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THE HELL GATE ARCH BRIDGE AND APPROACHES OF THE NEW YORK CONNECTING RAILROAD OVER THE EAST RIVER IN NEW YORK CITY*

By O. H. Ammann, M. Am. Soc. C. E.

With Discussion by Messrs. W. H. Breithaupt, Leon S. Moisseiff, Samuel T. Wagner, Charles Evan Fowler, Henry H. Quimby, Henry B. Seaman, Gustav Lindenthal, Clement E. Chase, and O. H. Ammann.

Synopsis.

The principal object of this paper is to present an account of the design and construction of the Hell Gate Arch Bridge over the East River in New York, a structure of imposing magnitude, with unusual features and details of unprecedented size which mark a decided advance in bridge engineering.

The Hell Gate Bridge, which forms part of the New York Connecting Railroad, is the greatest arch bridge built to date, having a span of 995 ft. 1⅜ in. between centers of bearings and 1 017 ft. between faces of abutments, and a total height of 305 ft. above mean high water. It carries four railroad tracks on a heavy ballasted floor. Its principal features, besides its great span and capacity, are the exceptional size and weight of its individual members and riveted connections, the use of special high-carbon steel, the unusual method of erection, and the monumental towers forming the abutments, one of which rests on a deep and difficult pneumatic caisson foundation.

* Presented at the meeting of November 21st, 1917.

The introductory part of this paper is devoted to the object, history, and a brief general description of the New York Connecting Railroad.

The Hell Gate Bridge is the result of many years of careful and laborious studies involving the design of different types of bridges. These various preliminary designs are briefly described and critically discussed, and the conditions which led to the final adoption of the spandrel-braced arch type are set forth.

The description of the Hell Gate Bridge, as built, is confined to the more important features and details, with particular reference to the reasons and considerations which governed their design, and with a further view to bring out their merits and illustrate the progress made in bridge design in recent years. Typical details are shown in the accompanying illustrations and therefore need no particular description.

With due consideration of the fact that the strength and durability of a bridge are gauged by the strength and efficiency of its details, most careful attention has been given to their design, and a number of important improvements over common practice can be recorded. Among the details deserving special mention are the compact closed section of the main arch rib, the extraordinary rigid bracing, the efficient latticing of compression members, the full splicing of compression joints, and the provision of heavy braking girders to relieve the floor-beams from stress due to the braking and traction forces.

It has been the aim of the administration of the Railroad Company to produce a first-class railroad in every respect. This principle has been observed throughout the design and construction of all its structures. The design has been governed by rules and specifications especially prepared by the consulting engineer. These differ in many respects from common specifications, and contain many original features. Extracts from these rules and specifications are given in this paper, with critical remarks on the selection of the material, assumptions of the various loads and forces, and of the permissible unit stresses.

Among the original features of these specifications may be mentioned a new formula for impact which, in combination with apparently high permissible unit stresses, is applicable to the design of bridges of any length of span or any capacity, and secures in each case well-proportioned structures.

The paper further contains a detailed account of the essential features of the fabrication and erection of the Hell Gate Bridge which called for the highest grade of workmanship, required special tools and machinery of exceptional size, and involved unusual erection problems. This account is intended to illustrate the advance in the fabrication and erection of large bridges of the riveted type. Especial mention may be made of the complete assembling of the arch trusses at the shop, and the extensive utilization of parts of the permanent structure for erection purposes.

The approaches to the Hell Gate Bridge, which comprise the East River Bridge Division of the New York Connecting Railroad, consist of several heavy truss bridges, a double-leaf bascule bridge, about 10 800 lin. ft. of plate-girder viaducts with concrete piers, and about 3 200 lin. ft. of embankment between high retaining walls and concrete arches. Space does not permit of their detailed description, but their unusual and original features, and the reasons and conditions which governed their design, are related.

The calculation of stresses in the arch trusses, as given in Appendix A, although presenting no new theory, will be found of value to those who may have to solve similar problems. It illustrates the application of the modern theory of elasticity in the shortest and most convenient form.

The writer is under obligation, for permission to present this paper and for valuable information, to Gustav Lindenthal, M. Am. Soc. C. E., the Consulting and Chief Engineer, to whose plans and direction the successful completion of the Hell Gate Bridge and Approaches is due.

For convenience of reference and discussion, the paper is written under the following principal headings:

1.—Object and History of the New York Connecting Railroad;
2.—General Description of East River Bridge Division;
3.—Development of Design of Hell Gate Bridge;
4.—General Arrangement, Proportions and Cost of Arch Bridge;
5.—Masonry Towers and Foundations;
6.—Details of Design, and Weight of Steel Superstructure of Hell Gate Bridge;
7.—Camber and Deformation of Arch Trusses;
8.—Material;
9.—Loads and Unit Stresses;

10.—Workmanship and Fabrication;
11.—Erection of Hell Gate Bridge;
12.—Approaches;
13.—Track Floor Construction;
14.—Engineering Organization.
Appendix A.—Calculation of Dead-Load, Live-Load, and Temperature Stresses in Arch Trusses of Hell Gate Bridge;
Appendix B.—Financing, and Franchise of the New York Connecting Railroad Company.

1.—Object and History of the New York Connecting Railroad.

Object.—The New York Connecting Railroad is a part of the comprehensive plan of the Pennsylvania Railroad, inaugurated 15 years ago by the late A. J. Cassatt, then President of that Company, and which had for its principal objects the extension of the line from New Jersey into New York City and direct rail connection, for passengers and freight, with Long Island and the New England States.

The parts of this general project which established entrance into New York City, and connection with the Long Island Railroad, comprising the tunnels under the North River, Manhattan Island, and the East River, the great passenger terminal in Manhattan, a large terminal yard, called "Sunnyside Yard", in Long Island City, a freight terminal yard, at Greenville, N. J., and various other improvements have been fully outlined and described in detail.* (See also the map, Fig. 1).

The New York Connecting Railroad establishes a physical connection between the Pennsylvania Railroad and the New York, New Haven, and Hartford Railroad Systems, and thus provides continuous rail communication, through New York City, between Canada, the New England States, and the South and West.

The connection with the New Haven Line is made at Port Morris in The Bronx. From there, the Connecting Railroad crosses the East River, *via* Wards and Randalls Islands, and joins the Pennsylvania Railroad at Sunnyside Yard in Long Island City, whence the trains pass through the East River Tunnels to the Pennsylvania Station at 34th Street, in Manhattan. This connection *via* Sunnyside Yard,

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* *Transactions,* Am. Soc. C. E., Volumes LXVIII and LXIX (1910).

however, is intended for passenger trains only, inasmuch as the tunnels under the North and East Rivers and the Manhattan Station can now accommodate only passenger traffic.

The freight connection will be made by another branch of the New York Connecting Railroad, which after crossing the East River at Hell Gate, passes through Long Island City, Woodside, Winfield Junction, and Middle Village, to Fresh Pond Junction, near the Brooklyn Borough line, where it connects with the Bay Ridge Branch of the Long Island Railroad. From Bay Ridge, the freight cars are transported on car-floats across New York Bay to the Greenville Yards of the Pennsylvania Railroad in New Jersey. Eventually, a tunnel may be driven under New York Bay from Bay Ridge to Greenville, which would provide also for freight by continuous-rail connection from New England to the South and West through New York City.

Besides this through freight traffic, the New York Connecting Railroad will accommodate the bulk of the freight transportation to and from the large manufacturing districts which have been developed during recent years in Brooklyn and Queens. For this purpose a number of freight yards have been established along the Bay Ridge Branch of the Long Island Railroad.

Although not doing away entirely with car-float and ferry service, the New York Connecting Railroad will greatly relieve the inner waters of New York Harbor of this traffic, particularly of the obstructive car-floats which now transfer freight from the Pennsylvania Railroad terminals on the Jersey shore of the Hudson River, up the East River to the New Haven Railroad freight terminals at Port Morris and Oak Point, in The Bronx.

Traffic.—The early plans contemplated a double-track line. Later, it was realized that the probable future development of an extensive passenger and freight traffic between the New Haven and the Pennsylvania Systems would soon require four tracks. Estimates also proved that it would be much more costly to add two tracks in the future than to build a four-track line in the first place. The entire portion from Port Morris to the point in Long Island City where the two passenger tracks branch off toward Sunnyside Yard was built therefore for four tracks, the remainder being for two tracks only.

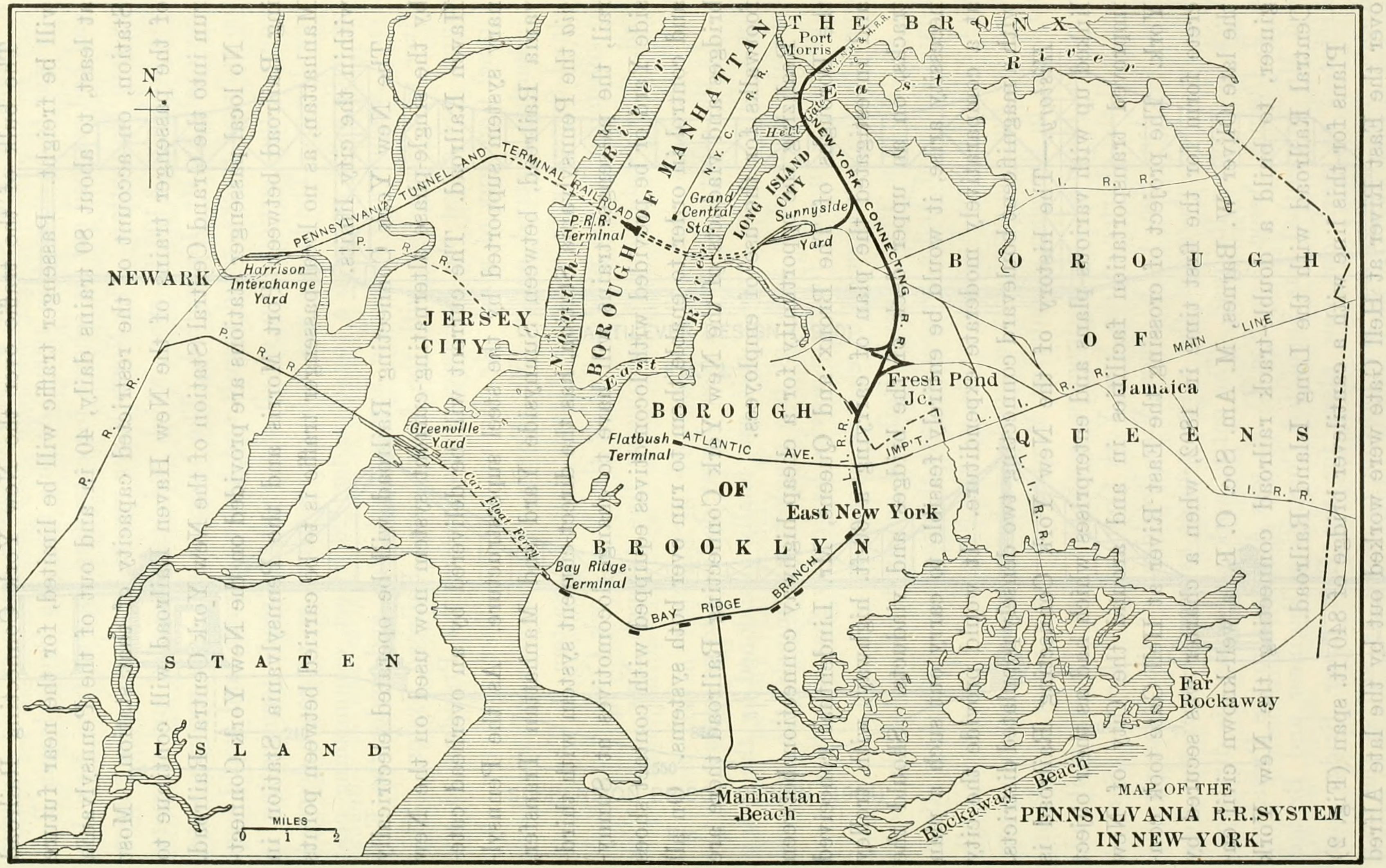

N
THE BRONX
Port Morris
N.Y.N.H. & H.R.R.
East River
Hell Gate
NEW YORK CONNECTING R.R.
N.Y.C. R.R.
BOROUGH OF MANHATTAN
Grand Central Sta.
P.R.R. Terminal
LONG ISLAND CITY
Sunnyside Yard
L. I. R. R.
PENNSYLVANIA TUNNEL AND TERMINAL RAILROAD
NEWARK
Harrison Interchange Yard
P. R. R.
JERSEY CITY
North River
West Shore R.R.
BOROUGH OF BROOKLYN
Fresh Pond Jc.
MAIN LINE
Jamaica
QUEENS
Greenville Yard
Car Float Ferry
Flatbush Terminal
ATLANTIC AVE.
IMP'T.
L. I. R. R.
East New York
Bay Ridge Terminal
BAY RIDGE BRANCH
STATEN ISLAND
Far Rockaway
Manhattan Beach
Rockaway Beach
MILES
0 1 2
MAP OF THE
PENNSYLVANIA R.R. SYSTEM
IN NEW YORK

FIG. 1.

