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

We can eliminate the unknown quantities, δ_1 and δ_2 from Equations (1), (2), and (3), and have the following resulting equation:

$$\frac{M_1}{E}\left(\frac{L}{3\,E\,I} + \frac{1}{h^2}\int\frac{x'^2\,dx'}{I}\right) + \frac{M^2}{E}\left(\frac{1}{h}\int\frac{x'\,dx'}{I} - \frac{1}{h^2}\int\frac{x'^2\,dx'}{I}\right) = \frac{\Delta}{h}. (4)$$

A second equation involving only the unknown M_1 and M_2 can be written by considering the deflection of A_2 with respect to the tangent at C, as follows:

$$\frac{M_1}{E}\left(\frac{1}{h}\int\frac{x\,dx}{I} - \frac{1}{h^2}\int\frac{x^2\,dx}{I}\right) + \frac{M_2}{E}\left(\frac{1}{h^2}\int\frac{x^2\,dx}{I}\right) = \frac{\Delta}{h} \ldots (5)$$

Numerical substitution in Equations (4) and (5) results readily in a solution of values of M_1 and M_2, the desired moments.

It might be of interest to explain the method used in ascertaining the values of the integrals, $\int \frac{x\,dx}{I}$, $\int \frac{x^2\,dx}{I}$, etc.

These integrals are plotted to scale on Fig. 5. The process of plotting is as follows:

1. Plot values of I, b represents the bottom of the girder—the deformation between A_2 and b was neglected, that is, I was assumed to be infinite.

2. Plot values of $\frac{1}{I}$.

3. Plot values of $\int \frac{dx}{I}$; to locate a point of the $\int \frac{dx}{I}$ curve, measure the area between the $\frac{1}{I}$ curve, the line, $A\,C$, and the ordinate at the point; for instance, f on the $\int \frac{dx}{I}$ curve represents the area, $bcde$.

4. Plot values of $\int \frac{x\,dx}{I}$ by measuring areas between the AX axis and the $\int \frac{dx}{I}$ curve; for instance, h on the $\int \frac{x\,dx}{I}$ curve represents the area $Abfg$.

5. The value of $\int \frac{x^2\,dx}{I}$ was obtained by measuring the whole area between the $\int \frac{x\,dx}{I}$ curve and the AX axis.

The differential areas are shown on Fig. 5 and make clear, without further explanation, the correctness of these methods.

Stresses Due to Wind Loads.—In calculating these stresses, we have a 2-story framed bent to consider; the fact that the columns have a variable moment of inertia complicates the problem, and it is necessary to use formulas taking this into account. Further complication results from the fact that the two columns in the bent are unlike, with different elastic properties.

To solve the problem and ascertain the desired moments in the columns and cross-girders, the theorem of three moments is applied.

The bent under consideration is shown in Figs. 6 and 7.

Under the influence of the wind loads, H_1 and H_2, the frame will deflect in some such manner as that shown in Fig. 7. In this diagram

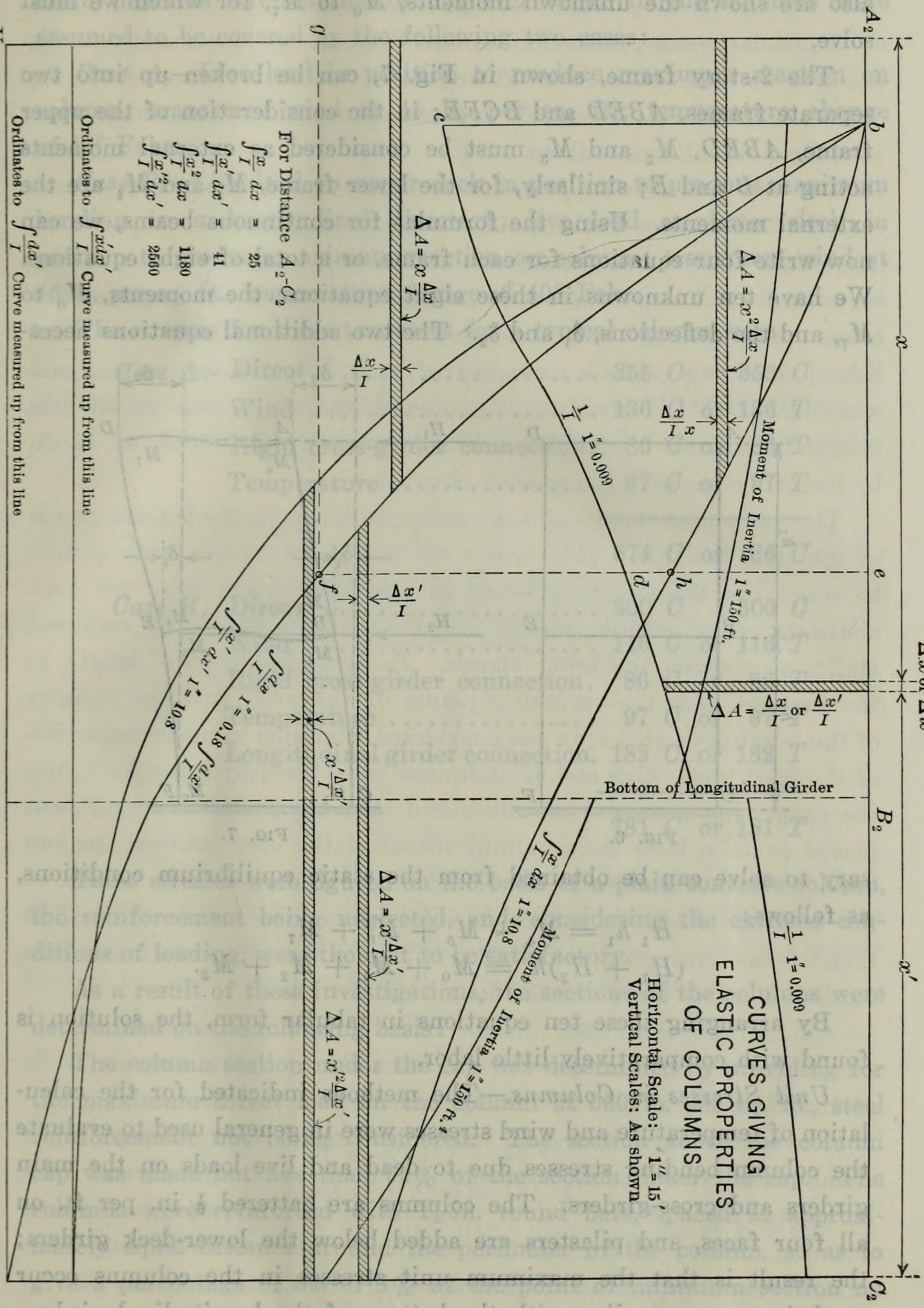

CURVES GIVING
ELASTIC PROPERTIES
OF COLUMNS
Horizontal Scale: 1″ = 15′
Vertical Scales: As shown
Moment of Inertia I″ = 150 ft.
Moment of Inertia I″ = 10.8
Bottom of Longitudinal Girder
For Distance A_2-C_2
$\int \frac{x}{I}\,dx$ = 25
$\int \frac{x'}{I}\,dx'$ = 41
$\int \frac{x^2}{I}\,dx$ = 1180
$\int \frac{x'^2}{I}\,dx'$ = 2560
Ordinates to $\int \frac{x\,dx'}{I}$ Curve measured up from this line
Ordinates to $\int \frac{dx'}{I}$ Curve measured up from this line

FIG. 5.

also are shown the unknown moments, M_0 to M_7, for which we must solve.

The 2-story frame, shown in Fig. 7, can be broken up into two separate frames, *ABED* and *BCFE*; in the consideration of the upper frame, *ABED*, M_2 and M_3 must be considered as external moments acting at *B* and *E*; similarly, for the lower frame, M_5 and M_4 are the external moments. Using the formulas for continuous beams, we can now write four equations for each frame, or a total of eight equations. We have ten unknowns in these eight equations, the moments, M_0 to M_7, and the deflections, δ_1 and δ_2. The two additional equations neces-

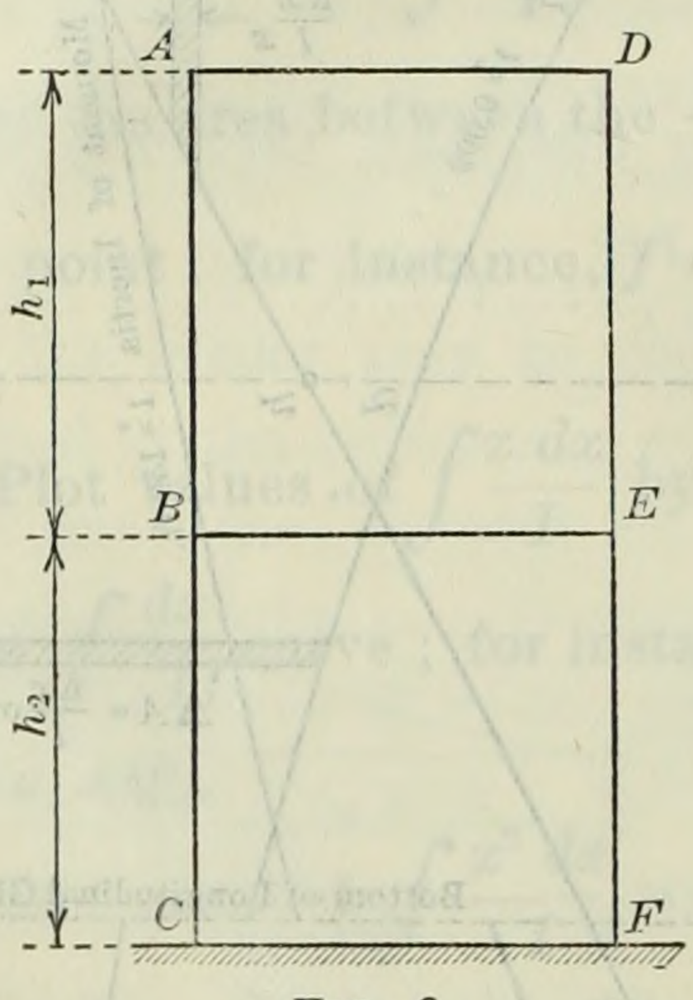


FIG. 6.

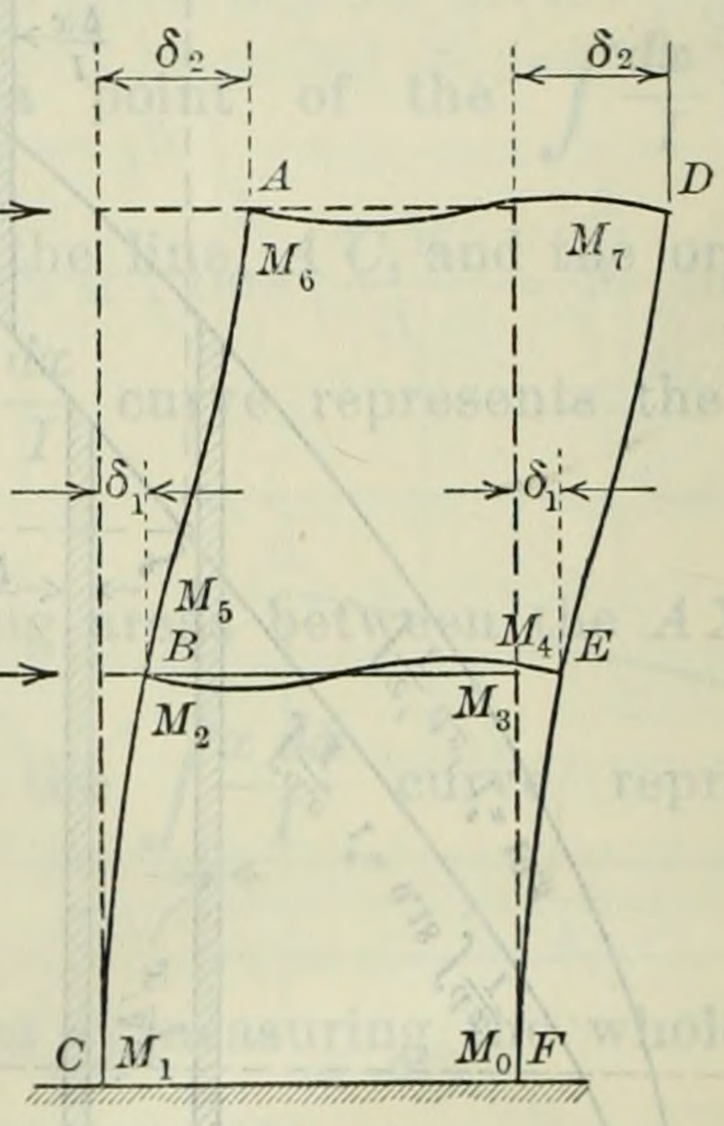


FIG. 7.

sary to solve can be obtained from the static equilibrium conditions, as follows:

$$H_1 h_1 = M_4 + M_5 + M_6 + M_7$$
$$(H_1 + H_2)h_2 = M_0 + M_1 + M_2 + M_3.$$

By arranging these ten equations in tabular form, the solution is found with comparatively little labor.

Unit Stresses in Columns.—The methods indicated for the calculation of temperature and wind stresses were in general used to evaluate the column bending stresses due to dead and live loads on the main girders and cross-girders. The columns are battered $\frac{1}{8}$ in. per ft. on all four faces, and pilasters are added below the lower-deck girders; the result is that the maximum unit stresses in the columns occur at their tops, on a line with the bottom of the longitudinal girders. Various probable combinations of loading were considered; the maxi-

mum probable stresses that would ever occur in the columns were assumed to be covered by the following two cases:

Case A.—Live load in position to produce maximum reaction on column; transverse wind at 15 lb. per sq. ft.; temperature change of 40° Fahr.

Case B.—Live load on upper deck in position to produce maximum bending in a longitudinal direction; live load on lower deck in position to produce maximum reaction on column; transverse wind at 15 lb. per sq. ft.; temperature change of 40° Fahr.

The stresses for these two cases for a typical column were:

Case A:	Direct	355 *C*		355 *C*
	Wind	136 *C*	or	136 *T*
	Rigid cross-girder connection	86 *C*	or	86 *T*
	Temperature	97 *C*	or	97 *T*
		674 *C*	or	36 *C*
Case B:	Direct	300 *C*		300 *C*
	Wind	116 *C*	or	116 *T*
	Rigid cross-girder connection	86 *C*	or	86 *T*
	Temperature	97 *C*	or	97 *T*
	Longitudinal girder connection	182 *C*	or	182 *T*
		781 *C*	or	181 *T*

These stresses were figured on the basis of a plain concrete column, the reinforcement being neglected, and, considering the extreme conditions of loading, were thought to be satisfactory.

As a result of these investigations, the sections of the columns were determined on the following basis:

The column section under the cap was determined by providing for the maximum direct load on the column at 350 lb. per sq. in., steel reinforcement not being considered. The section above the column cap was made not less than 92% of the section under the cap. The columns were reinforced with $1\frac{1}{4}$-in. round bars, spaced at approximately equal distance around the perimeter of the column, so as to give a percentage of 0.8 of 1% at the point of minimum section on the heavy column of any bent. The steel for the light columns was then determined by using the same spacing as for the heavy column.

B.—Design of Arch.

Other designing problems of special interest are in connection with the arch at Santa Fé Street. This span has a clear opening of 130 ft., and consists of two arch ribs transversely spaced so that the lower roadway passes between them.

The two arch ribs support spandrel columns which, in turn, carry cross-girders of the usual type and the cantilevers of the upper deck. The lower deck is suspended from the arch ribs. Because of the very limited distance between the surface of the roadway on the lower deck and the clearance required above the railroad tracks below, the lower deck is composed of steel I-beams encased in, and carrying a slab of, concrete. These cross-beams rivet into continuous longitudinal steel beams which are supported at intervals of 17 ft. by hangers composed of two steel channels.

The horizontal thrusts of the arches are taken by the bottom chords of the steel eye-bars. This avoids the use of abutments large enough to resist the thrust—there being in fact insufficient room for such abutments—and affords several advantages because of the resulting ability to shorten the lower chords. Because of the short length of the hangers near the springing points of the arch, and the rigidity of the lower-deck girder, it was considered advisable to make the arches of the fixed type. One end of each arch rib is fixed by being built into the supporting pier, and the other end rests on a tandem rocker bent placed so as to keep the resultant thrust of the arch between the two bearings for every condition of loading. The springing points of the arch are relatively on the same grade as the lower roadway, so that the east springing point is 3.5 ft. higher than the west one. This condition causes some unusual longitudinal thrusts.

All the structural steel members of the span are encased in concrete, and the smaller ones are surrounded by light concrete walls giving dimensions for satisfactory architectural appearance.

The arches were designed under the usual elastic theory. The calculations were carried through for both unit vertical and unit horizontal loads at the various points, vertical loads in this instance meaning those perpendicular to, and horizontal loads meaning those parallel to, the arch axis. These latter loads include those due to thrust from the street-car loading, as well as the components of the dead and live loads caused by the grade.

The maximum stresses were calculated at each springing point, at the crown, and at seven intermediate points on each half of the ring—a total of seventeen different points. All calculations were carried through in appropriate tabular form, and are hardly of sufficient interest to warrant their inclusion in this paper. The influence lines for maximum moments at various points, due both to vertical and to horizontal loads, were plotted and are shown on Plate XVII.

Considerable experience in actual designs recommends highly the influence-line methods for a check on calculations. Errors seem to be invariably made evident by the failure of the various ordinates to any influence line to show a smooth curve. In the diagrams, it will also be noted that the apices of the various moment curves themselves lie on a smooth curve, a further check on the numerical work.

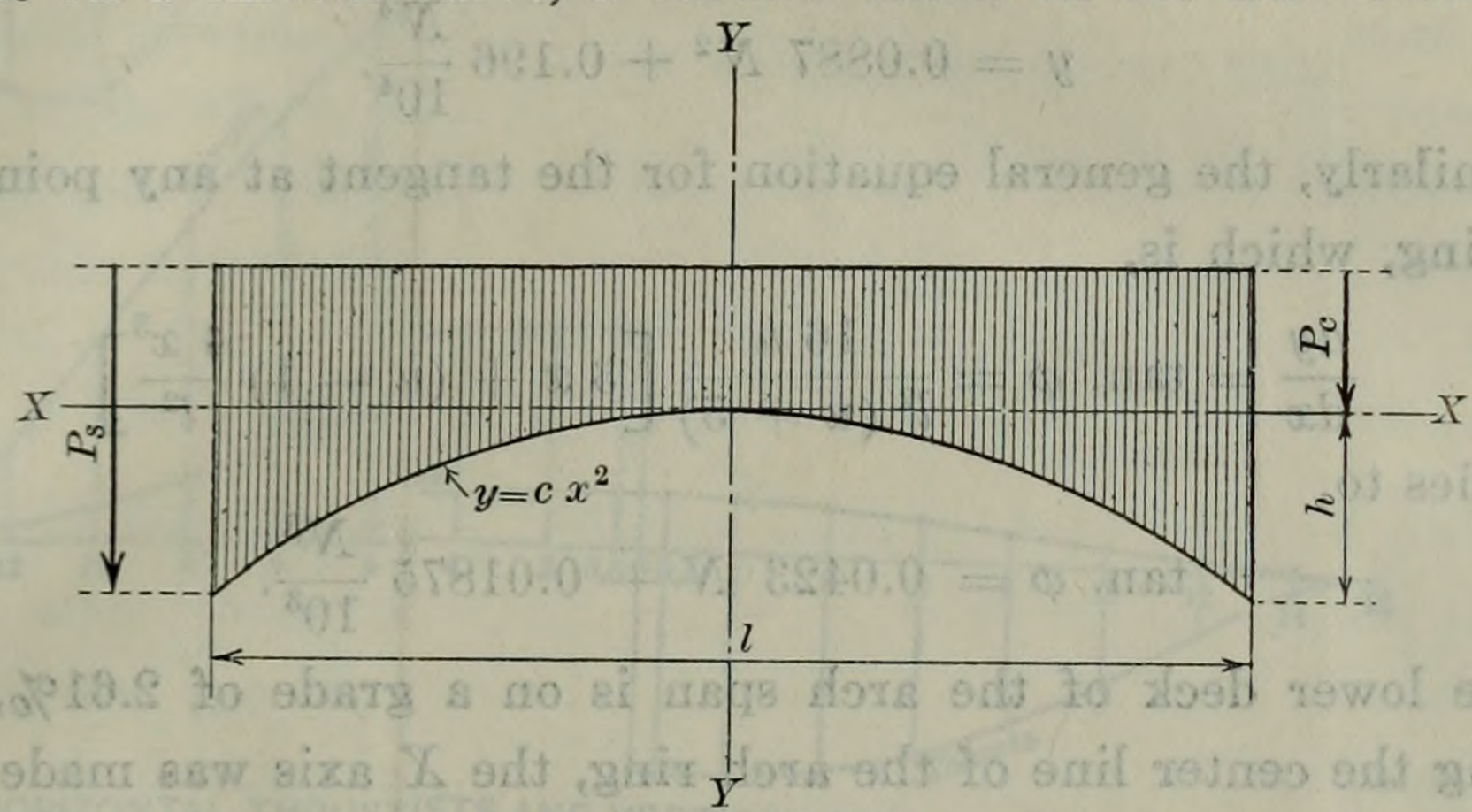


FIG. 8.

It was assumed that the intensity of loading on the arch ring, including the weight of the ring itself, would vary as the ordinates of a parabola, a condition of loading shown by Fig. 8; and the center line of the arch ring was located in the following manner:

Let P_c = equivalent uniform load at the crown;

P_s = equivalent uniform load at the springing;

$$u = \frac{P_s}{P_c};$$

h = rise of arch ring = 24 ft.;

l = span of arch ring = 134 ft.

Then, the equation of the funicular polygon results as follows:

$$y = \frac{4\,h}{l^2\,(u+5)}\left[6\,x^2 + (u-1)\,\frac{4\,x^4}{l^2}\right].$$

This equation was used as the equation for the center line of the arch ring, using dead load plus one-half live load in substituting

the numerical values of P_c and P_s. This substitution gave slightly different values of u for the two arch rings; and therefore an average value of $u = 1.34$ was used.

The equation was further simplified for use by dividing each half of the arch ring into sixteen equal horizontal sections, and numbering the division points successively from the crown to the springing.

Let N = number of any division point;
= 0 at crown;
= 16 at springing;

Then, $X = \frac{N\,l}{32}$.

Substituting the values of u, h, l, and x in the original equation, we have the following equation for the center line of the arch ring:

$$y = 0.0887\ N^2 + 0.196\ \frac{N^4}{10^4}.$$

Similarly, the general equation for the tangent at any point of the arch ring, which is,

$$\frac{dy}{dx} = \tan.\ \phi = \frac{16\ h}{l^2\ (u + 5)} \left[3\ x + (u - 1)\ \frac{4\ x^3}{l^2} \right]$$

simplifies to,

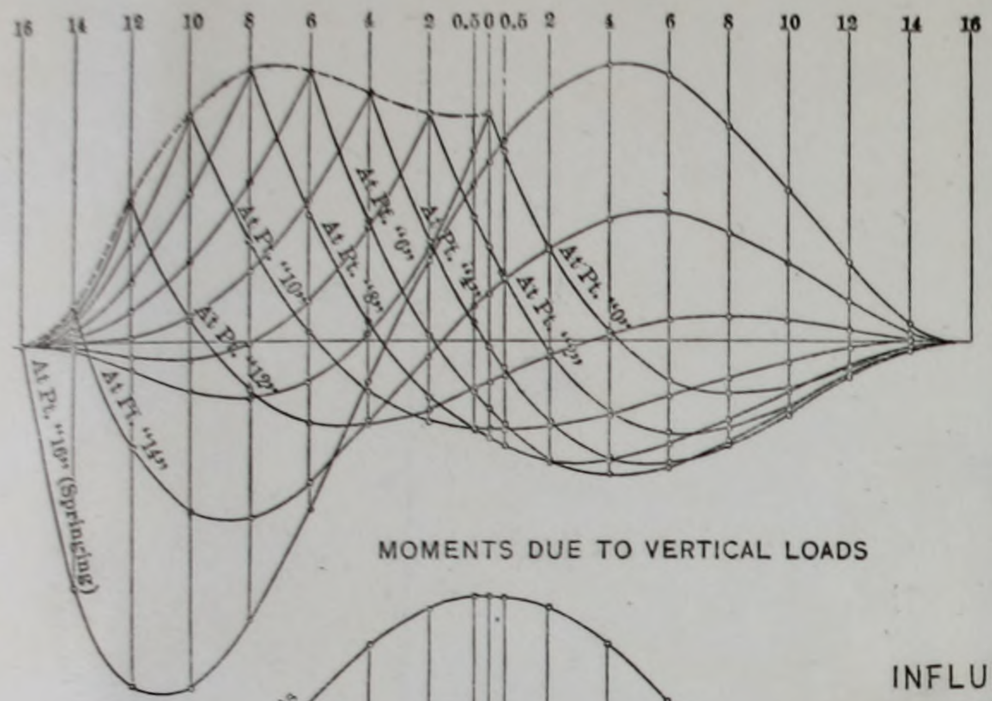
$$\tan.\ \phi = 0.0423\ N + 0.01875\ \frac{N^3}{10^3}.$$

The lower deck of the arch span is on a grade of 2.61%, and in locating the center line of the arch ring, the X axis was made parallel to the grade and the Y axis perpendicular to it. As the hangers supporting the lower deck are vertical, it is evident that the dead loads are not applied symmetrically with respect to the crown of the arch ring. For purposes of calculation, it was assumed that the loads were applied symmetrically; this resulted in an error of less than 1%, which was considered negligible.

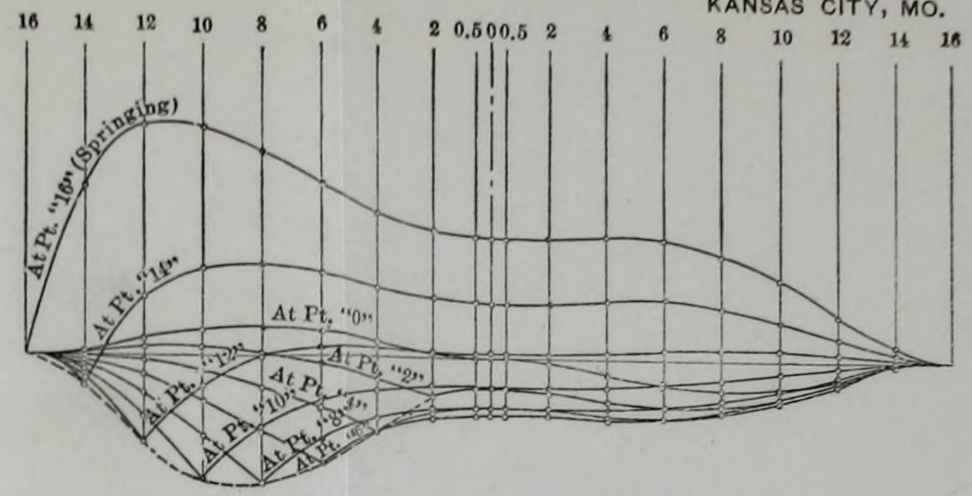
The use of the bottom chords of the eye-bars to take the thrust eliminates all temperature stresses in the arch rings, with the exception of those due to a possible variation in temperature between the arch ring and the bottom chord, which were considered negligible.

Toggle arrangements to shorten the lower chords were provided to accomplish the following:

1.—To take up the stretch in the bottom-chord eye-bars, due to dead load, the result being to leave the dead-load stresses in the arches the same as for fixed arches supported on rigid abutments.

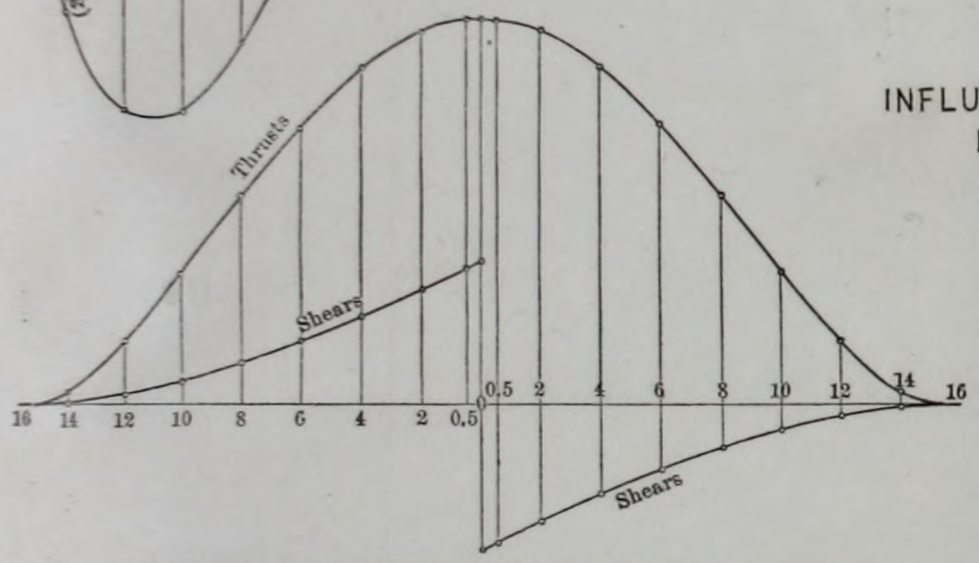


MOMENTS DUE TO VERTICAL LOADS

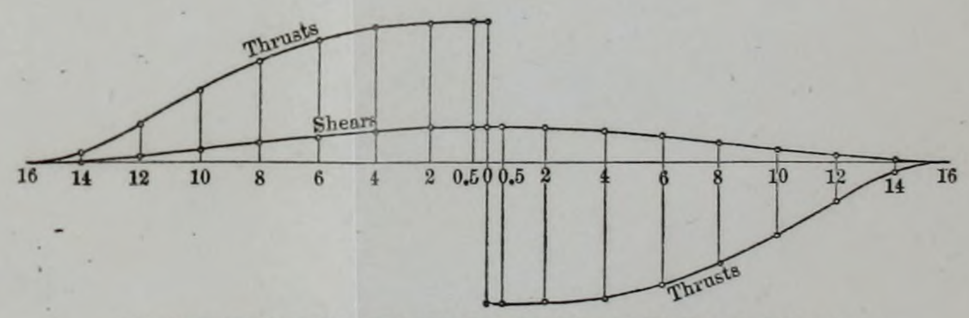


MOMENTS DUE TO HORIZONTAL LOADS

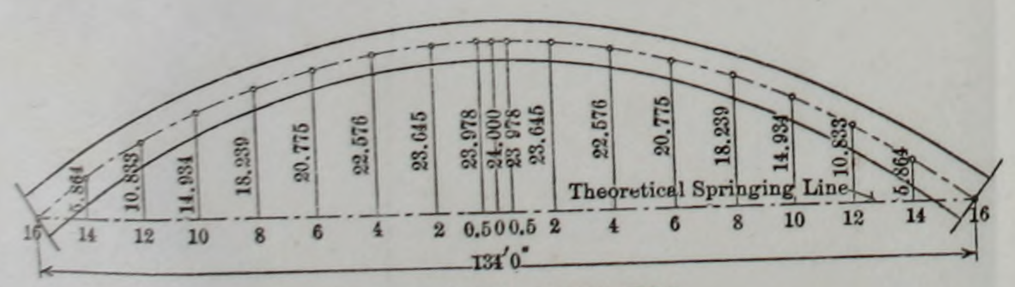
INFLUENCE LINE DIAGRAMS
FOR ARCH SPAN



HORIZONTAL THRUSTS AND VERTICAL SHEARS AT CROWN
DUE TO VERTICAL LOADS



HORIZONTAL THRUSTS AND VERTICAL SHEARS AT CROWN
DUE TO HORIZONTAL LOADS



ARCH RIB WITH ORDINATES

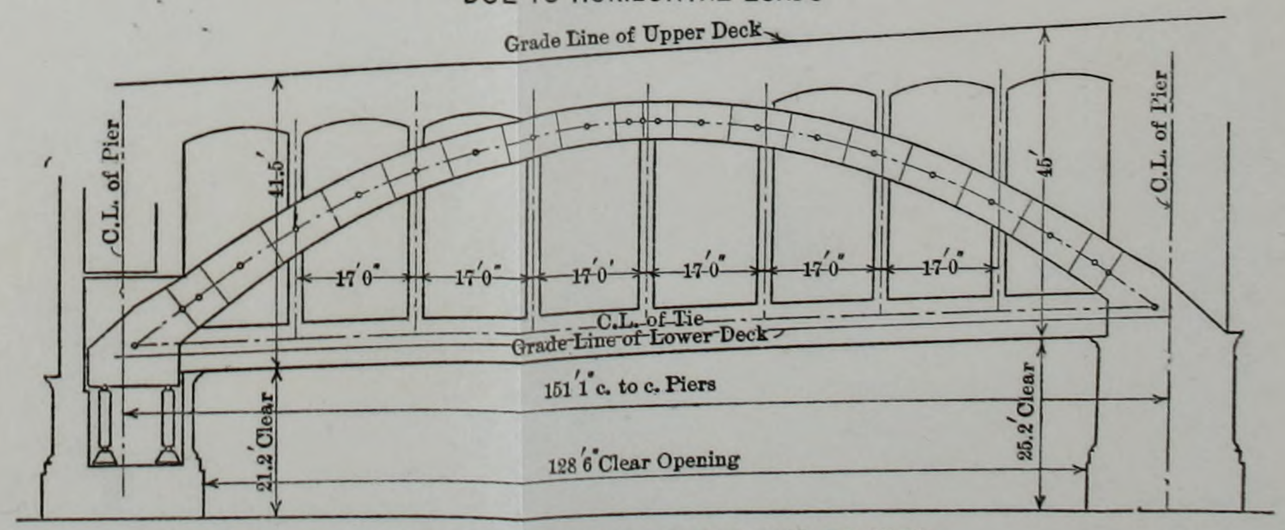


DIAGRAM OF GENERAL DIMENSIONS OF ARCH
SHOWING ASSUMED DIVISIONS OF ARCH RING

2.—To shorten the bottom chords by amounts sufficient to eliminate the stresses in the arch rings due to their shortening under dead load.

3.—To shorten the bottom chords by the amounts which they would stretch, and the arch ribs would shorten, under full live load, over one-half the span.

As indicated by the influence lines for maximum moments at various points of the arch ring, the loaded length for a maximum moment is generally about one-half the span length; the result of shortening the bottom chord as indicated under Paragraph 3 is, therefore, to give practically the same maximum stresses in the arch ring as would result in a fixed arch supported on abutments in the ordinary way, with the further advantage that the stresses due to rib shortening are eliminated.

All these provisions required a total shortening of the bottom chords by the toggles of $1\frac{1}{16}$ in., which meant that the distance between the springing points, after the dead load was placed on the arches, was $\frac{3}{8}$ in. shorter than when the arches were constructed on the falsework.

4.—Specifications for Materials and Workmanship.

The specifications were prepared with a view to letting a unit-price contract after competitive bidding. Every effort was made to have them full and precise in description, both of materials and of workmanship, eliminating as far as possible the uncertain characterization of work "satisfactory to the Engineer" by expressing, wherever feasible, the views of the engineers in advance, and attempting to make perfectly clear to a prospective builder just what would be required of him. Though definite and explicit in the description of results desired, the specifications left open as far as possible the methods by which such results were to be secured, in order that the City might receive the full benefit of any contractor's skill and ingenuity in laying out and handling the work, and enjoy a portion of the economies thus effected.

Approximate quantities of various units of the structure were prepared and tabulated by the engineers. The City received bids on a unit basis of price per cubic yard for concrete in various divisions of the structure, price per pound for reinforcing material, for structural steel, etc., and the total bids were compared on the basis of the approximate quantities.

Concrete.—The specifications required the concrete of the structure to be made of 1 part cement, of detailed requirements, conforming in most respects to the recommendations of the American Society for Testing Materials; of 2 parts sand, "particles of hard, clean stone in graded sizes which shall pass a sieve having holes ¼ in. square, and not less than 50% be retained on a No. 30 sieve"; and 4 parts broken stone, "pieces of hard and durable rock entirely free from clay, loam, or any other material than sound, hard stone—of crusher-run sizes—with all material that will pass a ¼-in. sieve removed", to pass a 1¼-in. ring.

For the hand-rails, stair risers, and treads, a proportion of 1 part cement, 1½ parts sand, and 3 parts broken stone, to pass a ¾-in. ring, was required. The proportions are stated to be by volume, with the ingredients measured loose, one 380-lb. bbl. of cement being taken as 4 cu. ft. The upper ¾ in. of the sidewalk slab is of mortar of 1 part cement to 1½ parts sand placed before the supporting slab had set. The use of a mechanical batch mixer is specified, to be operated long enough after the last ingredient is deposited in it thoroughly to mix and incorporate all ingredients.

Reinforcement.—The reinforcement is of corrugated bars, "deformed bars having their corrugations at right angles to the length of the bar" having been required, rolled from open-hearth billets of medium steel having an ultimate strength of from 60 000 to 75 000 lb., and a minimum elastic limit of 35 000 lb., and affording an elongation of not less than 22% in 8 in.

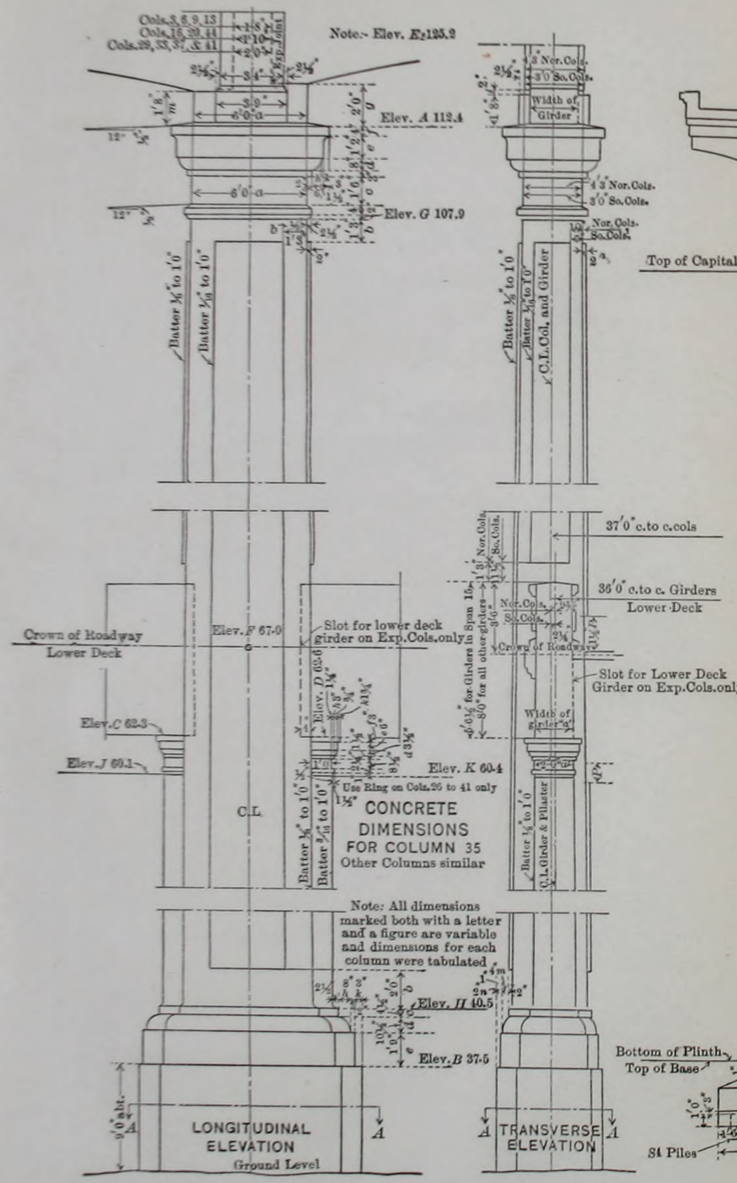
The specifications provide that the "reinforcement in the finished structure shall conform accurately in size and position to the requirements of the plans. Bars shall be firmly bound and tied together by wire where they lap or cross, or be fastened by clips or other devices. Each piece must be held rigidly and positively in position, so that there will be no displacement during the depositing of concrete".