The bulk of the traffic over the New York Connecting Railroad will be freight. Passenger traffic will be limited, for the near future at least, to about 80 trains daily, 40 in and out of the Pennsylvania Station, on account of the restricted capacity of that station. Most of the passenger trains of the New Haven Railroad will continue to run into the Grand Central Station of the New York Central Railroad.

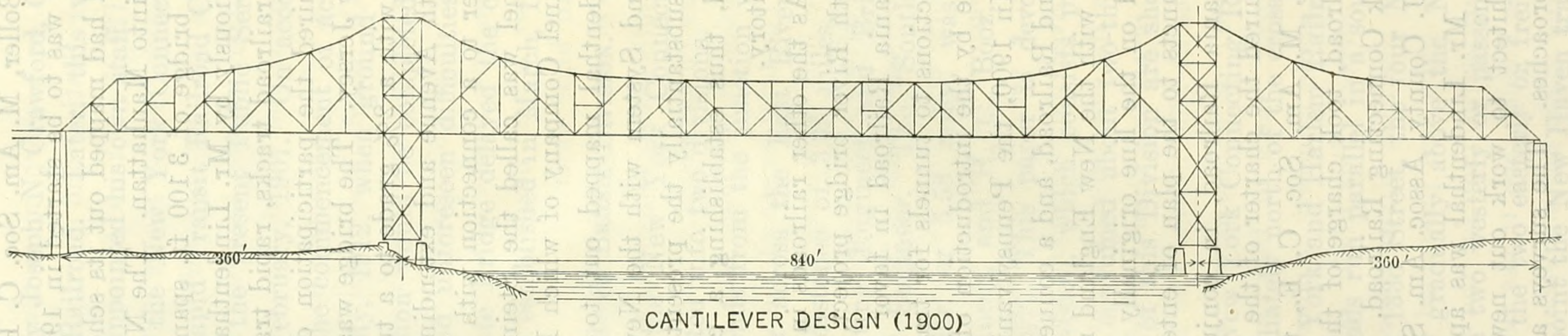
No local passenger stations are provided on the New York Connecting Railroad between Port Morris and the Pennsylvania Station in Manhattan, as no local passenger traffic is to be carried between points within the city limits.

The New York Connecting Railroad will be operated electrically by the single-phase alternating-current system now used on the New Haven Railroad. The current will be delivered by an overhead catenary system supported by the steel superstructure. As the Pennsylvania Railroad, between Sunnyside Yard and Manhattan Transfer, *via* the Pennsylvania Station, has the direct-current system with third-rail, the passenger trains will have to change locomotives at Sunnyside Yard or be provided with locomotives equipped with contact shoes and control in order to enable them to run over both systems. On all bridges and viaducts of the New York Connecting Railroad there are footwalks for the use of employees.

Realizing the opportunity for a cheap highway connection between the Boroughs of The Bronx and Queens, Mr. Lindenthal conceived and investigated the plan of carrying a 60-ft. highway, with trolley tracks, on an upper deck of the bridges and viaducts. Should the necessity arise, it would be entirely feasible to carry out such a plan at a comparatively moderate expenditure. It would provide the city with a magnificent boulevard connecting two densely populated districts.

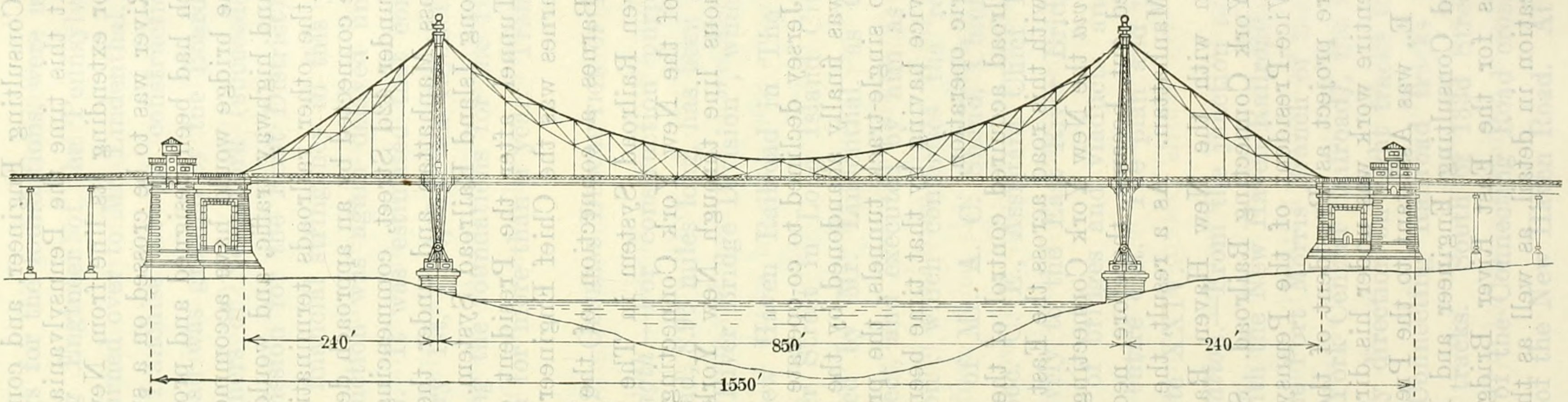
History.—The history of the New York Connecting Railroad is linked up with various plans and enterprises which had as their object improved transportation facilities in and around the City of New York. The project of crossing the East River at Hell Gate took concrete form for the first time in 1892, when a charter was secured by the late Oliver W. Barnes, M. Am. Soc. C. E., a well-known civil engineer, to build a double-track railroad connecting the New York Central Railroad with the Long Island Railroad.

Plans for this line with a cantilever bridge of 840 ft. span (Fig. 2) over the East River at Hell Gate were worked out by the late Alfred



CANTILEVER DESIGN (1900)

FIG. 2.



SUSPENSION BRIDGE DESIGN (1904)

FIG. 3.

P. Boller, M. Am. Soc. C. E., as Consulting Engineer, and construction was to be started in 1900. At this time the Pennsylvania Railroad had mapped out its scheme for extending its line from New Jersey into Manhattan. The North River was to be crossed on a suspension bridge of 3 100 ft. span, which had been designed and promoted previously by Mr. Lindenthal. The bridge would have accommodated ten railroad tracks, rapid transit and highway traffic, and would have required the participation of all the other railroads terminating in New Jersey. The bridge was to be connected by an approach descending with a 2% grade to a tunnel under 42d Street, commencing near Tenth Avenue and extending across Manhattan and under the East River to a connection with the Long Island Railroad System. This tunnel was called the Steinway Tunnel, after the President of the Tunnel Company of which Mr. Barnes was the Chief Engineer. Mr. Lindenthal mapped out to Mr. Barnes a connection of the Long Island System with the New Haven Railroad System in The Bronx on substantially the present line of the New York Connecting Railroad, thus establishing a continuous line through New York City territory.

As the other railroads in New Jersey declined to co-operate in the North River bridge project, this was finally abandoned by the Pennsylvania Railroad in favor of two single-track tunnels, the previous objections to tunnels for train service having by that time been overcome by the introduction of electric operation.

In 1900, the Pennsylvania Railroad acquired control of the Long Island Railroad, and a connection with this road across the East River, and with the New England roads, *via* the New York Connecting Railroad on the line originally mapped out, became, therefore, necessary adjuncts to the plan of entering Manhattan. As a result, the Pennsylvania Railroad, in conjunction with the New Haven Railroad, acquired the charter of the New York Connecting Railroad. Samuel Rea, M. Am. Soc. C. E., then Vice-President of the Pennsylvania Railroad, took charge of the entire project as President of the New York Connecting Railroad. The entire work was under his direction. A. J. County, Assoc. Am. Soc. C. E., was Assistant to the President, and Mr. Lindenthal was appointed Consulting Engineer and Bridge Architect to work out new plans for the East River Bridge and Approaches. The surveys and location in detail, as well as the first

borings for the foundations, were made by Joseph N. Crawford, Consulting Engineer of the Pennsylvania Railroad. Later, this work was turned over to Mr. Lindenthal, who appointed his own staff.

A franchise for the construction of the New York Connecting Railroad was granted by the Board of Rapid Transit Railroad Commissioners of New York (succeeded by the present Public Service Commission for the First District), in February, 1907. On account of the financial stringency in this year, the commencement of actual construction was again delayed until July, 1912, when ground was broken. It was estimated that construction would be completed by January 1st, 1916, but an injunction and unforeseen difficulties in building the foundations for the Hell Gate Bridge delayed the completion for more than a year. Traffic was established in March, 1917.

2.—General Description of East River Bridge Division.

Length.—For construction purposes, the New York Connecting Railroad, 8.96 miles long, has been separated into two divisions. The "East River Bridge Division", which extends from the junction with the New Haven Railroad in The Bronx across the East River to Stemler Street in Long Island City, a distance of 3.38 miles, was designed by Mr. Lindenthal, as Consulting Engineer and Bridge Architect, and executed by him as Chief Engineer. The "Southern Division", which comprises the remainder, or 5.58 miles, was in charge of Mr. A. C. Shand, Chief Engineer, and H. C. Booz, M. Am. Soc. C. E., Assistant Chief Engineer. This paper is devoted exclusively to the East River Bridge Division, which consists principally of bridges and viaducts, and has consumed about two-thirds of the entire cost. The plan and profile of this Division are shown on Plate XXIV.

Location.—From the junction of the New York Connecting Railroad with the New Haven Railroad, immediately north of the crossing over the Port Morris Branch of the New York and Harlem Railroad (New York Central Railroad), the two roads run parallel in a southwesterly direction on eight tracks to East 133d Street, the four New York Connecting Railroad tracks rising gradually above the New Haven tracks. South of 133d Street, the two westerly or passenger tracks of the Connecting Road cross over the two easterly or freight tracks of the New Haven Road. At this point the New Haven tracks

turn on a sharp curve toward the northwest, ending in the Harlem River Station and Yard, opposite 125th Street, Manhattan.

The New York Connecting Road continues in the southeasterly direction, crosses Bronx Kill on a double-leaf bascule bridge, and then skirts the east shore of Randalls Island on a viaduct 1 965 ft. long. Another arm of the East River, called "Little Hell Gate", between Randalls and Wards Islands, is bridged over by four skew deck truss spans, of an aggregate length of 1 154 ft.

On Wards Island the line turns to a southeasterly direction with a sweeping curve on a viaduct from 95 to 130 ft. high and 2 654 ft. long. The main channel of the East River is crossed at "Hell Gate" by an arch bridge having a single span of 1 017 ft. between abutment towers, a total height of 305 ft. and a clear head-room of 135 ft. above mean high water.

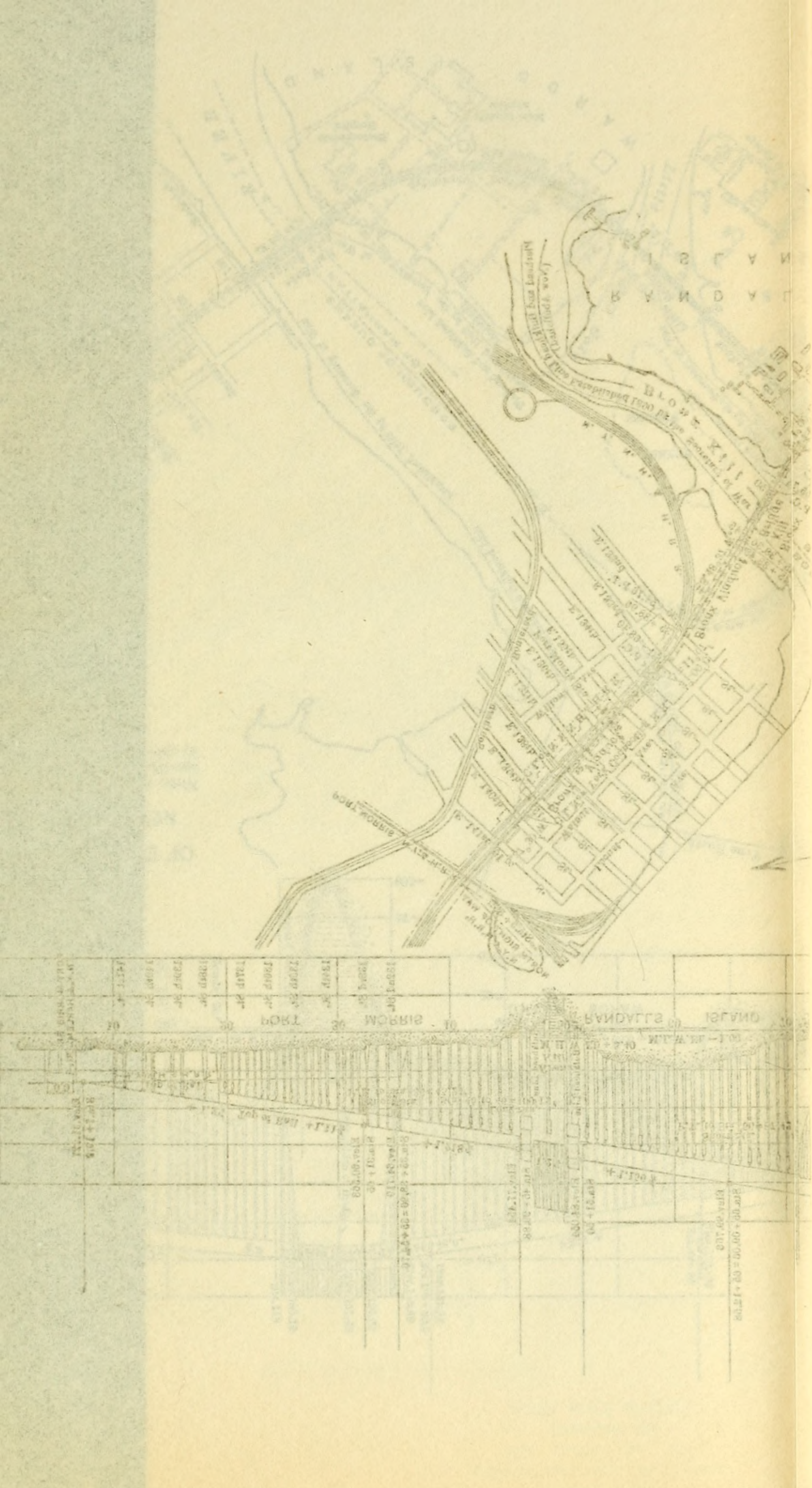
On the Long Island side of the East River, the road continues in a southeasterly direction, descending partly on a viaduct, and partly on an embankment, from a height of 110 ft. to 30 ft. above ground at Stemler Street, Long Island City, where the Southern Division begins.

The total length of the main line is 8.96 miles, of which 3.73 miles have four tracks and the remainder two tracks. A study of the map and of the river conditions as outlined herein will show at once that this route, crossing the East River at its narrowest point, was the natural one to follow. Any other route would have been much more expensive. A tunnel under the East River at this point would have been a very expensive and hazardous undertaking, and would have deprived the passengers of the picturesque and more comfortable ride over the elevated structure.

Curves and Grades.—The maximum curvature is 4° for a short distance at 133d Street, The Bronx. On Wards Island, the curvature is 3° 10′ for a length of 2 545 ft. All other curves are less than 1 degree. North of Hell Gate Bridge, or against south-bound traffic, the grade is nearly uniform, 1.2% (maximum 1.218%), compensated on curves. This grade was governed by the elevation of the tracks over the East River, where the clear height of 135 ft. above mean high water was prescribed by the War Department; this is the same clearance as that for the other East River bridges. South of Hell Gate Bridge, the tracks descend on a uniform grade of 0.72%, the ruling grade for north-bound traffic.

MAP AND PROFILE
OF EAST RIVER BRIDGE DIVISION, NEW YORK CONNECTING R.R.

Note- All elevations given refer to datum of the New York Connecting R.R., which is equal to mean low water at the Battery = Elev. 0.0



Quantities and Cost.—The New York Connecting Railroad will be one of the most expensive railroad lines ever built. Its total cost, inclusive of right of way, will be about $27 000 000. Of this, the East River Bridge Division consumed approximately $18 500 000, or $5 500 000 per mile of four-track line. This division required about 500 000 cu. yd. of granite and concrete masonry and 90 000 tons of steelwork.

3.—Development of Design of Hell Gate Bridge.

Introductory.—A great work of art evolves from an idea in the mind of its creator. It is brought on paper or into a more contemplative form and then changed and remodeled. Not until the plans have passed through changes and corrections, and have been submitted to an almost endless series of finishing touches, does the great work attain its perfection.

A great bridge in a great city, although primarily utilitarian in its purpose, should nevertheless be a work of art to which Science lends its aid. An elaborate stress sheet, worked out on a purely economic and scientific basis, does not make a great bridge. It is only with a broad sense for beauty and harmony, coupled with wide experience in the scientific and technical field, that a monumental bridge can be created. Fortunately, the Hell Gate Bridge was evolved under such conditions, and therefore may well be said to be one of the finest creations of engineering art of great size which this century has produced.

As mentioned heretofore, under "History of the New York Connecting Railroad", the first design for the Hell Gate Bridge was made in 1900 by the late Alfred P. Boller. It was a cantilever design with a central span of 840 ft., supported on braced steel towers. (Fig. 2.) The bridge was designed for two tracks only, for a light live load, approximately equivalent to Cooper's E-40, and for open tie flooring.

From 1904, when Mr. Lindenthal was appointed by the Pennsylvania Railroad to work out new plans on more modern lines, until 1912, when actual construction was started under his direction, the design of the Hell Gate Bridge received almost continuous and thorough study, involving the working out of complete designs of several types of bridges and a number of modifications of the type finally adopted.

River Conditions.—The East River is an estuary or tidal stream forming the eastern entrance to New York Harbor from the Atlantic Ocean by way of Long Island Sound. That part immediately east of its confluence with the Harlem River, between Wards Island and the Long Island shore, is the so-called "Hell Gate," which name is due, evidently, to the great dangers and trying conditions to which navigation was formerly exposed at this spot. On account of the presence of many protruding rocks and reefs, the sharp bend of the channel just below Hell Gate, and the rapid tidal currents, which even now attain velocities of 7 miles per hour, collisions and disasters in this locality were of frequent occurrence. These conditions have been greatly improved by the removal of the most dangerous reefs, some of which required the blasting away of enormous quantities of rock. The most famous operation was the removal of "Flood Rock", in 1885, when nearly 300 000 cu. yd. of rock were broken up in one blast.

The river at the Hell Gate Bridge is 850 ft. wide between shore lines and 700 ft. between bulkhead lines, as established by the War Department. Both shores fall rapidly to a greatest depth of 105 ft. below mean high water. The mean tide is 5.7 ft. At present, the channel has a minimum depth of 26 ft., but, eventually, it is to be dredged to a depth of 35 or 40 ft. so as to permit the safe passage of deep-draft vessels. The river traffic is quite considerable, consisting to a great extent of car-floats and tows which are difficult to control.

Types to be Considered.—The river conditions, as outlined, and the great height of the tracks above the water, prohibited physically and economically the construction of any permanent or even temporary support in the river channel, and called for a single river span of at least 850 ft., and of a type that could be erected without the use of falsework in the river. The only types which can be taken into consideration under such conditions are the cantilever or its relative, the continuous truss, the stiffened suspension bridge, and the arch (hingeless, two, or three-hinged).

Judging from prevailing tendencies, most engineers undoubtedly would have considered the cantilever type as best suited to the existing conditions, and it is not surprising, therefore, that the first design was of that type.

The suspension type for railroad bridges is considered by many engineers as unsuitable for spans of less than 2 000 ft., and, perhaps,

very few would have looked to the arch as a suitable type in this case, because it is usually associated with steep rocky shores which afford natural solid abutments and cheap anchorages for erection back-stays.

The span length, required clearance, character of soil, and other local conditions at Hell Gate are such that, in a broad sense, there is little if any difference in cost between the several types mentioned. Whatever differences in cost may be found by comparative designs are largely due to the individual judgment of the designer in the selection of the truss system, material, permissible unit stresses, foundations, and architectural features.

A real economy of the suspension type over the others comes in with spans greater than 850 ft.; an appreciable economy of the arch over the cantilever and suspension types would have been realized with more favorable configuration and character of ground, particularly if the required clearance had permitted the same span length for the arch as for the other types. There was the more reason, therefore, for selecting the type for the Hell Gate Bridge on broader than mere economic principles.

Mr. Lindenthal conceived the bridge as a monumental portal for the steamers which enter New York Harbor from Long Island Sound. He also realized that this bridge, forming a conspicuous object which can be seen from both shores of the river and from almost every elevated point of the city, and will be observed daily by thousands of passengers, should be an impressive structure. The arch, flanked by massive masonry towers, was most favorably adapted to that purpose.

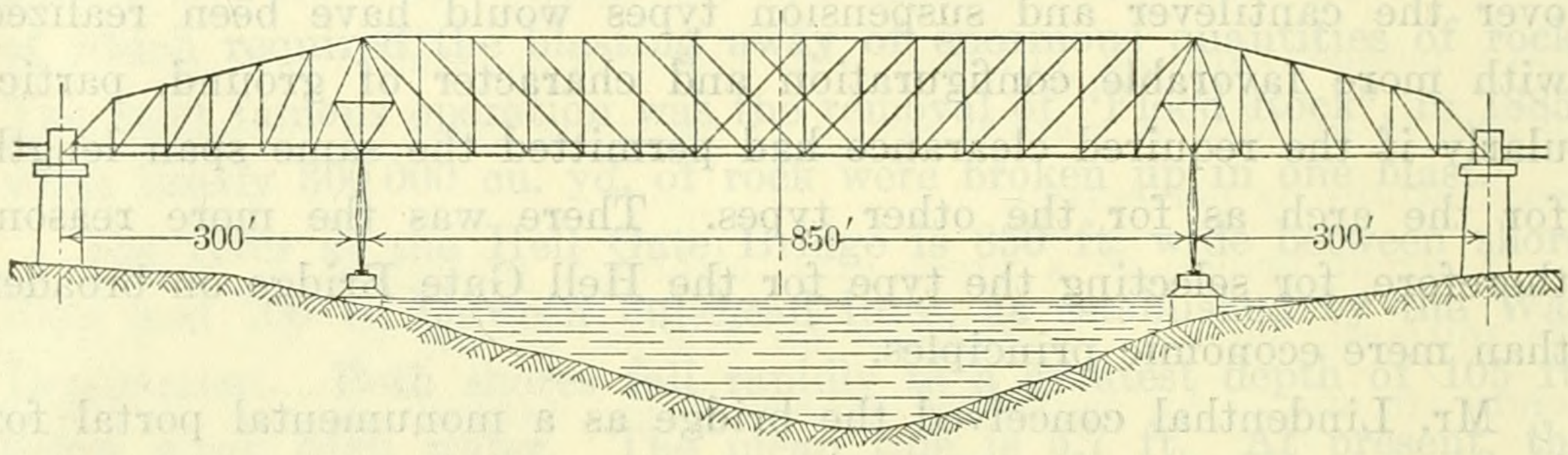
Comparative Designs of Cantilever, Continuous, and Suspension Types.

In 1904 Mr. Lindenthal made comparative designs of the three types comprising the stiffened suspension type with eye-bar chains (Fig. 3), the three-span continuous truss (Fig. 4), and the three-span cantilever (Fig. 5), all with a central span of 850 ft. and a total length varying from 1 450 to 1 550 ft. The designs were made both for two and four tracks, and open tie flooring. The live load assumed was the Pennsylvania Railroad standard loading of 1904, which is approximately equivalent to Cooper's E-50.

Nickel steel was assumed for the trusses in the four-track designs only, and ordinary carbon steel for the floor system, bracing, and towers

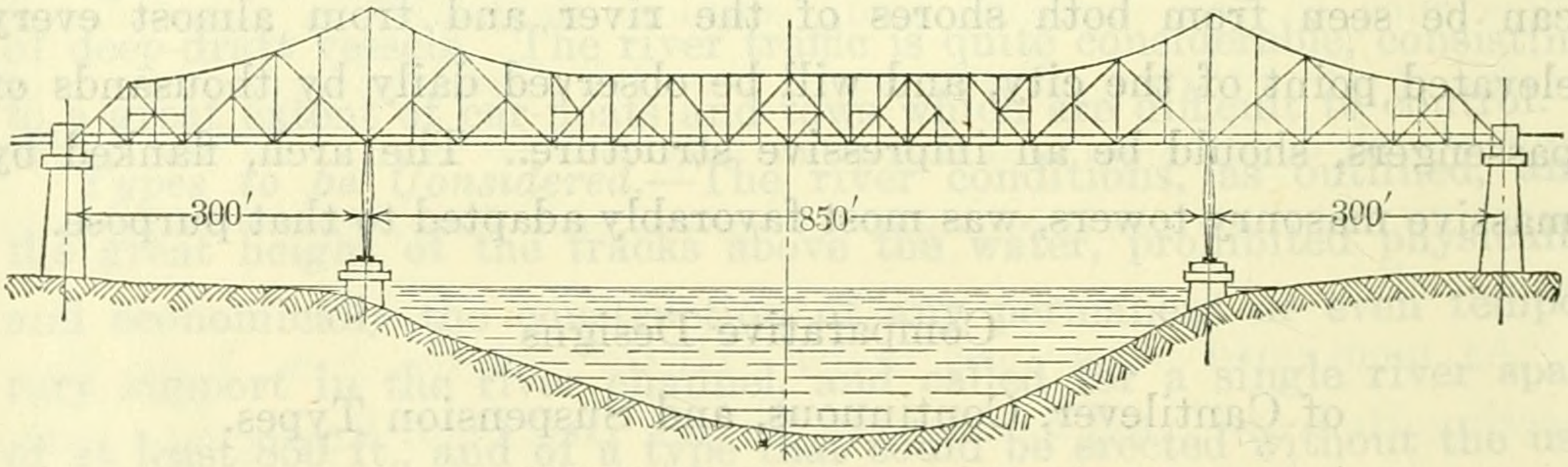
in the four-track designs and throughout for the two-track designs. The estimated weights of steelwork varied from 7 000 to 8 500 tons for double-track and from 11 200 to 14 200 tons for four tracks (if carbon steel had been assumed these weights would have been 14 000 and 17 000 tons, respectively), being least for the suspension bridge and greatest for the cantilever.

The saving in steelwork in the suspension design was partly offset by the greater cost of the anchorage piers, but, under assumed favorable soil conditions, the estimates showed a saving in cost in favor of the latter design. Under the more unfavorable soil conditions actually found on the Wards Island side, the total cost would be more nearly alike for the different designs.