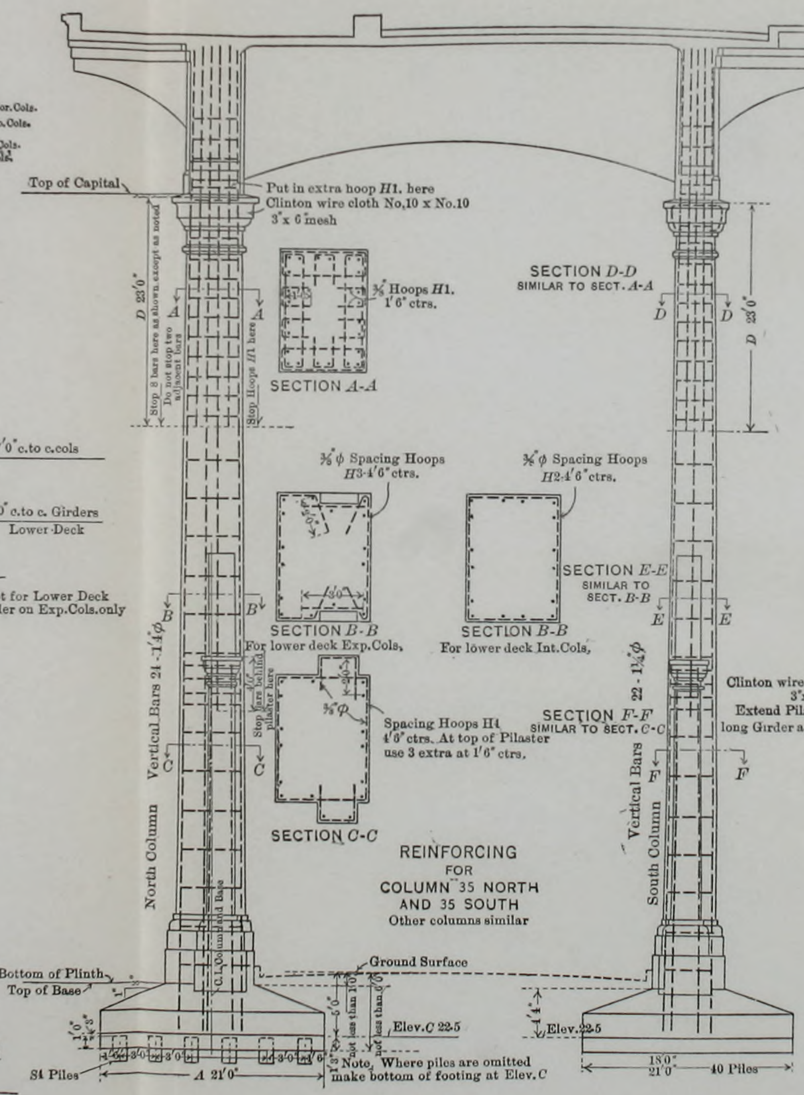
Concrete Piles.—Precast reinforced concrete piles were designed, and were detailed on the drawings, but the specifications left some option to the contractor as to the exact type to be used, excluding, however, "piles in which fresh or unset concrete is placed against the soil". The contractor finally elected to use Raymond piles, which were accepted by the engineers.

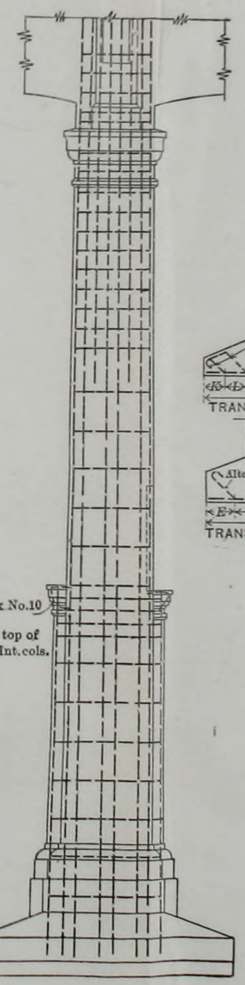
A description of this well-known concrete pile is hardly necessary, but it may be said briefly to consist of a sheet-steel shell, spirally

TYPICAL DETAILS OF COLUMNS, CAPS, AND BASES TWELFTH STREET TRAFFICWAY VIADUCT

KANSAS CITY, MO.







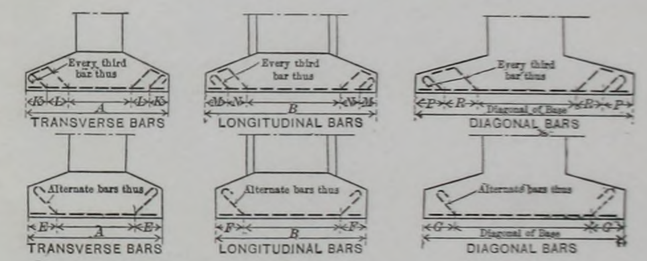
LOADS ON FOUNDATIONS

BASES WITH PILES

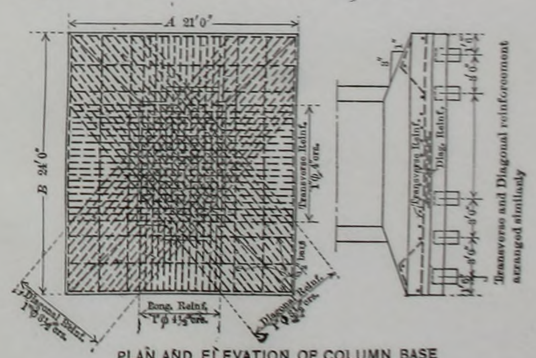
Average load on piles in each base, 25 tons per pile
Maximum load, considering effect of wind, temperature changes, and deflection of cross-girders, 35 tons per pile

BASES WITHOUT PILES:

Pier No.	Loads in Pounds per Square Foot	
	Av.	Max.
39,40,41,43,45	10 000	14 000
42	8 000	11 000
44	6 500	9 000



BENDING DIAGRAMS FOR BASE REINFORCEMENT



PLAN AND ELEVATION OF COLUMN BASE

reinforced, driven into the ground on a collapsible steel core. After reaching a satisfactorily resisting penetration, the core is withdrawn and the shell is filled with concrete. No reinforcing rods in the concrete were required, and as the concrete was not to be disturbed after placing, a mixture 1 part cement, 3 parts sand, and 5 parts broken stone was permitted.

Pavement.—The specifications for the creosoted wood block pavement for the lower roadway and on the street contain no unusual features. For steeper grades the following requirements are of interest:

"Creosoted block pavements, for roadways having a gradient exceeding 4%, are to be constructed in all parts of material and as specified for level roadways with the following exceptions: Creosoted laths about 1¼ in. wide, ⅜ in. thick, and 4 ft. long, surfaced on one side, of uniform thickness, are to be furnished and placed on edge between every row of blocks continuously across the roadway, leaving a space about ⅜ in. wide and 2¾ in. deep between every row of blocks. The spaces between the blocks shall be filled with hot gravel which has passed through a sieve having holes ¼ in. square, and from which all sand and fine material which will pass a No. 20 sieve has been removed." * * * "Immediately after the gravel is in place, before it has cooled, paving pitch, as herein specified, shall be poured into it in such manner and quantity as entirely to fill all interstices."

This is practically the form of block pavement used in Vancouver, B. C., and elsewhere, on heavy grades. It affords a good grip for horses, and has proved very satisfactory.

Water-proofing.—The surfaces of both decks are covered with water-proofing in an effort to prevent the dripping of water on the roadways below. Under the ballast of the street-car area, this water-proofing consists of a two-ply burlap mat, laid down in water-proofing pitch, on which is placed a layer of asphaltic felt, and this, in turn, is covered with 1 in. of asphaltic mastic, and given a sand finish. Under the creosoted block pavement, one layer of burlap and one layer of asphaltic felt are laid in water-proofing pitch and given a sand finish. Indentations are formed in the concrete curbs some 6 in. below their tops, and the water-proofing mats are brought up, turned in, and finished in them. It happened that just before the contractor was to place his order for the burlap, the supply was entirely cut off owing to the European war conditions, so that felt was substituted, at some reduction of cost.

Lighting System.—The requirements for the lighting system were given as follows:

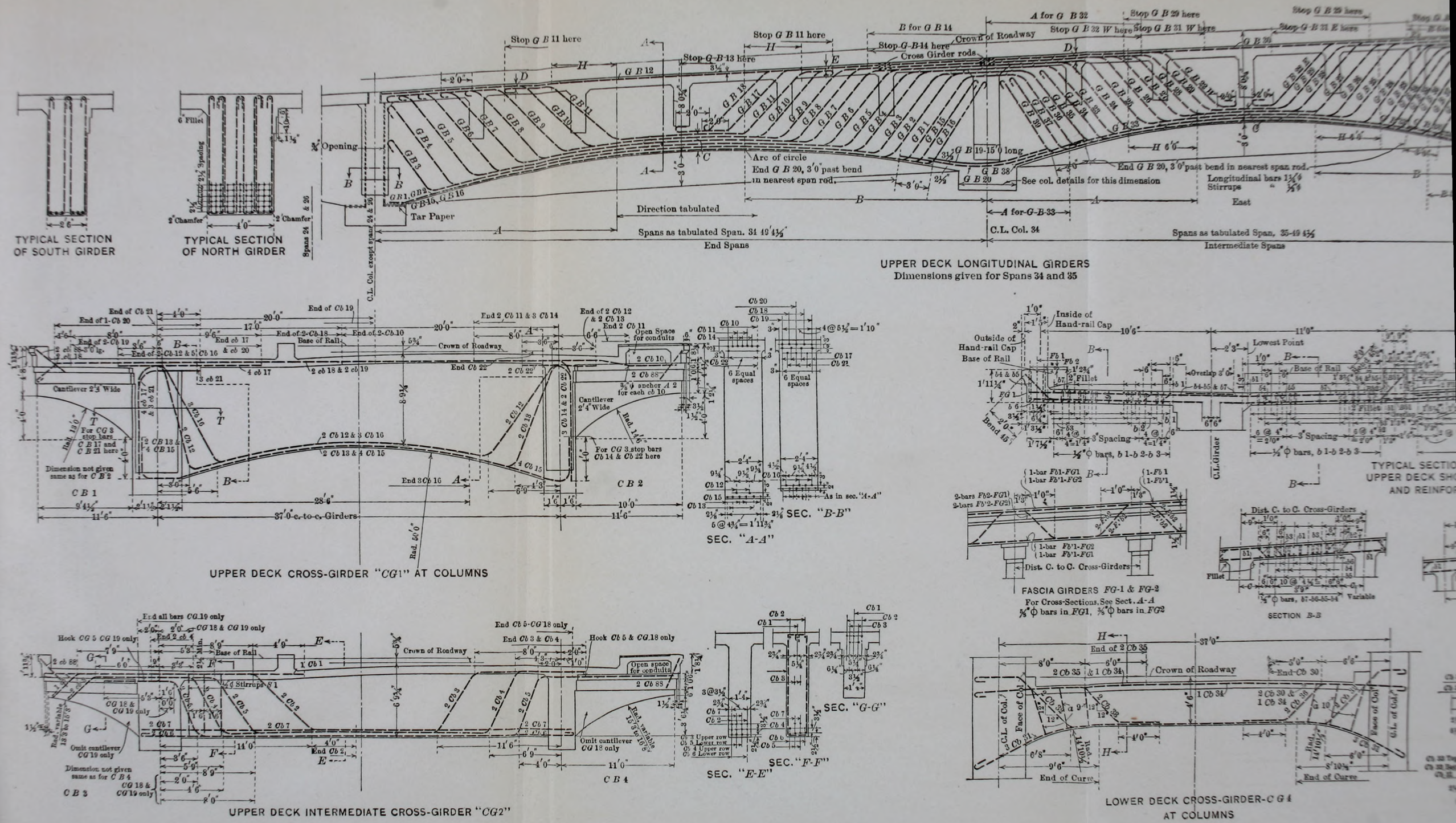
"The structure is to be illuminated by an electric lighting system. There shall be furnished and installed, all lamps, globes, conduits, wiring, and all other apparatus and appurtenances necessary for a complete series-Tungsten lighting system, taking current from the Kansas City Electric Light Company's 6.6-ampere, constant-current feeders at the east end of the structure. There will be two circuits. The circuit for lighting the upper roadway will have sixty 125-watt, 6.6-ampere, constant-current series-Tungsten lamps. Each lamp will be supported on a cast-iron standard, and will be surrounded by an opal glass globe, 14 in. in diameter. The circuit for lighting the lower roadway and the stairways will have fifty-six 75-watt, 6.6-ampere, constant-current, series-Tungsten lamps. Each lamp will be supported on a cast-iron standard or bracket, and will be surrounded by an opal glass globe, 12 in. in diameter. All wiring shall be drawn into Sherardized steel-pipe conduits of a satisfactory type, so as to be free from all flaws or mechanical injuries. All wires shall be No. 6, high-tension, lead-covered, Okonite, stranded, copper wire of the best quality. The conduit shall be encased in the concrete as the latter is placed, and shall be properly supported and arranged so that, later, the wire may be drawn into place. Suitable expansion joints shall be provided, and all connections shall be made in the conduits so that they will conform properly to the requirements of the structure. All joints between pipes and between pipes and boxes shall be made water-tight. The entire lighting system shall be constructed in a thoroughly workmanlike manner and to the satisfaction of the engineers."

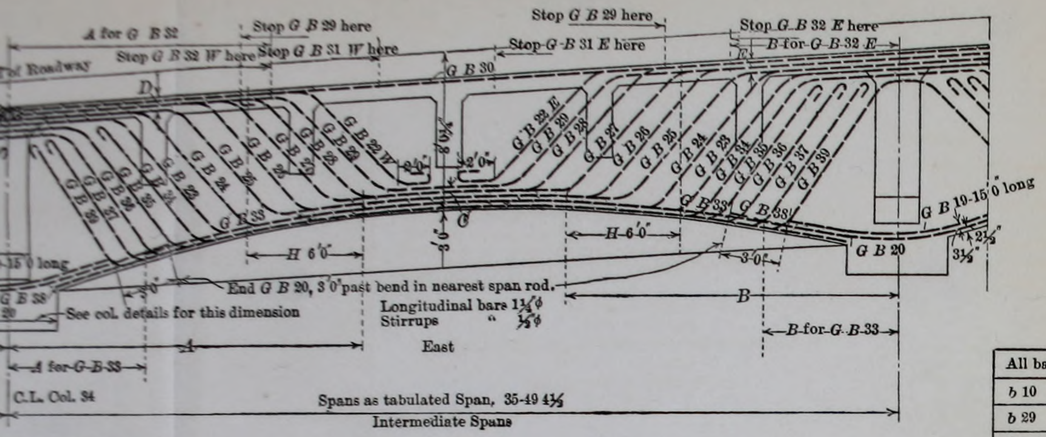
Other features of the specifications will be evident from the descriptions of methods of construction.

5.—Equipment for, and Methods of, Construction.

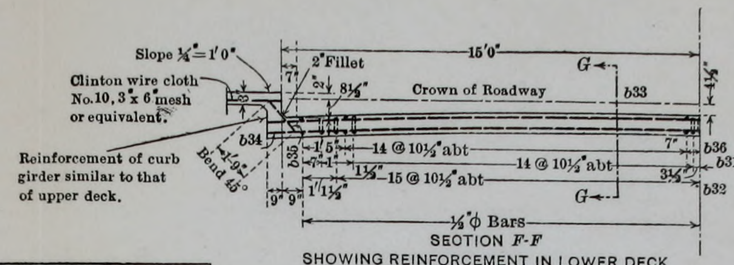
Yard Space.—The contractor's plant was at the west or low end of the viaduct, where some storage space, switch tracks, sidings, and facilities for handling materials, were more easily available than elsewhere on the right of way. The working space comprised practically all the ground from Liberty Street to Hickory Street, and from the north curb line of Eleventh Street to the alley south of Twelfth Street, an area of about 35 000 sq. ft.

Three standard-gauge spur tracks were provided, one track paralleling Twelfth Street in the alley south of it—used for the delivery of timber—and two tracks crossing Twelfth Street at right angles—one for the delivery of sand, stone, and steel, and the other for cement,

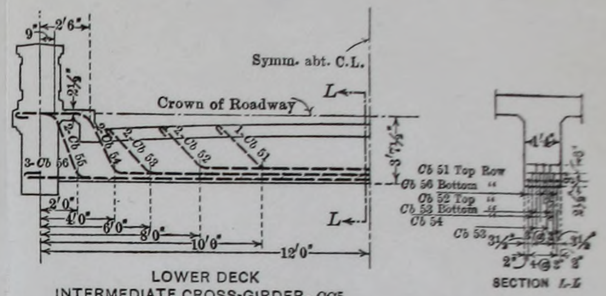

TYPICAL SECTION OF SOUTH GIRDER
TYPICAL SECTION OF NORTH GIRDER
UPPER DECK LONGITUDINAL GIRDERS
Dimensions given for Spans 34 and 35
Direction tabulated
Spans as tabulated Span. 34 49'4½"
End Spans
C.L. Col. 34
Spans as tabulated Span, 35-49 4½
Intermediate Spans
Tar Paper
Arc of circle
End G B 20, 3'0" past bend in nearest span rod.
See col. details for this dimension
Longitudinal bars 1¼ ϕ
Stirrups ½ ϕ
East
UPPER DECK CROSS-GIRDER "CG1" AT COLUMNS
SEC. "A-A"
SEC. "B-B"
Cantilever 2'3" Wide
Cantilever 2'4" Wide
For CG 3 stop bars C B 17 and C B 21 here
For CG 3 stop bars Cb 14 & Cb 22 here
Dimension not given same as for C B 2
37'0" c. to c. Girders
UPPER DECK INTERMEDIATE CROSS-GIRDER "CG2"
SEC. "G-G"
SEC. "E-E"
SEC. "F-F"
Omit cantilever CG 18 only
Omit cantilever CG 19 only
Dimension not given same as for C B 4
FASCIA GIRDERS FG-1 & FG-2
For Cross-Sections, See Sect. A-A
⅝" ϕ bars in FG1, ⅜" ϕ bars in FG2
SECTION B-B
LOWER DECK CROSS-GIRDER-C G 4
AT COLUMNS
End of Curve



GITUDINAL GIRDERS
for Spans 34 and 35

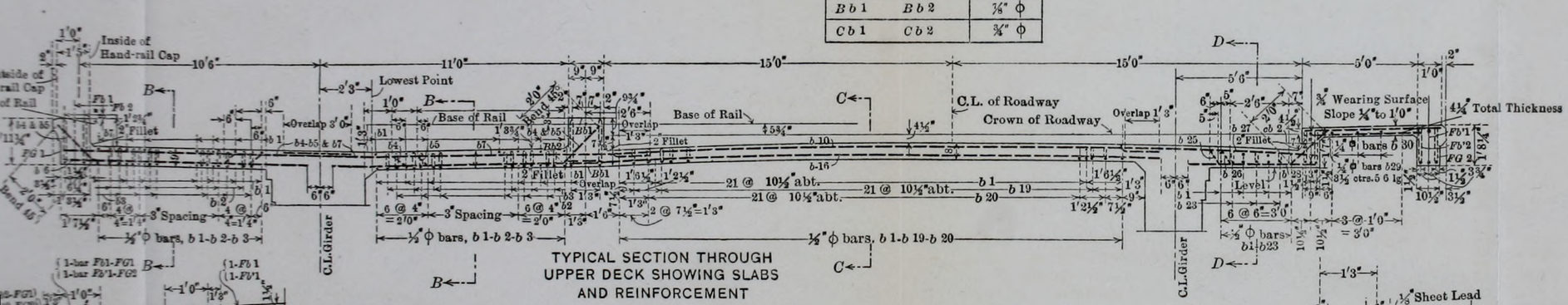


SECTION F-F
SHOWING REINFORCEMENT IN LOWER DECK
TYPICAL DETAILS OF
LOWER DECK FLOOR-SLABS

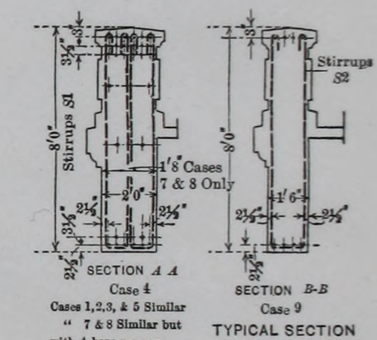


LOWER DECK
INTERMEDIATE CROSS-GIRDER - CG5
SECTION L-L

All bars ½" ϕ, except as follows,		
b 10	b 16	⅜" ϕ
b 29	b 30	¼" ϕ
F b 1	F b 2	⅝" ϕ
F b'1	F b'2	⅜" ϕ
B b 1	B b 2	⅞" ϕ
C b 1	C b 2	¾" ϕ

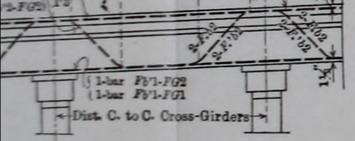


TYPICAL SECTION THROUGH
UPPER DECK SHOWING SLABS
AND REINFORCEMENT

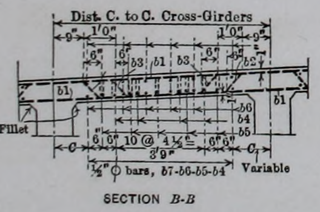


SECTION A A
Case 4
Cases 1,2,3, & 5 Similar
" 7 & 8 Similar but with 4 bars per row
TYPICAL SECTION
OF NORTH GIRDER

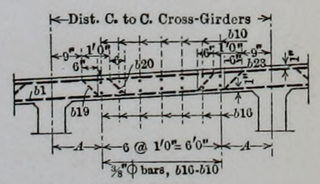
SECTION B-B
Case 9
TYPICAL SECTION
OF SOUTH GIRDER



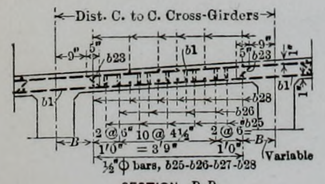
FASCIA GIRDERS FG-1 & FG-2
For Cross-Sections, See Sect. A-A
⅜" ϕ bars in FG1, ⅜" ϕ bars in FG2



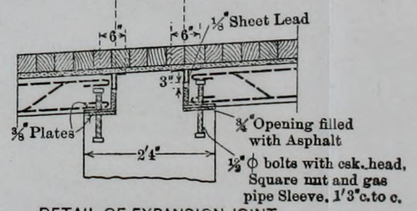
SECTION B-B



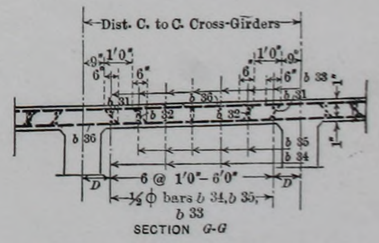
SECTION C-C



SECTION D-D

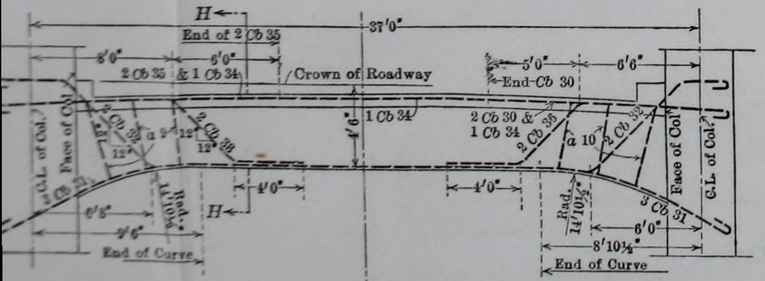


DETAIL OF EXPANSION JOINT
FOR ROADWAY SLABS

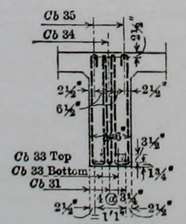


SECTION G-G

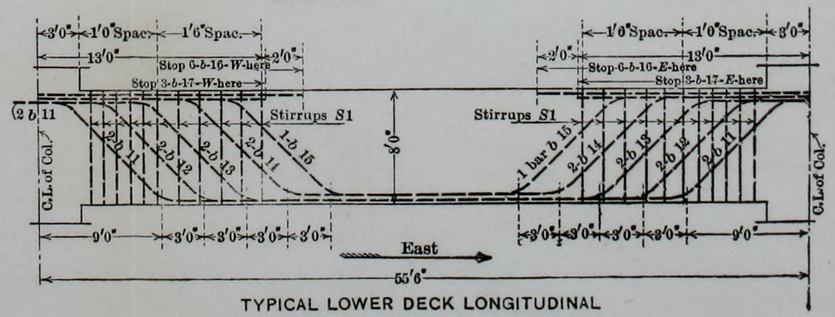
TYPICAL DETAILS OF
SLABS, GIRDERS, AND
LONGITUDINAL GIRDERS
FOR BOTH DECKS
TWELFTH STREET
TRAFFICWAY VIADUCT,
KANSAS CITY, MO.



LOWER DECK CROSS-GIRDER-CG4
AT COLUMNS



SECTION H-H



TYPICAL LOWER DECK LONGITUDINAL
GIRDER INTERMEDIATE SPAN

Concrete Plant.—The mixing plant, placed on the line of the viaduct, consisted of elevated bins for sand and broken stone arranged to discharge into measuring hoppers, below which a 33-cu. ft. Chicago-cube mixer was installed. The mixer dumped the concrete into hopper cars operating on a 30-in. gauge track passing underneath. Stone and sand were delivered in bottom-dump cars and unloaded into a receiving pit alongside the bunkers. A 22-in. Jeffery, belt-bucket elevator, of a total height of 70 ft., raised the stone and the sand from the pit to the top of the bins, where a baffle-board directed each to a proper bin. One elevator thus handled both sand and stone, unloading first one and then the other. The cement was hauled up an inclined track from the cement storage shed on a car, self-dumping at the top, to a platform under the sand bin, where it was filled into a sheet-iron tube with a bottom gate, and so discharged into the mixer. The tube was of just the proper dimensions to hold six sacks of cement, the normal charge. The limited room for tracks made necessary considerable shifting of the cars, as only the first car of a train could be placed over the pit. The operations of unloading and mixing were not always simultaneous, but when all operations were going forward, there were required, severally, two men for unloading and handling the elevator, three men for unloading cement from cars, one in the storage shed, two opening sacks and feeding to the mixer, and one man attending to charging and operating the mixer. Thus, the services of nine men were needed for the unloading and mixing of concrete ingredients.

During severe weather, the stone, sand, and water for the concrete were heated by passing steam from a 60-h.p. boiler into the bottoms of the sand and stone bins, and into the water, thoroughly and satisfactorily heating all the materials.

Equipment for Steel and Timber Reinforcing.—The steel-storage yard was adjacent to one of the switch tracks, and was equipped with a working platform and a power bar-bender of ingenious design. This bender was devised on the job, and included a motor connected by a worm gear to a vertical shaft which had at its upper end a clutch with a jaw to engage the bars to be bent, thus bending the bars in a horizontal plane on a protractor plate to any desired angle. The lumber-storage yard was beside the third siding. From this lumber yard, the material was passed into a wood-saw shop, thence into the carpenter shops, and then forward to the structure. The wood shop was equipped

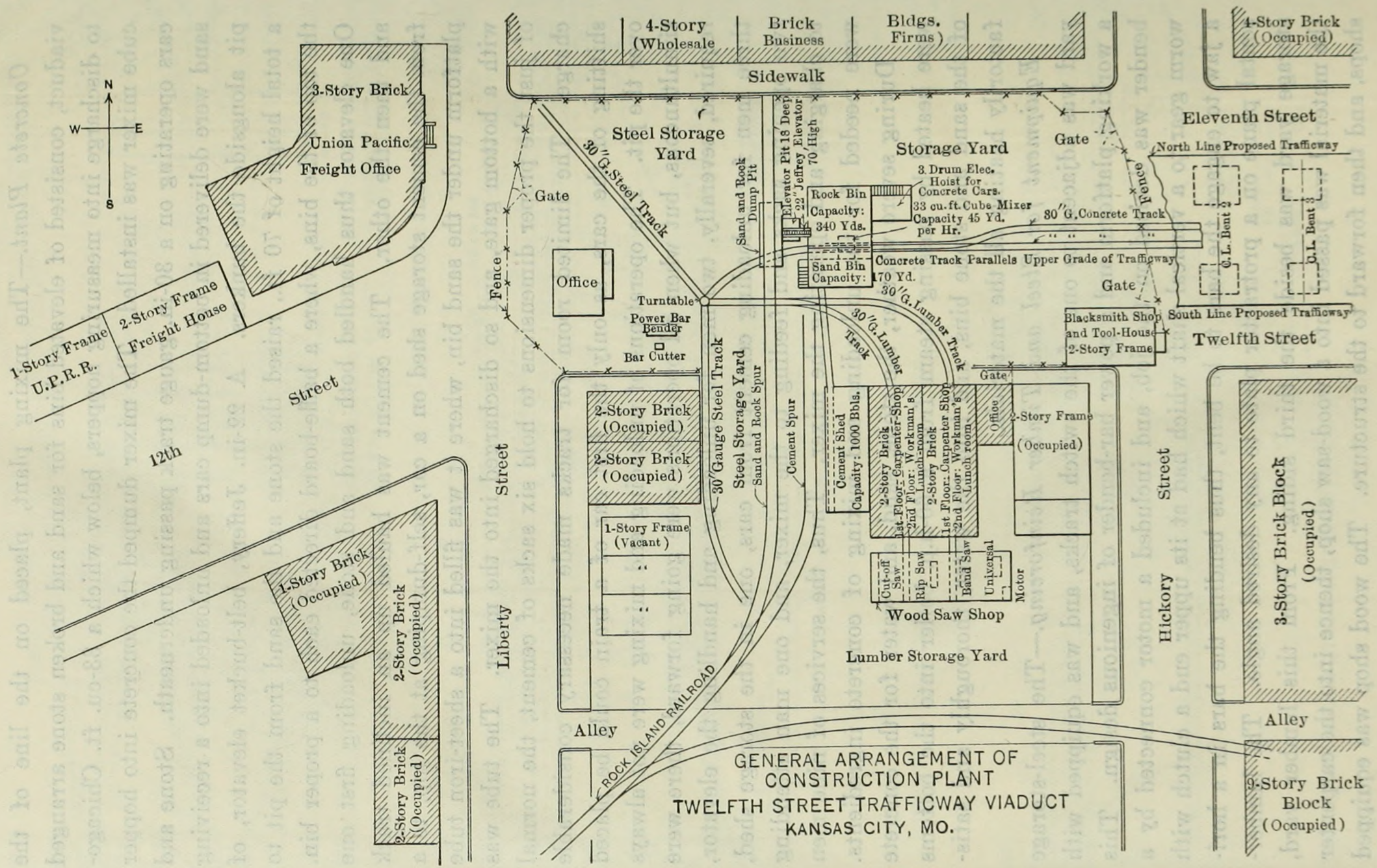
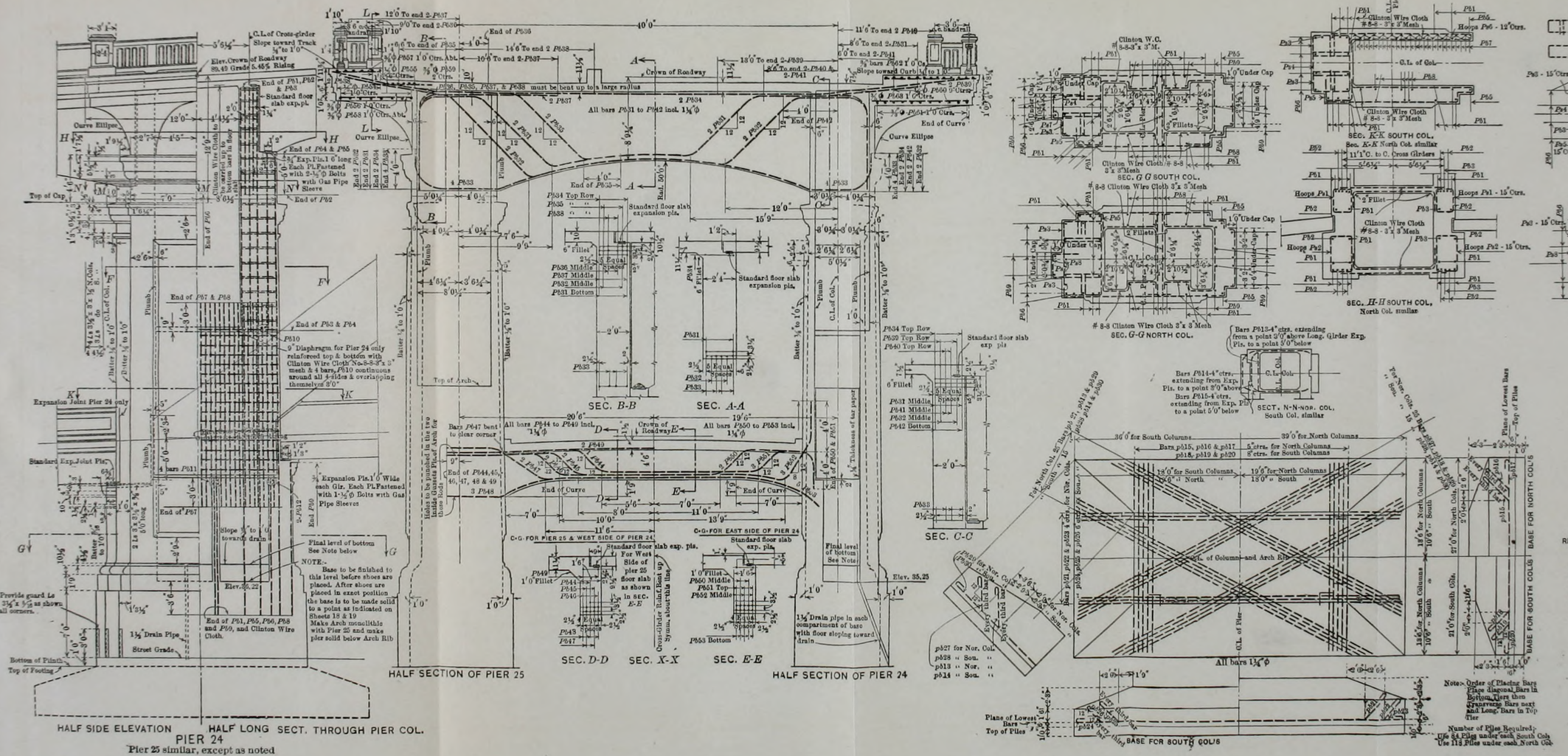

N
W
E
S
3-Story Brick
Union Pacific
Freight Office
1-Story Frame
U.P.R.R.
2-Story Frame
Freight House
Street
12th
1-Story Brick
(Occupied)
2-Story Brick
(Occupied)
2-Story Brick
(Occupied)
Liberty
Street
4-Story
(Wholesale
Brick
Business
Bldgs.
Firms)
Sidewalk
4-Story Brick
(Occupied)
Steel Storage
Yard
30" G. Steel Track
Gate
Fence
Office
Turntable
Power Bar
Bender
Bar Cutter
2-Story Brick
(Occupied)
2-Story Brick
(Occupied)
1-Story Frame
(Vacant)
Sand and Rock
Dump Pit
Elevator Pit 18' Deep
22" Jeffrey Elevator
70' High
Storage Yard
3 Drum Elec.
Hoist for
Concrete Cars.
Rock Bin
Capacity:
340 Yds.
33 cu. ft. Cube Mixer
Capacity 45 Yd.
per Hr.
80" G. Concrete Track
Concrete Track Parallels Upper Grade of Trafficway
Sand Bin
Capacity:
170 Yd.
30" G. Lumber Track
30" G. Lumber
Track
30" Gauge Steel Track
Steel Storage Yard
Sand and Rock Spur
Cement Spur
Cement Shed
Capacity: 1000 Bbls.
2-Story Brick
1st Floor: Carpenter-Shop
2nd Floor: Workman's
Lunch-room
2-Story Brick
1st Floor: Carpenter Shop
2nd Floor: Workman's
Lunch room
Office
2-Story Frame
(Occupied)
Cut-off
Saw
Rip Saw
Band Saw
Universal
Motor
Wood Saw Shop
Lumber Storage Yard
Gate
Eleventh Street
North Line Proposed Trafficway
C. L. Bent 2
C. L. Bent 3
South Line Proposed Trafficway
Twelfth Street
Blacksmith Shop
and Tool-House
2-Story Frame
Hickory
Street
3-Story Brick Block
(Occupied)
Alley
9-Story Brick
Block
(Occupied)
ROCK ISLAND RAILROAD
GENERAL ARRANGEMENT OF
CONSTRUCTION PLANT
TWELFTH STREET TRAFFICWAY VIADUCT
KANSAS CITY, MO.

FIG. 9.



Mark	Size	Mark	Size
P61	1" ϕ	P613	⅝" ϕ
" 2	"	P614 & 15	⅝" ϕ
" 3	"		
" 4	"		
" 5	"		
" 6	"		
" 7	⅝" ϕ		
" 8	1" ϕ		
" 9	"		
Hooping P-1 to P-8	⅝" ϕ		
P-10, 11, & 12	⅝" ϕ		

REINFORCEMENT IN COLUMN SHAFTS OF PIER 24

Reinforcement in Col. Shafts of Pier 25 Same as in Col. Shafts of Pier 24 except omit P67, P68, P69 P610 & P611 Carry the other bars to 3'6" below top of Arch Ring except the bars in the circumference, which bars are to be carried to a point 8'0" below crown of lower dock roadway.