CONTINUOUS TRUSS DESIGN (1904)

FIG. 4.



CANTILEVER DESIGN (1904)

FIG. 5.

Suspension Design.—The system of trusses adopted for the suspension design (Fig. 3) is that of an inverted three-hinged spandrel-braced arch suspended from hinged towers. The upper chord or chain of eye-bars follows very nearly the equilibrium polygon for dead load. The web members and lower chord form, with the main chain, the stiffening trusses.

Owing to the hinge at the center, the system is statically determinate and immune to settlements of the foundations. A similar system,

but without the center hinge, was used by Mr. Lindenthal in his design for the bridge over the St. Lawrence River, at Quebec, made in 1898, and for the Manhattan Bridge over the East River in New York City. This system has been fully described and discussed by him,* and, therefore, will not be further explained here.

The system used in Mr. Lindenthal's design for the Quebec Bridge, made in 1910,† although similar in form, differs from the previously mentioned system in that its two chords or chains form intersecting catenaries equi-distant from the line of equilibrium for dead load. This system is applicable to very long spans only, where the tension in the chains from dead load cannot be reversed by the live load.

In the design for the Hell Gate Bridge a hinge was provided at the center, as it was not known whether solid rock foundations could be obtained at reasonable depth. Wherever the piers rest on unyielding foundations, it is preferable to omit the hinge. In comparison with the bridge types shown in Figs. 4 and 5, the suspension type, with its graceful outlines, possesses unquestionably the advantage of more pleasing and monumental appearance. The anchorage masonry and the main towers give opportunity for architectural treatment. The appearance would be improved by a slight increase in the length of the end span.

In point of rigidity, the cantilever, in general, is superior to the suspension bridge. However, with the system of stiffening trusses selected for this design and the great sag of the chains (one-sixth of the span length), the deflections of the suspension bridge are reduced to about one and one-half times those of the cantilever. This greater deflection, however, is no serious disadvantage in a bridge of such size and capacity, in which the dead load is more than twice as much as the maximum assumed live load and more than four times the live load under average traffic conditions. Moreover, in a suspension bridge, the deflections from live load are free from that jerkiness which is the disagreeable characteristic of deflection in the cantilever bridge system.

Although unusual, the erection of the eye-bar chain suspension bridge, without falsework in the center span, is entirely feasible, and does not present more serious difficulties than that of a cantilever.

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* *Transactions,* Am. Soc. C. E., Vol. LV (December, 1905).

† *Engineering News,* November 23d, 1911.

It was particularly for its monumental character, together with its apparently smaller cost, that the stiffened suspension bridge was recommended by the designer for adoption in preference to the other types, and it would undoubtedly have been executed had construction been started at that time and had not certain conditions developed later which led to the study and final adoption of the arch type.

Continuous Truss.—In point of rigidity, the continuous truss (Fig. 4) is superior to the cantilever and suspension type. As regards appearance, however, this design is the least satisfactory. Both for economy and appearance the design could be improved by depressing the top chord so as to make the height at the middle of the center span about two-thirds of that over the main piers, and by using a single instead of the antiquated double web system. In any case, however, the appearance would be that of a utilitarian structure.

The common objection to a bridge of this type is that the trusses are statically indeterminate and may be affected seriously by settlements of the foundations. Where solid foundations can be reached at reasonable depth, this objection, however, is not valid, and this type may be found to be cheaper and more suitable than other types if esthetic considerations are of no importance. The erection of a continuous truss bridge, being similar to that of a cantilever, does not present unusual difficulties.

Cantilever Design.—The cantilever design (Fig. 5) is similar to that shown in Fig. 2, except that slender hinged towers take the place of the unsightly braced towers of the latter design. Although superior in appearance to many existing cantilever bridges, both designs produce the effect of utilitarian structures inherent to cantilever bridges. There is no opportunity for monumental towers or abutments at the ends, because the absence of a large horizontal thrust or pull does not justify a large mass of masonry at those points, as in the case of an arch or a suspension bridge.

The hinged towers in the design (Fig. 5) are a distinct improvement over the braced towers, in that they eliminate dangerous secondary stresses which are caused in the case of the latter. A further decided improvement as regards appearance, rigidity, and permanence, and, therefore, fully justifying the slightly greater expense, would be the substitution of solid masonry piers for the steel towers below the bottom chord of the trusses. Such piers have the further advantage

that the longitudinal forces from braking and traction can be transmitted through them to the foundations on the shortest way and without increasing appreciably the size of the piers. In the design (Fig. 5) the anchor arms have the proper length, whereas in the design (Fig. 2), they are too long, both as regards economy and as the ends are subject to reversal of reaction from live load which causes objectionable "hammering."

Conditions Which Led to Investigation of Arch Type.

In 1905 the line of the railroad on Wards Island was moved farther north, in order to keep it a greater distance from the State Hospital buildings there. This resulted in a sharp 3° 10′ curve extending at both ends almost to the shore line of the island. A long shore span such as would have been required for a cantilever or suspension bridge would have necessitated a still sharper curve, which was not desirable on account of the heavy grade.

Moreover, in the case of a cantilever or suspension bridge, it would have been necessary, in order to keep the span length down to 850 ft., to place the main piers close to the shore lines. On the Long Island side this would have necessitated, at considerable additional expense, the shifting landward of the boulevard which runs along the river shore, and would undoubtedly have been objected to by the City authorities.

These conditions induced Mr. Lindenthal to investigate a single-span arch bridge design. For an arch, the location of the Long Island abutment on the land side of the boulevard was the proper one, the span length of about 1 000 ft. being determined by the clear height of 135 ft. which was required by the Government for navigation and had to be maintained for the full width between the established bulkhead lines. By that time, also, preliminary borings had been completed, giving more accurate information as to the soil conditions. These borings indicated that solid foundations, which are necessary to resist the tremendous thrust of an arch of such great span, could be had at reasonable depth.

Comparison of Arch with Cantilever and Suspension Types.

Comparison in cost with the suspension and cantilever designs indicated a saving in favor of the arch. The estimated weight of steelwork of the arch, using carbon steel only, was about 13 000 tons, as

compared with 14 000 and 17 000 tons for the suspension and cantilever designs, respectively. As actually built, the arch is probably no cheaper than a cantilever or suspension bridge under the same conditions, because the saving in steelwork was practically offset by the greater cost of the more elaborate towers adopted in the final design, and by the greater cost of foundations, due to the unfavorable soil conditions encountered on the Wards Island side.

Had the arch abutments been designed to satisfy only the static requirements, a substantial saving in favor of the arch would have resulted, and, of course, still more so, if it had been possible to give the arch a span of only 850 ft. These conditions, together with the considerations for appearance, finally led to the adoption of the arch type.

The two masonry towers placed at each end of the bridge are an architectural necessity. Without them, the arch would lose much of its monumental character and be reduced to a utilitarian structure similar to the cantilever. The towers, however, have their static function also. With their great weight, they steepen the resultant arch thrust and thereby limit the size of the deep foundations to a minimum. They also facilitated and cheapened the erection of the arch to a considerable extent. Aside from the considerations which favored it in this case, the arch type, as finally adopted, possesses over the suspension and cantilever types the advantages of greater rigidity, its vertical deflection under live load being only about two-thirds of that of the cantilever. The deflections, which are maximum at the quarter-points of the arch span, are not greater than those at the center of a simple span, which is well known as the stiffest type of bridge.

The secondary stresses, also, are small in an arch of the adopted type, and very much smaller than in most cantilever bridges, although this depends very much on the truss system and the method of erection.

The economy of an arch depends largely on the method of erection. An arch is erected either on falsework (which was out of question in this case) or by the cantilever method, being, in the latter case, held by the temporary back-stays. These back-stays may require considerable extra material, and may thus impair the economy of the arch, unless the configuration of the ground is such that they can be made of short ties and anchored cheaply in solid rock.

This was not possible in the case of the Hell Gate Bridge. However, the adjoining viaduct spans, the floor system, suspenders, and

other parts of the arch bridge proper afforded in this case ample material to make up the temporary back-stays and anchorages, so that only little extra material had to be used. This cheapened the erection very considerably. The erection on the cantilever principle presents no more serious problems than that of a cantilever proper; on the contrary, the final erection adjustments are simpler in the case of the arch.

Two Comparative Designs of Arch Type.

Two designs for an arch bridge were made by Mr. Lindenthal in 1905. Fig. 6 represents the two-hinged crescent arch, as used, for instance, for the railroad bridges over the Garabit Valley, in France, and over the Douro River, in Portugal, with spans of 541 and 525 ft., respectively. Fig. 7 represents the two-hinged, spandrel-braced arch, similar to the bridges over the Rhine at Düsseldorf and Bonn, which have spans of 595 and 614 ft., respectively. The two designs were made for the same loads and specifications as the cantilever or suspension designs, but ordinary carbon steel was assumed throughout, because, at prevailing prices, nickel steel ($40 per ton higher than carbon steel) did not seem to offer any saving. The estimated weight of steelwork was slightly in favor of the crescent arch, but the spandrel-braced arch offered greater advantages for the cantilever erection. Although both designs are pleasing in appearance, the spandrel arch, owing to its height increasing from the center toward the ends, is more expressive of rigidity than the crescent arch, the ends of which appear to be unnaturally slim in comparison with the great height at the center.

The three-hinged arch was not considered. It is not cheaper than the two-hinged arch, and is not as rigid. The fact that the two-hinged arch is statically indeterminate is frequently cited as an objection to that type. This is not justified. There is no more uncertainty in the stress distribution in that type than in a so-called statically determinate structure with riveted connections, and even pin connections do not remove the uncertainty, as is now well recognized. Moreover, if desired, it is always possible, as has been done in the case of the Hell Gate Bridge, to erect the two-hinged arch so that it is statically determinate for dead load. This, however, is a convenience in erection rather than an advantage as regards stress action. Of course, the above characteristic of the two-hinged arch is a serious objection where no solid foundations can be obtained, because of the uncertainty of

stresses which may be produced by a spreading of the foundations. A two-hinged arch requires unyielding abutments.

Modifications in Adopted Arch Design.

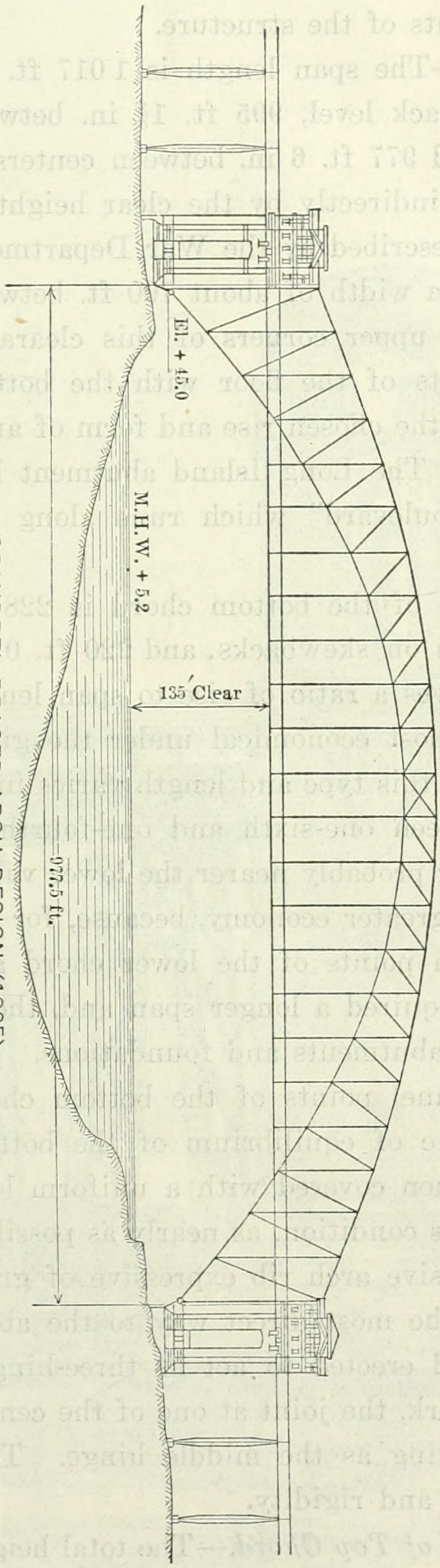
The spandrel-braced arch type was finally adopted. Before the design was worked out in detail, the top chord was changed by giving it a slight reversal of curve toward the ends, in order to provide for a stout portal and wind bracing for the top chords, and also improve the silhouette of the arch. The towers were increased in height, and their architectural features modified.

In 1907, the design of the tower shown in Fig. 8 was submitted to the Municipal Art Commission of New York. This Commission, although not objecting to the design as a whole, disapproved of the decorative features of the towers and their bases. The towers then received several further modifications until, in 1912, shortly before construction started, the design illustrated in Fig. 9 was finally adopted. The tower in this design represents a great improvement over that shown in Fig. 8, being more impressive in outline and simpler in architectural ornament and, therefore, more in harmony with the simple, imposing forms and lines of the bridge proper.

It should also be mentioned that, in 1910, the steel superstructure was re-designed to carry Cooper's E-60 loading and a solid ballasted floor. This loading had already been adopted by a number of railroads, including the New York, New Haven and Hartford Railroad, which is to use the bridge. High-carbon steel was adopted in place of the ordinary structural steel. This last design, moreover, was based on special rules of design prepared by Mr. Lindenthal, as abstracted hereinafter. These resulted in a heavier floor system, a stronger connection, and heavier details throughout. Four lines of stringers and floor-beam brackets, strong enough to carry trolley traffic, were provided outside of the trusses. These modifications increased the total weight of the steelwork to 18 900 tons.

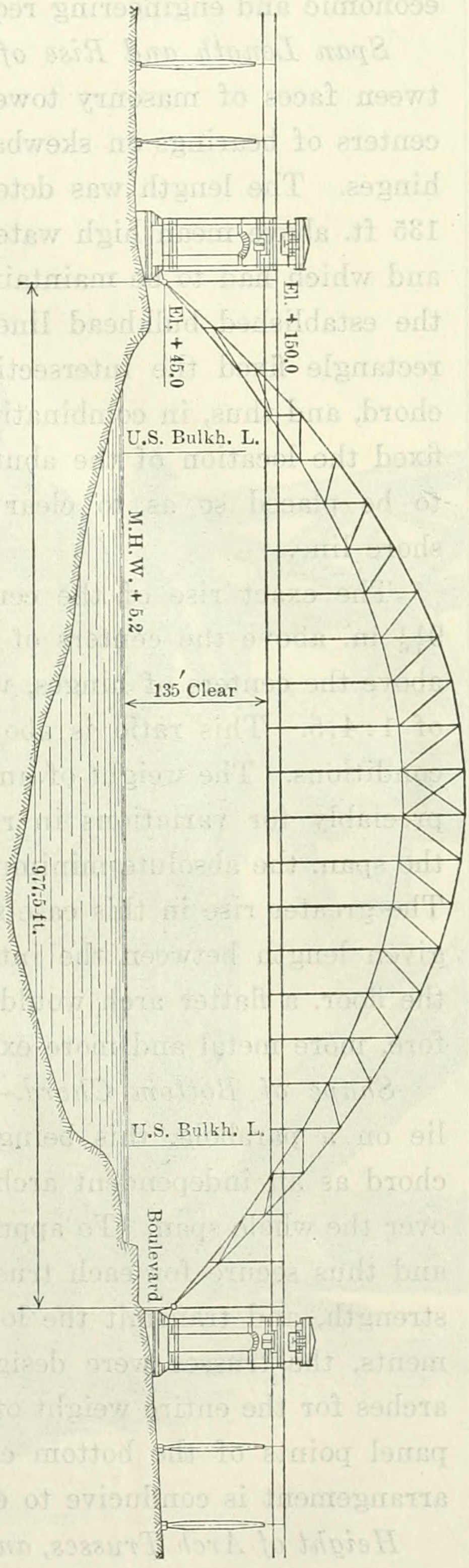
4.—General Arrangement, Proportions, and Cost of Arch Bridge.

The Hell Gate Bridge, as built (Plate XXV), is a two-hinged spandrel-braced arch, carrying four tracks. Its general proportions were dictated partly by local conditions and partly by the requirements for economy and rigidity. The artistic outlines of the steel



SPANDREL BRACED ARCH DESIGN (1905)

FIG. 7.



CRESCENT ARCH DESIGN (1905)

FIG. 6.

superstructure are the result of the proper interpretation of the economic and engineering requirements of the structure.

Span Length and Rise of Arch.—The span length is 1 017 ft. between faces of masonry towers at track level, 995 ft. $1\frac{3}{8}$ in. between centers of bearings on skewbacks, and 977 ft. 6 in. between centers of hinges. The length was determined indirectly by the clear height of 135 ft. above mean high water, as prescribed by the War Department, and which had to be maintained for a width of about 700 ft. between the established bulkhead lines. The upper corners of this clearance rectangle fixed the intersection points of the floor with the bottom chord, and thus, in combination with the chosen rise and form of arch, fixed the location of the abutments. The Long Island abutment had to be placed so as to clear the "boulevard" which runs along the shore line.

The exact rise of the center line of the bottom chord is 228 ft. $9\frac{11}{16}$ in. above the centers of bearings on skewbacks, and 220 ft. 0 in. above the centers of hinges, which gives a ratio of rise to span length of 1 : 4.5. This ratio is about the most economical under the given conditions. The weight of an arch of this type and length varies inappreciably for variations in rise between one-sixth and one-fourth of the span, the absolute minimum being probably nearer the lower value. The greater rise in this case secured greater economy, because, for the given length between the intersection points of the lower chord and the floor, a flatter arch would have required a longer span and, therefore, more metal and more expensive abutments and foundations.

Shape of Bottom Chord.—The panel points of the bottom chord lie on a parabola, this being the line of equilibrium of the bottom chord as an independent arch rib, when covered with a uniform load over the whole span. To approach this condition, as nearly as possible, and thus secure for each truss a massive arch rib expressive of great strength, and transmit the loads in the most direct way to the abutments, the trusses were designed and erected to act as three-hinged arches for the entire weight of steelwork, the joint at one of the center panel points of the bottom chord acting as the middle hinge. This arrangement is conducive to economy and rigidity.

Height of Arch Trusses, and Shape of Top Chord.—The total height of the arch trusses from center of hinge to center of top chord at the middle of the span is 260 ft. $2\frac{5}{8}$ in., and the total height of the steel

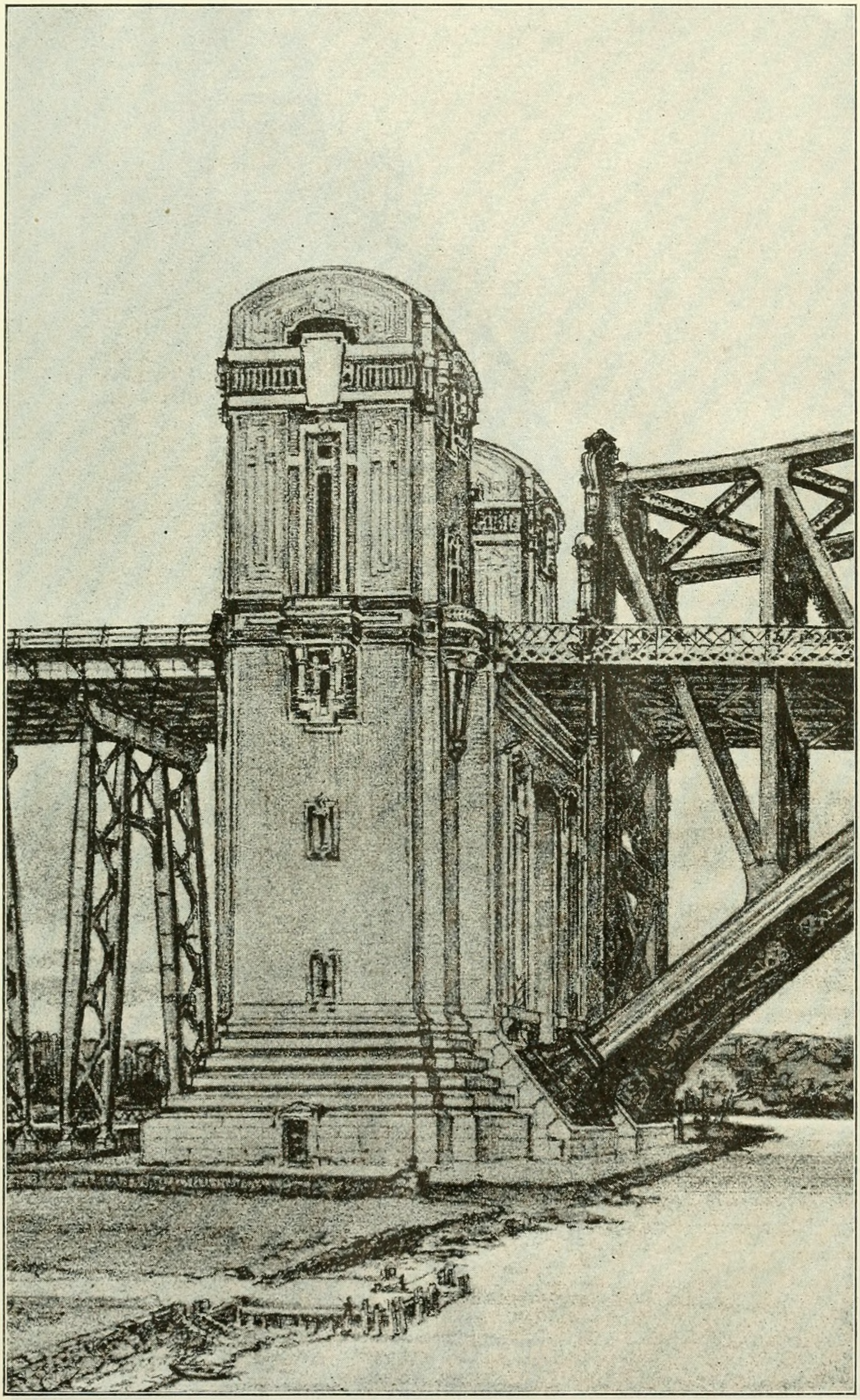
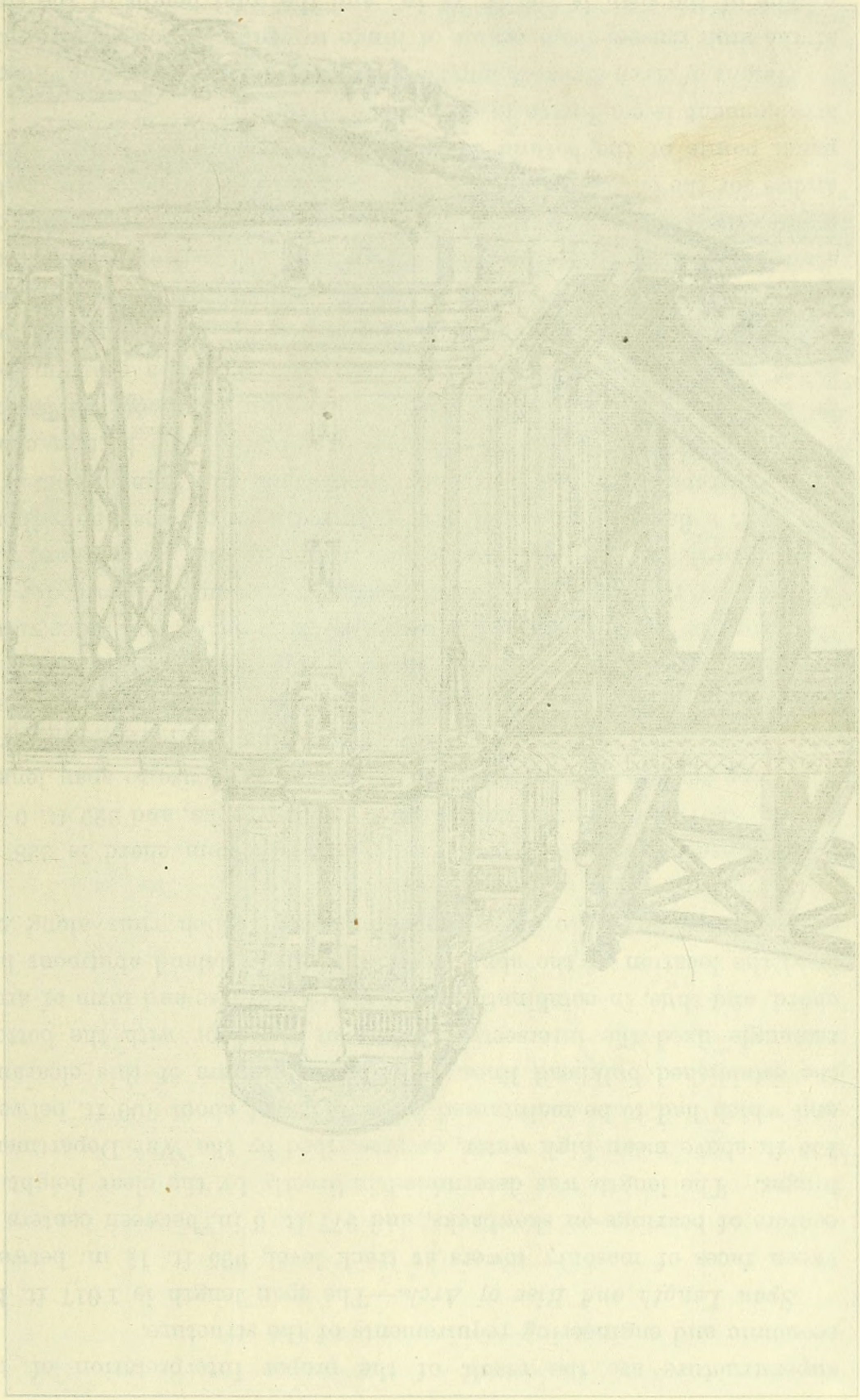


FIG. 8.—PERSPECTIVE VIEW OF ARCH DESIGN (1906), HELL GATE BRIDGE.