DETAILS OF PIERS
OF ARCH SPAN
AT SANTA FE STREET
TWELFTH STREET
TRAFFICWAY VIADUCT
KANSAS CITY, MO.

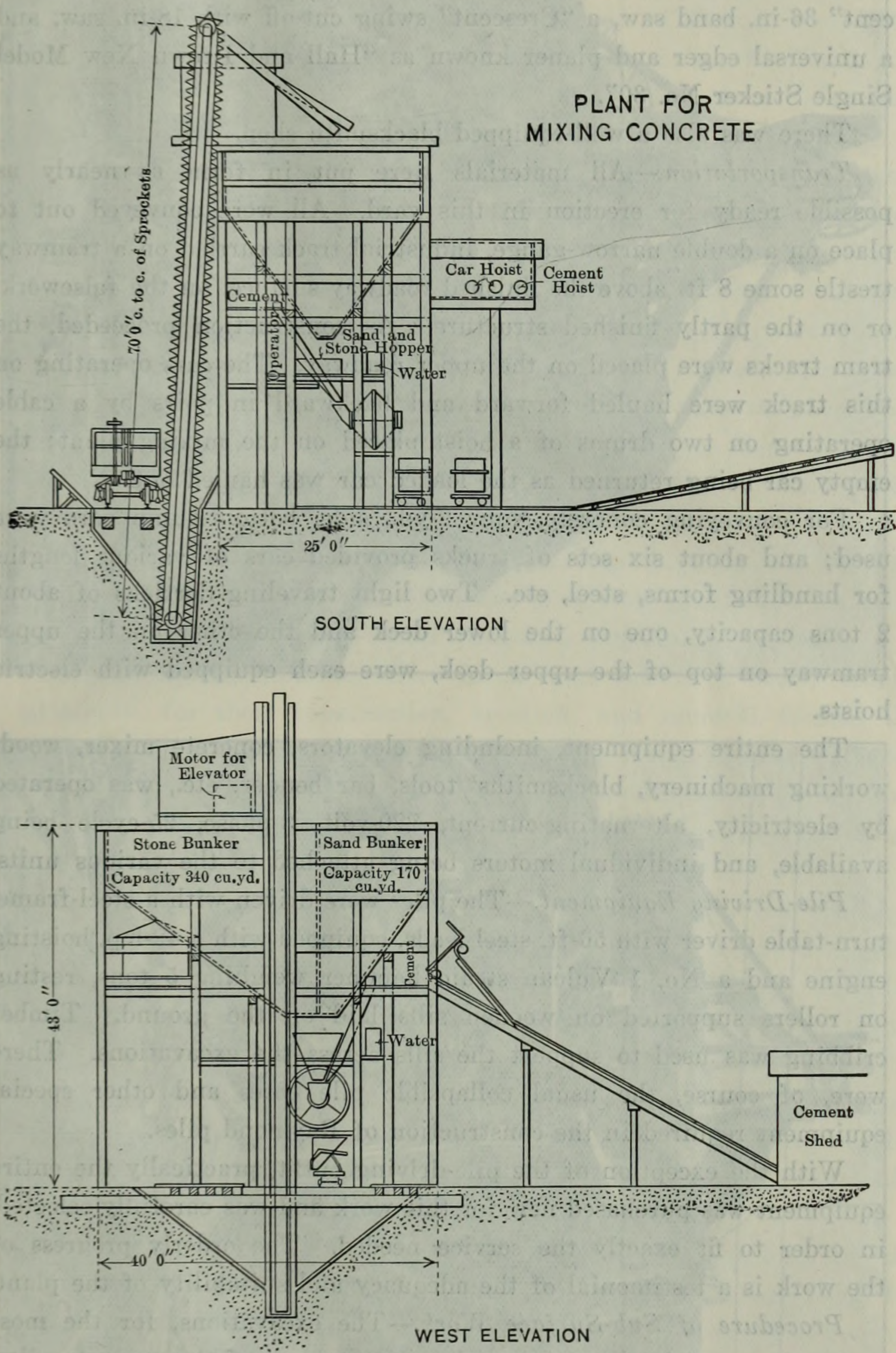

PLANT FOR
MIXING CONCRETE
70′0″ c. to c. of Sprockets
Car Hoist
Cement
Hoist
Cement
Operator
Sand and
Stone Hopper
Water
25′ 0″
SOUTH ELEVATION
Motor for
Elevator
Stone Bunker
Capacity 340 cu.yd.
Sand Bunker
Capacity 170
cu.yd.
Cement
Water
43′ 0″
Cement
Shed
40′ 0″
WEST ELEVATION

FIG. 10.

with a circular rip-saw, a "Hall and Brown rip-saw No. 131", a "Crescent" 36-in. band saw, a "Crescent" swing cut-off with 18-in. saw, and a universal edger and planer known as "Hall and Brown New Model Single Sticker No. 30".

There was also a well-equipped blacksmith shop.

Transportation.—All materials were put in form as nearly as possible ready for erection in this yard. All were conveyed out to place on a double narrow-gauge, industrial track carried on a tramway trestle some 8 ft. above the finished roadway surface, on the falsework, or on the partly finished structure. As construction proceeded, the tram tracks were placed on the upper roadway. The cars operating on this track were hauled forward and backward in pairs by a cable operating on two drums of a hoist placed on the mixing plant; the empty car being returned as the loaded car was hauled out.

For conveying concrete, four 2-yd., side, bottom-dump care were used; and about six sets of trucks provided cars of various lengths for handling forms, steel, etc. Two light traveling derricks of about 2 tons capacity, one on the lower deck and the other on the upper tramway on top of the upper deck, were each equipped with electric hoists.

The entire equipment, including elevators, concrete mixer, woodworking machinery, blacksmiths' tools, bar bender, etc., was operated by electricity, alternating-current, 220-volt, 3-phase, 25-cycle being available, and individual motors being attached to the various units.

Pile-Driving Equipment.—The piles were driven with a steel-frame, turn-table driver with 56-ft. steel leads, equipped with a 42-h.p. hoisting engine and a No. 1 Vulcan steam hammer weighing 5 tons, resting on rollers supported on wooden sills laid on the ground. Timber cribbing was used to support the sills across the excavations. There were, of course, the usual collapsible pile cores and other special equipment required in the construction of Raymond piles.

With the exception of the pile-driving outfit, practically the entire equipment was purchased new for this work and was carefully selected, in order to fit exactly the service needed. The orderly progress of the work is a testimonial of the adequacy and suitability of the plant.

Procedure of Sub-Surface Work.—The excavations, for the most part, were very simple, not very deep, on account of the concrete piles, and in a sandy loam, easily dug, and standing without much shoring

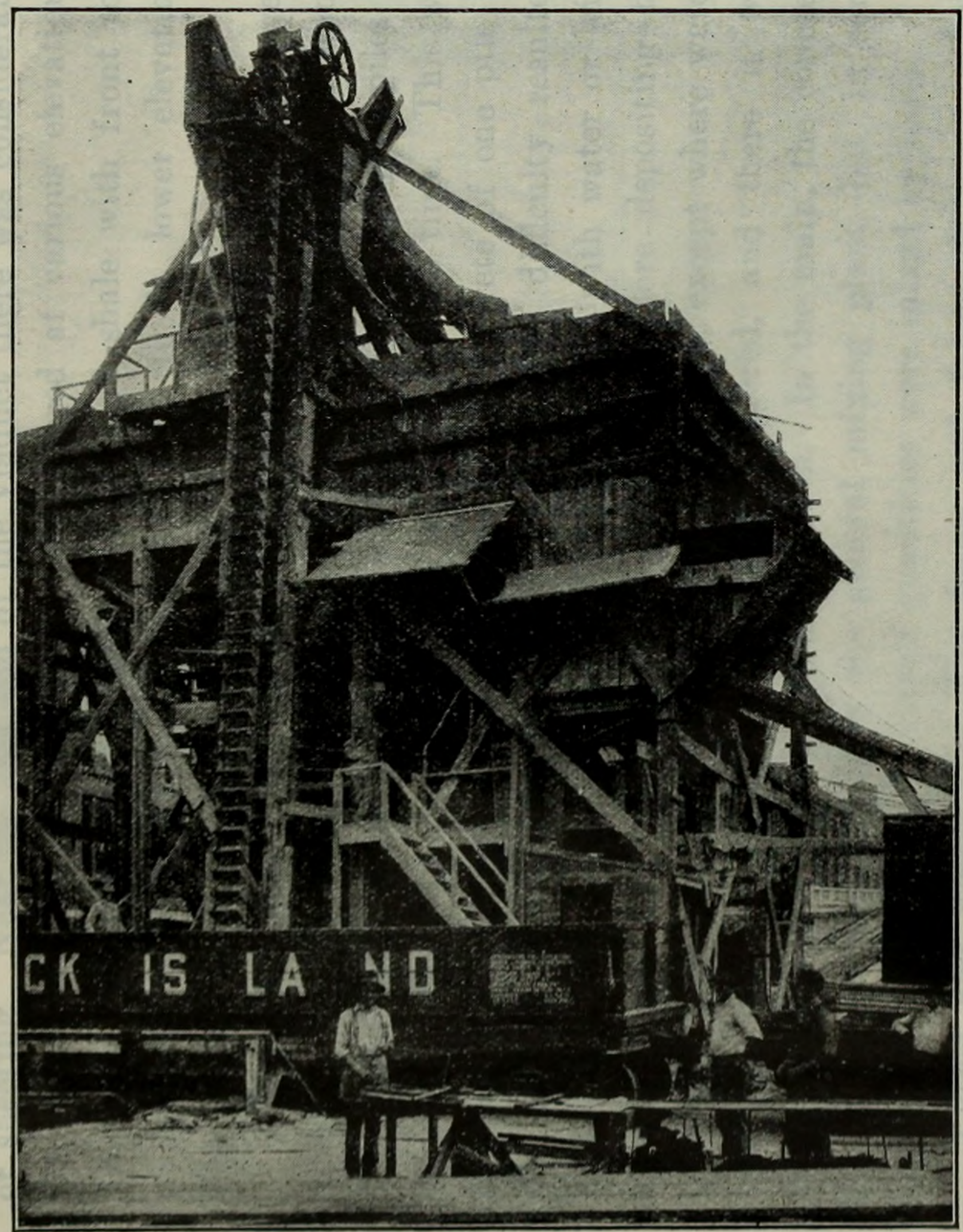


FIG. 11.—BACK VIEW OF MATERIAL BINS AND MIXING PLANT, SHOWING PIT AND BELT BUCKET ELEVATOR.

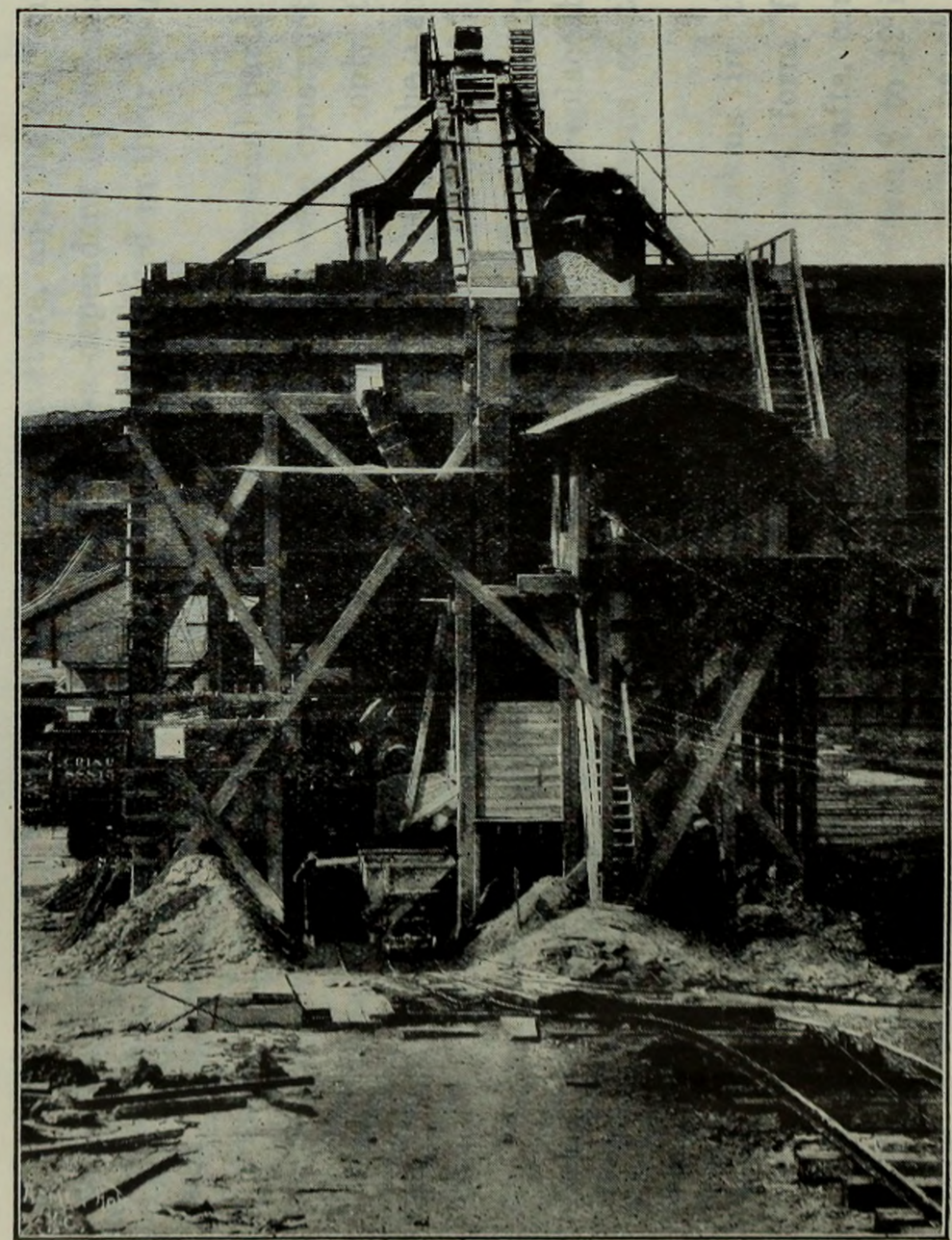


FIG. 12.—FRONT VIEW OF MATERIAL BINS AND MIXING PLANT, SHOWING MIXER, CONCRETE CAR, HOIST, ETC.

or sheeting. Near the east end of the viaduct there was considerable old, filled material; and rock was encountered at various elevations. The east abutment rests in part on clay and shale with front toe-walls, 5 ft. thick, carried to rock at about 10 ft. lower elevation. The work was simple, open excavation.

After excavations were made, the Raymond concrete piles were driven, in the usual manner, to the refusal the apparatus would stand. The engineers required that all the steel shells for a group of piles be driven and then the concrete for all be placed at one time. This was to prevent any possible damage to the unset concrete of one pile by the driving of another near it. No particular difficulty resulted, except that the shells would sometimes partly fill with water or with sand and silt, which had to be pumped out before depositing the concrete. The shells stood open without collapse, except where water-bearing, so-called "quicksand" was encountered, and there it was found necessary to fill each shell as driven. In the main, the concrete for the piles was mixed in the general mixing plant, but, in some instances, for convenience, small quantities were mixed by hand.

Forms and Falsework.—The forms and falsework merit special attention, for their construction, erection, and removal formed the principal items of labor cost of the whole structure, and the efficiency with which the work was carried forward was dependent in no small degree on the thoroughness and care the contractor used in their design and construction.

Plans and sketches were prepared dimensioning all special parts of all forms, and, as far as possible, controlling portions were constructed in the carpenter shops and taken out entire for erection; only the principal portion of the sheathing was put on in place in the usual manner. Though the structure has an effect of duplication, there is, in reality, very considerable variation of dimensions within small limits, so that there were few parts of forms removable and replaceable without alteration. For instance, there are no two columns of the entire structure alike.

The forms for the columns were built to dimensions in the carpenter shops in three principal parts; a ready-to-erect form for plinth and for capital, and partly finished forms for the shafts, comprising four side frames of correct height and batter, from 8 to 12 in. wide at each corner, or rather extending back to the raised panel of

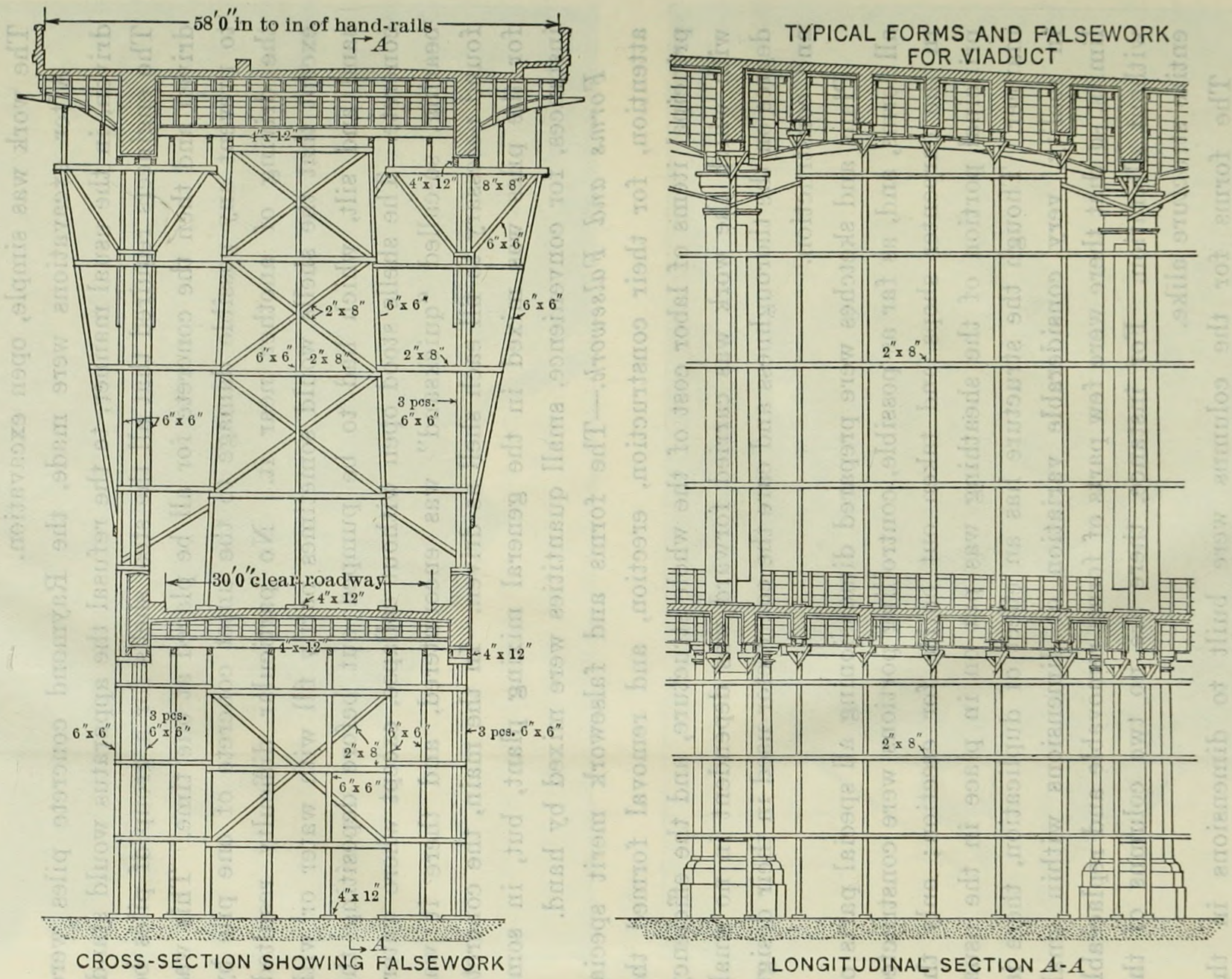
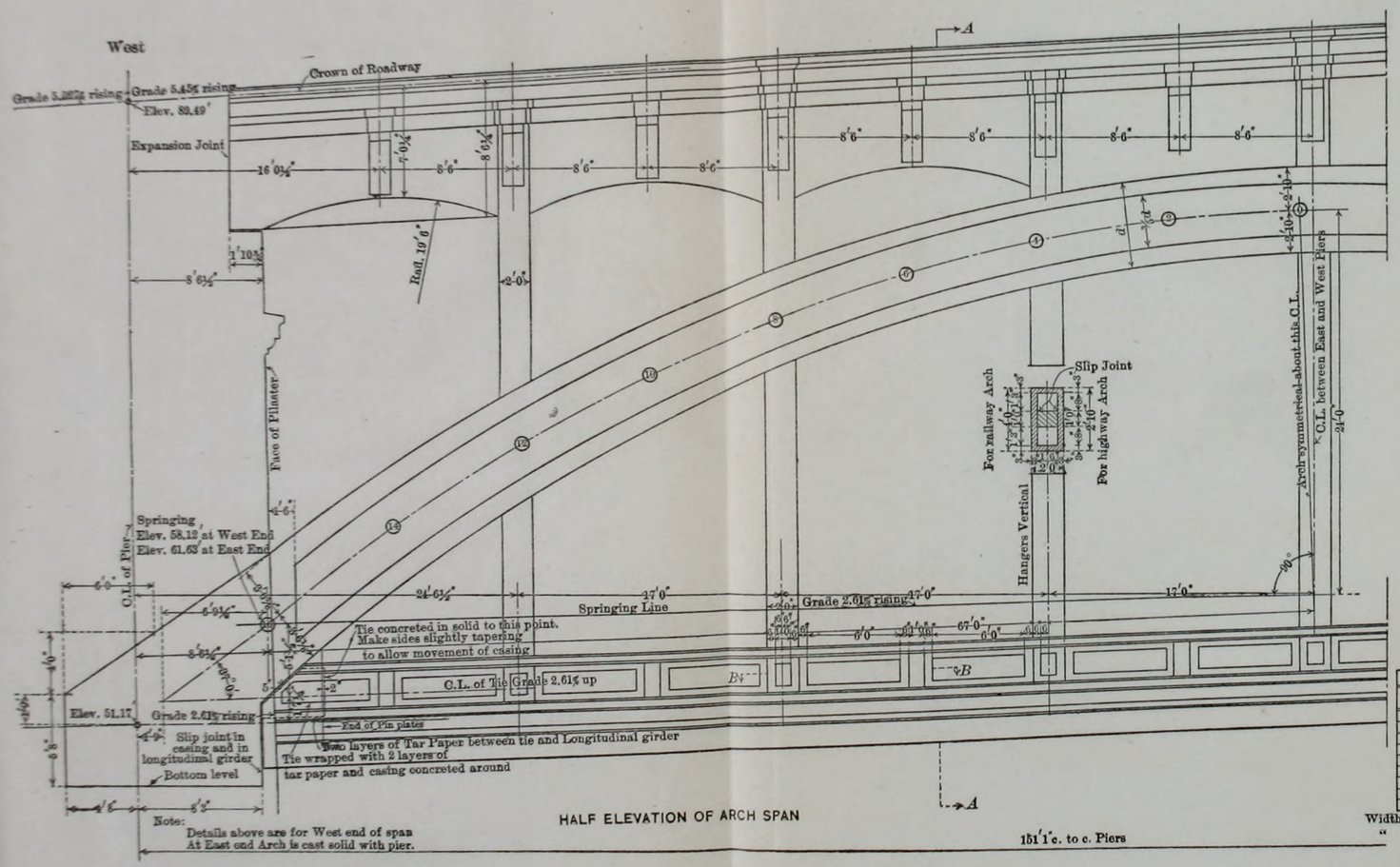

58′0″ in to in of hand-rails
A
1″x 12″
4″x 12″
8″x 8″
6″x 6″
2″x 8″
6″x 6″
6″x 6″
6″x 6″
2″x 8″
2″x 8″
3 pcs.
6″x 6″
6″x 6″
30′0″ clear roadway
4″x 12″
4″x 12″
4″x 12″
3 pcs.
6″x 6″
6″x 6″
6″x 6″
2″x 8″
3 pcs. 6″x 6″
6″x 6″
4″x 12″
A
CROSS-SECTION SHOWING FALSEWORK
TYPICAL FORMS AND FALSEWORK FOR VIADUCT
2″x 8″
2″x 8″
LONGITUDINAL SECTION A-A

FIG. 13.



Note:
Details above are for West end of span
At East end Arch is cast solid with pier.

HALF ELEVATION OF ARCH SPAN

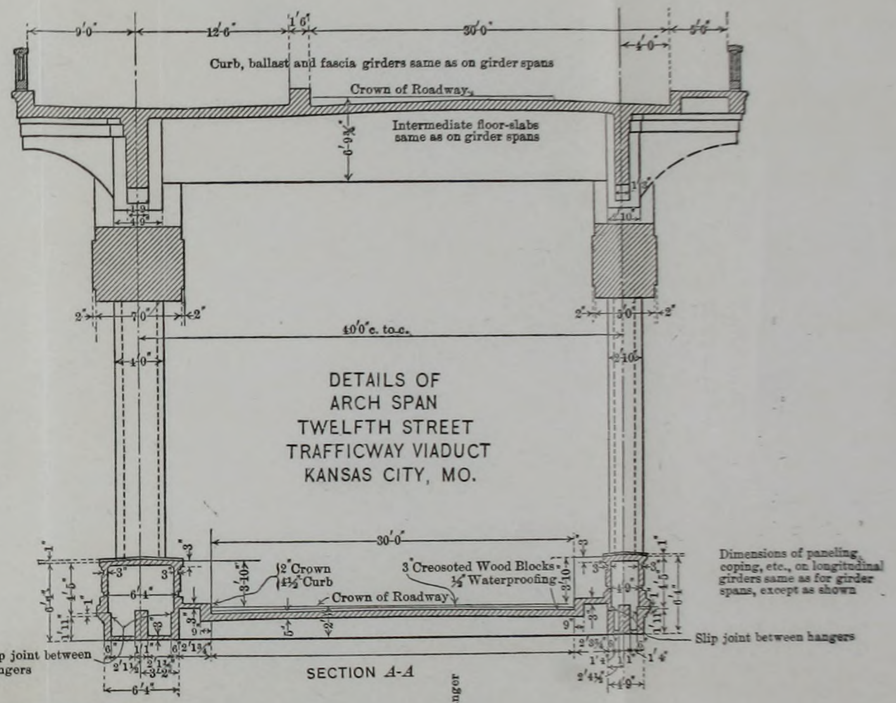


TABLE FOR ARCH RIB			
Point	Coordinates X in ft.	Coordinates Y in ft.	Depth in ft.
0	0	0	5.67
2	8.38	0.86	5.69
4	16.76	1.42	5.70
6	25.14	3.23	5.87
8	33.52	5.76	6.02
10	41.90	9.07	6.22
12	50.28	13.17	6.45
14	58.66	18.14	6.72
16	67.00	24.00	1.02

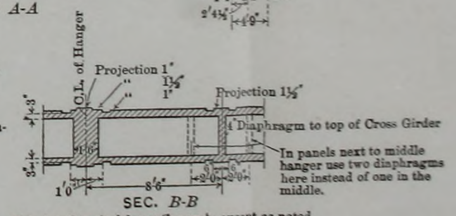
Width:- Highway Arch, 5' 0" throughout
" Railway " 7' 0" "

Note:-

Abscissae measured from crown, in both directions on line parallel to lower deck grade line.

Ordinates measured downward, perpendicular to abscissae.

The depth is measured in radial direction.



This detail typical for entire arch, except as noted

the column. The sheathing over the panels was prepared and placed after the side frames were adjusted to position. Yokes surrounding the column, of the usual four timbers bolted at the corners, and tied across with a central bolt surrounded by a tin tube, were placed at intervals and given bearing by wooden wedges.

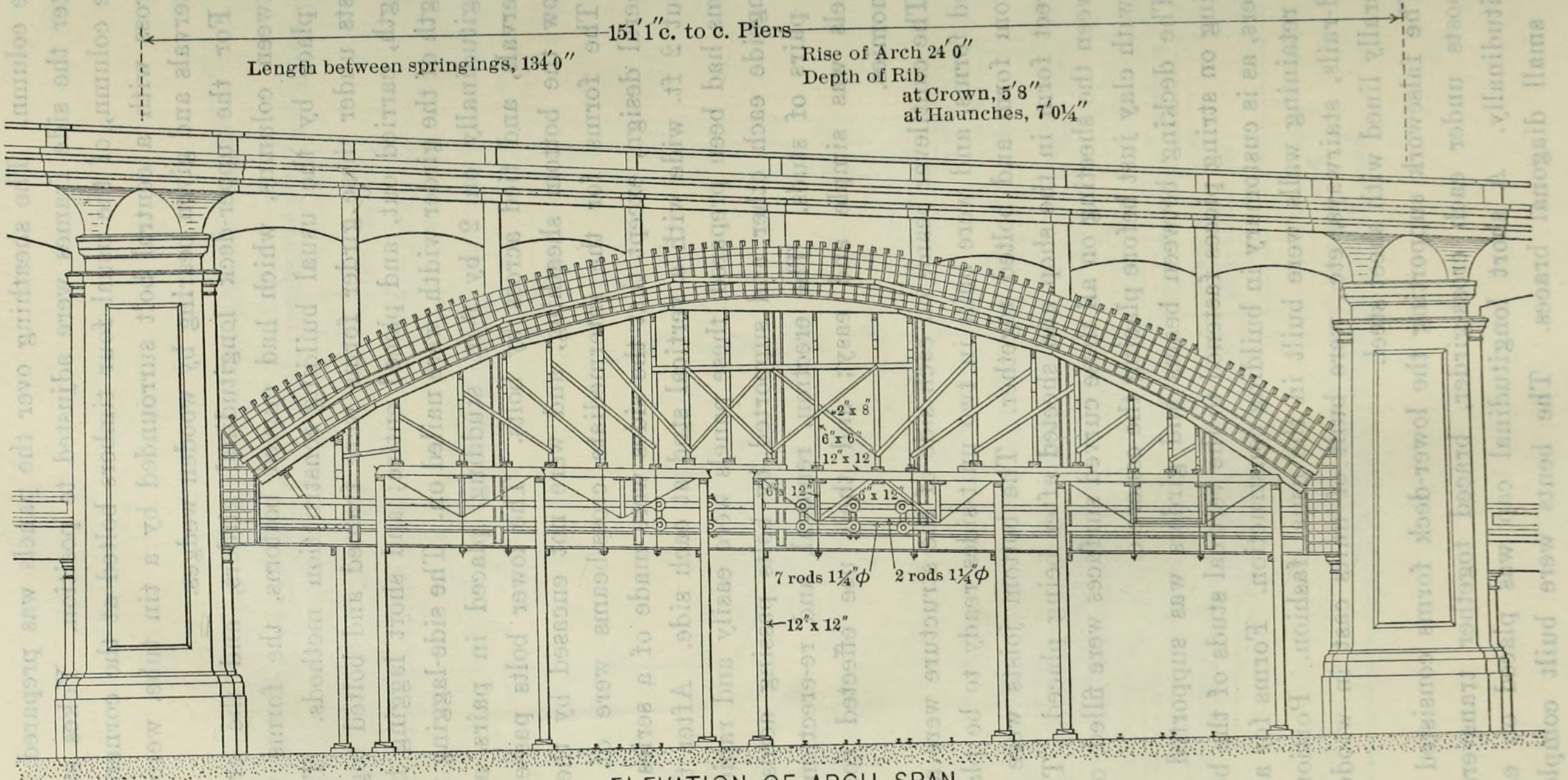
For the upper-deck longitudinal girders, and the cross-girders between columns, which had curved bottoms, the forms were built in place by the usual building-construction methods. The curved joists under these girder forms were framed and bolted together full length, carried out, and placed entire; and short lagging boards, the length of the girder width, were nailed on. The side-lagging was placed longitudinally on 2 by 6-in. studding, placed in pairs at suitable intervals, and tied across by bolts. The lower bolts passed through below the bottom sheathing, and were not encased by the concrete.

The forms for the intermediate cross-beams were of the same general design, except that the sides were made of a series of panels about 2 ft. wide with a vertical stud at each side. After the bottom forms had been prepared, these panels were easily and rapidly set up alongside each other and supported by bolts passing across between the pairs of studs. The erection, removal, and re-erection of these panels was simple and easy; and their use effected considerable economies.

The cantilever beams on each side of the structure were also duplicated forms, and were built in two unit-sides ready to be placed on a bottom form and bolted together. The bottom joists were sawed to correct form in the shop and sheeted after being placed. The cracks between the sheeting on all these curved surfaces were filled, or pointed up, with clay just before placing the concrete.

The decking between beams and girders was supported on joists resting on string pieces fastened to the vertical studs of the beams and girders, as is customary in building construction. Forms for abutments and retaining walls were built in the usual fashion. Portions of the hand-rails, stairways, etc., were built of units cast in wooden forms generally lined with sheet steel.

The falsework supporting the lower-deck forms consisted of bents of posts under each cross-girder, braced together transversely and longitudinally. A short longitudinal cap was placed on each post with small diagonal braces. The bents were built complete with



ELEVATION OF ARCH SPAN
SHOWING
FALSEWORK FOR ARCH RIB

Fig. 14.

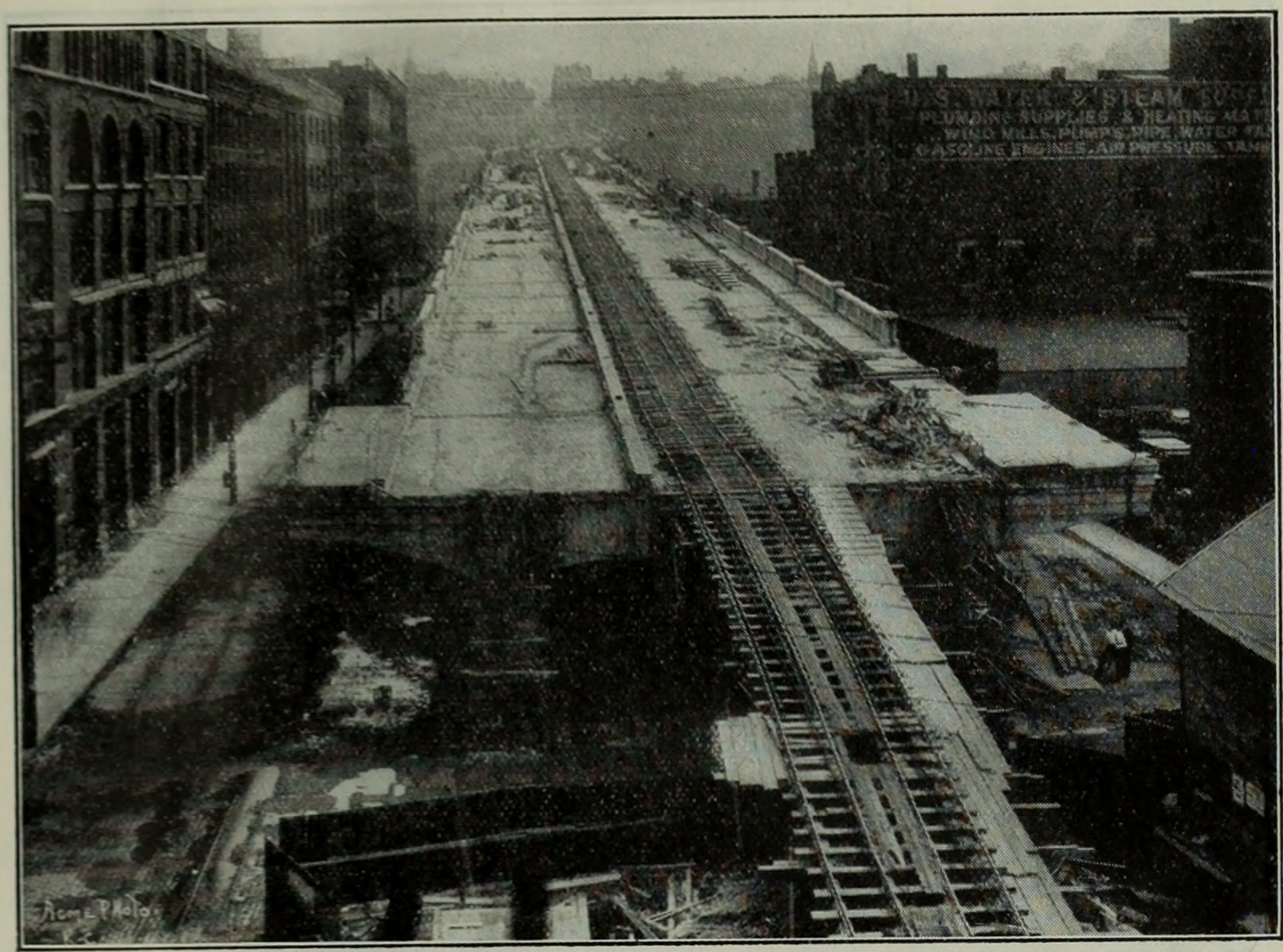


FIG. 15.—UPPER DECK, TWELFTH STREET TRAFFICWAY VIADUCT, DURING CONSTRUCTION.

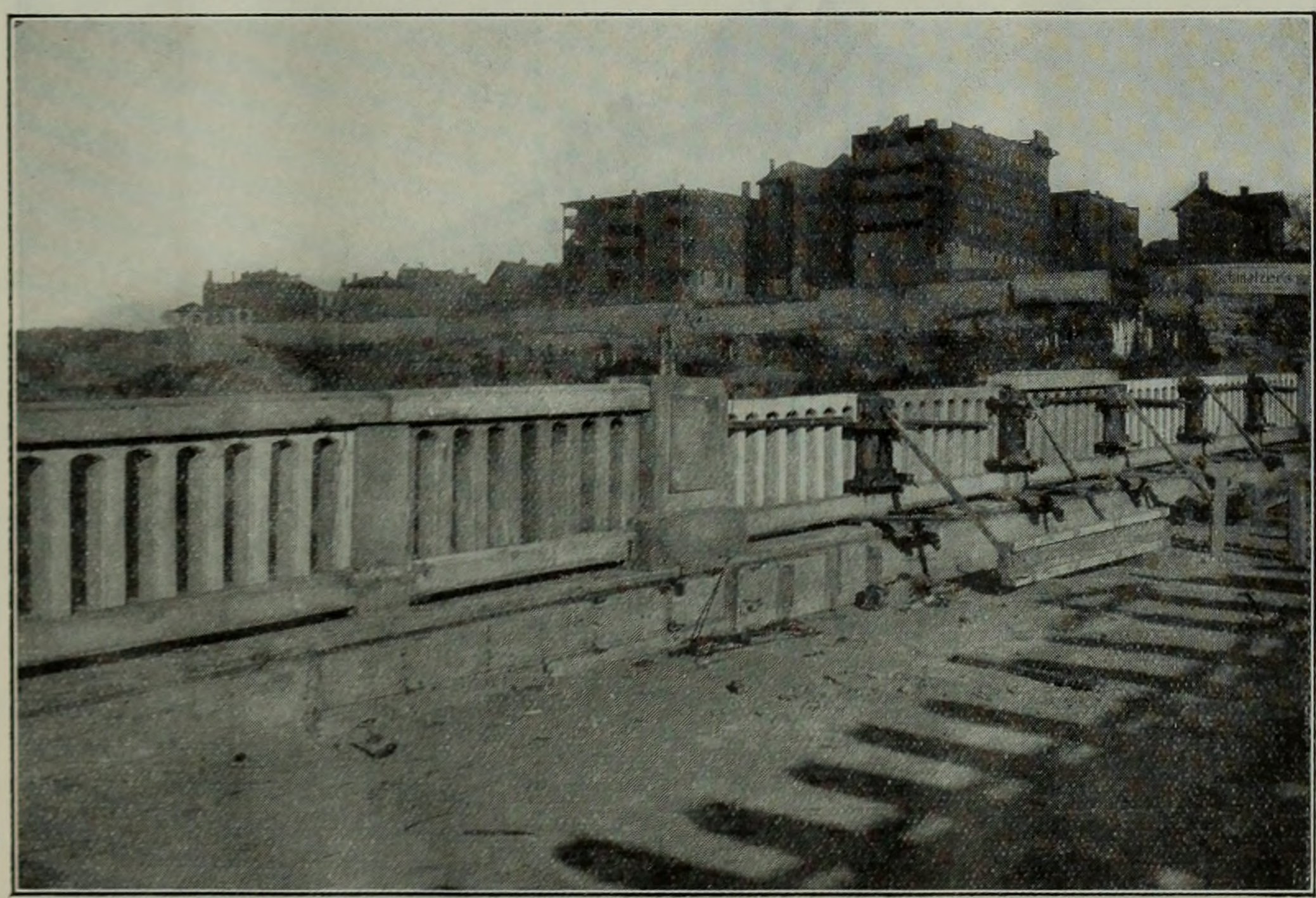


FIG. 16.—METHODS OF BUILDING HAND-RAIL, TWELFTH STREET TRAFFICWAY VIADUCT.

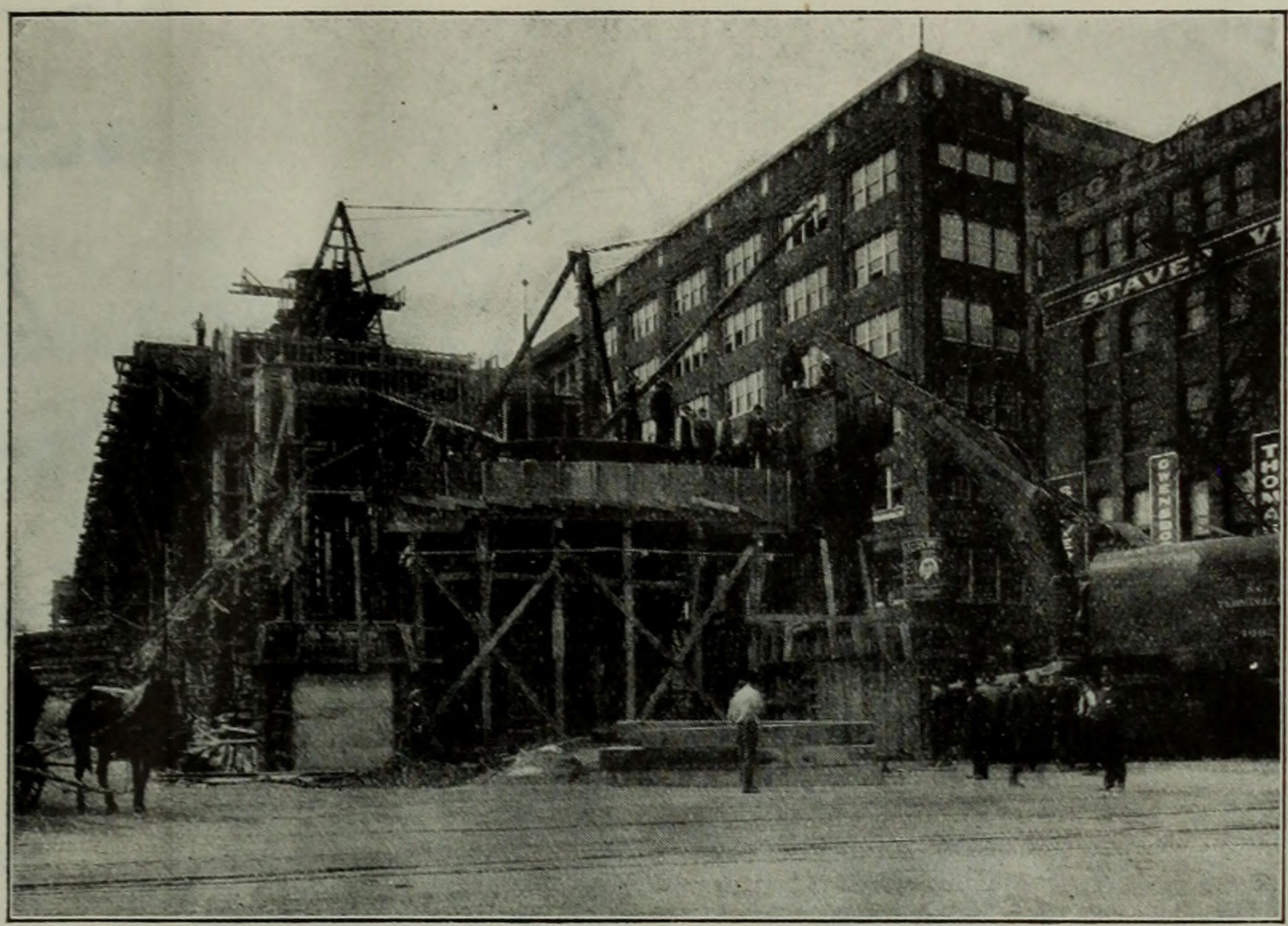


FIG. 17.—SANTA FÉ ARCH SPAN: ROCKER BENTS AND ARCH-RIB GUSSET-PLATE. TWELFTH STREET TRAFFICWAY VIADUCT.

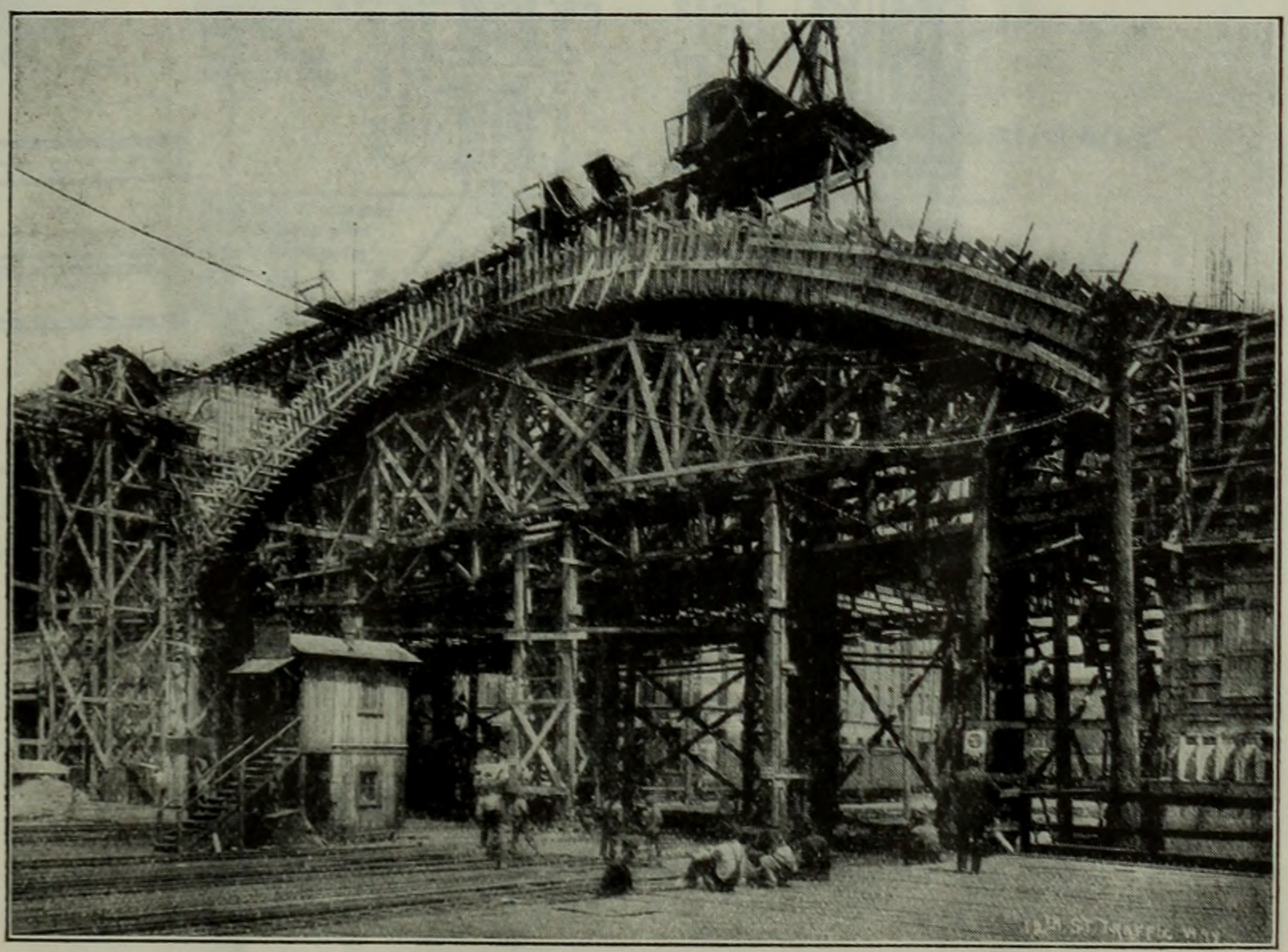
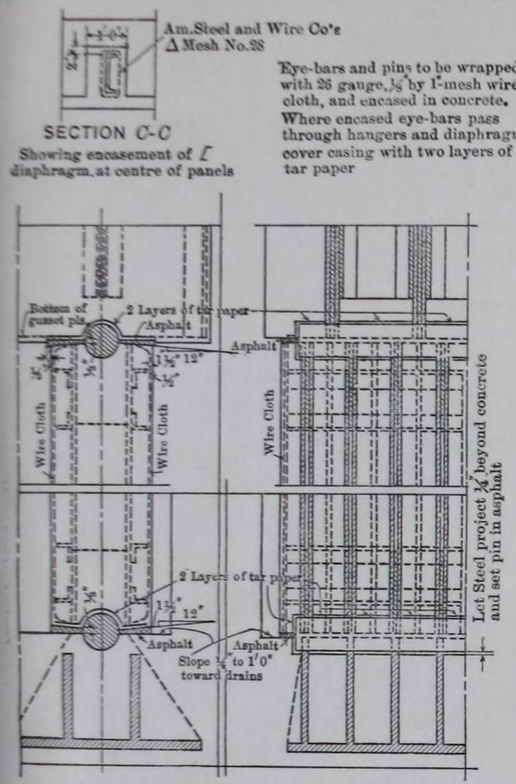
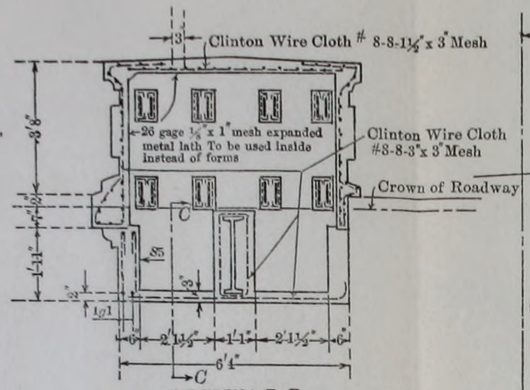


FIG. 18.—FALSEWORK AND FORMS, SANTA FÉ ARCH SPAN, TWELFTH STREET TRAFFICWAY VIADUCT.

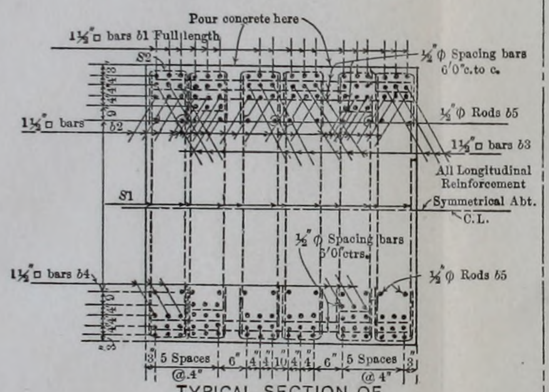


SKETCH SHOWING CONCRETING AROUND PINS

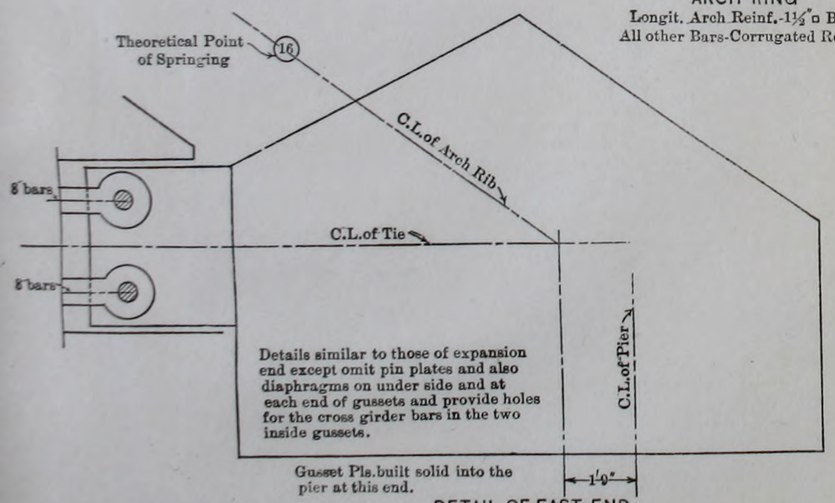
Eye-bars and pins to be wrapped with 26 gauge, ⅛" by 1" mesh wire cloth, and encased in concrete. Where encased eye-bars pass through hangers and diaphragms, cover casing with two layers of tar paper



SECTION B-B
Showing tie and Longitudinal Girder



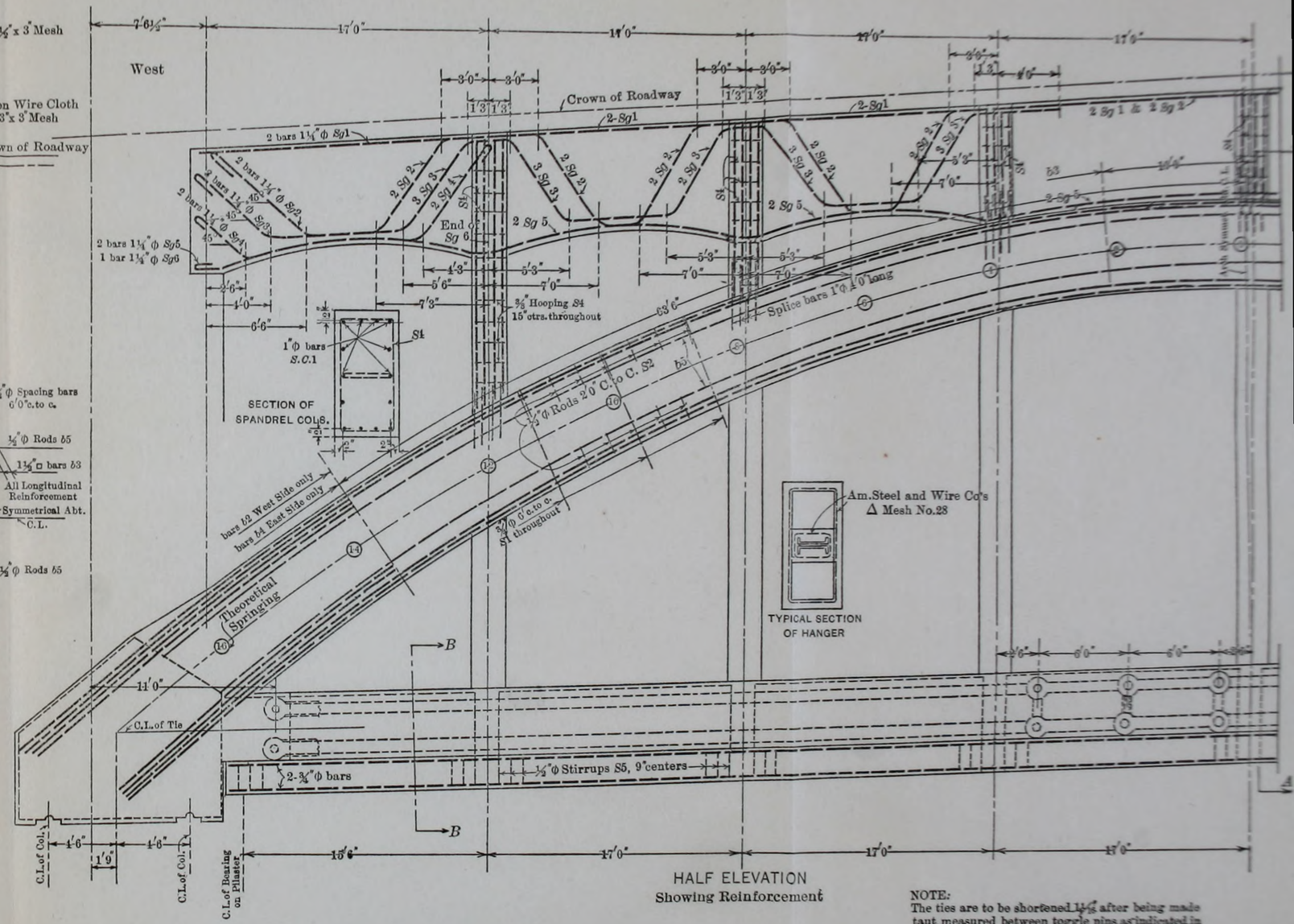
TYPICAL SECTION OF ARCH RING
Longit. Arch Reinf.-1½"□ Bars
All other Bars-Corrugated Rounds



Details similar to those of expansion end except omit pin plates and also diaphragms on under side and at each end of gussets and provide holes for the cross girder bars in the two inside gussets.

Gusset Pls. built solid into the pier at this end.

DETAIL OF EAST END



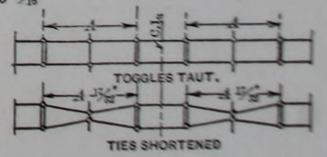
HALF ELEVATION
Showing Reinforcement

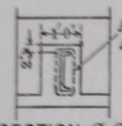
GENERAL NOTES:-

Except where otherwise noted the minimum spacing shall be 3 diameters of the bar
" " " " " edge distance shall not be less than 2 dia. of the bar. (C.L. of bar to edge of concrete)
" " " " " bars shall be bent to a radius of not less than 15 dia.
" " " " " hooks must be of dimensions as shown in this sketch
$R = 2\frac{1}{2}$ diameters of the bar $A = 5$ diameters of the bar

For splicing, the bars must be overlapped 40 diameters, and wired securely together
The splices must be alternated, so that in no case more than half the number of bars are spliced at the same point. Where possible the splices should be located at points of minimum tension

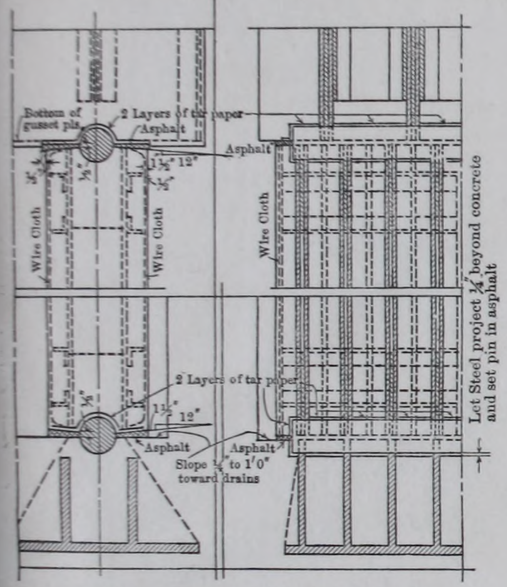
NOTE:
The ties are to be shortened $1\frac{1}{16}$" after being made taut, measured between toggle pins as indicated in sketches, or such an amount that the distance between springing points, after dead load is on the arch, is ⅜" shorter than constructed.
Rivets ⅞" φ — All holes sub punched and reamed to $\frac{15}{16}$"



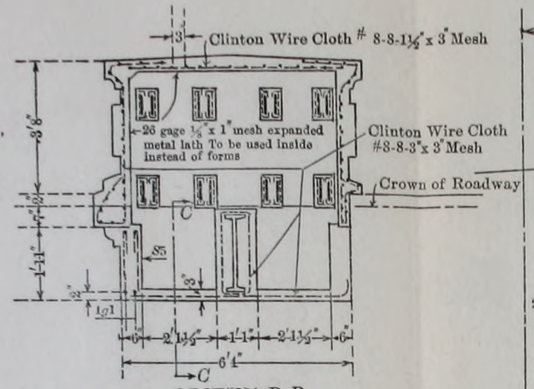


SECTION C-C
Showing encasement of [diaphragm at centre of panels

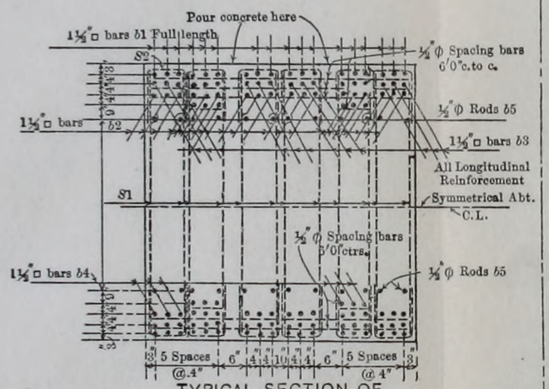
Eye-bars and pins to be wrapped with 26 gauge, ½" by 1"-mesh wire cloth, and encased in concrete. Where encased eye-bars pass through hangers and diaphragms, cover casing with two layers of tar paper



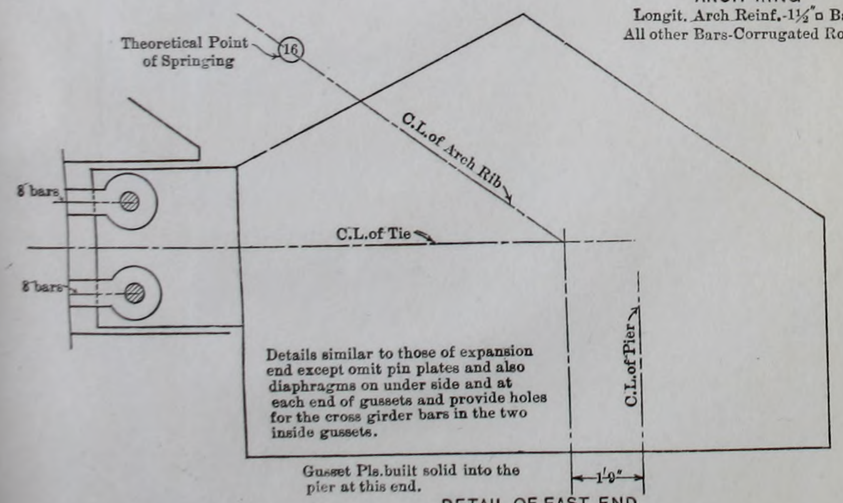
SKETCH SHOWING CONCRETING AROUND PINS



SECTION B-B
Showing tie and Longitudinal Girder



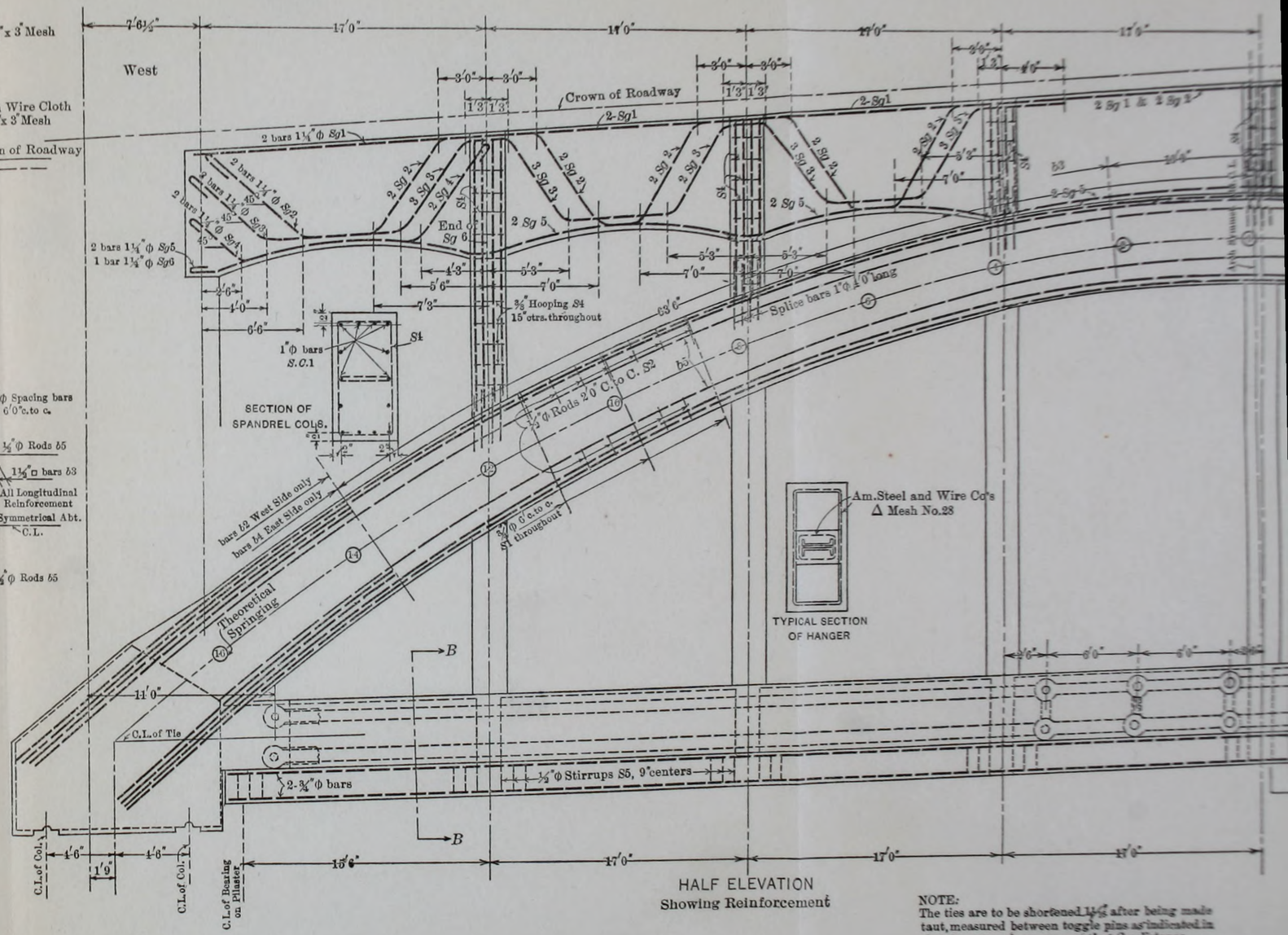
TYPICAL SECTION OF ARCH RING
Longit. Arch Reinf.-1½"□ Bars
All other Bars-Corrugated Rounds



Details similar to those of expansion end except omit pin plates and also diaphragms on under side and at each end of gussets and provide holes for the cross girder bars in the two inside gussets.

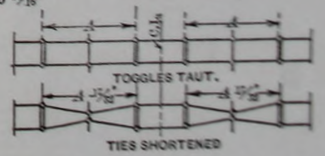
Gusset Pls. built solid into the pier at this end.

DETAIL OF EAST END



HALF ELEVATION
Showing Reinforcement

NOTE:
The ties are to be shortened 1 1/16" after being made taut, measured between toggle pins as indicated in sketches, or such an amount that the distance between springing points, after dead load is on the arch, is ⅜" shorter than constructed.
Rivets ⅞"φ — All holes sub punched and reamed to 15/16"



GENERAL NOTES:-

Except where otherwise noted the minimum spacing shall be 3 diameters of the bar
" " " " " edge distance shall not be less than 2 dia. of the bar. (C.L. of bar to edge of concrete)
" " " " " bars shall be bent to a radius of not less than 15 dia.
" " " " " hooks must be of dimensions as shown in this sketch
R = 2½ diameters of the bar A = 5 diameters of the bar
For splicing, the bars must be overlapped 40 diameters, and wired securely together
The splices must be alternated, so that in no case more than half the number of bars are spliced at the same point. Where possible the splices should be located at points of minimum tension

bracing on the ground and set up with the derrick traveling on the lower deck. The posts were generally 6 by 6-in. timbers, supported on ordinary ground-sills, and the bracing was of 3 by 6-in. pieces bolted in place.

The falsework supporting the upper deck forms was of similar bents resting on the lower-deck floor-slabs, with the posts under the main girder supported on the main girders below. These were made in three parts; a central bent of three posts and bracing supporting the central portion of the cross-beam, and two side bents of three posts each with a cap cantilevered to each side, one end of the cap supporting the cantilever beam and the other extending far enough under the cross-beam to carry a corresponding load, balancing the entire cap load over the posts. These three parts of each bent were prepared on the lower deck near position, or were carted out to position on wagon trucks and raised by the derrick on the upper deck.

The requirements of horizontal clearance for the railroad tracks on Santa Fé Street made necessary the use of three pairs of small wooden truss spans to support the forms for the arch ribs. These trusses rested on bents of posts set in available spaces between the tracks. The arch centering was placed on top of them, and the lower deck forms were suspended from them.

Falsework and forms for about seven girder spans were provided and carried forward as it was removed, the posts of the falsework being spliced as the height increased.

Procedure.—The falsework, forms, and pouring were each, relatively, on the lower deck about two span lengths ahead of the upper deck, as the falsework for the upper deck could not be placed on the lower-deck concrete until it had hardened somewhat. The whole construction thus moved forward in sequence, maintaining about the same relation of lower deck advance. This sequence of operations is shown by Fig. 20.

The reinforcing bars were delivered, as far as possible, in lengths correct for use. Diagrams were prepared showing the shape and dimensions for bending each bar and assigning a mark to it. The bars were bent in the yard, and each special bar was tagged with its erection mark on a linen tag. Groups of plain bars, etc., were not tagged individually.

As the forms neared completion, the prepared bars were carried out on the tram cars and set in place. Metal clips, separators, and wire ties at the crossing of every two bars, supported them in place. Use was made of small cement briquettes or blocks in some cases to hold the bars up from the forms.

The placing of bars and depositing of concrete in the columns was carried forward in customary routine. For the upper deck, all the bars were placed for main girders, cross-girders, cantilevers, and slabs complete from the last division to a bulkhead placed across the structure at a convenient point. Concrete was poured continuously across the whole width of the structure, filling the girders, beams, slabs, etc., without a stop, for the longitudinal distance selected.

The concrete was transported in cars, as described, hauled by cable, discharged into a movable distribution hopper, and from this into Insley chutes, which conveyed it to final position in the forms. A trussed steel chute about 80 ft. long carried the concrete from the receiving hopper on the upper deck forward to position in the lower portions of the structure. The concrete was mixed to a soft consistency so that it flowed readily, but it was not so wet as to have water standing on its surface. It was spaded and sliced at the forms and around the reinforcing bars to insure complete contact, and to work forward the mortar.

The maximum capacity of the concrete plant delivering 1 300 ft.—or, say, to the mid-point of the structure—was about 30 cu. yd. per hour. The largest quantity placed in continuous run was about 450 cu. yd. in 15 hours. A trip one way, from the plant to the east abutment, required 3½ min.

The hand-rail units were cast on the deck in metal-lined wooden forms, near the place where they were to be used. The lower rail was cast in place, the separately cast balusters were set in it in rows, and the pre-cast top rail was set on them, extending into the posts. The forms for the posts, except for the capital, were clamped on, and the posts were cast in place. The stairs were cast in units of one riser and one tread. These were supported on falsework and projected into the forms for the supporting girders, which were built in place.

Materials and Wages, General Observations.—The total quantity of timber used on the job was about 1 500 000 ft. b.m. This may be

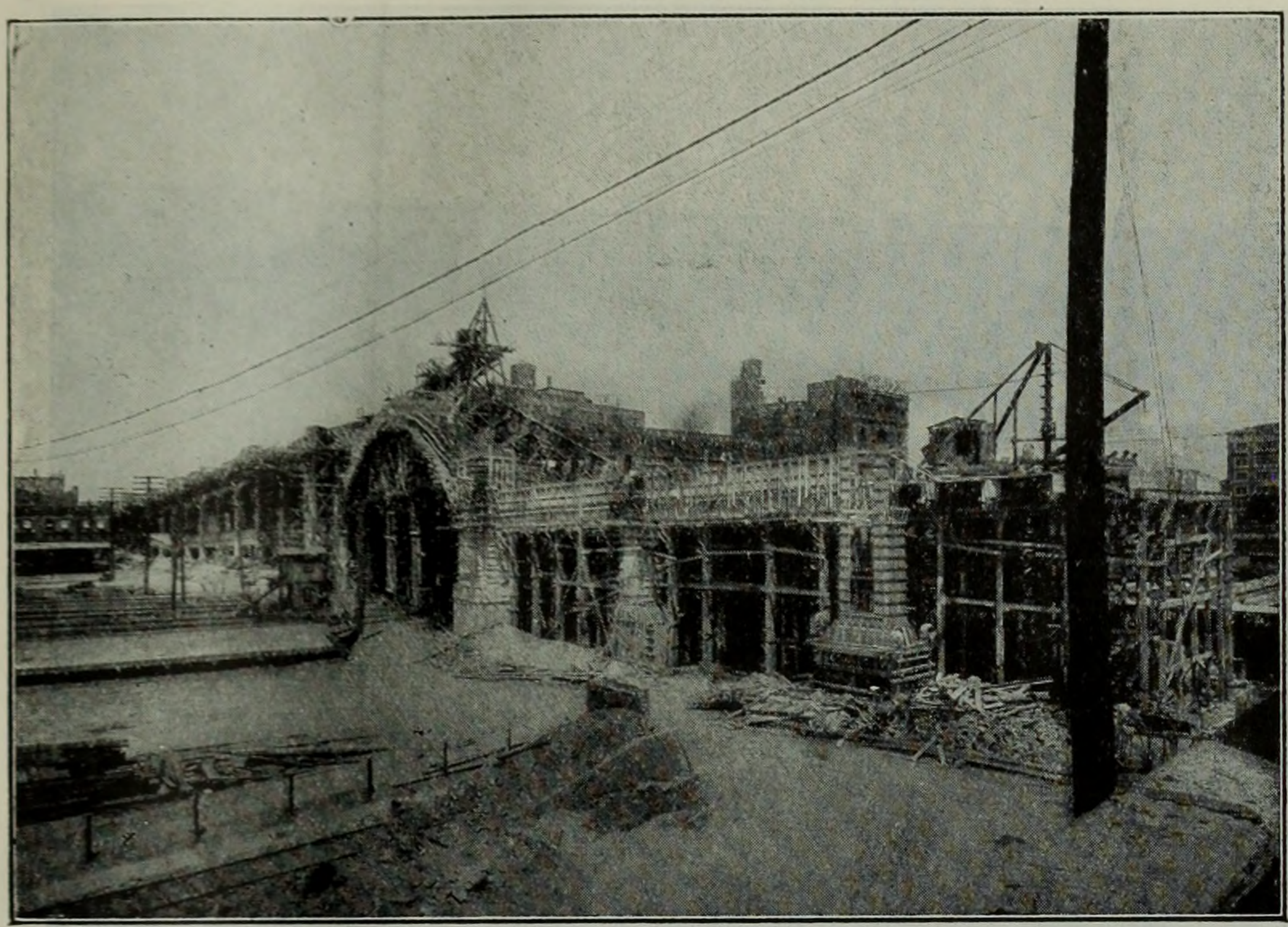


FIG. 19.—LOWER DECK ADJACENT TO ARCH SPAN, TWELFTH STREET TRAFFICWAY VIADUCT, DURING CONSTRUCTION.

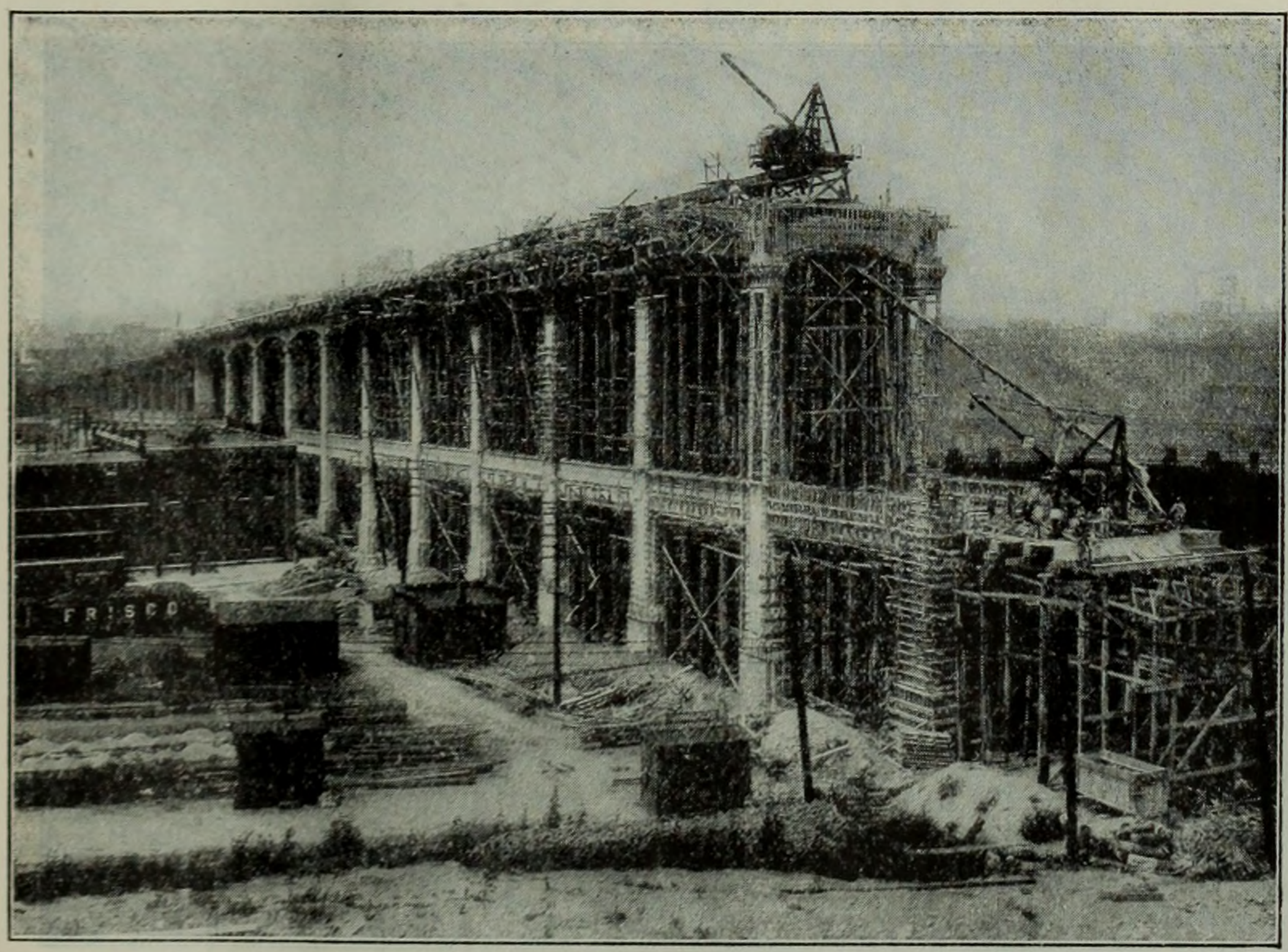


FIG. 20.—TWELFTH STREET TRAFFICWAY VIADUCT: SEQUENCE OF OPERATIONS DURING CONSTRUCTION.