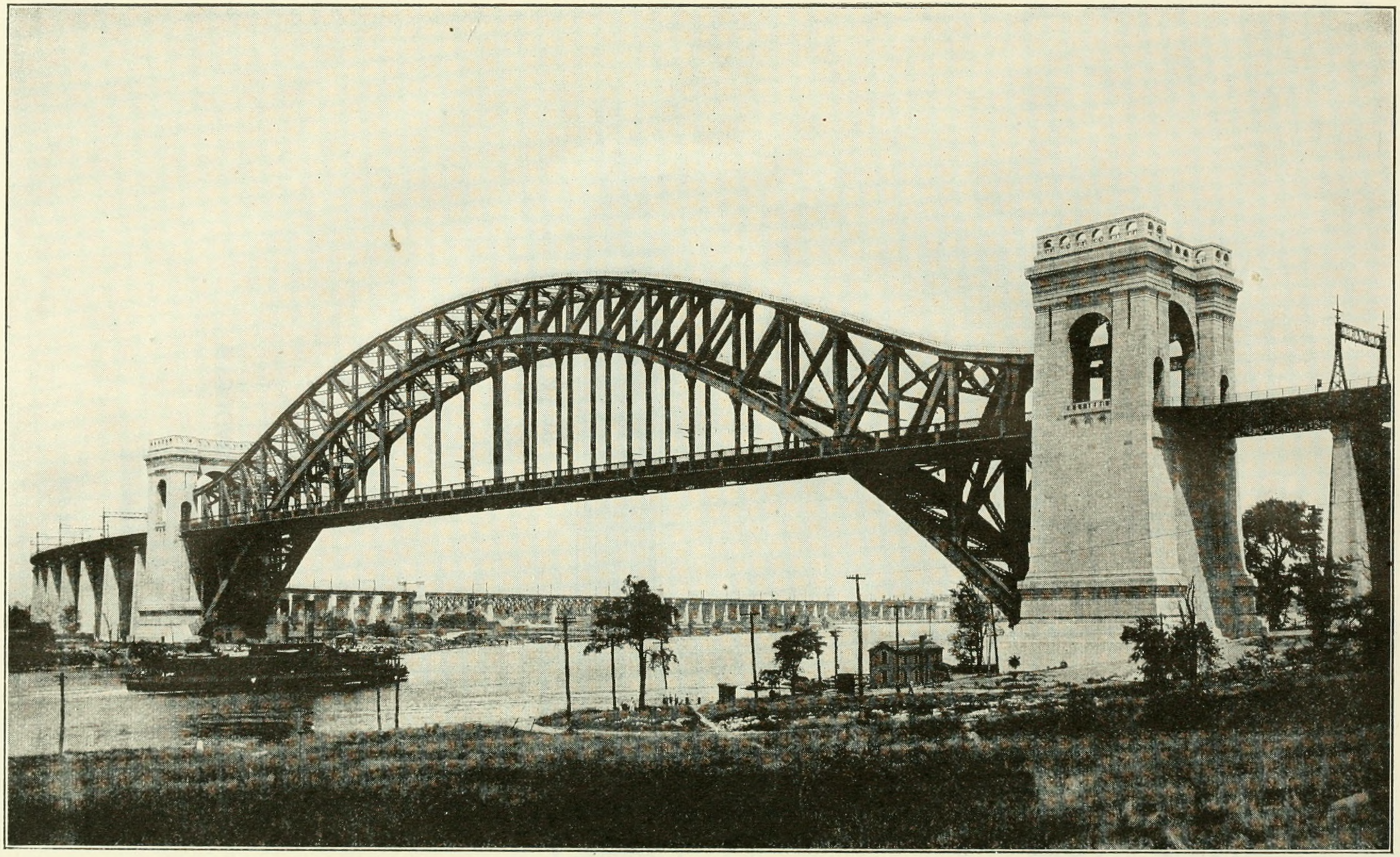
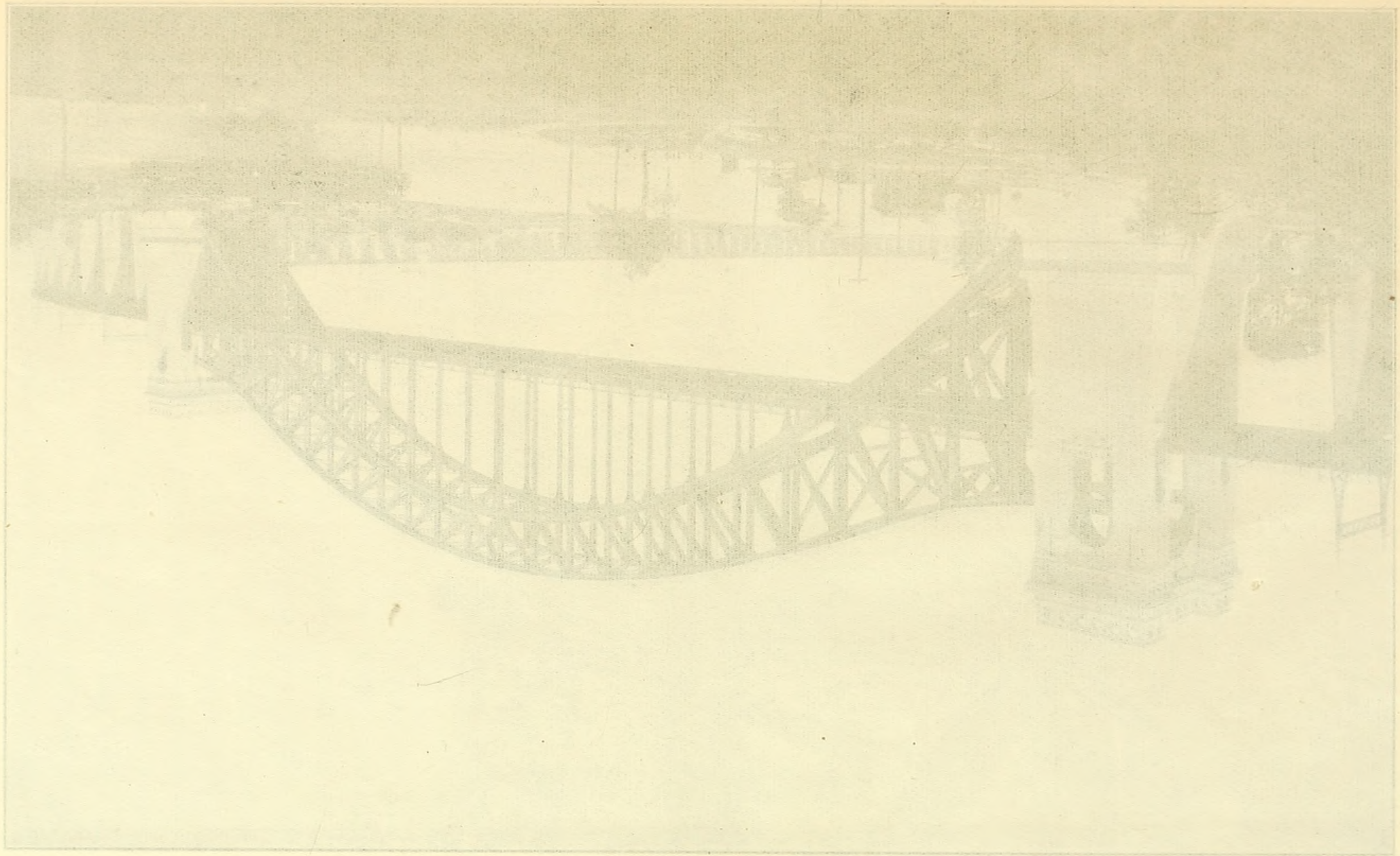
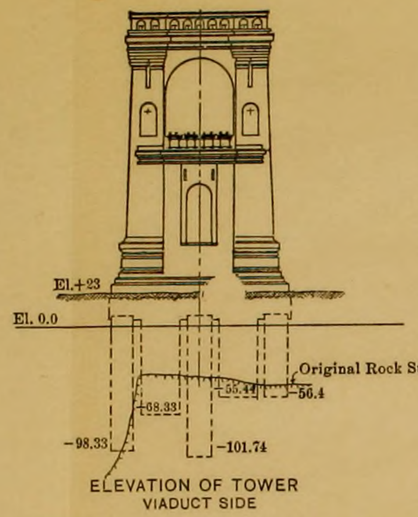


FIG. 9.—PERSPECTIVE VIEW OF FINAL DESIGN (1912), HELL GATE BRIDGE.

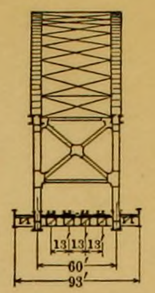
FIG. 3.—PERSPECTIVE VIEW OF FINAL DESIGN (1915), HELL GATE BRIDGE.



GENERAL PLAN OF HELL GATE BRIDGE AND PORTIONS OF WARDS AND LONG ISLAND VIADUCTS

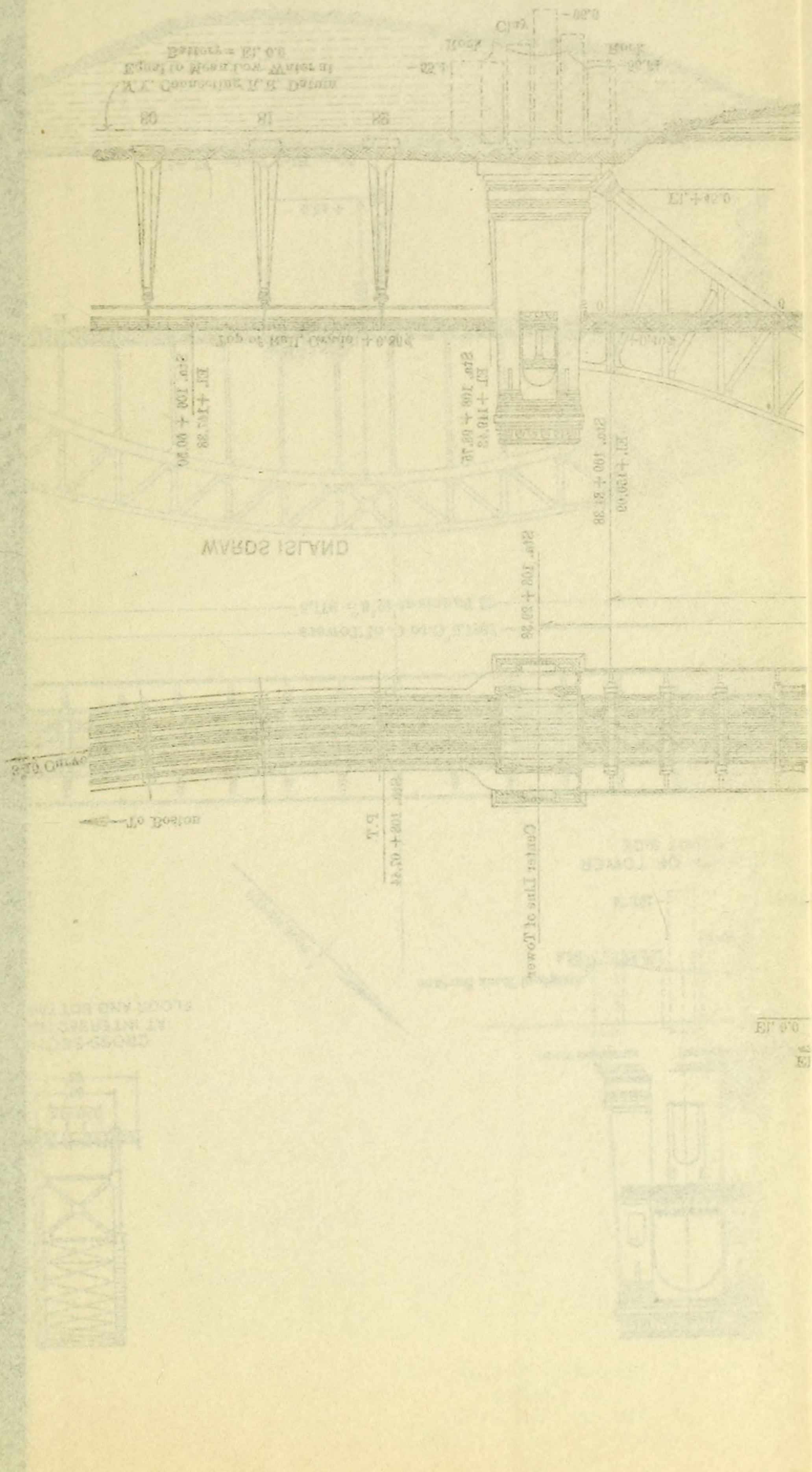


ELEVATION OF TOWER
VIADUCT SIDE



CROSS-SECTION
AT INTERSECTION OF
FLOOR AND BOTTOM CHORD





superstructure above mean high water is 305 ft. The height of the trusses at the quarter points of the span, where the greatest deflections occur, was chosen at 60 ft., or slightly more than one-fourth of the rise of the bottom chord. This proportion of height to rise insures ample rigidity. Besides, it is sufficient to keep the maximum live-load stresses in the bottom chord approximately within the stresses from live load covering the whole span, and, therefore, no extra material is required in the bottom chord for the otherwise large stresses from partial live load.

On the other hand, for a greater proportion of height to rise than that chosen, the aggregate weight of the top chord and web members increases, and the weight of the bottom chord remains nearly constant.

The height of the trusses was decreased toward the center to 40 ft. $2\frac{5}{8}$ in., or approximately one-twenty-fourth of the span length, so as to reduce the temperature stresses, which are greatest at the center. The height of 140 ft. at the ends of the trusses was determined by the necessity for rigid portals between the end posts above the track floor. These assumptions for height resulted in the slightly reversed curve of the top chord, which produces a very pleasing sky line.

Width.—The width of 60 ft. between centers of trusses resulted from the required clear width of 53 ft. for the four tracks (the distance between centers of tracks being 13 ft.) and an allowance of 7 ft. for the width of the bottom chord at its intersection with the floor. This width, being one-sixteenth of the span length, was sufficient for lateral stability and rigidity, and therefore it was not necessary to spread the trusses or to place them in inclined planes. To obtain great lateral rigidity of the suspended floor and an economical floor wind truss, however, the latter is made 93 ft. wide, its chords being placed $16\frac{1}{2}$ ft. outside of the main trusses and carried by cantilever extensions of the floor-beams.

Web System and Panel Length.—The web system consists of a single line of verticals and diagonals, the latter falling toward the center of the span, as commonly used in arch bridges of this type. The system is simple, economical, and free from large secondary stresses. There are twenty-three equal panels, each $42\frac{1}{2}$ ft. long. For a two-hinged arch, an odd number of panels produces a better appearance than an even number. The panel length was chosen with a view to obtain the most economical floor and truss web system. The latter was secured by

making the average inclination of the diagonals about 45° with the horizontal.

General Arrangement of Arch Bracing.—The transverse bracing between the two trusses comprises a lateral system along the top chords, a lateral system along the bottom chords, and sway-frames and portals in the planes of the first five verticals at each end of the span. Sway-frames between the other truss verticals and between the floor suspenders have been omitted purposely, as they are not needed and would have to be very heavy to resist the stresses from unequal deflection of the two trusses under one-sided load. The top lateral truss is assumed to transmit its reactions to the portals between the end posts and through these and the sway-frames below the floor to the bearings. The wind forces which act along the bottom chords are transmitted through the bottom lateral truss directly to the arch bearings. Owing to the polygonal shape of the chords, components of the lateral stresses are transmitted into the main trusses at each panel point, which, although small, had to be considered in proportioning the truss members. At the intersections of the floor with the bottom chord, the lateral bracing between these chords had to be interrupted to provide the necessary head-room above the floor. Stiff portals were substituted for the laterals at these points, and the chords were proportioned for the additional bending stresses.

General Arrangement of Floor System.—The floor system (Plates XXVIII and XXXII) comprises the following parts:

1.—A floor-beam at each panel point, rigidly framed into the trusses at the first four verticals at each end of the span and hung from the trusses by suspenders in the middle portion of the span.

2.—Eight lines of railroad stringers, 6 ft. 6 in. apart, framed into the floor-beams and braced together in pairs for each track by top and bottom laterals and sway-frames. Each pair carries a concrete trough which supports the ballasted track.

3.—Four lines of sidewalk stringers, one pair outside of each track, framed into cantilever extensions of the floor-beams. These stringers support only a light sidewalk, but are made strong enough to carry the trolley line which was contemplated.

4.—Two lines of lattice girders, one on each side of the floor, placed 16½ ft. outside of the center line of the main truss, and carried at the end of the floor-beam extensions. The function of these girders is to

screen the floor system and thus secure a more uniform and neat appearance. At the same time, these girders act as railings, and their bottom chords form the chords of the floor lateral truss.

5.—A floor lateral truss to resist the wind and lateral forces which act on the trains and floor.

6.—Two "braking girders", one at each intersection of the floor with the main trusses. These girders transmit the longitudinal forces from braking and traction from the stringers to the main trusses, and thus eliminate serious horizontal bending of the floor-beams.

Provision for Expansion of Floor.—The floor had to have at least one expansion joint at or between its intersections with the bottom chord (Panel Points 6, Plates XXV and XXVIII), so as not to be strained by temperature changes or deformation of the arch trusses.

The expansion of the floor for a change in temperature of $\pm$ 72° Fahr. is $\pm$ 4.1 in., but this is partly offset by an increase of $\pm$ 1.6 in. in the distance between Points 6, due to the temperature deformation of the arch truss, leaving a movement from the normal position, of $\pm 4.1 \mp 1.6 = \pm 2.5$ in., which had to be provided for at the expansion joint. The effect of a maximum live load covering the entire span is to open the joint by 0.1 in., which is negligible.

In deciding on the location of the expansion joint, the following conditions had to be taken into consideration:

First.—To secure the greatest lateral rigidity, the floor lateral system should be such as to cause the least lateral deflections.

Second.—To avoid large stresses in the stringers and their connections from the longitudinal force, the distance between the expansion joint and the braking girder should be as small as possible.

Third.—The floor suspenders should be subject to the least possible bending in the plane of the truss from the expansion of the floor.

To meet all these conditions, the expansion joint was placed at Panel Point 12, six panels from the Wards Island end. At the corresponding Point 12 on the Long Island side, the floor laterals are rigidly connected at the center of the floor-beam, but the wind chords are cut, so as to secure hinge action of the floor-lateral truss. The latter, therefore, forms a three-span cantilever truss with a suspended span between Points 12, Cantilever Arms 12—6, and Anchor Arms 6—0.

The suspended span delivers its reactions to the cantilever arms by a fixed connection of the diagonals at the center of Floor-beam 12 (Long Island side) and a sliding bearing at the center of Floor-beam 12 (Wards Island side). The reactions at Points 6 are transmitted to the bottom chord lateral system and, through this, to the arch bearings. The reactions of the floor-lateral truss at the ends are transmitted to the sway-frames between the end posts and, through these, to the bearings.

The longitudinal force from Part 0—12 (Wards Island side) is transmitted to the braking girder at 6 (Wards Island side), and the force from Part 12 (Wards Island side) to 0 (Long Island side) is transmitted to the braking girder at 6 (Long Island side). The stringer connections to the floor-beams are made strong enough to resist safely the longitudinal force in addition to the vertical shear.

Cost.—The Hell Gate Bridge contains approximately 110 000 cu. yd. of masonry in the towers and foundations and 19 400 tons of steel in the steel superstructure. (Detailed quantities are given under the respective headings.)

The cost of construction is approximately as follows:

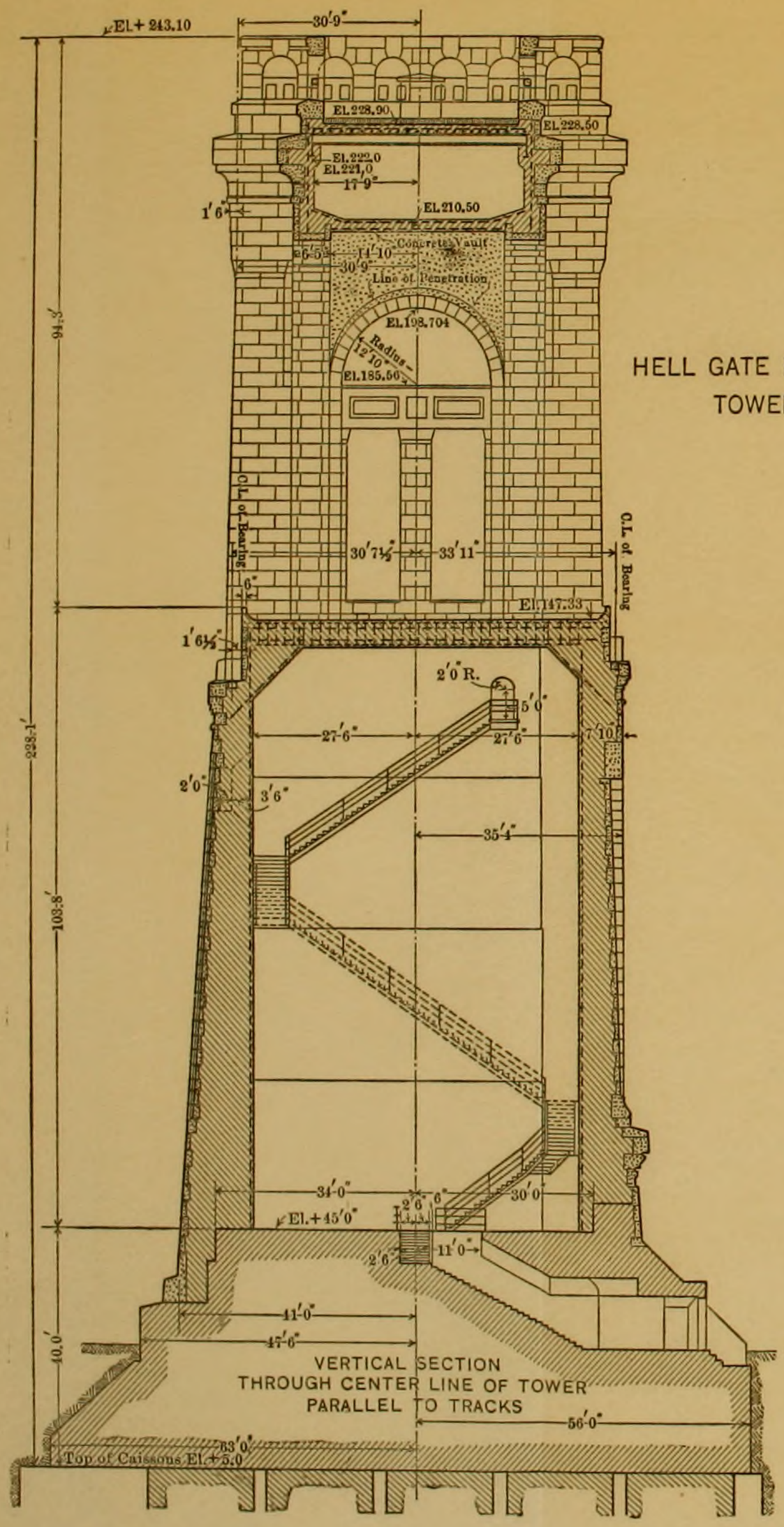
Towers and foundations	$1 700 000
Steel superstructure	1 900 000
Concrete flooring and tracks	100 000
Total	$3 700 000

5.—Masonry Towers and Foundations.

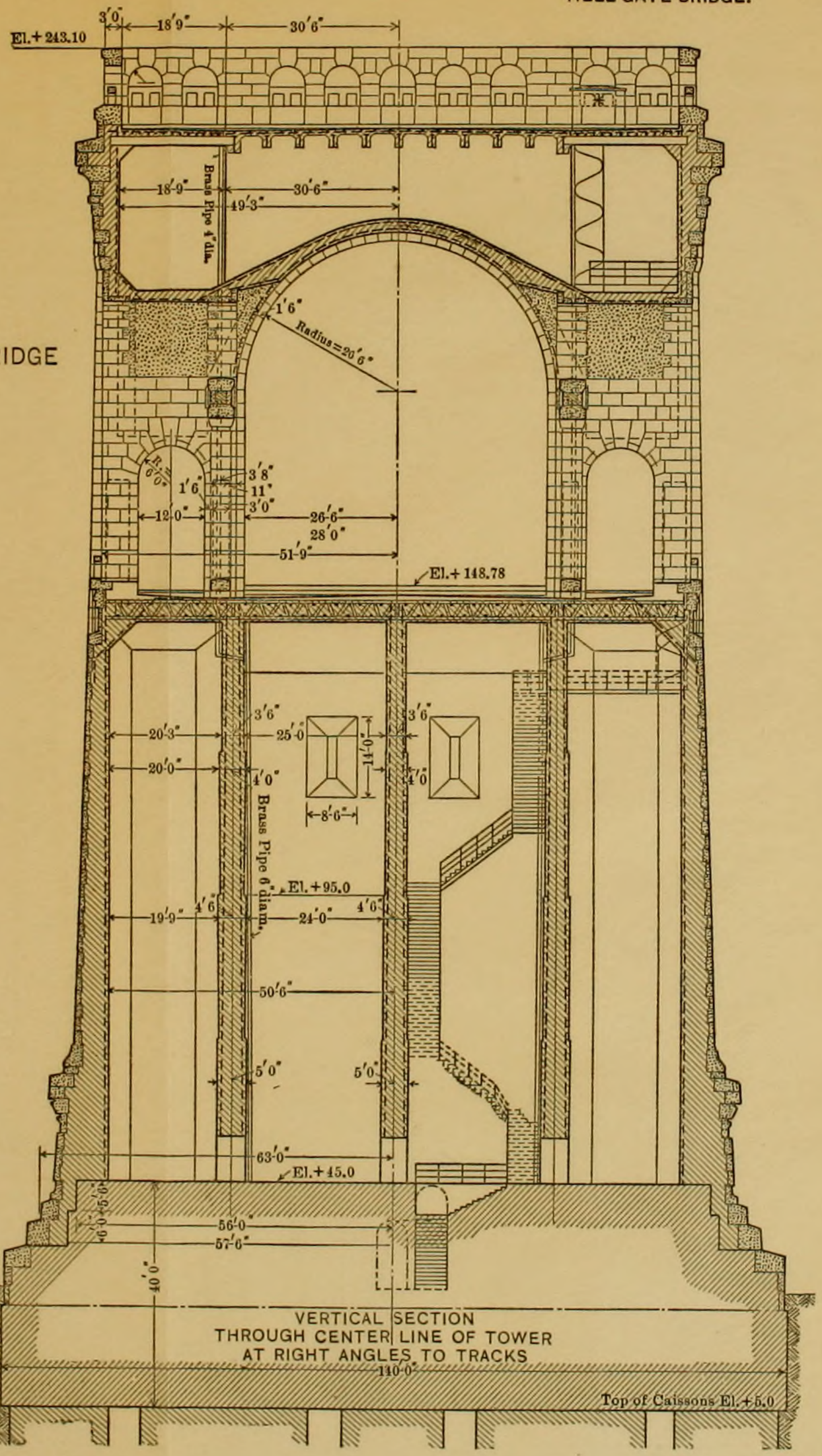
The massive masonry towers which flank the steel arch greatly enhance the appearance of the bridge and give it its monumental character. They also give expression to the solidity of the abutments to resist the great thrust of the arch. Without the towers, the statically trained eye would want that expression of stability, because of the comparative flatness of the shores.