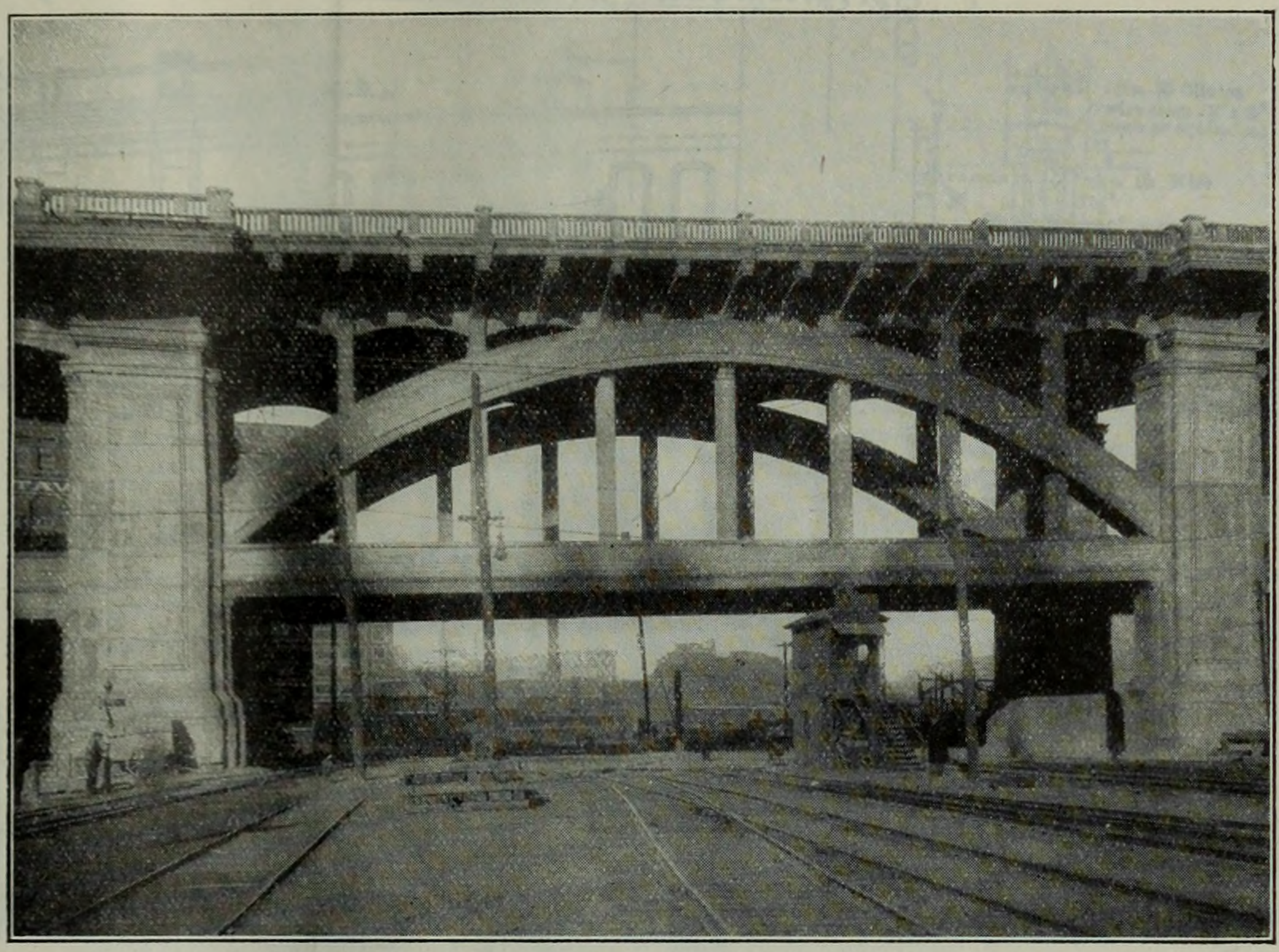


FIG. 21.—SANTA FÉ ARCH SPAN AFTER COMPLETION, TWELFTH STREET TRAFFICWAY VIADUCT.

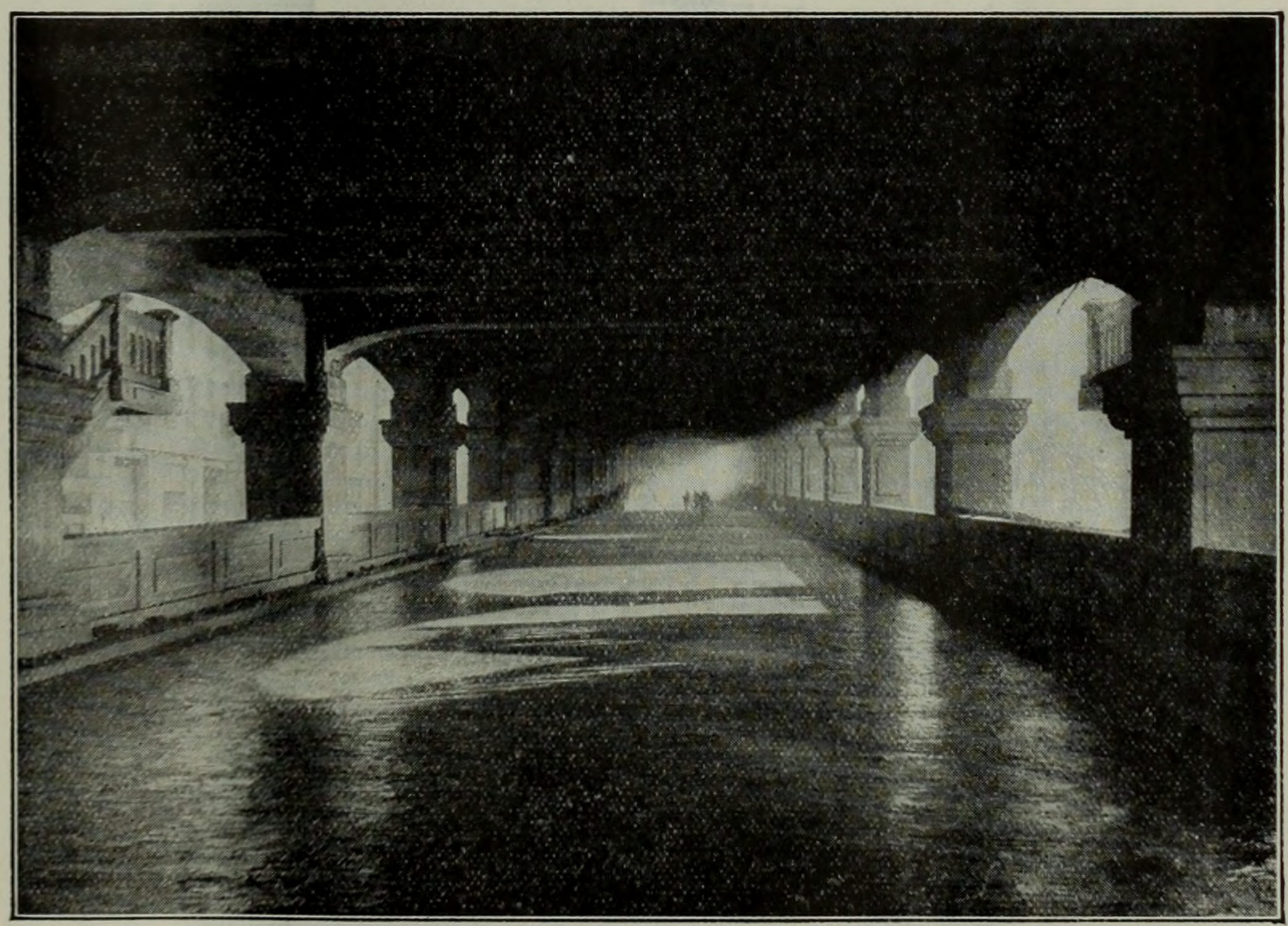
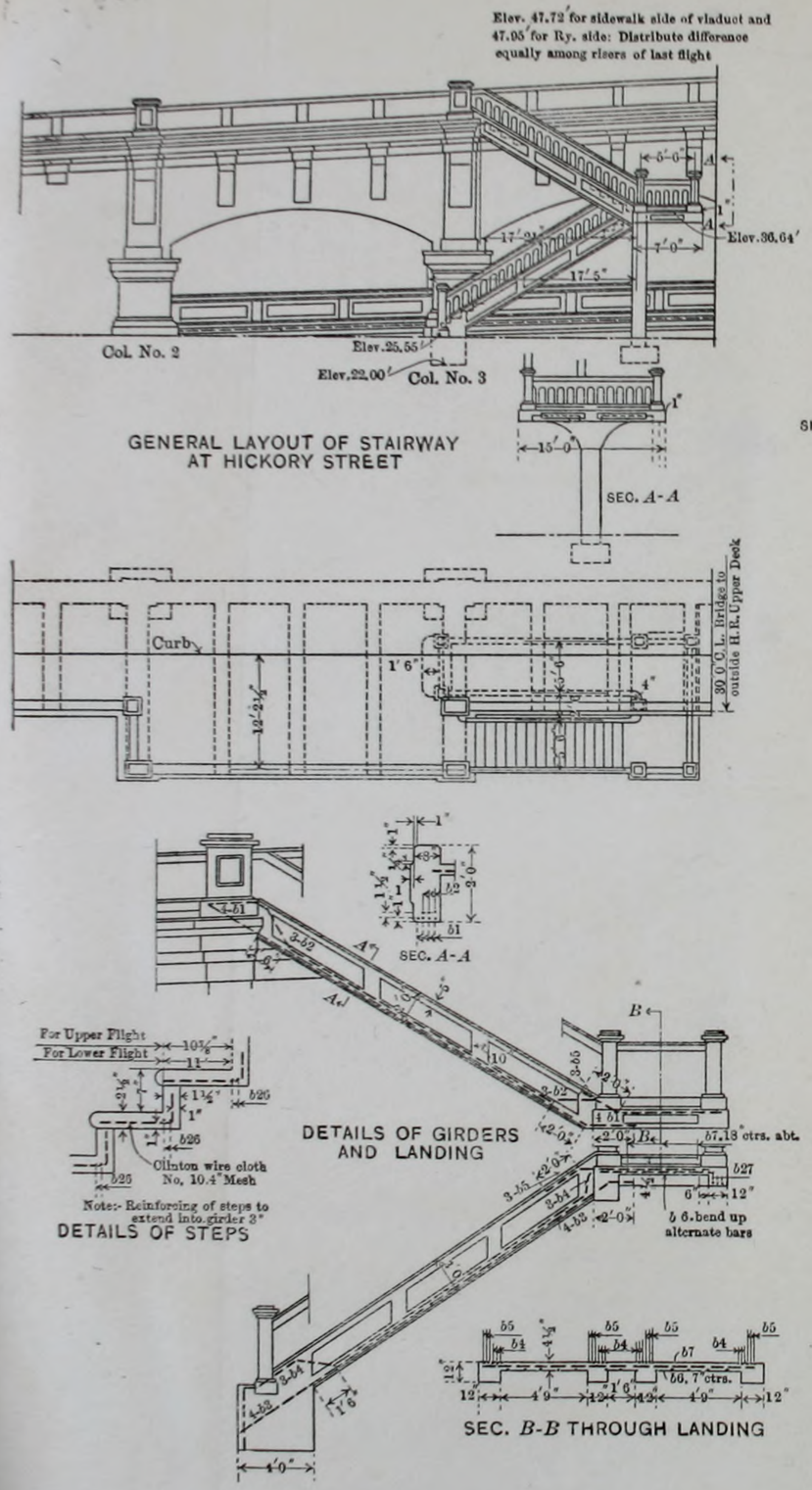
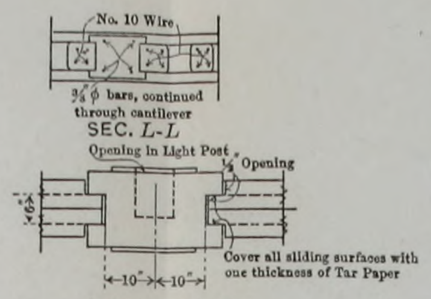
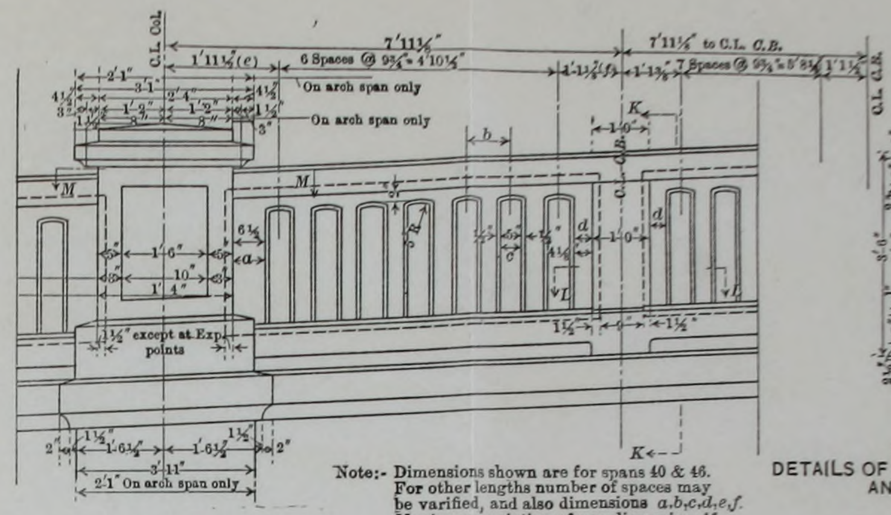
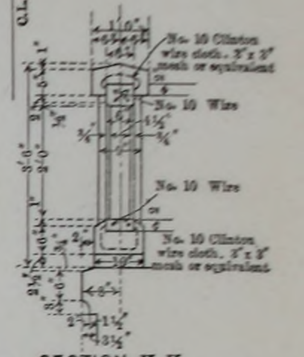


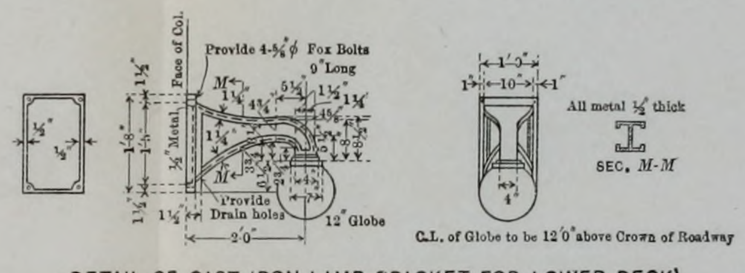
FIG. 22.—TWELFTH STREET TRAFFICWAY VIADUCT, LOOKING EASTWARD ON THE LOWER DECK.

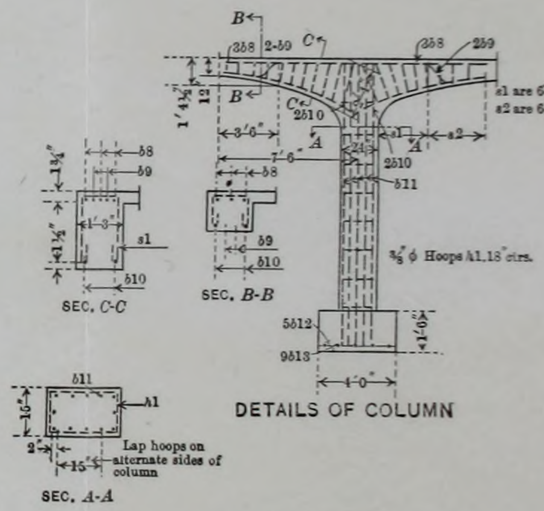


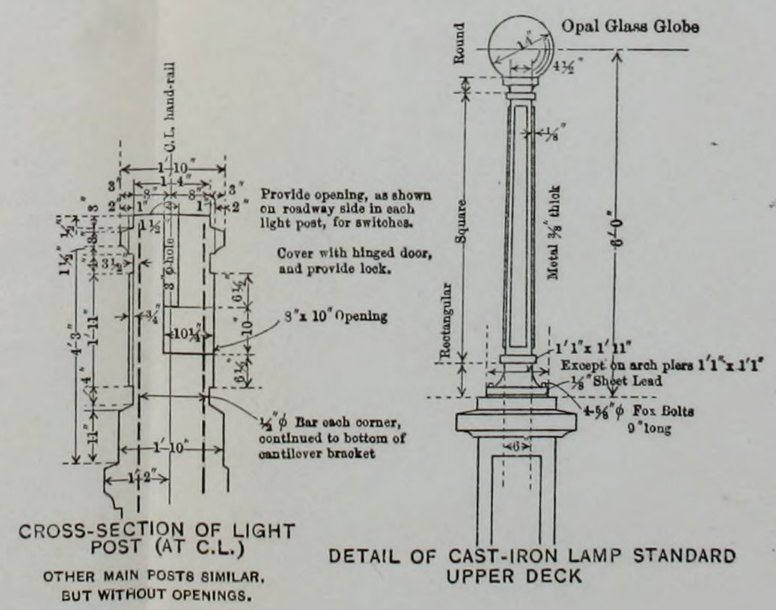












DETAILS OF
HAND-RAILS, STAIRWAYS,
LIGHT BRACKETS. ETC.
TWELFTH STREET TRAFFICWAY VIADUCT
KANSAS CITY, MO.

roughly divided into 600 000 ft. of form lumber, 850 000 ft. of falsework, and 50 000 ft. in general use, including bins and mixing plant. The average cost of this material was about $20 per thousand, or about $1 per yd. applied to the whole concrete, or $1.25 per yd. applied to concrete above the bases.

The number of men employed varied from 150 to about 325, an average of about 200 men, of whom, perhaps, 150 were carpenters and helpers. The wages prevailing were, for common labor, $2.00, for carpenters' helpers, $2.50, for carpenters, $3.50, for finishers, $3.50, and for derrick and hoist operators, $4.00; all for 9 hours per day.

The conduct and execution of the work merit special comment. The viaduct was built substantially in the allowed time of 15 months, moving forward in logical and orderly manner, largely due, of course, to systematic handling of labor and materials. On the first day of every week, a schedule of work to be accomplished each day of that week was prepared in writing, and the foremen were instructed, not only what to do each day, but also how much had to be done. If the amount assigned was not completed at the usual stopping time, the men kept at work until it was done. This applied particularly to placing concrete, the day's run being completed more than once after midnight. The conditions surrounding the work permitted it to be handled as a manufacturing proposition, and with factory-like precision and system.

There was some difficulty in securing stone as clean as desired, for the limestone strata in this region are overlaid with clay and other objectionable materials; and, especially during a rainy season, it is impossible to prevent some clay and soft shale from mingling with the stone in the crusher bins. In fact, the more careful crushers shut down during rainy weather to avoid this difficulty. The stone cost $1.15 per cu. yd., delivered in cars at the site. The sand supply from the Kaw River is clean, sharp, and well graded, testing about 52% retained on a No. 30 sieve. Its cost delivered in cars at the site was about 65 cents per cu. yd. The cement was Kansas City-Portland, manufactured in a mill about 15 miles from the site, and costing, delivered in cars at the site, $1.20 per bbl. (plus sack loss). These delivered prices include somewhat complex switching charges.

Reinforcing steel delivered cost about 1.75 cents per lb.

6.—Quantities and Costs.

The contract assigns unit prices for forty classified items, including many classifications of concrete with regard to location in the structure. The following summarized figures were prepared from such classifications, and an attempt has been made to give the salient features of the final quantities of the structure, as built. These costs are the amounts paid by the City, and include the contractor's profit.

There are about 32 500 cu. yd. of concrete, distributed about as follows:

	Cubic yards.
Concrete piles	1 360
Columns, piers at Santa Fé Street, and footings	11 300
Upper deck, including girders, cross-girders, cantilevers, floor-slabs, etc	11 300
Lower deck, including girders, cross-girders, floor-slabs, etc.	3 600
Retaining walls and abutments	3 000
Ribs of arch at Santa Fé Street	600
Hand-rails	500
Stairways, sidewalks, curbs, paving base on fills, and miscellaneous	800

No special payment was made for excavation, but the amount paid for concrete included its cost. The total excavation was about 10 000 cu. yd. The amounts paid for concrete in place varied from about $8.00 to $13.50 per cu. yd. for different classifications, averaging $9.45 for the whole structure, omitting only the piles and the hand-rails, as both of these were paid for at linear foot prices.

There were driven 2 610 piles, having a total length below bottoms of footings of about 47 000 lin. ft. and an average length each of 18 ft., costing in place $1.20 per lin. ft.

The approximate quantities of metal, and the prices paid for it in place are as follows:

Reinforcing steel, ¾ in. in diameter and greater	2 900 000 lb. at 2.95 cents per lb.
Reinforcing steel, less than ¾ in. in diameter	1 000 000 lb. at 3.25 cents per lb.
Unfabricated metal, such as bed and bearing plates, guard-angles, etc	100 000 lb. at 4.40 cents per lb.
Fabricated structural metal	620 000 lb. at 4.60 cents per lb.
Cast-iron lamp-posts and brackets	25 000 lb. at 5 cents per lb.

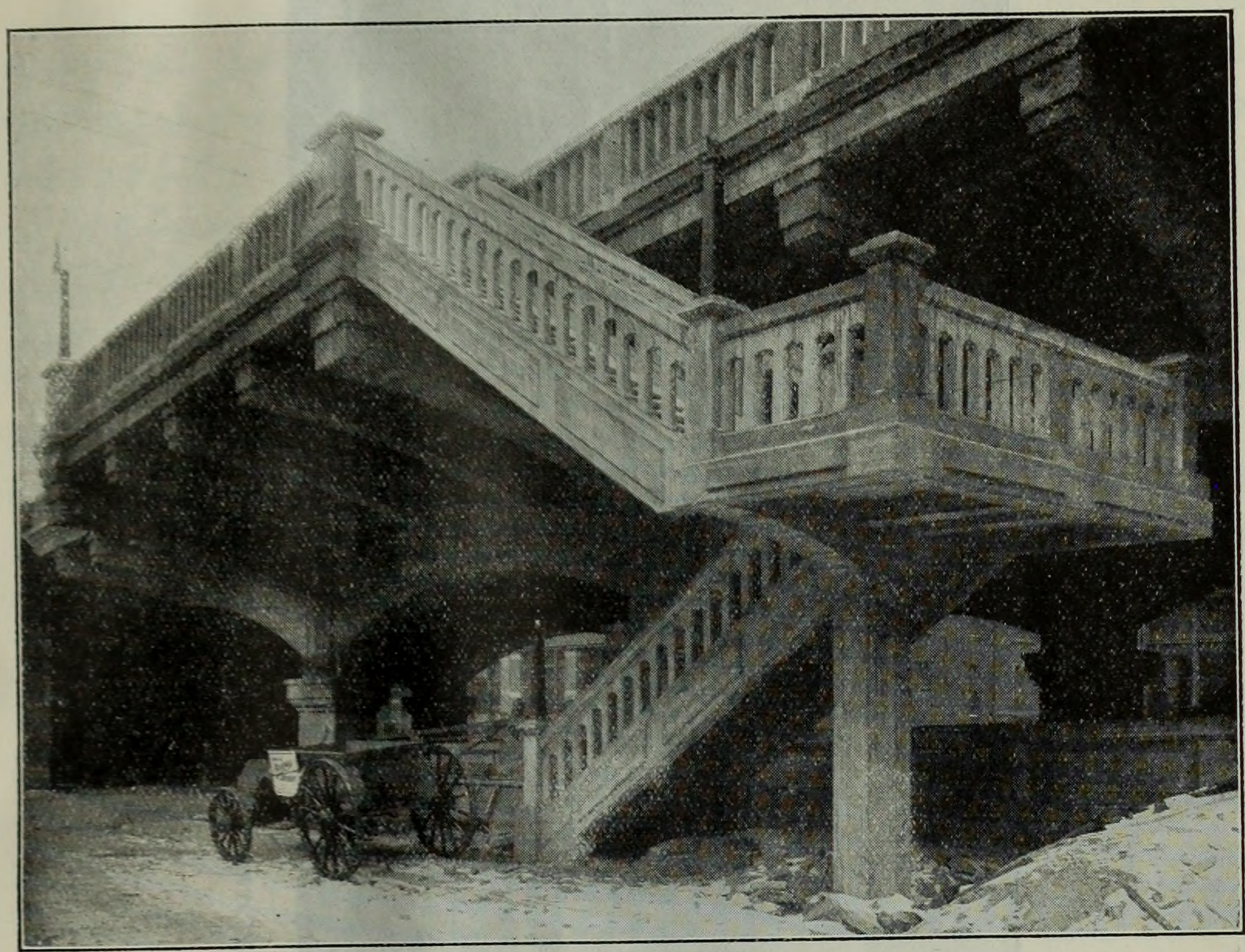


FIG. 23.—STAIRWAY AT HICKORY STREET, TWELFTH STREET TRAFFICWAY VIADUCT.

FIG. 24.—STAIRWAY AT MULBERRY STREET, TWELFTH STREET TRAFFICWAY VIADUCT.

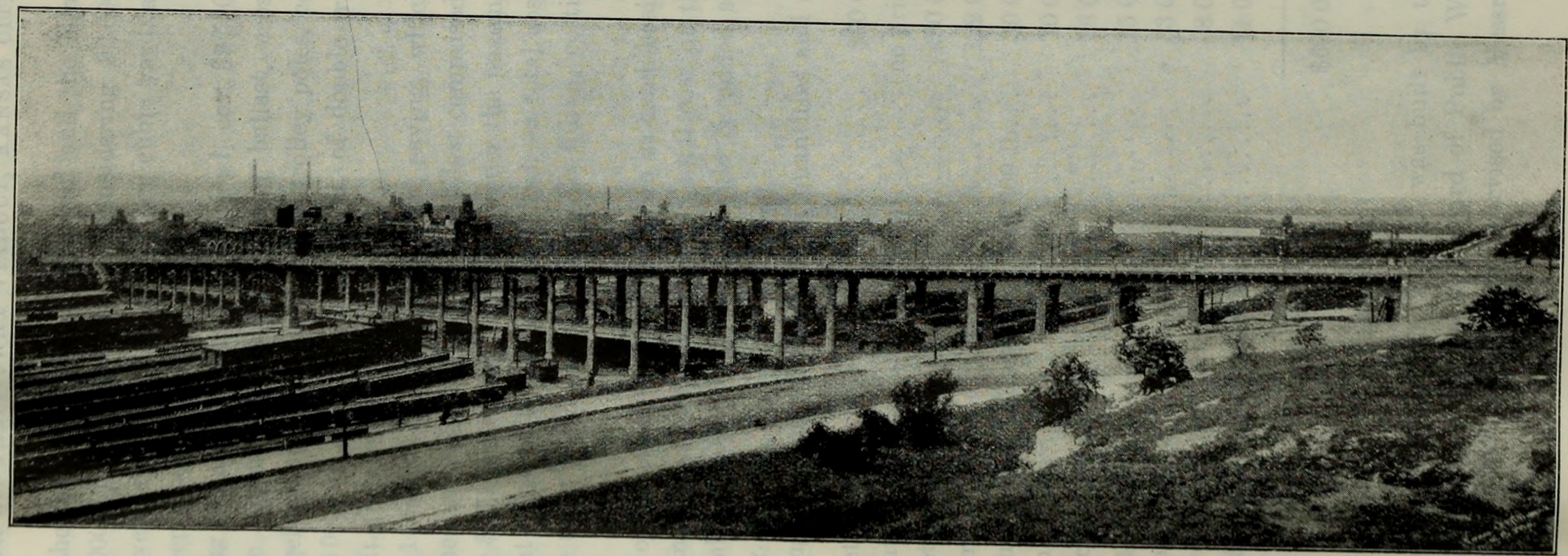


FIG. 25.—TWELFTH STREET TRAFFICWAY VIADUCT, KANSAS CITY, MO.

Omitting the fabricated structural metal and the lamp-posts gives a total of 4 000 000 lb. of metal, or, considering 30 600 cu. yd. of concrete, an average of 130 lb. of metal per cubic yard, at an average price of 3.06 cents per lb.

The lighting system, complete as described, cost $9 500.

The double street-car tracks, including ballast, cross-ties, track rails, inner steel guard-rails, bonding, and trolley poles, cost, in place, a total of $18 000 for a length of 2 600 ft. of double track, or, say, $3.50 per lin. ft. for single track.

There are 14 500 sq. yd. of wood-block paving, which cost $2.25 per sq. yd. The water-proofing under the ballast comprised about 5 000 sq. yd., costing $1.71 per sq. yd., and that under the pavement included 11 500 sq. yd. at a cost of 58.5 cents per sq. yd.—a total cost of $15 000.

There were some 40 000 cu. yd. of earth filling, costing 28 cents per cu. yd.

There were other miscellaneous items such as catch-basins, drainage pipes, sewer connections, etc., bringing the total cost of the structure to the city to about $600 000, to which should be added some $40 000 for engineering.

The following is a summarized table of quantities and costs:

Concrete piles (2 160)	$60 000
Excavation, about 10 000 cu. yd	(no price)
Concrete, 30 600 cu. yd	290 000
Reinforcing metal, 4 000 000 lb	122 000
Structural metal and castings, 645 000 lb	30 000
Lighting system	10 000
Street-car tracks, etc	18 000
Water-proofing	15 000
Block pavement	33 000
Hand-rails	8 000
Earth filling and miscellaneous	14 000
	$600 000

7.—Personnel.

The Twelfth Street Trafficway Viaduct was built by the City of Kansas City, Mo., under control of the Board of Public Works. The plans were made and the construction supervised by Messrs. Waddell

and Harrington, Consulting Engineers, under the personal supervision of Mr. Harrington. The writer prepared the general preliminary studies and estimates, directed the design, and wrote the specifications. L. R. Ash, M. Am. Soc. C. E., was City Engineer at the time the plans were adopted, and after joining the firm, directed the construction work. The details of design were largely the work of Messrs. A. E. Slettum and C. W. Yelm. Mr. M. J. Maher was Resident Engineer, in charge of the entire construction work.

The structure was built under one general contract by The Graff Construction Company, of Seattle and Kansas City. Mr. Graff gave the work his constant personal attention, and was ably assisted by his superintendent, Mr. Harold E. Ketchum, in handling the building operations and in the design of the contractor's plant, falsework, forms, and other temporary structures; Mr. George Hockensmith was General Foreman.

For their courteous assistance in the preparation of these data, the writer desires to express his cordial appreciation to the above gentlemen and to H. C. Tammen, Assoc. M. Am. Soc. C. E., at present Designing Engineer of Harrington, Howard, and Ash.

DISCUSSION

F. W. Green,* Assoc. M. Am. Soc. C. E. (by letter).—The author has made a noteworthy contribution to engineering literature. Unique seems to be the word best adapted, adjectively, to apply to a structure which is neither straight as to alignment, level as to gradient, symmetrical as to transverse loading, nor orthodox as to the tenet of design which requires that indeterminate stresses must be solved by being avoided. Mr. Green.

Moreover, it is unique in that the embellishments of art have been invoked, with very gratifying success, to commend to the esthetic emotions a structure primarily intended to be useful. This, too, under circumstances most unpropitious—an ugly *situs,* an unbalanced subject, and an inauspicious environment; and, as the author states, "without any extra funds for adornment".

A consideration of the problem creates a feeling of dismay, which is superseded later by one of admiration for the ingenuity shown and the resource developed in mastering the many baffling features. To the mind mathematically inclined, the graphical method for solving the integrals shown on pages 494 and 495 will commend itself both for elegance and simplicity. The method for obtaining moments and deflections for the rectangular frames (given on page 496) by the solution of ten simultaneous equations, though no laughing matter, offers opportunity for mirth: "When a structure is so formed that the stresses in it cannot be determined by statics, it is said to be a redundant system; and the principle of least work applicable to it is as follows, etc."† Do we not have here a paradox? Is it strictly proper to say that the solution of these ten simultaneous equations involves the principle of least work? The author says: "By arranging these * * * in tabular form, the solution is found with comparatively little labor." Did he intend "little" in the sense of *"multum in parvo"*?

Seriously speaking, however, the impression uppermost in the writer's mind, after reading this paper, is that this work marks another important instance of the present tendency in American development to pass from the stage of pioneer civilization, with its temporary, but economical and expedient, types of buildings, works, ways, and systems, to an era of permanence, beauty of line and mass, sufficiency, and efficiency.

The design of this structure is a bold departure in many respects from the established canons of design, and the novice should be admonished not to undertake liberties of this kind unless and until he is absolutely sure of his ground. For example, a comparatively slight settlement of the foundation under one of the 120-ft. columns conceivably might have disastrous consequences.

* Stamps, Ark.

† Merriman's "Mechanics of Materials", 10th Ed., p. 321.

Mr. Mensch. L. J. MENSCH,* M. AM. SOC. C. E. (by letter).—Reinforced concrete seems to enlist the brightest engineers under its colors. Though viaducts like that described in this paper have often been built of steel, one would search American literature in vain for a description of such viaducts in which the attempt was made to design the structure for temperature and other so-called secondary stresses, as was done for the Twelfth Street Trafficway Viaduct of Kansas City. Because such refined—but often very necessary—computations are rare in American practice, we need not be surprised that, in one of the first attempts, such computations lack the simplicity, elegance, and surety of the

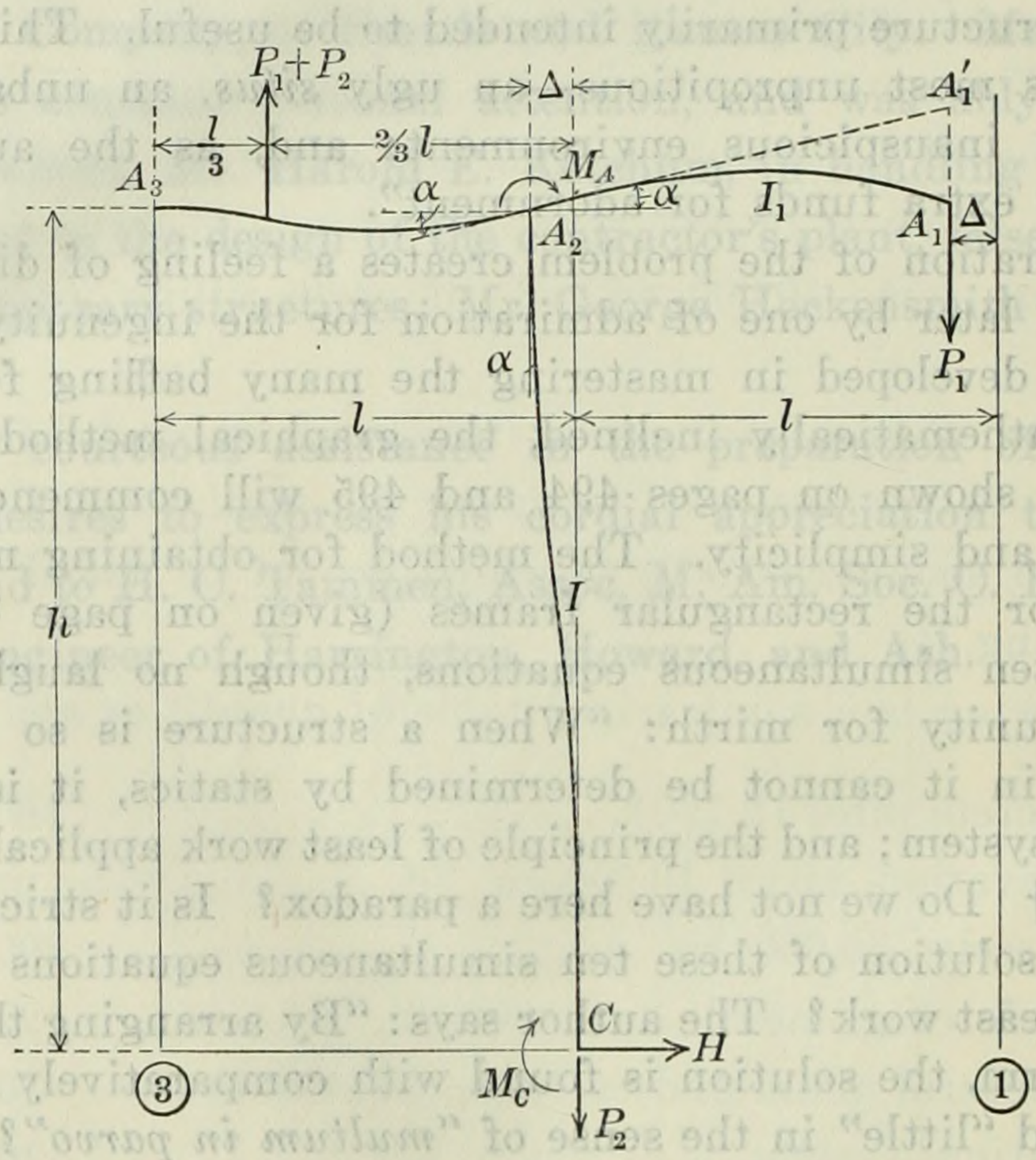


FIG. 26.

schooled designer. Not until a great many of our students take post-graduate courses in the design of so-called "higher structures" can a material change be expected in this matter. Take, for example, Fig. 2 and its explanation. The writer has designed indeterminate structures for more than twenty years, yet he can follow the reasoning only with great difficulty. There is no need for such lack of clearness.

The problem can be solved entirely, with much less effort, by finding the unknown conditions of the supports at A_1 and C (Fig. 26). A sliding joint was assumed at A_1, hence the only unknown that can act there is the vertical force, P_1. The point, C, is fixed, hence the support will exert on the column, $A_2\ C_1$, a thrust, H, a pull, P_2, and a moment, M_c.

* Chicago, Ill.

Mr. Mensch.

These unknown reactions can easily be found by the rules for the change of angles and for the deflection of a beam:

$$\Delta\,\alpha = \int \frac{M ds}{E\,I} = \frac{1}{E\,I}\text{ (area of diagram of moment)} \ldots\ldots\ldots\ldots (1)$$

$$\Delta\,y = \int \frac{M x ds}{E\,I} = \frac{1}{E\,I}\text{ (static moment of diagram of moment about the point, the deflection of which is sought)} . (2)$$

For the force, P, acting at the end of a cantilever,

$$\Delta\,\alpha = \int \frac{P x dx}{E\,I} = \frac{P l^2}{2\,E\,I}$$

$$\Delta\,y = \int \frac{P x \times x dx}{E\,I} = \frac{P l^3}{3\,E\,I}, \text{ provided } I \text{ is constant.}$$

$A'_1\,A_1$ can be considered the deflection of the point, A_1, due to P_1, or

$$l\,\alpha = \frac{P_1\,l^3}{3\,E\,I_1},$$

or

$$\alpha = \frac{P_1\,l^2}{3\,E\,I_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

It is true, as the author has stated, that the point of inflection of $D\,B$ is $\frac{1}{3}\,l$ from D, and, in order to preserve the equilibrium, we can imagine that P_1 and P_2 are acting at this point, producing a moment, $(P_1 + P_2)\,\frac{l}{3}$, at A_3, and $(P_1 + P_2)\,\frac{2}{3}\,l$, at A_2.

$$\text{Then}\quad \alpha = \frac{(P_1 + P_2)\,\frac{2\,l}{3} \times \frac{2\,l}{3} \times \frac{1}{2} - (P_1 + P_2)\,\frac{l}{3} \times \frac{l}{3} \times \frac{1}{2}}{E\,I_1}$$

$$= (P_1 + P_2)\,\frac{l^2}{6\,E\,I_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

From Equations 3 and 4, $\frac{P_1 + P_2}{6} = \frac{P_1}{3}$,

or

$$P_1 = P_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

We can determine, also, from the column, $A_2\,C$,

$$\alpha = \frac{M_c\,h - \frac{H\,h^2}{2}}{E\,I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and from Equations 3 and 6, $\frac{P_1\,l^2}{3\,E\,I_1} = \frac{M_c\,h - \frac{H\,n^2}{2}}{E\,I}$

Mr. Mensch. let $n = \frac{h}{l} \frac{I_1}{I}$, $\qquad M_c n - \frac{H h n}{2} = \frac{P_1 l}{3}$ .....(7)

From $A_2 C$,

$$\Delta = \frac{\frac{M_c h^2}{2} - \frac{H h^3}{6}}{E I}, \text{ or } M_c - \frac{H h}{3} = \frac{2 E I}{h^2} \Delta \text{ .....(8)}$$

By taking moments about A_2,

$$(P_1 + P_2) \frac{2}{3} l + P_1 l + M_c = H h \text{ .....(9)}$$

Equations 7, 8, and 9 have three unknowns, and we easily obtain

$$\left.\begin{aligned} H h &= \frac{2.4\, E I}{h^2} \Delta \frac{1 + 7 n}{1 + 2.8 n} \\ M_c &= \frac{2.4\, E I}{h^2} \Delta \frac{1 + \frac{7}{2} n}{1 + 2.8 n} \\ P_1 l &= 3.6\, E \frac{I}{h^2} \Delta \frac{n}{1 + 2.8 n} \\ M_A &= 8.4\, E I \times \frac{\Delta}{h^2} \times \frac{n}{1 + 2.8 n} \end{aligned}\right\} \text{ ..... (10)}$$

If the structure is cast monolithically, without any joints, we can place $I = \frac{b \times d^3}{12}$.

The moment of resistance $= \frac{b \times d^2}{6}$, and

$$\text{the stresses at } C = \frac{M_c}{\frac{b \times d^2}{6}} = \frac{2.4\, E \frac{b \times d^3}{12}}{\frac{b \times d^2}{6}} \frac{\Delta}{h^2} \frac{1 + \frac{7}{2} n}{1 + 2.8 n}$$

$$= 1.2\, E\, d \frac{\Delta}{h^2} \frac{1 + \frac{7}{2} n}{1 + 2.8 n} \text{ .....(11)}$$

The moment of inertia of the columns varies, and, assuming the section of the base only to govern, $n = 4.46$; of the top only to govern, $n = 2.76$, which produces hardly any difference in the value of the fraction, $\frac{1 + \frac{7}{2} n}{1 + 2.8 n}$, as the respective quotients are 1.23 and 1.233.

Assuming $d = 7$ ft., $l = 48$ ft., $h = 89$ ft., $E = 3 \times 10^6$, $\Delta = \frac{l}{2\,000}$, then $S_c = 94$ lb. per sq. in.

Mr. Mensch.

Where the bents are $h = \frac{89}{4} = 22$ ft. 6 in., S_c is nearly sixteen times as great, or 1 504 lb. (the value of $\frac{1 + \frac{7}{2} n}{1 + 2.8\, n}$ changing only to 1.2), which is not counterbalanced by the dead load of the structure, and large cracks will be prevented only by steel reinforcement.

In this case, $E\,I$ must be replaced in Equation 10 by $E_s\, A_s d'^2$, where E_s is the modulus of elasticity of the steel, A_s is the steel reinforcement, in square inches, on the tension side, and $d' = jd$, the moment of resistance being represented by $A_s\, jd$.

The stress in the reinforcement at C

$$= \frac{2.4\, E_s\, A_s\, d'^2 \frac{\Delta}{h^2} \frac{1 + \frac{7}{2} n}{1 + 2.8\, n}}{A_s\, d'} = 2.4\, E_s \frac{d'\, \Delta}{h^2} \frac{1 + \frac{7}{2} n}{1 + 2.8\, n} \ldots\ldots (12)$$

In this latter case, M_c is of considerable magnitude, and will cause an unequal loading of the piles, the latter being symmetrically arranged around the columns.

Evidently the author forgot to assume a thrust at C, or he would have found that M_2, in Fig. 3 must be negative, if he considers M_1 positive.

It is absolutely unnecessary to resort to the summation methods, where the expression, $\frac{d\,s}{I}$, varies.

In most cases we can put $\frac{d\,s}{I} = \frac{d\,x}{I}\left(1 + C\frac{x}{l}\right)$, which will allow of easy integration of $\frac{M\,d\,s}{I}$ and of $\frac{M\,x\,d\,s}{I}$.

It is extremely rare, where, instead of a straight-line variation, a parabolic, or hyperbolic variation of $\frac{d\,s}{I}$ must be adopted.

The advantage of integration over summation is evident by the fact that in the first case we can obtain a formula of general application, but in the latter we obtain the solution of one particular problem only.

The use of the formulas for continuous beams for solving stresses in framed structures is accompanied with great difficulties, and is not the shortest or most lucid solution of the problem.

Any one reading the author's method of finding the wind stresses in a two-story frame, would think it requires the enormous labor of solving a six times indeterminate structure, even if both columns have the same moment of inertia. Such a structure is only twice indeterminate. On account of the symmetry of construction, the point of inflection of the girders must be the center of the span. By the same

Mr. Mensch. method as the writer used for the temperature stresses, the following equations may be found to solve for P_1 and P_2:

$$\frac{I_1}{I_2} P_2 = P_1 + 6\, n_1 \times P_1 - 6\, \frac{M'\ n_1}{l} \quad \ldots\ldots\ldots\ldots (13)$$

$$0 = P_2 + 6\, (P_1 + P_2)\, n_2 - 6\, \frac{M''\, n_2}{l} \quad \ldots\ldots\ldots\ldots (14)$$

When $n_1 = \frac{h_1}{l} \frac{I_1}{I}$, $n_2 = \frac{h_2}{l} \frac{I_2}{I'}$, and $M' = \frac{H_1 h_1}{2}$, and $M'' = H_1 \left(h_1 + \frac{h_2}{2}\right) + H_2 \frac{h_2}{2}$.

Where the moments of inertia of the columns are not alike, the exact solution, as above stated, is very laborious, and mostly time and men are lacking to do such work; but, fortunately, we can find an approximate solution, by considering that P_1 and P_2 vary only slightly from the values obtained from Equations 13 and 14, and that the point of inflection of the girders moves only very slowly from the center toward the weaker column. Where the ratios of the moments of inertia of the columns is infinite, the point of inflection would be at the connection of the girder and the weaker column; in the Twelfth Street Trafficway Viaduct it is about $\frac{3}{10}\, l$ from the weaker column. The assumption of the point of inflection is easily checked by calculating the deflection at the top of the frame. The deflection of the left column must be equal to that of the right, and if at the first trial they are not alike, a good guess will serve to make them so.

By comparing Fig. 27 with Fig. 7 it should be noted that the columns show one point of inflection at B and one midway between B and C, which were omitted by the author.

The equation for the funicular line of an arch, as shown in Fig. 8, saves considerable time over the graphical method generally used in arches with earth fill, but it is not a very fair approximation in an arch of open spandrel construction. In the latter case it is more correct to assume that $P_s - P_c$, the difference of weight per linear foot at abutment and crown, respectively, decreases to zero according to a straight line from the abutment to the crown, in which case the equation of the funicular polygon is

$$y = \frac{4\, h}{l^2} \frac{3\, x^2 + 2\, (u - 1) \frac{x^3}{l}}{2 + u},$$

and the corresponding thrust $= \frac{P_c\, l^2}{8\, h} \left(1 + \frac{(u - 1)}{3}\right)$.*

* The equations for both cases might be found in the writer's "Reinforced Concrete Pocketbook", where also the ordinates of the curves are calculated in fractions of h for various values of u.

Mr. Mensch.

The summation method of finding the statically unknown value of the thrust, reaction, and moment of the abutment, requires very lengthy computations, with great likelihood of numerical and other errors. The writer can state positively that such errors must have been made when finding the influence lines, as the apex of the influence line for moments due to vertical loads at Point 6 should have been higher than at Point 8, on Plate XVII; and the apex at Point 0 should have been at least 8% lower than at Point 2.

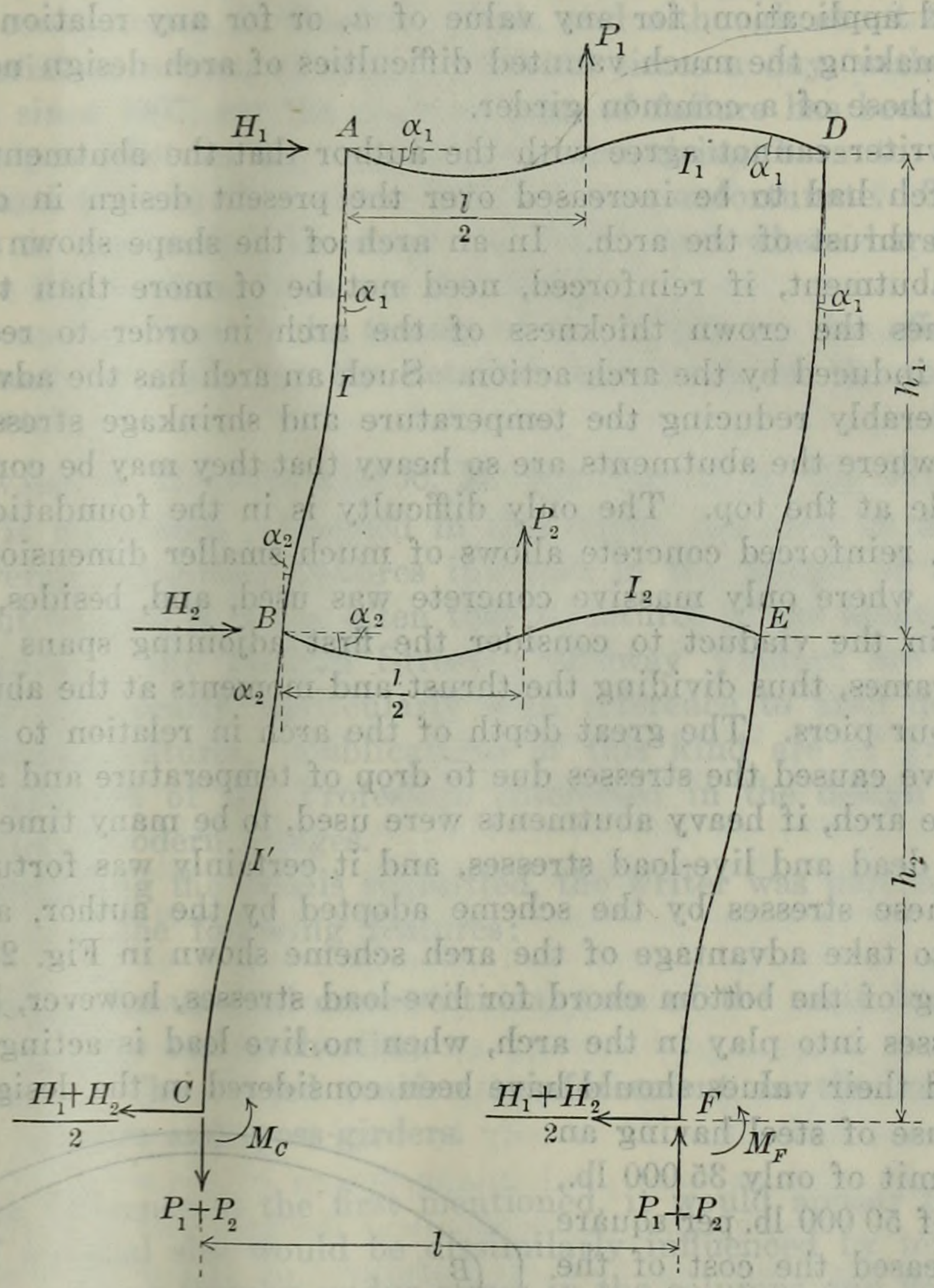


FIG. 27.

It is well known that Howe and many other writers have given very simple formulas for the statically indeterminate values of a parabolic arch with constant $\frac{ds}{I}$; but there nearly all writers have stopped. The question is often asked: can these formulas be applied to arches with varying $\frac{ds}{I}$, and of correct shape to suit the earth fill?

Mr. Mensch. In most cases they can be applied directly for live loads, but they fail widely for temperature and shrinkage stresses. As indicated before, $\frac{ds}{I}$ may be put $= \frac{dx}{I}\left(1 + C\frac{x}{l}\right)$ and, instead of a parabolic curve, the one given by the author accompanying Fig. 8 can be used. The work necessary to integrate the elastic equations is not greater than the numerical work of the summation method, and the results are formulas of general application, for any value of u, or for any relation of rise to span, making the much vaunted difficulties of arch design nearly as small as those of a common girder.

The writer cannot agree with the author that the abutment of the 130-ft. arch had to be increased over the present design in order to resist the thrust of the arch. In an arch of the shape shown in Fig. 28, the abutment, if reinforced, need not be of more than twice or three times the crown thickness of the arch in order to resist the moments induced by the arch action. Such an arch has the advantages of considerably reducing the temperature and shrinkage stresses over a design where the abutments are so heavy that they may be considered immovable at the top. The only difficulty is in the foundations, but also here, reinforced concrete allows of much smaller dimensions than formerly, where only massive concrete was used, and, besides, it was possible in the viaduct to consider the first adjoining spans at each side as frames, thus dividing the thrust and moments at the abutments among four piers. The great depth of the arch in relation to the rise would have caused the stresses due to drop of temperature and shortening of the arch, if heavy abutments were used, to be many times larger than the dead and live-load stresses, and it certainly was fortunate to reduce these stresses by the scheme adopted by the author, after he omitted to take advantage of the arch scheme shown in Fig. 28. The shortening of the bottom chord for live-load stresses, however, brought new stresses into play in the arch, when no live load is acting on the arch; and their values should have been considered in the design.

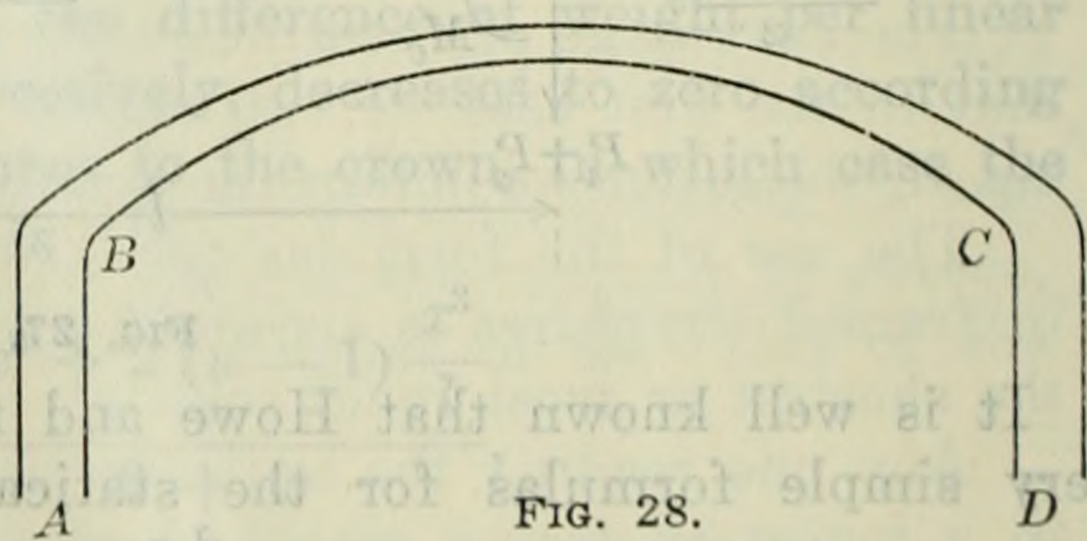


Fig. 28.

The use of steel having an elastic limit of only 35 000 lb., instead of 50 000 lb. per square in., increased the cost of the structure about $40 000. The writer cannot understand the prejudice of engineers against the use of a higher grade of steel, made of billet stock, in reinforced concrete construction. All tests made by the best-known authorities show that the ultimate carrying capacity is increased by its use by more than 50% over low-carbon steel. There certainly is a very small chance that shock

will affect the steel in columns or footings, or that the small steel bars in the slabs will be affected by shocks. The only place where an engineer may properly doubt the advisability of using high-carbon steel reinforcement is in the bent bars of girders, because the bars are often injured by careless bending. Where careful bending is done, with large radii, say about six to ten times the diameter, no engineer need be afraid of any unfavorable results. The writer used high-carbon bars for the reinforced floors of the Utah copper concentrator, in Garfield, Utah, and although about 8 000 tons of material have been vibrating 1 000 000 times a day, without interruption since 1907, not the slightest sign of failure has been detected. Mr. Mensch.

The provision in the specifications requiring bars to be firmly bound and tied together where they lap was unfortunate. The stress in bars is transmitted by shear and bond, and where bars are tied together the bond surfaces are decreased.

The author deserves the thanks of the Profession for offering this paper, and especially for the detailed description of the methods of construction.

Howard W. Holmes,* Assoc. M. Am. Soc. C. E. (by letter).—The writer is very much interested in this paper, not only on account of the several uncommon features involved in the design of the structure, but because it is not often that structures of the magnitude and importance of the Twelfth Street Trafficway Viaduct are described as fully and clearly, particularly with reference to arch design and construction features. Publications of this kind are of great value to all members of the Profession interested in the design and construction of modern bridges. Mr. Holmes.

In reviewing the details submitted, the writer was particularly impressed with the following features:

First.—The unequal cross-sectional area of the main longitudinal girders and arch ribs;

Second.—The curved tension reinforcement in the longitudinal girders and cross-girders.

With reference to the first mentioned, it would appear that members of unequal size would be dissimilarly influenced by temperature changes, causing thereby undue stress in the columns and floor system which would combine with the secondary stresses induced by the rigid connections of the upper and lower decks. The arch ribs, owing to their unequal size, would undoubtedly, when subjected to temperature changes, exert a like influence on the spandrel columns and supported deck, resulting in strains and cracks in the floor system and weaker members, though granting certain elastic deformation through-

* Portland, Ore.

Mr. Holmes. out the system. The stress induced as above, combined with the possible variation in temperature between the arch ribs and lower chords, should, in the writer's opinion, merit consideration. The possibility of the dissimilar temperature influence is further emphasized by the fact that the lighter members are all on the south, or more exposed side of the structure. The effects of these stresses might be modified, but not eliminated, by constructing expansion joints at the intersection of the arch axis and the line of temperature thrust, or at the spandrel columns nearest this intersection.

It is gratifying to note the thorough and systematic methods followed in the arch analysis, and that the method of influence lines is coming into more general use, as it is certainly the only method that can be relied on for structures subjected to the severe concentrations imposed by modern traffic. Under the common loadings of several years ago, it was thought sufficient to consider the arch under two or three conditions of loading. This method, though still used to a great extent, is entirely inadequate, and, if applied, gives no assurance that the maximum stresses in the arch have been determined. Failures have been averted only by providing an excess of material; but, at the present day, when economy of design is considered a factor of major importance, a more exact method of loading must be adopted. Since the position of the live load producing maximum moment at any point will vary in arches of different proportions and form, modern practice makes it imperative, in the interests of safety and economy, that the more complete and exact method of analyzing an arch for a load of unity at each load point be used. Influence lines may then be drawn for both the upper and lower fibers, and the exact maximum values due to both vertical and horizontal loads, either uniform or concentrated, may be determined, as the influence lines will show clearly the extent and general position of the loads for maximum stress. Undoubtedly, the design of an elastic arch is one of the most delicate problems of structural engineering, but if the designer is sufficiently familiar with his work, it may be performed without great difficulty. It is only by applying rational and systematic methods, such as were used in the design of the structure under discussion, and combining the economical design evolved with faultless construction, that the field of usefulness of this esthetic form of structure will become more generally recognized.

The author has used the method of constructing influence lines for moment and thrust. By constructing these lines for stress in extreme fiber, a great deal of time may be saved by applying a modification of the stress equation for homogeneous beams,

$$S = \frac{M}{\frac{I}{c}},$$

Mr. Holmes.

where S = stress in extreme fiber,
M = maximum sectional bending moment,
I = moment of inertia,
and c = distance to neutral axis.

When the fiber stress is wholly compression, and considering the additional factors,

r = least radius of gyration,
N = thrust normal to the section,
A = area of transformed section,
I' = moment of inertia of transformed section,
and e = eccentricity,

the unit stress in the concrete will be $\frac{N}{A}$, and the unit flexural will be $\frac{M\,c}{I'}$. Considering the combined section, we have

$$(M\,c\,A) + (N\,I') = S\,A\,I',$$

$$\text{or } S = \frac{N}{A} + \frac{M\,c}{I'}.$$

Equating $A\,r^2 = I'$, and $M = N\,e$, we have

$$S = \frac{M'\,c}{I'}, \text{ where } M' = N\left(e + \frac{r^2}{c}\right).$$

Computing the values of $\frac{r^2}{c}$ for the various sections, and considering this distance as the moment arm, and the fiber stress as varying as the moment, an influence line drawn for the moments thus computed will serve directly as an influence line for fiber stress. The writer has found that this direct method eliminates many chances for error, and saves considerable time. Of course, when the tensile stress in the concrete is ignored, as it should be, and the resultant stresses are tensile, it will be necessary to plat the influence lines for moment and thrust, and compute the stress in extreme fiber in the usual manner.

In connection with the discussion of stress in an arch of this type, a most important feature is the method of construction. The writer believes that cracks and initial stresses, as often developed, may be traced directly to the construction methods, and a description of the latter would complete this valuable paper.

With reference to the curved tension reinforcement in the longitudinal and cross-girders, it may not be unreasonable to ask why the main tension reinforcement was not placed in a horizontal position, and lighter and shorter lengths of steel bars used to reinforce the haunches. It appears to the writer that the method used

Mr. Holmes. does not accomplish the results in the most economical way. Although the distribution of metal throughout the girders is ideal, it would appear that the same results could be accomplished with a smaller quantity of steel.

As shown by the following demonstration (see Fig. 29):

Let P = total stress in bar;
p = normal pressure per linear unit;
ds = single element.

Then the vertical component $= p\,ds$; $\cos.\,\theta = pr\,d\,\theta$; $\cos.\,\theta = X$.

$$\Sigma\,X = \int_c^B pr,\ \cos.\,\theta,\ d\,\theta + c = pr\ \sin.\,\theta + c.$$

$$\Sigma\,X = 0 \text{ when } \theta = 0, \text{ therefore } c = 0.$$

$$\Sigma\,X = pr\ \sin.\,\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$P = pr, \text{ or } p = \frac{P}{r},$$

substituting in Equation (1)

$$\Sigma\,X = \frac{P}{r}\,r\ \sin.\,\theta = P\ \sin.\,\theta.$$

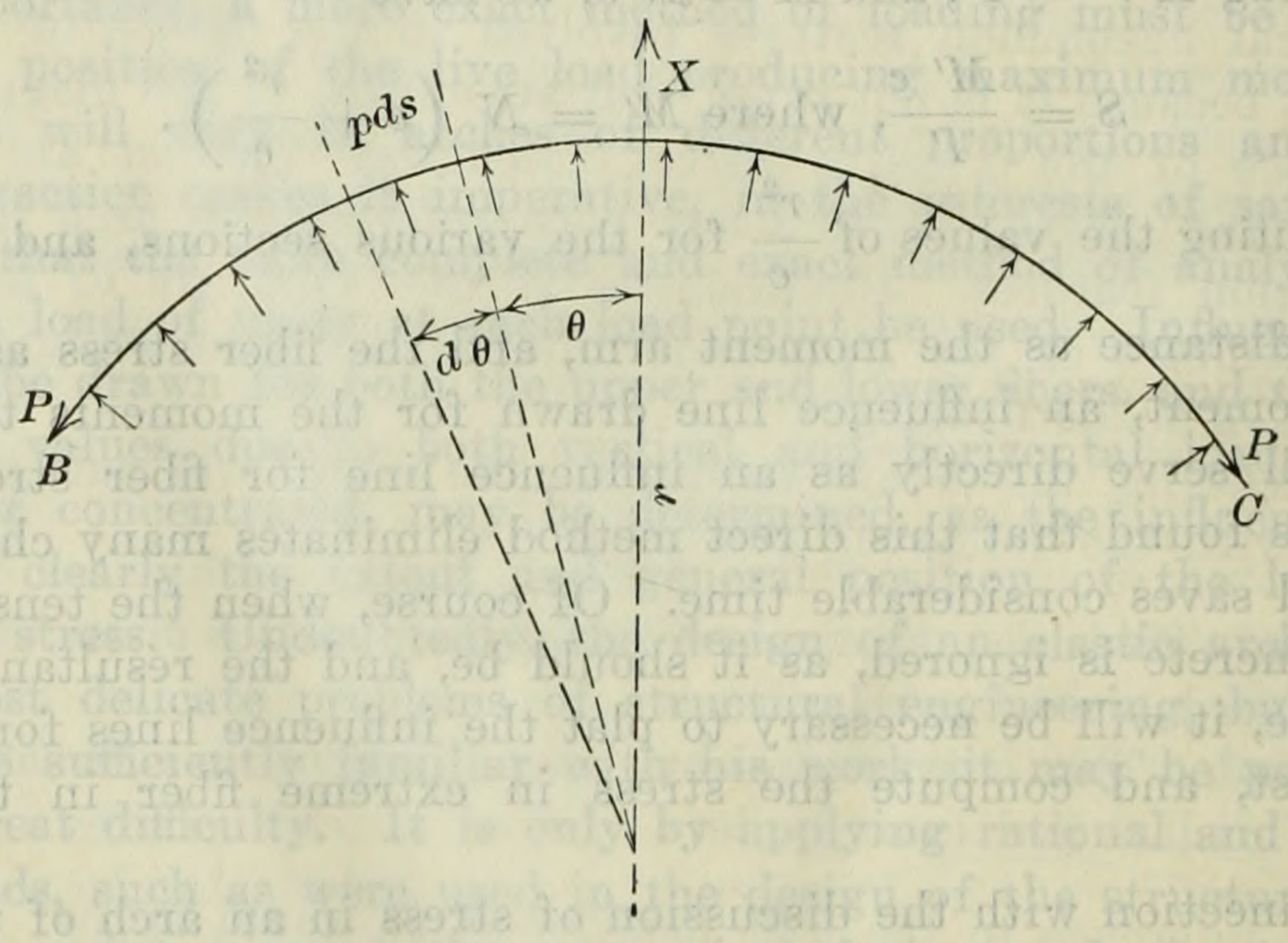


FIG. 29.

The steel with its convex curvature must resist a tensile stress induced by the bending moment caused by a vertical load equivalent to the summation of the various vertical components of force, in addition to the bending moment caused by the dead and live loads. It would seem logical to conclude, therefore, that if just sufficient steel is used to satisfy the stress induced by the live and dead loads, the working stress would be exceeded by a quantity equal to the increment of induced stress indirectly caused by the convex curvature.

Mr. Holmes.

On the other hand, it would not be economical to compensate for the effect of the vertical components of force by increasing the sectional area of metal at the critical section.

The distribution of the summation of the vertical forces referred to would follow the sine curve and result in considerable increase of moment at the center of the girder.

The painstaking care exercised by the designers in the close attention to details is to be commended, as there is very little to be guessed at by the prospective builder in figuring his estimates of costs.

Another important feature, in the writer's opinion, was the exclusion of piles in which fresh or unset concrete is placed against the soil. Considerable experience in connection with concrete piles under various subsoil conditions has demonstrated conclusively, to the writer's satisfaction, that reliable results in this most important feature of foundation work may be expected only by using piles in which the unset concrete is not allowed to come in contact with the subsoil. The many failures resulting from the use of concrete piles constructed in immediate contact with the soil should fully justify the statement that those of the latter type are not to be recommended where permanent and reliable results are desired.

The writer is pleased to note that the conventional sand cushion under the wood block pavement has been omitted. Considerable experience on the writer's part in constructing wood block pavements on bridge floors and reconstructing the floor systems of various bridges has demonstrated that the blocks are bound to heave and buckle, due to the shifting of the sand cushion, particularly when laid on a grade. The sand cushion will also give trouble where the blocks are laid with open joints, owing to the possibility of water leaching through the joints and displacing the cushion. The method used by the author has many advantages, and effectively water-proofs the bottom of the blocks.

Owing to the great variety of traffic that will no doubt be carried by this viaduct, it will be interesting to note the results obtained, under various climatic conditions, by constructing a wood block pavement on a grade of 5.5% and using the $\frac{3}{8}$-in. lath separators between the rows. This method has been tried before, but it would seem that observation of the great variety of types of street users that may be expected to pass over this structure will result in interesting and valuable data.

In reviewing the paper, one cannot help being most favorably impressed with the fact that, in drafting the specifications, an effort was made to eliminate as far as possible the uncertain characterization of work "satisfactory to the engineer", to make perfectly clear to the prospective builder just what was to be required, and to express the views of the engineer in advance. The writer does not wish it

Mr. Holmes. to be inferred, however, that the engineer should have no discretion whatever in regard to carrying out the contract, but it may be unhesitatingly stated that this feature is one of the most important matters before the Engineering Profession to-day. The "satisfaction" of an engineer is a factor over which the contractor has no control, and it would be possible for a contractor to perform work absolutely above criticism, and yet not be able to compel the engineer to be "satisfied". Most contractors take pride in their work, and, as a matter of business policy and to maintain a reputation as a reliable contractor, make conscientious efforts to perform their work to the satisfaction of the engineer. Reliable contractors realize that competent supervision on the part of the engineer is absolutely essential. The making of a contract to the effect that work shall be performed to the "satisfaction of the engineer" and the letting of contracts without advance engineering information have been responsible for considerable pooling and combination in the contract business. The very idea of contracting to perform certain work is that such work can be planned and estimated in advance, as a result of experience with similar work, under like conditions, so that the contractors may have full knowledge of what is to be required and be able to fix with reasonable accuracy the cost of performing it. Contract work should be confined to that which involves quantities and conditions which may with reasonable certainty be determined in advance by experienced engineers. Work which cannot be determined in this way should be let on a basis of reasonable cost, plus a just percentage for profit. It should be clearly understood, however, in this latter case, just what items should be included in the cost.

In defense of the Profession, however, it should be considered that many of the weaknesses of specifications governing, or supposed to govern, public work may not be the fault of the engineer, as it is undoubtedly true that many of the absurd and unjust requirements, which are injurious both to contractors and to the public, are provided by law and by city charters. That many of the clauses found in public work specifications are not necessary is shown by the faithful and satisfactory completion, by contractors all over the country, of contracts with business firms where no such clauses are considered necessary, and where there exists recognition of the relation between the responsibility of the contractor and the requirements for the bond and retained percentage.

One of the main reasons that public work is often more expensive and less satisfactory than work contracted for by private firms is the fact that most of our laws compel the award to the lowest bidder. This, combined with the many other uncertain requirements, has in many cases proved most injurious to the public welfare.

Mr. Holmes.

It is quite generally supposed that profits from public work contracts are so great that anything that can be imposed on the contractor is so much clear gain. Needless risk should not be imposed on a contractor. Any one possessing an elemental knowledge of business principles should know that no competent and financially responsible contractor will take a gambling risk without charging enough on his bid to save himself from loss. Of course, there are contractors who will bid low in order to obtain the work, but, in the majority of cases, they are either financially irresponsible, or shrewd enough to make up their loss by slighting the work. Work let to the contractor of this latter class is bound to cost in the long run a good deal more than had it been let to a competent and reliable man who understands his business.

The writer has had considerable experience in contract work, both public and private, and has learned to appreciate the integrity and desire of contractors to perform work creditable to their reputation. The Profession should stand as a unit in making an effort to overrule the many unfair clauses of almost universal use in specifications, as these clauses have their part in limiting competition, in lowering the standard of honesty among contractors, and in lessening the dignity and standing of the Profession. By cultivating the confidence of reputable contractors and providing for competitive bidding on public contracts, immense sums may be saved to the taxpayers. A continuation and more wide-spread adoption of the policy followed by the engineers in connection with the contract for the structure under discussion will accomplish the desired results. The author deserves the thanks of every member of the Profession for his effort.

Mr. Upson.

M. M. Upson,* Assoc. M. Am. Soc. C. E. (by letter).—As is usual in a descriptive paper of this character, limited space has prevented Mr. Howard from taking up in detail the reasons for selecting the component parts of the structure. In particular, a short account of the selection of Raymond piles by the contractor, and their acceptance by the engineers, will show the care and forethought which characterized this entire work, and can now be seen to have saved both time and money.

Up to 1912, with one exception, the only concrete piles used in Kansas City were pre-cast. Mr. Howard had used pre-cast piles on a somewhat similar viaduct within half a mile of the present structure. It was natural, therefore, that the specifications should allow pre-cast as well as cast-in-place piles, the latter being limited to those in which "fresh or unset concrete is not placed against the soil." Pre-cast piles 25 ft. long were estimated as being necessary, it being provided that a variation in length could be ordered if required when the

* New York City.

Mr. Upson. work began, and it was specified that such piles were to be jetted into place.

The soil of the site is a typical Missouri River fill overlying a very fine and comparatively clean sand, admirably suited to jetting, and into which the contractor would have experienced no difficulty in placing the 25-ft. pre-cast pile. On the east end of the viaduct, where it approaches the rock cliff, erosion subsequent to the initial placing of the soil had evidently taken place, as the sand and rock were covered with a very soft muddy material which necessitated piles at least 30 ft. long.

It is apparent that had pre-cast piles been used, about 10% of their total number would have been increased fully to 30 ft. in length on this account. The remainder, by the use of a jet, would have penetrated 25 ft., so that the 2 610 piles would have totaled 66 555 lin. ft.

After a careful study of the conditions, Raymond piles were selected. They were driven without jetting to a sufficient penetration to support safely the maximum load of 35 tons each. By driving to an agreed penetration to the last inch, a uniform bearing value was obtained, the lengths varying to meet the different soil conditions. They ranged from 12 to 30 ft., and in some individual piers there was a length variation of as much as 5 or 6 ft. By the use of the Raymond system, it was unnecessary to predetermine the lengths of the piles, and the city paid for only the actual number of feet driven. In this instance, the 2 610 piles totaled 46 625 lin. ft., which showed a saving of 19 930 ft. over the prescribed 66 550 lin. ft. of pre-cast piles. At the contract price, $1.20 per ft., this saved the city about $24 000, or 43% of the total cost of the piling.

It may be well to point out one or two reasons why, in the writer's opinion, a less length of pile may safely carry the same load. In this instance, as in most pile installations, the piling element does not penetrate to rock or hardpan, but carries its load by surface friction. The friction per square foot of surface in a given soil is dependent on several factors:

1.—The angle between the soil and the surface. It is obvious that the friction on a vertical surface can be no more than the shear of the adjacent soil, though the friction on a perfectly horizontal surface will be equivalent to the compressive strength of the soil. The vertical surface friction varies between these two limits. On that account a tapered friction pile has a great advantage over the straight cylindrical pile. The writer has frequently tested tapered piles to a surface friction of more than 2 000 lb. per sq. ft. without rupture.

2.—The degree of compression of the soil which is attained by the driving. This depends on the amount of abuse to which the driven

pile or form may be subjected. The necessary cushion to protect the concrete of a pre-cast pile limits its effectiveness in this function. Mr. Upson.

3.—The maintaining of the compression after it has been attained through the driving. This is effectively accomplished in the pre-cast pile, as it is also in the Raymond type, with its spirally reinforced and corrugated shell, which is left in the ground.

In addition to the saving of cost by reason of the shorter lengths required, the use of Raymond piles avoided a difficulty which would undoubtedly have proved both costly and troublesome to the contractor. This is the operation of jetting on the site.

The work was in a congested district, with unusually heavy street and railroad traffic. Had pre-cast piles been used, the disposal of the water from the jets would have been a great problem. A filter of some kind would have been required to separate the sand from the water, as the sewers in that part of Kansas City have a very small fall, which would prevent them from carrying water containing any large quantity of sand.

The jetting of piles in a city always entails many difficulties, especially when fine and light soil, easily carried in suspension, is encountered. It is, of course, quite impossible to carry the water on the surface, because of streets and crossings which occur at close intervals. It will be seen, therefore, that, by avoiding this difficulty, the contractor saved himself much annoyance and expense.

Time is another factor of economy affected by the use of Raymond concrete piles on this work. Had pre-cast piles been used, most of them would have been cast during the winter. A minimum time of 30 days was specified, and under ordinary winter conditions this time would have been doubled before the piles had cured sufficiently to be safely handled. As a matter of fact, the weather during this work was unusually mild, so that the temperature delay would not have been serious. However, the necessary 30 days after the receipt of the reinforcing rods, which usually requires several weeks for delivery on the ground, necessitates an interval of from 6 to 8 weeks between the signing of the contract and the placing of the footings. Furthermore, a slight variation in the underlying soil conditions may involve the contractor in a further delay of from 30 to 60 days, in order to provide longer piles to meet the varied requirements.

The actual money saving by cutting down the time is frequently lost sight of by the contractor as well as by the owner. After a construction project is determined on, and the preparations are made, the money loss to the owner usually can be computed accurately, and often amounts to several hundred dollars a day, and all wise contractors readily realize the fact that their profit, under ordinary conditions, is inversely proportional to the time required to complete the work. Sev-

Mr. Upson. eral instances have come to the writer's attention where the loss to the owner and a proportional amount to the contractor, on account of the delay incident to choice of improper foundation methods, has involved a sum greater than the total cost of the foundation itself.

To summarize: Raymond piles were selected, first, because of the saving in length, which, in this instance, amounted to 43%, or $24 000, on the total expenditure of $56 000; second, the saving of time, which may be estimated at not less than 30 days, as the footings and superstructure followed very closely on the driving of the piles; third, the avoidance of annoying and expensive pumping equipment and methods for taking care of waste water incident to the jetting.

Mr. Slettum. E. A. Slettum, Esq.* (by letter).—The investigation of the columns for the Twelfth Street Trafficway Viaduct was made by the writer, then in the employ of Waddell and Harrington, the designers of the structure, as Assistant Engineer. On account of the great number of columns and their size, it was of the greatest importance, in order to obtain an economical and safe structure, to determine, with as great precision as practicable, the various bending moments that might occur. The most difficult problem was the determination of

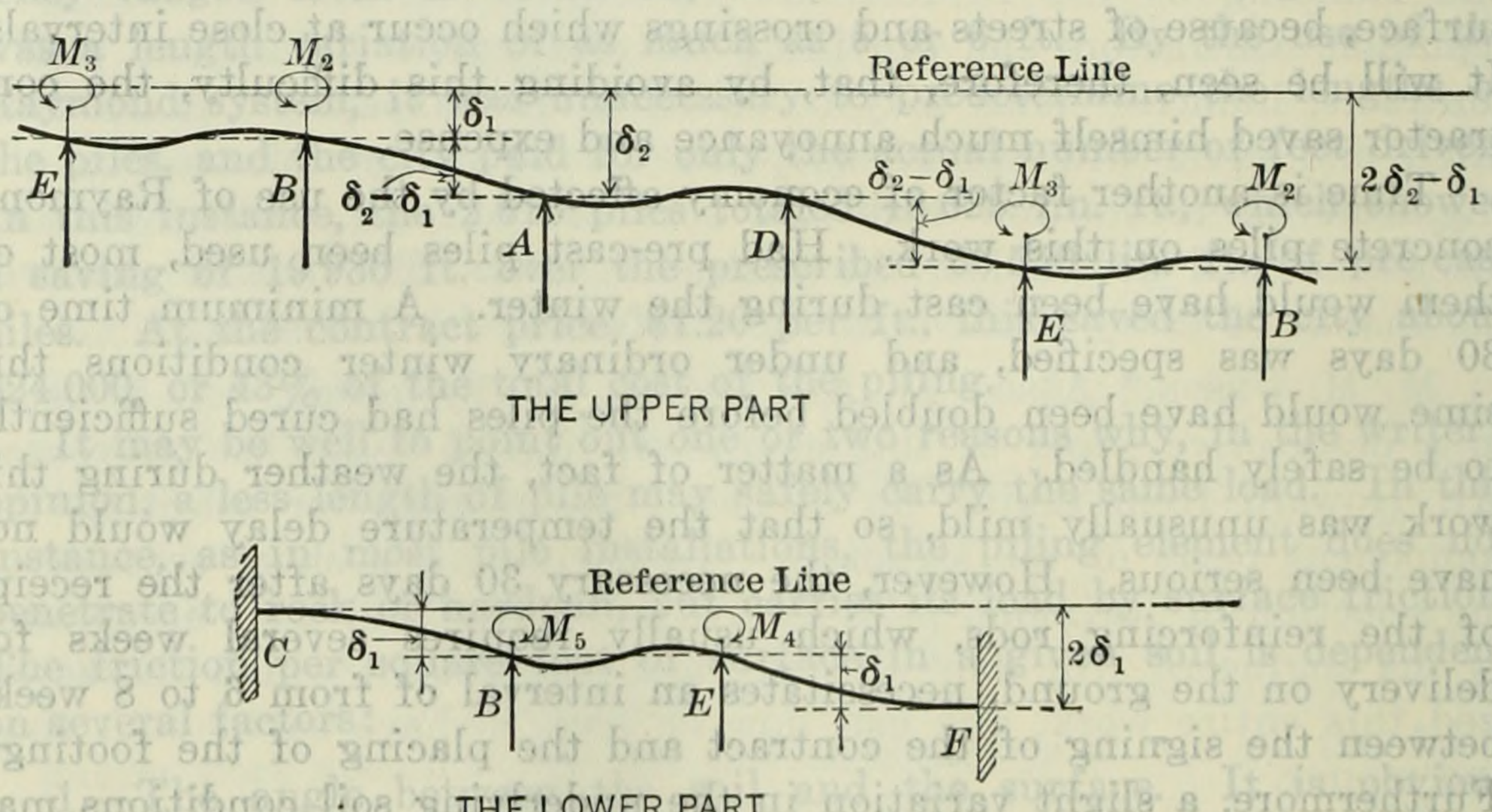


Fig. 29.

bending moments due to wind loads. A typical bent, as shown by Fig. 6, was chosen, and the problem was solved, as mentioned, by breaking the frame up into two parts, one upper part, *ABED*, and one lower part, *BCFE*, the cross-girder, *BE*, being common for both. The two parts were then straightened out, as shown in Fig. 29, and calcu-

* Christiania, Norway.

lated as continuous beams on supports at different levels, the internal moments, M_2 and M_3, of the lower part acting as external moments on the upper part, and, *vice versa,* the internal moments, M_4 and M_5, of the upper part acting as external moments on the lower part. Mr. Slettum.

The frame was assumed to be rigidly fixed at the bases, C and F. All members of the frame, except the cross-girder, BE, had variable cross-sections, and, therefore, the three-moment equation for beams with variable moments of inertia was used. This equation, not being published in any book known to the writer, was developed by him in connection with this work, and may possibly be of some interest.

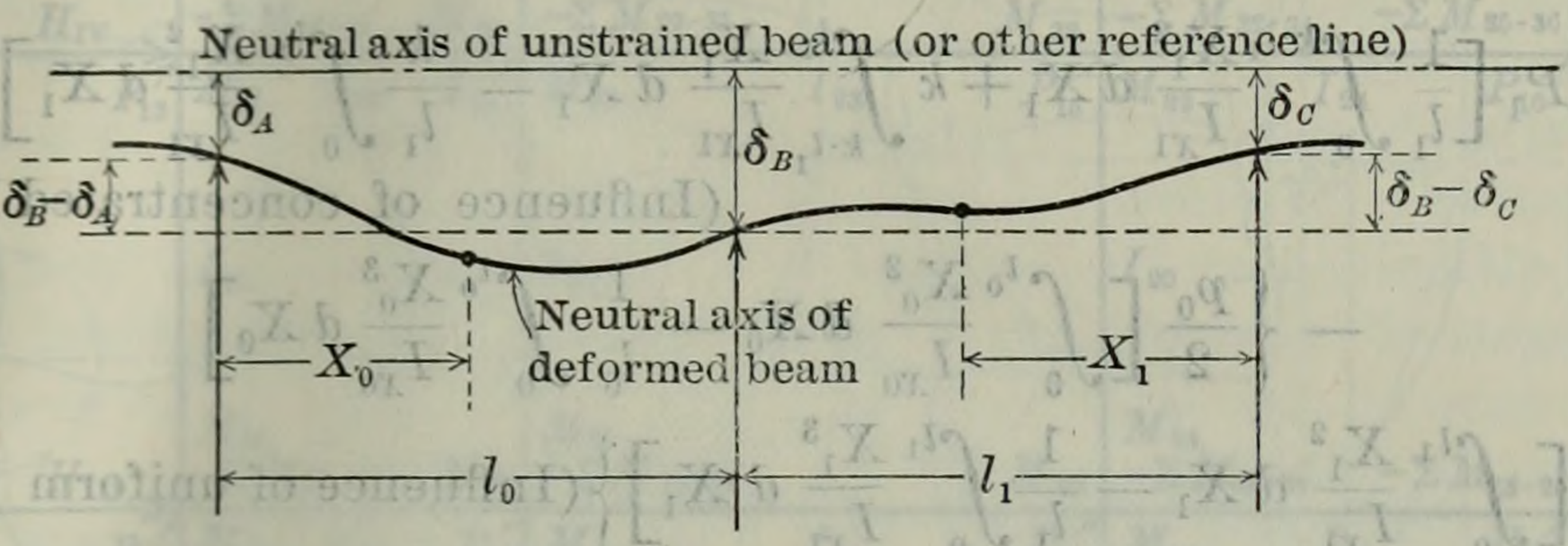


FIG. 30.

Let ABC be any three consecutive supports in a continuous beam;

I_{x0} be the general value of the moment of inertia in the left-hand panel (Panel AB);

I_{x1} be the general value of the moment of inertia in the right-hand panel (Panel BC);

X_0 be the distance from the left-hand support to any point in the left-hand panel;

X_1 be the distance from the right-hand support to any point in the right-hand panel;

P_0, p_0, and M_0, respectively, be any concentrated load, uniform load per unit, and external bending moment, in the left-hand panel.

P_1, p_1, and M_1, respectively, be any concentrated load, uniform load per unit, and external bending moment, in the right-hand panel;

kl_0 and kl_1 be the distances, respectively, from the left-hand support in the left-hand panel and from the right-hand support in the right-hand panel, to any concentrated load or external bending moment;

E be the modulus of elasticity of the material; and

M_A, M_B, and M_C be the internal moments (continuity-moments) at A, B, and C, respectively.

Mr. Slettum. Then, with the notation shown in Fig. 30, the three-moment equation for beams with variable moments of inertia is written thus:

$$M_A\left[\frac{1}{l_0}\int_0^{l_0}\frac{X_0}{I_{X0}}\,d\,X_0-\frac{1}{l_0^2}\int_0^{l_0}\frac{X_0^2}{I_{X0}}\,d\,X_0\right]$$

$$+M_B\left[\frac{1}{l_0^2}\int_0^{l_0}\frac{X_0^2}{I_{X0}}\,d\,X_0+\frac{1}{l_1^2}\int_0^{l_1}\frac{X_1^2}{I_{X1}}\,d\,X_1\right]$$

$$+M_C\left[\frac{1}{l_1}\int_0^{l_1}\frac{X_1}{I_{X1}}\,d\,X_1-\frac{1}{l_1^2}\int_0^{l_1}\frac{X_1^2}{I_{X1}}\,d\,X_1\right]$$

$$=-\Sigma\Big\{P_0\left[\frac{1}{l_0}\int_0^{k\cdot l_0}\frac{X_0^2}{I_{X0}}\,d\,X_0+k\int_{k\cdot l_0}^{l_0}\frac{X_0}{I_{X0}}\,d\,X_0-\frac{k}{l_0}\int_0^{l_0}\frac{X_0^2}{I_{X0}}\,d\,X_0\right]$$

$$+P_1\left[\frac{1}{l_1}\int_0^{k\cdot l_1}\frac{X_1^2}{I_{X1}}\,d\,X_1+k\int_{k\cdot l_1}^{l_1}\frac{X_1}{I_{X1}}\,d\,X_1-\frac{k}{l_1}\int_0^{l_1}\frac{X_1^2}{I_{X1}}\,d\,X_1\right]\Big\}$$

(Influence of concentrated loads)

$$-\Big\{\frac{p_0}{2}\left[\int_0^{l_0}\frac{X_0^2}{I_{X0}}\,d\,X_0-\frac{1}{l_0}\int_0^{l_0}\frac{X_0^3}{I_{X0}}\,d\,X_0\right]$$

$$+\frac{p_1}{2}\left[\int_0^{l_1}\frac{X_1^2}{I_{X1}}\,d\,X_1-\frac{1}{l_1}\int_0^{l_1}\frac{X_1^3}{I_{X1}}\,d\,X_1\right]\Big\}$$ (Influence of uniform loads)

$$-\Sigma\Big\{M_0\left[\frac{1}{l_0^2}\int_0^{l_0}\frac{X_0^2}{I_{X0}}\,d\,X_0-\frac{1}{l_0}\int_{k.l_0}^{l_0}\frac{X_0}{I_{X0}}\,d\,X_0\right]$$

$$+M_1\left[\frac{1}{l_1^2}\int_0^{l_1}\frac{X_1^2}{I_{X1}}\,d\,X_1-\frac{1}{l_1}\int_{k.l_1}^{l_1}\frac{X_1}{I_{X1}}\,d\,X_1\right]\Big\}$$

(Influence of exterior bending moments or pairs of forces)

$$+E\left[\frac{\delta_B-\delta_A}{l_0}+\frac{\delta_B-\delta_C}{l_1}\right]$$ (Influence of yielding supports, or supports at different levels).

In this equation, P and p are introduced with positive sign when producing tension on the under side of a beam freely supported on two supports, and M is introduced with positive sign when producing tension on the under side of the beam between the left-hand support and the point of application in the left-hand span (that is, when acting counter clockwise), and between the right-hand support and the point of application in the right-hand span (that is, when acting clockwise).

The work was somewhat simplified in this case by having no other exterior loads with which to deal, other than the bending moments at B and E. The quantities, $\int\frac{X}{I}\,d\,X$, $\int\frac{X^2}{I}\,d\,X$, etc., were determined graphically in the manner indicated by Fig. 5.

On the basis of these calculations, the writer has devised a more general application of the theory of the continuous beam to problems of this nature, which is set forth in the following discussion.

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He does not believe that the method will be used to any great extent, except for rather simple frames, the labor of solving the many equations being enormous, on account of the calculations having to be carried through with the most rigid accuracy and to the last decimal point, in order to obtain correct results, the slightest rounding off resulting in an error which increases rapidly with the number of equations.

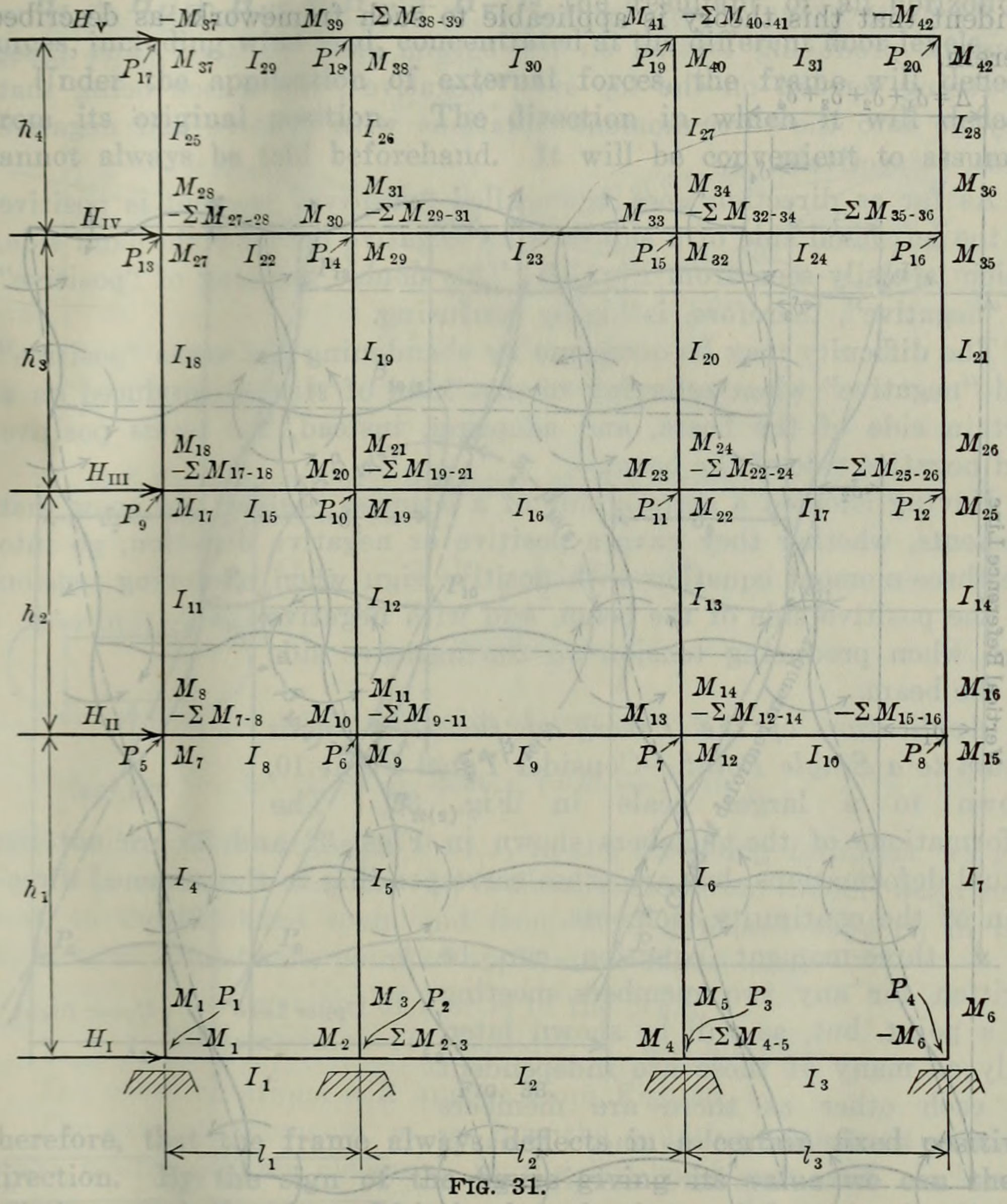

FIG. 31.

For problems, such as the case in hand, having not more than eight or ten statically indeterminate quantities, the writer has used the method with entire success, although even such simple cases require careful calculations.

The Application of the Theory of the Continuous Beam in Calculating the Stresses in Frames with Rigid Connections. A frame with rigid connections is one in which all members are connected at

Mr. Slettum. their intersection points in such a way that the angle between any two members meeting at a point will remain constant under all conditions of loading. A change in the direction of the end tangent of one member, therefore, will cause the same change in the direction of the end tangents of all other members meeting at the point. As this is the basis of the theory of the ordinary continuous beam, it is self-evident that this theory is applicable to such frameworks as described herein.

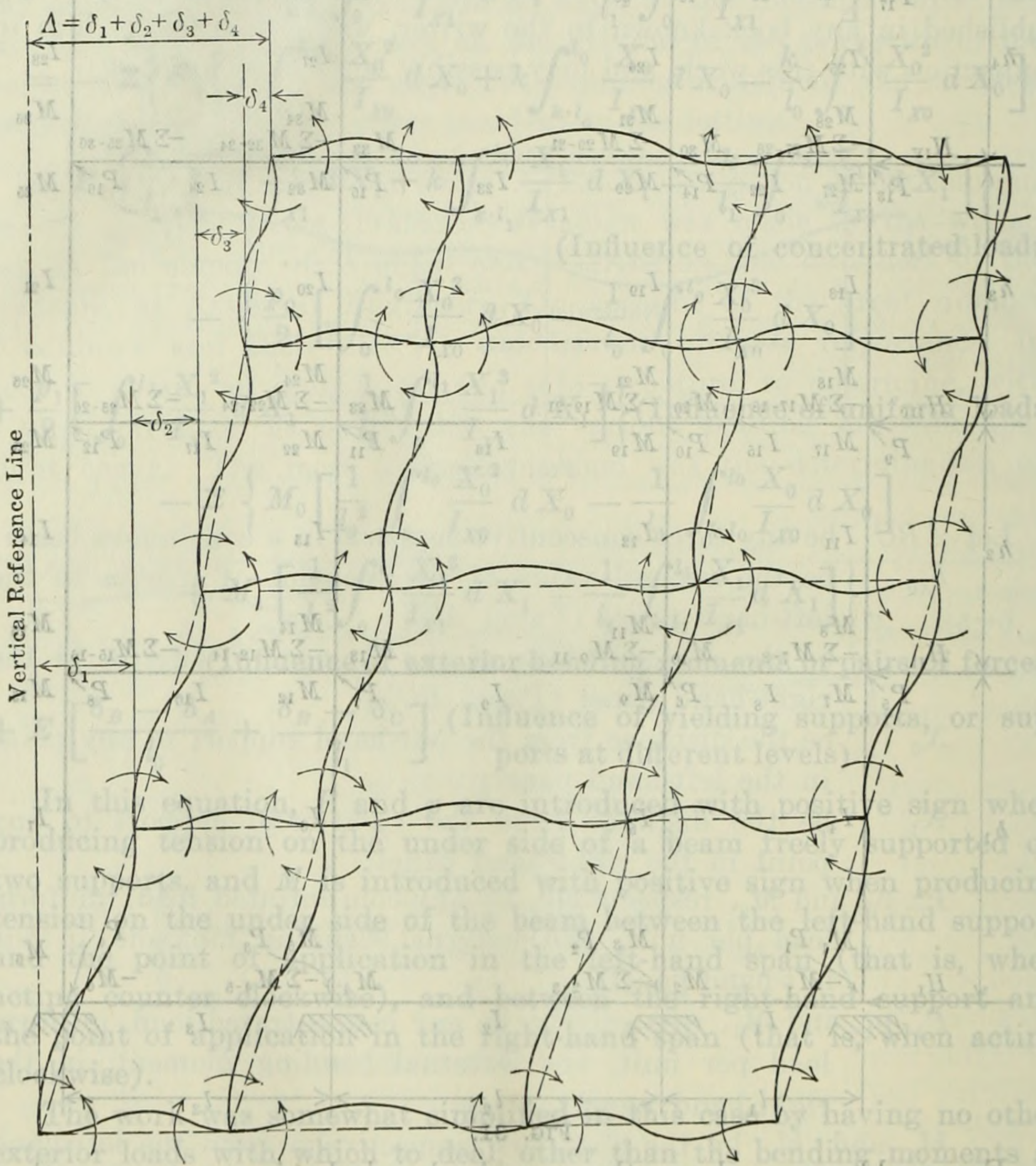

FIG. 32.

To illustrate the general application of the theory of the continuous beam for calculating the stresses in such frames, take the framing bent of a four-story structure, as shown in Fig. 31.

The moments of inertia have been assumed to be different for different members, but constant throughout the length of each separate member. The only difference in the case of a variable moment of

inertia is that the coefficients in the three-moment equation change as shown under the theory of the continuous beam. Mr. Slettum.

The frame will be assumed to be loaded in any manner, horizontally or vertically, by forces acting in its plane.

The influence of axial stresses on the deformation of the frame will be neglected.

$H_I + H_{II} + H_{III} + H_{IV} + H_V$ is the resultant of all horizontal forces, including wind load, concentrated at the different floor levels.

Under the application of external forces, the frame will deflect from its original position. The direction in which it will deflect cannot always be told beforehand. It will be convenient to assume,

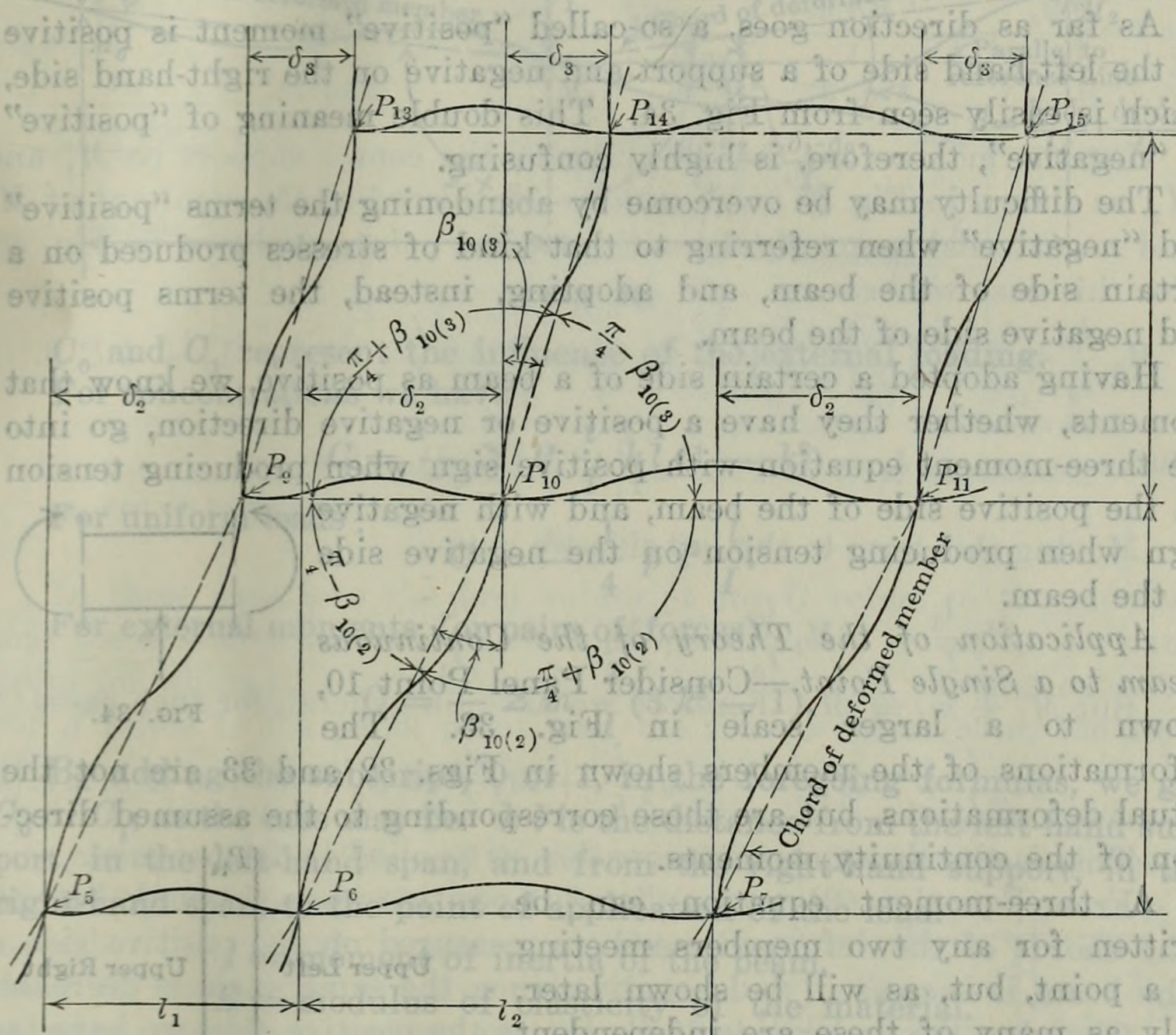

FIG. 33.

therefore, that the frame always deflects in a certain fixed positive direction. By the sign of the figure giving its value we can then tell the actual direction. If it is plus, the frame deflects in the direction assumed; if it is minus, it deflects in the opposite direction. This also applies to the unknown continuity moments.

Assume that from left to right is the positive direction for deflections.

Assume a clockwise direction as positive for moments when applied to the panel points, and counter clockwise when applied to the members.

Mr. Slettum. In Fig. 32 is shown the deflected frame with the moments applied to the panel points. As each panel point is in equilibrium, the algebraic sum of the moments about any point must equal zero; hence, if all moments but one about a point are assumed to act in a positive direction, this latter moment will equal the sum of the others, and will be negative, as shown in Fig. 32.

When speaking about beams, it is the usual practice to term moments producing tension on the bottom side "positive", and those producing tension on the top side "negative", the reason being that they go into the three-moment equations with positive and negative signs, respectively.

As far as direction goes, a so-called "positive" moment is positive on the left-hand side of a support and negative on the right-hand side, which is easily seen from Fig. 34. This double meaning of "positive" or "negative", therefore, is highly confusing.

The difficulty may be overcome by abandoning the terms "positive" and "negative" when referring to that kind of stresses produced on a certain side of the beam, and adopting, instead, the terms positive and negative side of the beam.

Having adopted a certain side of a beam as positive, we know that moments, whether they have a positive or negative direction, go into the three-moment equation with positive sign when producing tension on the positive side of the beam, and with negative sign when producing tension on the negative side of the beam.

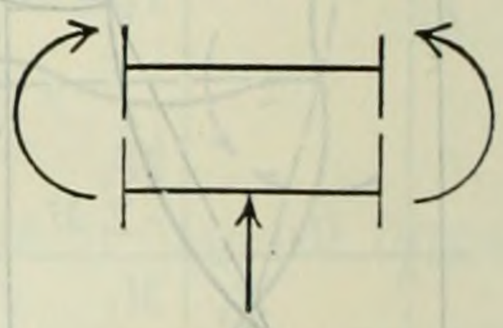

FIG. 34.

Application of the Theory of the Continuous Beam to a Single Point.—Consider Panel Point 10, shown to a larger scale in Fig. 33. The deformations of the members shown in Figs. 32 and 33 are not the actual deformations, but are those corresponding to the assumed direction of the continuity moments.

A three-moment equation can be written for any two members meeting at a point, but, as will be shown later, only as many of these are independent of each other as there are members meeting at the point, less one. Thus, if there are four members meeting at a point, we can write only $4 - 1 = 3$ independent equations.

For convenience, therefore, we shall confine ourselves to members forming either a lower left, upper right, or lower right, corner, as shown in Fig. 35.

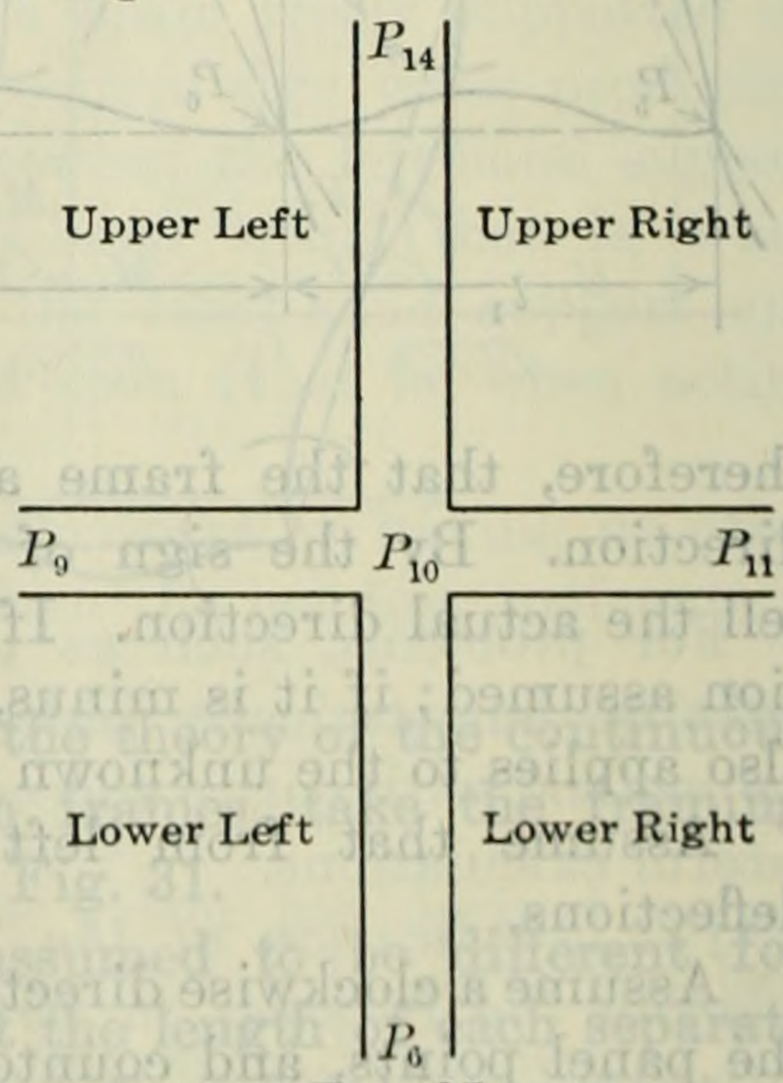

FIG. 35.

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The three-moment equation for the ordinary continuous beam is:

$$M_0 \frac{C_0}{I_0} + 2\, M_1 \left(\frac{l_0}{I_0} + \frac{l_1}{I_1} \right) + M_2 \frac{l_1}{I_1}$$

$$= C_0 + C_1 + 6\, E \left(\frac{\delta_1 - \delta_0}{l_0} + \frac{\delta_1 - \delta_2}{l_1} \right) \ldots\ldots\ldots (1)$$

The subscript, 0, refers to the left-hand span, and the subscript 1, to the right-hand span. (See Fig. 36.)

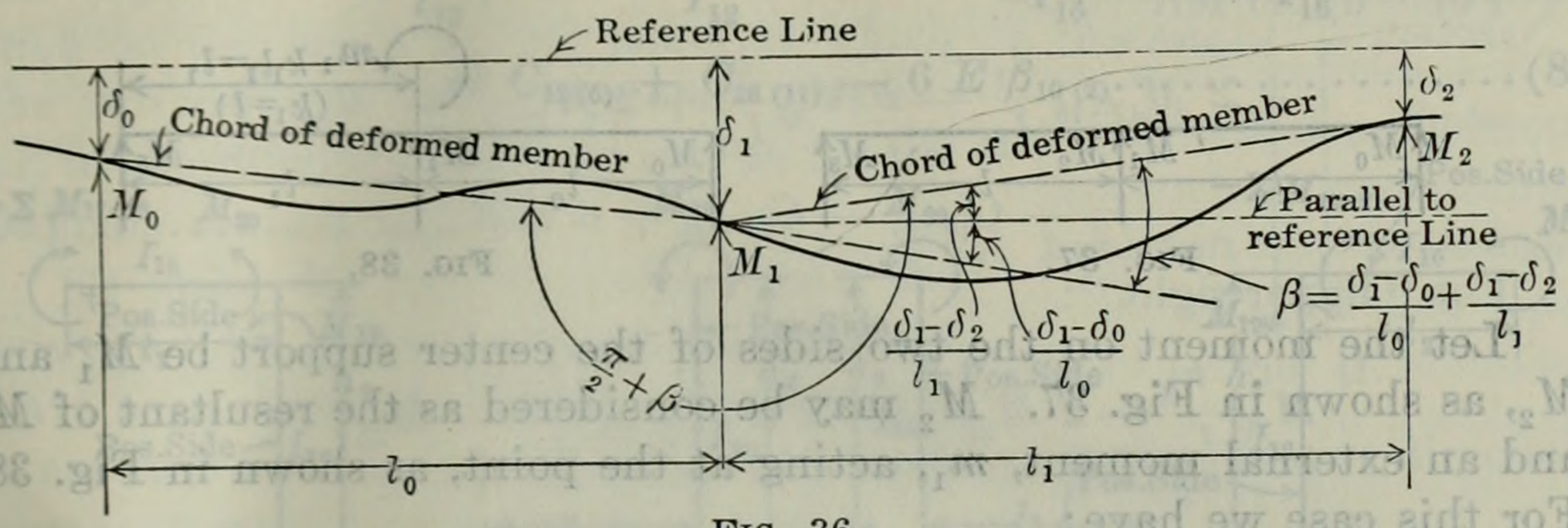


FIG. 36.

C_0 and C_1 represent the influence of the external loading. For concentrations we have:

$$C = - \Sigma\, P \frac{l}{I}\, k\, l\, (1 - k^2).$$

For uniform loads:

$$C = - \frac{1}{4}\, p\, l^2 \frac{l}{I}.$$

For external moments (or pairs of forces):

$$C = - \Sigma\, m \frac{l}{I} (3\, k^2 - 1).$$

By adding the subscript, 0 or 1, in the foregoing formulas, we get C_0 or C_1, as the case may be. $k\, l$ is the distance from the left-hand support, in the left-hand span, and from the right-hand support, in the right-hand span, to the point of application of the load.

I = moment of inertia of the beam.
E = modulus of elasticity of the material.

The other notations will appear from Fig. 36.

The external moments, m, go into the equation with positive sign when acting counter clockwise in the left-hand span and clockwise in the right-hand span.

The unknown continuity moments, M_0, M_1, and M_2, are assumed to act in such a direction that they produce tension on the positive side of the beam, which is the under side.

From Fig. 36 it is seen that the angle, β, which represents the change in angle between the two members, equals:

$$\beta = \frac{\delta_1 - \delta_0}{l_0} + \frac{\delta_1 - \delta_2}{l_1} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Mr. Slettum.

Therefore, the three-moment equation (1) can be written:

$$M_0 \frac{l_0}{I_0} + 2\, M_1 \left(\frac{l_0}{I_0} + \frac{l_1}{I_1}\right) + M_2 \frac{l_1}{I_1} = C_0 + C_1 + 6\, E\, \beta \ldots\ldots (3)$$

Now, a frame differs from an ordinary continuous beam therein, that, instead of having the same moment, M_1, on both sides of the center support, it has different moments on the two sides, due to the action of the other members meeting at the point.

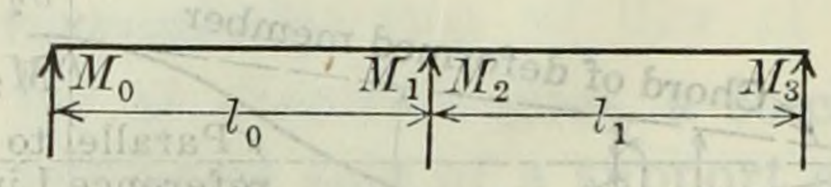


FIG. 37.

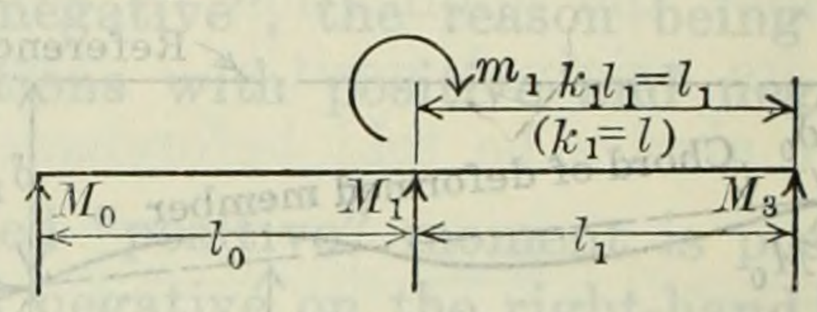


FIG. 38.

Let the moment on the two sides of the center support be M_1 and M_2, as shown in Fig. 37. M_2 may be considered as the resultant of M_1 and an external moment, m_1, acting at the point, as shown in Fig. 38. For this case we have:

$$M_0 \frac{l_0}{I_0} + 2\, M_1 \left(\frac{l_0}{I_0} + \frac{l_1}{I_1}\right) + M_3 \frac{l_1}{I_1} = -m_1 \frac{l_1}{I_1} (3\, k_1^2 - 1) + 6\, E\, \beta.$$

But $\quad k_1 = 1$, and $m_1 \dfrac{l_1}{I_1} (3\, k_1^2 - 1) = 2\, m_1 \dfrac{l_1}{I_1}$.

Moving this term to the left side we get:

$$M_0 \frac{l_0}{I_0} + 2\, M_1 \frac{l_0}{I_0} + 2\, (M_1 + m_1) \frac{l_1}{I_1} + M_3 \frac{l_1}{I_1} = 6\, E\, \beta.$$

But $M_1 + m_1 = M_2$, therefore we have, in general, for such cases:

$$M_0 \frac{l_0}{I_0} + 2\, M_1 \frac{l_0}{I_0} + 2\, M_2 \frac{l_1}{I_1} + M_3 \frac{l_1}{I_1} = C_0 + C_1 + 6\, E\, \beta \ldots\ldots (4)$$

This is the three-moment equation as applied to frameworks.

From Equation (2) we see that β is positive when the angle between the chords of the deformed members measured on the positive side of the beam is increased, and negative when the same angle is decreased.

Choosing the under side of $P_9\, P_{10}$ as the positive side, we have (see Fig. 39 and also Fig. 33):

For "lower left":

$$\Sigma\, M_{17\text{-}18} \frac{l_1}{I_{15}} + 2\, M_{20} \frac{l_1}{I_{15}} - 2\, M_{19} \frac{h_2}{I_{12}} + M_{11} \frac{h_2}{I_{12}}$$
$$= C_{15\,(9)} + C_{12\,(6)} - 6\, E\, \beta_{10\,(2)} \ldots\ldots\ldots\ldots (5)$$

For "upper left":

$$\Sigma\, M_{17\text{-}18} \frac{l_1}{I_{15}} + 2\, M_{20} \frac{l_1}{I_{15}} - 2\, M_{21} \frac{h_3}{I_{19}} + M_{29} \frac{h_3}{I_{19}}$$
$$= C_{15\,(9)} + C_{19\,(14)} - 6\, E\, \beta_{10\,(3)} \ldots\ldots\ldots\ldots (6)$$

Mr. Slettum.

For "upper right":

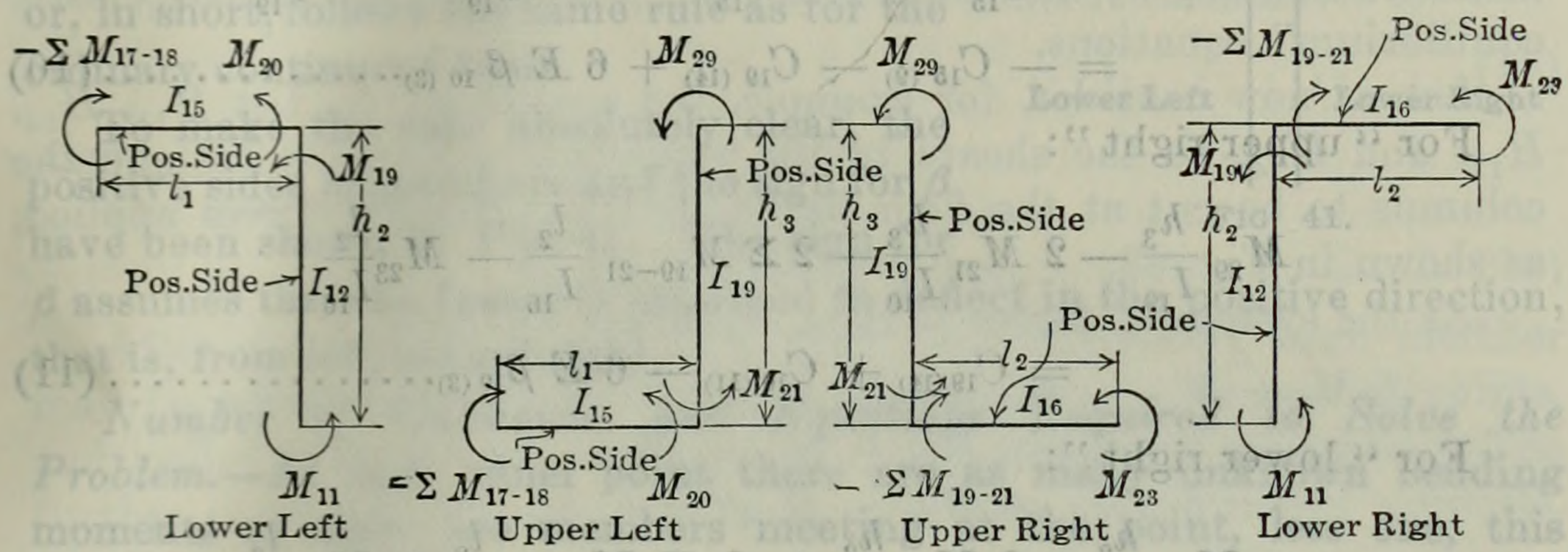
$$M_{29}\frac{h_3}{I_{19}} - 2\,M_{21}\frac{h_3}{I_{19}} - 2\,\Sigma\,M_{19\text{-}21}\frac{l_2}{I_{16}} - M_{23}\frac{l_2}{I_{16}}$$

$$= C_{19\,(14)} + C_{16\,(11)} - 6\,E\,\beta_{10\,(3)} \quad \ldots\ldots\ldots\ldots (7)$$

For "lower right":

$$M_{11}\frac{h_2}{I_{12}} - 2\,M_{19}\frac{h_2}{I_{12}} - 2\,\Sigma\,M_{19\text{-}21}\frac{l_2}{I_{16}} - M_{23}\frac{l_2}{I_{16}}$$

$$= C_{12\,(6)} + C_{16\,(11)} - 6\,E\,\beta_{10\,(2)} \quad \ldots\ldots\ldots\ldots (8)$$



Note that in these sketches the moments are shown applied to the members, and not to the panel points.

FIG. 39.

In these equations the first subscript for C refers to the member, and the second subscript, which is in parentheses, refers to the panel point for which C is to be figured. In the same way, the first subscript for β refers to the panel point, and the second subscript to the story of the frame. (See Fig. 33.)

From Fig. 40 it is seen that the positive side of the members in one panel is the inside, and in the next panel the outside, the inside of one panel being the outside of its neighbor. It would be necessary, therefore, to assume the positive side of one member and then go through the whole frame and mark the positive side, as shown in Fig. 40.

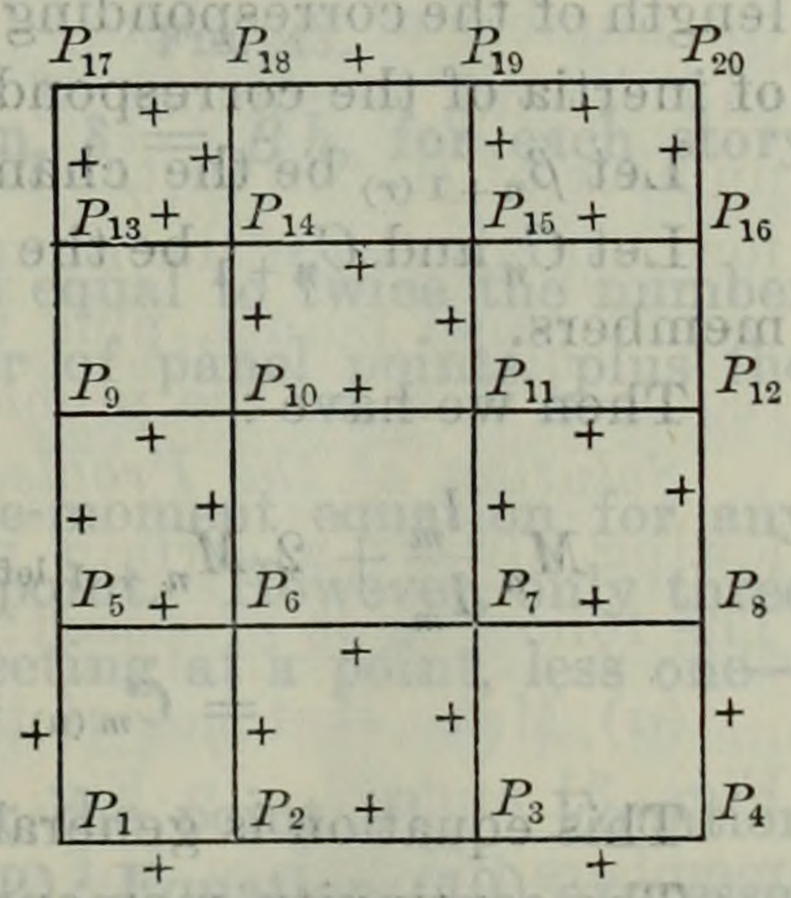


FIG. 40.

This would be very likely to cause some confusion, but the difficulty can be overcome simply by changing the signs throughout in the equations for "upper left" and "lower right". It is easily seen that this is equivalent to changing the positive side in "upper left" and "lower right", so that the inside of the panel to which the two members

Mr. Slettum. belong, in all cases, will be the positive side. The resulting equations for the point, P_{10}, therefore, are:

For "lower left":

$$\Sigma M_{17-18} \frac{l_1}{I_{15}} + 2 M_{20} \frac{l_1}{I_{15}} - 2 M_{19} \frac{h_2}{I_{12}} + M_{11} \frac{h_2}{I_{12}}$$

$$= C_{15\,(9)} + C_{12\,(6)} - 6 E \beta_{10\,(2)} \ldots\ldots\ldots\ldots\ldots (9)$$

For "upper left":

$$- \Sigma M_{17-18} \frac{l_1}{I_{15}} - 2 M_{20} \frac{l_1}{I_{15}} + 2 M_{21} \frac{h_3}{I_{19}} - M_{29} \frac{h_3}{I_{19}}$$

$$= - C_{15\,(9)} - C_{19\,(14)} + 6 E \beta_{10\,(3)} \ldots\ldots\ldots\ldots\ldots (10)$$

For "upper right":

$$M_{29} \frac{h_3}{I_{19}} - 2 M_{21} \frac{h_3}{I_{10}} - 2 \Sigma M_{19-21} \frac{l_2}{I_{16}} - M_{23} \frac{l_2}{I_{16}}$$

$$= C_{19\,(14)} + C_{16\,(11)} - 6 E \beta_{10\,(3)} \ldots\ldots\ldots\ldots\ldots (11)$$

For "lower right":

$$- M_{11} \frac{h_2}{I_{12}} + 2 M_{19} \frac{h_2}{I_{12}} + 2 \Sigma M_{19-21} \frac{l_2}{I_{16}} + M_{23} \frac{l_2}{I_{16}}$$

$$= - C_{12\,(6)} - C_{16\,(11)} + 6 E \beta_{10\,(2)} \ldots\ldots\ldots\ldots\ldots (12)$$

As the rule of signs now is the same for all panels, the positive sign always being the inside of the panel, we can write the following general equation: Let P_n, P_{n+1}, and P_{n+2} be the consecutive supports from left to right of any two members of a frame forming a lower left, upper left, upper right, or lower right corner. Let M_n, $M_{n+1\,\text{left}}$, $M_{n+1\,\text{right}}$, and M_{n+2} be the corresponding moments. Let l_m and l_{m+1} be the length of the corresponding members. Let I_m and I_{m+1} be the moments of inertia of the corresponding members.

Let $\beta_{n+1\,(r)}$ be the change in angle between the two members.

Let C_n and C_{n+1} be the influence of external load on the corresponding members.

Then we have:

$$M_n \frac{l_m}{I_m} + 2 M_{n+1\,\text{left}} \frac{l_m}{I_m} + 2 M_{n+1\,\text{right}} \frac{l_{m+1}}{I_{m+1}} + M_{n+2} \frac{l_{m+1}}{I_{m+1}}$$

$$= C_{m\,(n)} + C_{m+1\,(n+2)} + 6 E \beta_{n+1\,(r)} \ldots\ldots\ldots\ldots\ldots (13)$$

This equation is general, and the rule of signs is as follows:

The continuity moments, M_n, M_{n+1}, etc., enter with positive sign when producing tension on the positive side of the member, that

Mr. Slettum.

is, the inside of the panel to which the member belongs. The quantities, C_m and C_{m+1}, enter with positive sign when the external loads act in such a way as to produce tension on the negative side of the member, that is, the outside of the panel to which the member belongs.

The quantity, $\beta_{n+1(r)}$, is positive when the angle between the two members, measured on the inside of the panel to which the two members belong, becomes greater; or, in short, follows the same rule as for the ordinary continuous beam.

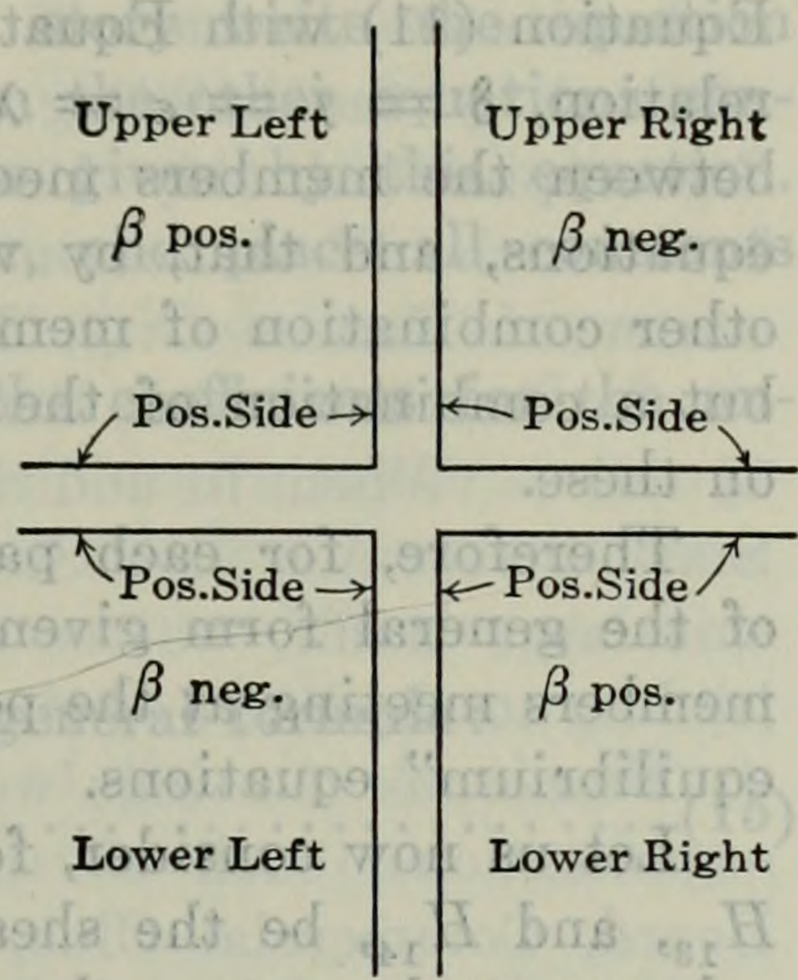


FIG. 41.

To make the case absolutely clear, the positive sides of members and the sign for β have been shown in Fig. 41. The sign for β assumes that the frame is supposed to deflect in the positive direction, that is, from left toward right.

Number of Unknowns and Equations Required to Solve the Problem.—At each panel point there are as many unknown bending moments as there are members meeting at the point, less one, this latter being equal to the algebraic sum of the others and opposite in direction.

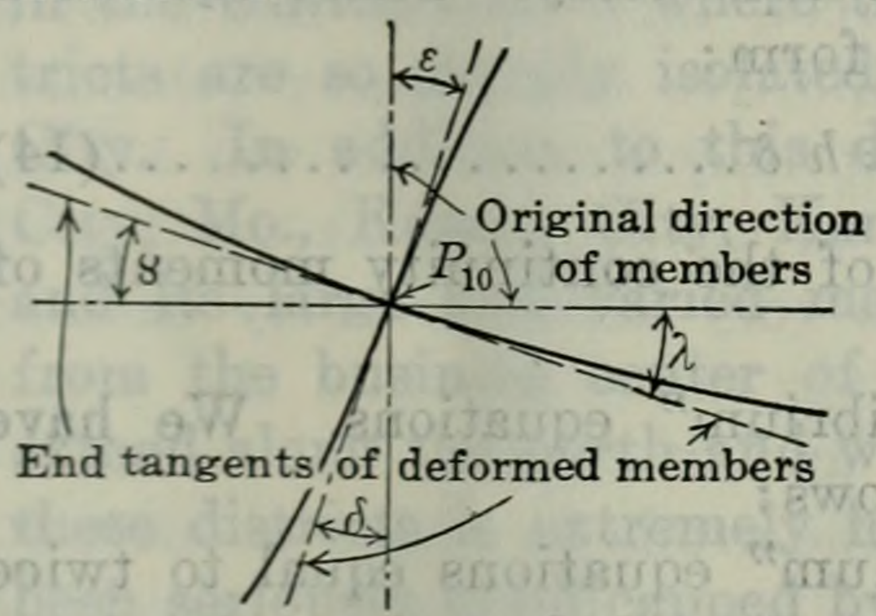


FIG. 42.

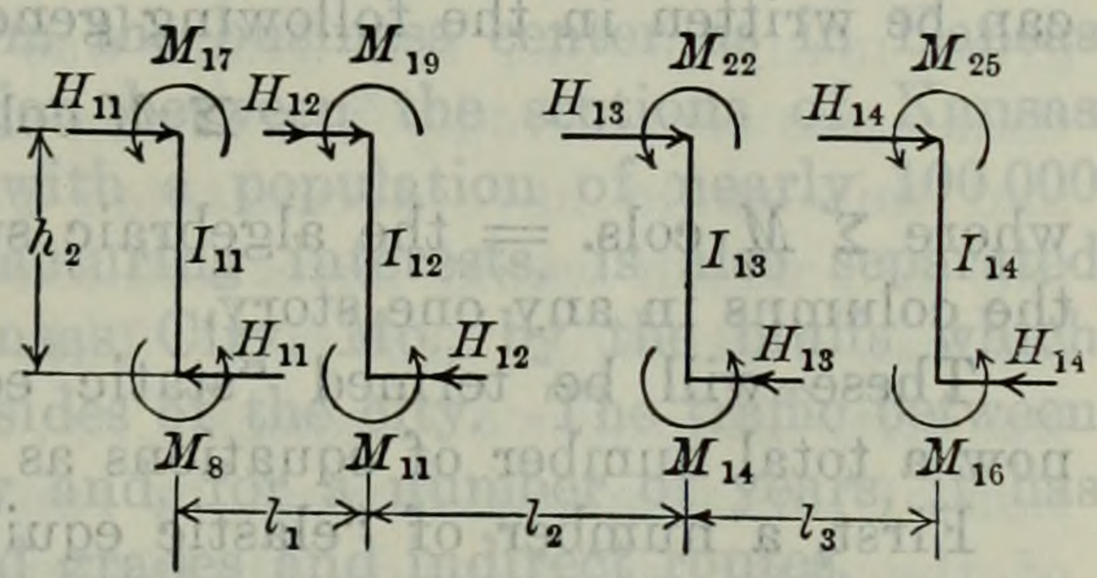


FIG. 43.

Besides, we have an unknown deflection, $\delta = \beta h$, for each story of the frame.

This gives a total number of unknowns equal to twice the number of members in the frame, less the number of panel points, plus the number of stories.

As already stated, we can write a three-moment equation for any combination of two members meeting at a point. However, only three —or in general the number of members meeting at a point, less one— are independent of each other.

To illustrate this, let us again consider the point, P_{10}. Equation (9) expresses the relation, $\delta = j$ (see Fig. 42); Equation (10) expresses the relation, $j = \epsilon$; and a combination of the two will express the relation, $\delta = j = \epsilon$.

Mr. Slettum. Equation (11) expresses the relation $\epsilon = \lambda$. A combination of Equation (11) with Equations (9) and (10), therefore, will express the relation, $\delta = j = \epsilon = \lambda$. Thus it is seen that the elastic relation between the members meeting at a point is fully expressed by three equations, and that, by writing the three-moment equation for any other combination of members, we write an equation which is nothing but a combination of the three equations, and accordingly dependent on these.

Therefore, for each panel point, we have a number of equations, of the general form given by Equation (13), equal to the number of members meeting at the point, less one. These will be termed "elastic equilibrium" equations.

Let us now consider, for example, the second story. Let H_{11}, H_{12}, H_{13}, and H_{14}, be the shears in the respective columns. Assume the columns to be cut at the floor levels, and the internal forces applied, as shown in Fig. 43.

As the system of forces is in equilibrium, we have:

$$M_8 + M_{11} + M_{14} + M_{16} + M_{17} + M_{19} + M_{22} + M_{25}$$
$$= h_2 (H_{11} + H_{12} + H_{13} + H_{14})$$
$$= h_2 (H_V + H_{IV} + H_{III}) = h_2 \delta_{II},$$

where δ_{II} is the external shear in the story.

Similar equations can be written for each story. These equations can be written in the following general form:

$$\Sigma M \text{ cols.} = h \delta \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where ΣM cols. = the algebraic sum of the continuity moments of the columns in any one story.

These will be termed "static equilibrium" equations. We have now a total number of equations as follows:

First, a number of "elastic equilibrium" equations equal to twice the number of members in the frame, less the number of panel points; and second, a number of "static equilibrium" equations equal to the number of stories in the frame. The total number of equations, therefore, equals twice the number of members in the frame, less the number of panel points, plus the number of stories, which is the number required to solve the problem.

Solution of the Problem.—Considerable time can be saved by following always a certain method in solving problems of this nature. The following is proposed:

(*a*) Make sketches of the structure, and show the quantities, as in Figs. 31 and 32. In Fig. 32 it is not necessary to show the deformed members themselves, but only their chords.

(*b*) Tabulate the values of l, h, I, $\frac{l}{I}$, $\frac{h}{I}$, and C.

(*c*) Write the equations with coefficients for the unknowns and the constants given by letters. For each story write the equation containing the term $6\ E\ \beta$ by itself, and in the other equations containing that term, substitute its value, as given by this equation. Arrange the unknowns in numerical order, and place all constants on the right-hand side of the equation. Mr. Slettum.

(*d*) Tabulate the numerical values of the coefficients for the unknowns and of the constants.

(*e*) Eliminate the unknowns.

(*f*) Find the unknowns by substitution.

The deflections are calculated from the general formula:

$$\delta = \beta h \qquad (15)$$

where δ = deflection of any single story.

The total deflection of the frame is:

$$\Delta = \Sigma\ \delta = \Sigma\ \beta\ h \qquad (16)$$

After the continuity moments have been calculated, each member may be assumed to be cut and freely supported at the panel points, and loaded with the external loads and the continuity moments. The further calculation is then as for an ordinary beam on two supports.

L. R. Ash,* M. Am. Soc. C. E. (by letter).—There are few cities in the United States where the freight-house and manufacturing districts are so sharply isolated from the business center as in Kansas City. In addition to this division between the sections of Kansas City, Mo., Kansas City, Kans., with a population of nearly 100 000 and its large and varied manufacturing interests, is also separated from the business center of Kansas City, Mo., by the bluffs which extend along the north and west sides of the city. The traffic between these districts is extremely heavy and, for a number of years, it has been seriously handicapped by bad grades and indirect routes. Mr. Ash.

The apparent need of a trafficway between Kansas City, Mo., and Kansas City, Kans., and to the West Bottoms, was great enough to enlist private capital in the enterprise of building the Sixth Street or Inter-city Viaduct some 8 or 10 years ago. This structure was built as a toll viaduct, and although it has proved to be a failure, from a financial standpoint, this failure in no way indicates that such a structure was not needed, but demonstrates the reluctance of the American public to pay tolls. During the times that the Inter-city Viaduct was open for traffic without charge, the structure was crowded with travel, both to and from Kansas City, Kans., as well as to and from the West Bottoms.

The question of building a viaduct at 12th Street, connecting with the West Bottoms, has been discussed for the last 15 years, and

* Kansas City, Mo.

Mr. Ash. although every one realized its great need, it seemed almost impossible to harmonize the various interests which in one way or another were affected by the building of the structure. The old viaduct at 12th Street provided only for street-railway traffic. All vehicular traffic between the West Bottoms and Kansas City, Kans., and the business center of Kansas City, Mo., had no thoroughfare from 6th Street on the north to 24th Street on the south, a distance of more than 1½ miles. When, in addition to this, we consider the fact that neither Sixth Street nor 24th Street led in the direction of the terminus for this large traffic, we see more clearly the great need of a trafficway leading to the heart of the business portion of the city.

Numerous meetings were held with representatives of the various organizations, such as the Team Holders Association, Real Estate Board, Commercial Club, etc., in an attempt to determine the most practicable structure, both as regards the requirements of traffic and the elimination of damages. These discussions were frequently very acrimonious, and politics cut no small figure in the handling of the proposition. The management of the whole enterprise was a shining example of the need of a more responsible and centralized authority in municipal affairs.

One of the great problems which confronted the promoters of the 12th Street Viaduct was that of grades. About 5% seemed to be the only practicable grade on 12th Street, and, in the minds of a great many people, this grade was prohibitive when heavy vehicular traffic was considered. The grade, as originally established, was 5.09%, and this would have necessitated a cut of about 17 ft. at the top of the hill, which would have resulted in a very heavy property damage estimated at about $250 000. This matter came up while the writer was City Engineer of Kansas City, Mo., and he suggested the elimination of this property damage by raising the grade to within about 4 ft. of the established grade at Washington Street. This increased the viaduct grade to 5.52%, a very slight change from the old one; and the opposition to the structure, thus avoided, abundantly justified the change.

The advent of the motor truck has decreased the necessity for lower grades, except where reasonably attainable, but, as a concession to the demand for a low-grade trafficway between the Bottoms and the business section, a lower deck was provided on the viaduct with a maximum grade of about 2½ per cent. It is expected to utilize this portion of the viaduct by building roadways along the bluff leading to the north and to the south from the east end of this lower deck. In the writer's opinion, the building of these roadways will not be justified, as they will lead to the old trafficway at 6th Street on the north, or to 17th Street on the south, neither of which points is in the line of travel, and from which there will be bad grades and other

conditions not inviting to heavy vehicular traffic. It would seem that a better solution of the problem would be to tunnel from the east end of the lower deck to about 14th and Wyandotte Streets, thus insuring a very easy grade and a direct route for the distribution of the traffic from the West Bottoms. Mr. Ash.

Now that the structure has been completed and thrown open to the public, the great need of it is proved by the very great traffic, both vehicular and by street cars, using it; and this is drawn to the structure in spite of the fact that the approaches to the viaduct have not been paved, as they await the settlement of the embankments at each end. The structure saves several minutes to each of the many people who go each day from the eastern portion of the city to the West Bottoms, and the 5.52% grade has proved to be no serious handicap, either to motor or horse-drawn vehicles.

It is interesting to note that the building of the viaduct has resulted in enhancing real estate values and rentals on East 12th Street and the adjacent streets for a distance of 3 or 4 miles eastward from the structure. This is because the opening of 12th Street provides a direct way to the West Bottoms for the large number of people employed there, without having to transfer or travel several blocks out of their way. The 12th Street cars, although greatly increased in number, are crowded with passengers, and the building of the viaduct has fixed 12th Street as the principal east and west artery of the city.

Mr. Howard has gone very fully into the design and construction features, and there is no additional comment to be made except to remark on the very satisfactory result, as shown by the finished structure. The surface finish of the concrete is not all that could be desired, but it is believed that it is in keeping with the character of the structure and, when conditions in the neighborhood are considered, it is thought that the more expensive methods of treating concrete surfaces would not have been justified in this case.

The specifications as drawn would permit the engineer to require the contractor to put a cement-gun surface on all columns and cantilevers, as well as on the outside surfaces of the longitudinal girders, but, in the writer's opinion, the attempt to place this finish on surfaces of the character in this structure, would not result satisfactorily, and it is believed that the surface as produced by the forms, with the rough places smoothed off with hammer and chisel and in certain places filled with mortar, has been the best treatment practicable.

The treatment of concrete surfaces is receiving a great deal of attention, and various methods have been advanced for producing a surface which is smooth and, at the same time, does not present the expressionless effect of a smooth mortar surface. The writer believes

Mr. Ash. that specifications can most easily provide for the finish of concrete surfaces by requiring the bidders to tender on two or three different kinds of treatment at a stated price per unit of surface, leaving the engineer to choose that method which, in his opinion, most nearly suits the condition when the forms are removed. Of course, all specifications should require forms to be built of smooth lumber, with tight joints and true to line.

The character of surface desired should be determined by the general characteristics of the structure, its location, type of construction, etc. No attempt should be made to give the concrete the appearance of stone or other material, and it is thought that, with a few exceptions, a reasonably smooth surface which is produced by good forms and well-spaded concrete, is better than can be secured by special treatments.

On the 12th Street Viaduct 22 ft. of the upper deck is set aside for street-car tracks, and a roadway, 30 ft. wide on the south side, is left for vehicles, a sidewalk 5 ft. wide extending along the south side of the structure. Although this arrangement has some advantages over that of placing the street railway in the center of the roadway, it would seem that the space between the hand-rails would be used more efficiently with the tracks in the center, as is the case in an ordinary city street. About the only justification for placing the tracks in a space independent of the roadway, is the length of the structure and the grade. It is believed that a sidewalk, 8 ft. wide, and a clear roadway of about 50 ft. between the curbs, with street-railway tracks in the center, would have resulted in a structure which would have cost somewhat less and would have been better suited to the general traffic conditions. As it is, vehicles on the north side of 12th Street, at each end of the viaduct, must cross the street railway tracks.

The use of wood blocks for paving the roadway portion has proved entirely satisfactory. It was feared that the grade would cause the pavement to be slippery, but the blocks were laid with a lath between each row, and the remaining space was well filled with broken stone. The blocks are 3 in. wide, and the resulting surface undoubtedly provides a better footing for horses than stone block, asphalt, or brick. The writer has observed heavily loaded teams turning out of the street railway portions of certain streets where there is stone block pavement, to the wood block pavement at the sides, because of the better footing secured on the latter.

The laying of the wood blocks on the viaduct without a sand cushion has been entirely satisfactory, and demonstrates the wisdom of omitting one of the great sources of trouble with pavements of this type. With care in finishing the concrete base, the surface can

be brought to true crown and grade without serious trouble, and when this is done there is no justification whatever for the use of a sand cushion. Mr. Ash.

The structure represents a great amount of careful study of the architectural treatment as well as the engineering features, and it is becoming more and more apparent that engineers should give consideration to matters pertaining to the appearance of structures of this character. There seems to be little excuse for absolutely neglecting the esthetic treatment of engineering structures, and although the Profession has been prone to do this, it is gratifying to note a marked change in its attitude in this matter.

Mr. Harrington.

John Lyle Harrington,* M. Am. Soc. C. E. (by letter).—So much of the Twelfth Street Trafficway Viaduct is of usual construction, and the author has exposed so fully the elements which are of interest to the engineer who is thoroughly experienced in the design of reinforced-concrete structures, that few matters pertaining to the viaduct require further comment.

Some designers of reinforced-concrete structures consider the work so simple that their plans consist of a few general drawings showing principal dimensions, the number, size, and approximate position of reinforcing bars, and with a few notes pertaining to ties, constituents of concrete, and character of finish; and their specifications consist of little more than references to standard specifications for materials, a brief description of the structure, and the clauses relating to bids. Those of another class, generally of European training, go to the opposite extreme, and make extremely full plans and all the detailed calculations necessary for the exact determination of the stresses in continuously-framed structures, composed of uniformly perfect materials, moulded and placed exactly in accord with the plans. Both are in error. Ill-considered, carelessly-made plans and loosely-drawn specifications are responsible for a large number of defective structures and for many costly failures. Lax supervision is almost sure to follow such designing; and, if the engineer has such small regard for thorough work, the contractor and his foremen and workmen surely cannot be expected to give the construction the greater thought and care essential to satisfactory workmanship.

On the other hand, the extremist who bases his plans on the expectation of perfect materials and workmanship, and allows nothing for imperfections in both, produces designs which, though carried out with all reasonable care, cannot secure satisfactory results. At the same time, he wastes much labor in substantially useless calculation, for considerations of convenience and of uniformity of design do not permit of sectioning exactly in accord with the calculated stresses,

* Kansas City, Mo.

Mr. Harrington. and knowledge of the imperfections of materials, of the shortcomings of workmen, and of the difficulties of construction, leads to the making of many assumptions and approximations.

Competent and experienced designers, who know and respect the theories involved in the design, but who also give careful consideration to the imperfections of the materials and the practical difficulties of construction, will secure the satisfactory and enduring structures. Mathematical analysis must be supported by sound judgment.

The advocates of the use of high-carbon steel for reinforcing concrete rarely understand its fragility. Any engineer who has had much experience in the shops knows that even medium steel, when dropped from the skids or otherwise roughly handled, is likely to be fractured; and in many instances which have come to the writer's attention, the breakage of high-carbon steel bars in unloading and handling has been so great as to cause serious doubt as to their quality. The bending of reinforcing bars is not a nice job: It is done in haste, and generally by unskilled workmen, often with unsuitable tools; hence breakage, or what is worse, incipient cracks, which escape notice but enlarge under stress in the structure, is sure to result from the use of this fragile material.

In the writer's opinion, the saving in cost resulting from the use of high-carbon steel does not warrant its adoption. The saving is generally estimated to be proportional to the difference between the unit stresses permissible in the various grades of steel, but this is not true, for, as every experienced designer knows, members cannot be proportioned exactly in accord with the stresses in them, and a large quantity of ties, hoops, and spacers are necessary, regardless of the unit stresses used. The saving is not worth the risk, particularly in structures subject to moving loads.

The finish of concrete structures is one of the most serious moot questions relating to them. In choosing a finish, the improvement in appearance is but one of the considerations; the added cost, the greater absorption of water, and the fouling by smoke resulting from the breaking of the cement skin, must be carefully considered. If the structure is situated so that few will see it at close range, or that it will be blackened by smoke, the expense of treating the surface is not warranted. The writer recently examined a viaduct over a railway yard, and it was covered with soot to such an extent that neither the character of the surface nor the material of the structure could be readily determined. In parks, on boulevards, and in other exposed positions, however, where many view the structure daily, bush-hammering, rubbing, floating, or some other satisfactory method of finishing the surface should be used in order to remove ugly form marks and secure uniformity of color and texture. There should be

no attempt to make concrete resemble stone, but flat surfaces should be broken. Plastering is almost certain to scale off and leave a surface uglier than that left by the forms; and, except possibly by using steel-lined forms, it is practically impossible to secure satisfactory surfaces, except by some mechanical treatment. Mr. Harrington.

No amount of surface dressing, however, will compensate for ugly lines and details, and the use of brick, tile, or plaster ornaments is rarely if ever commendable. The designer of such structures as this must rely for appearances chiefly on his ability to give them form and line suitable to the service to be rendered by the structure and in harmony with its dimensions and surroundings. The selection of the central or distinguishing features and the general treatment of the principal forms and parts of the structure are of vastly more importance than ornamentation or finish of surface.

The writer had too large a part in the design and construction of this viaduct to permit him to extol its appearance. The situation and conditions were unusually difficult, hence careful study was required to secure such esthetic results as the general views of the viaduct disclose.

E. E. Howard,* M. Am. Soc. C. E. (by letter).—It is of interest to observe that the mathematical demonstrations by Mr. Mensch and Mr. Slettum alike warrant the conclusion that the approximate methods by which the columns were actually proportioned give results within safe limits. There are refinements beyond which ordinary designing calculations for elastic structures need not be carried, provided there are occasional demonstrations that the approximate methods will vary only within safe limits. The type of arch and abutments suggested by Mr. Mensch has been considered and analyzed for similar cases, although not for this specific one, and the additional material in the arch proper to provide for the greater stresses, the adjustment for foundations, etc., never seemed to be compensated for by the saving in abutments. The foundation difficulty referred to may easily become quite serious. Mr. Howard.

To answer Mr. Mensch regarding high-carbon steel, the writer can suggest only that the same conservatism which leads most engineers to use medium steel in structures wholly of steel, would prompt the use of the same material in structures partly of steel. The writer has seen high-carbon steel bars, actually by the carload, which had been inspected and accepted at the mill, broken by merely being thrown from the car in unloading. Evidently, the sentence concerning the fastening of bars was not wholly clear, for the bars were tied together principally to hold them in correct position while placing the concrete. Any joining of bars intended to transmit the load from one bar to

* Kansas City, Mo.

Mr. Howard. another was made by some definite splice, such, for instance, as bolted rope clips.

Mr. Holmes' suggestions for constructing influence lines for extreme fiber stress brings forward another of the frequent cases in which individual preference may enter. In this special instance, the designers felt that to carry out the equations another step and plat the influence in lines of extreme fibers would require more work than the method followed. It is easy to agree with Mr. Holmes that in certain cases the other method would be most desirable.

The writer is most heartily in accord with Mr. Holmes' views concerning the relations between engineers, contractors, and public officials. The absurd and ridiculous laws sometimes controlling public construction have been cause for exasperation to many engineers, although, after all, the rabbit-like timidity of public officials, in fearing to do what they believe to be right, may be equally at fault. Although supposedly representing a whole community, public officials are often prone to listen to the vociferous requirements and the threats or promises of a small group of those who have personal interests to promote, and to forget the great silent majority who are not present. Fortunately, there are occasional very refreshing exceptions.

Mr. Upson's figures concerning the piles are substantially correct, but, as a just concession to the increased percentage of overhead cost due to the reduced quantities, a payment of $2 462 was made to the Contractor. This still left the City about $20 000 ahead of the probable cost of pre-cast piles. The Raymond piles were well suited to the foundation conditions and to the season in which the work had to be done.

The rather unusual requirements of a cement-gun finish, referred to by Mr. Ash, were included as one alternative method of surface finish only after the manufacturers of the cement-gun equipment gave assurances that they could provide such work, and gave an approximate price for it. The experiences of other users of the cement gun, especially on viaducts of the Kansas City Terminal Company, cast serious doubt on the practicability of producing an even, uniform, $\frac{1}{8}$-in. mortar coat over irregular surfaces. It was evident that the extruding corners would not be coated at all, that re-entrant corners and moulds would be filled and rounded, and that the clean lines of the structure would be lost. It is the writer's opinion that the cheap and satisfactory surface finish for ordinary concrete will some day be supplied by some kind of lime-cement wash or paint which can be brushed on, which will have characteristics so similar to the concrete that it will not flake off. Many industrial chemists are studying cements, limes, and mortars, and such a wash should be available at no distant day.

In conclusion, it has been a pleasure to the writer on frequent occasions to hear expressions of pride and satisfaction, from residents

of Kansas City, who provided the money for the viaduct, in the pleasing appearance and evident excellency of construction. The public verdict is that the structure is a success. The old excuse that engineers have been prone to bring forward, that their principals would not provide money for beautiful structures and therefore such structures were ugly, belonged to pioneer conditions, and, as Mr. Green suggests, the United States may fairly be said to have passed that stage. It would be hard to find any one, even of those who paid special assessments for this viaduct, who would ask the return of the small additional percentage which it cost, over the expense of a series of posts and beams which would have carried the load. If the Engineering Profession will lead the way for substantial and beautiful bridges, public sentiment will follow fast enough. Mr. Howard.

The writer desires to express his cordial appreciation for the courteous words of approval of his paper by those who discussed it; and, on behalf of all the engineers connected with the design and construction of the Twelfth Street Trafficway Viaduct, for their compliments in regard to the structure.