This static requirement, however, is not merely an apparent one. Preliminary wash-borings indicated that the foundations had to go to considerable depth, at least on the Wards Island side. There having been doubt as to the reliability of the wash-borings, the depth of rock was established later by core-borings at from 55 to 140 ft. below mean high water line. To restrict the size of the foundation to a minimum,

HELL GATE BRIDGE
TOWER



VERTICAL SECTION
THROUGH CENTER LINE OF TOWER
PARALLEL TO TRACKS



VERTICAL SECTION
THROUGH CENTER LINE OF TOWER
AT RIGHT ANGLES TO TRACKS

PLATE XXVI.
AM. SOC. CIV. ENGRS.
LXXXII, No. 1417.
AMMANN ON
L GATE BRIDGE.

it was necessary to provide above the ground a mass of masonry, the weight of which, combined with the inclined reaction of the arch, would give a steep resultant, passing well within the middle third of the foundation area, so that the edge pressure could be kept within permissible limits.

To be expressive of their purpose, the towers were designed architecturally as massive masonry blocks with simple outlines and plain structural ornamentation (Fig. 10). In working out the architectural form and details of the towers, Mr. Lindenthal had the valuable assistance and advice of Mr. Henry Hornbostel as Consulting Architect.

Towers Above Foundations.—Plate XXVI shows the type of construction of the towers above the foundations. The dimensions of the towers are 103 by 139 ft. at the ground level and diminish along parabolic lines to 61 by 105 ft. at the top. The total height above ground is 220 ft., and the extreme height above bottom of foundation is 345 ft. Each tower has a solid base, which acts as an abutment for the arch bearings and distributes the pressure over the foundations. On the Long Island side, the base, with the foundation course, forms a monolithic slab, 140 by 104 ft., with an average thickness of 49 ft., and rests on gneiss bed-rock, which was encountered at from 15 to 38 ft. below the surface, and was reached by open excavation. The maximum foundation pressure is $8\frac{1}{2}$ tons per sq. ft. On the Wards Island side, the base is 140 by 119 ft. and 40 ft. thick, and rests on the caisson foundation described subsequently.

Above this base, and up to the track floor, the towers are of hollow cellular construction, consisting of the four exterior walls and three interior walls parallel to the tracks. Above the track floor, the transverse walls are pierced by a main arch over the four tracks, and two smaller side arches over the footwalks. The longitudinal walls have also an arch opening for architectural reasons. The towers are topped by a flat roof surrounded by an ornamental balustrade. Stairs lead from an entrance at the ground level through the base and interior vaults to the track floor and roof, and also to the ends of the top chords of the steel arch. The towers are of concrete with granite facing. The concrete is well reinforced with vertical and horizontal steel rods in order to prevent temperature and shrinkage cracks. The track floor and roof are heavily reinforced with steel girders. The Snare and

Triest Company was the contractor for the towers above the bases, and the Ryan Construction Corporation for the bases.

Wards Island Tower Foundation.—On the Wards Island side, the tower base rests on twenty-one concrete caissons, all sunk by the pneumatic process to depths varying from 37 to 107 ft. below mean high water, which is 20 ft. below the ground surface. (Plate XXVII.) The caissons are arranged in five rows parallel to the axis of the bridge. Each outer row and the middle one consists of five cylindrical caissons, 18 ft. in diameter, which are calculated to carry only vertical pressure. Each of the two other rows consists of three rectangular caissons, 30 by 41 ft., which are interlocked by concrete keys extending nearly the full depth of the caissons. These two rows of caissons thus form two rectangular blocks, 30 by 125 ft., which are calculated to resist entirely the horizontal pressure from the arch. They exert a maximum edge pressure on the rock foundation of 20 tons per sq. ft., if skin friction and buoyancy are neglected. The space for the keys was excavated and filled with concrete, partly with and partly without the use of air pressure, after the caissons had been sunk to their final depth.

The dissection of the foundation into twenty-one individual caissons was advisable on account of the large size of the tower base and the greatly varying depth to solid rock. The formation of the bed-rock below Wards Island is very peculiar. One of the lines of cleavage between the dolomitic limestone formation and the gneiss rock which run parallel with the East River, appears to pass right under the Wards Island tower. About 45% of the foundation on the river side is on limestone and about 25% on the land side is on gneiss. Between these two rock formations there is a crevasse of unknown depth and of from 15 to 60 ft. width. Its sides are almost vertical, corresponding to the vertical stratification of the rock. The crevasse is filled mostly with red clay and some boulders. The clay is practically impervious to water, which is shown by the fact that the excavation was in part carried on with an air pressure of only 18 lb. at a depth of 100 ft. below water level, and, for some caissons, to 18 ft. below the cutting edge. In its natural state, the clay is very hard and has a high bearing capacity, but, in water, it dissolves readily.

Three of the fifteen cylinder caissons rest entirely on this clay, at depths of from 94 to 123 ft. below the surface, where there is no danger of disturbance. Under the rectangular caissons, the crevasse was

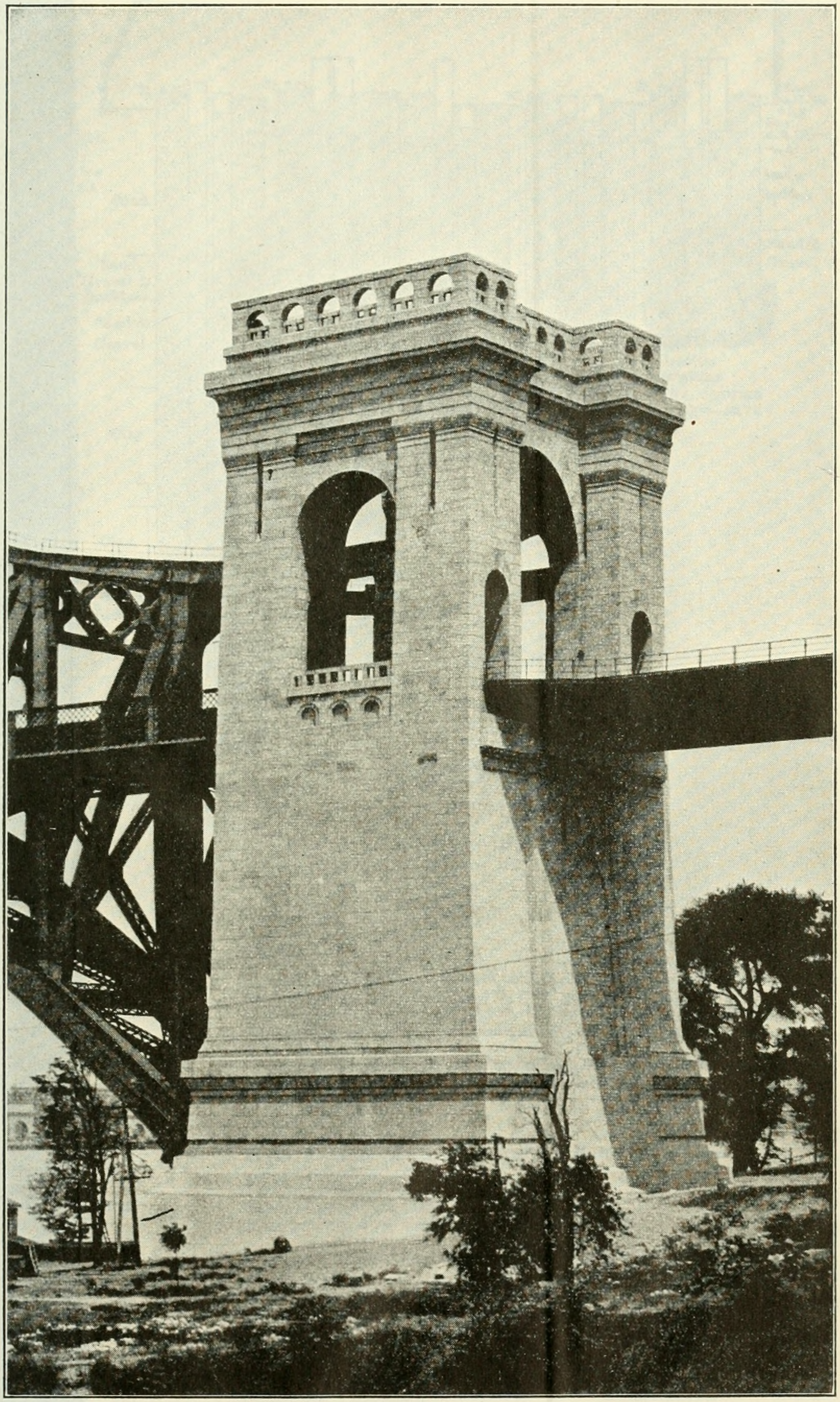
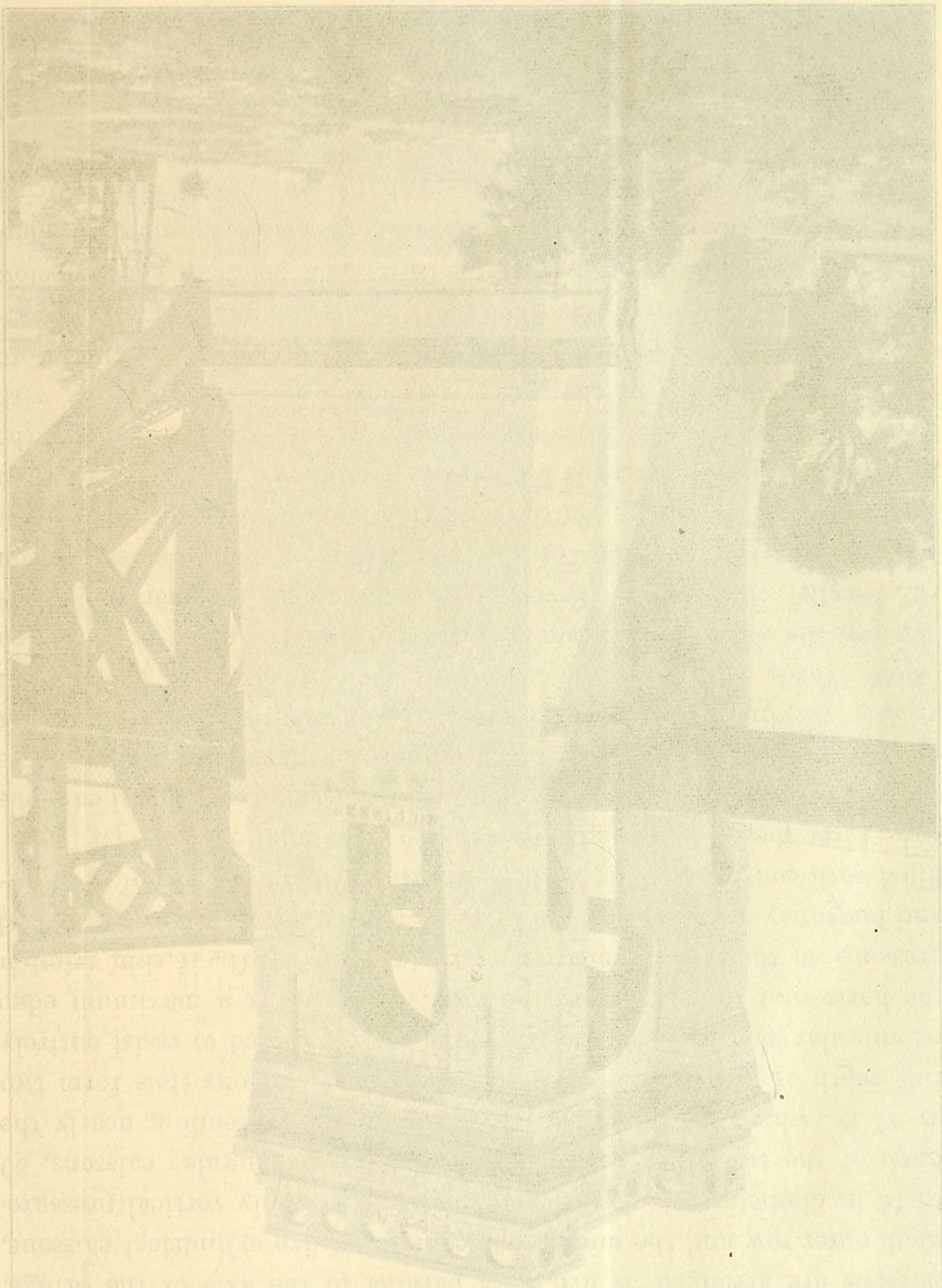
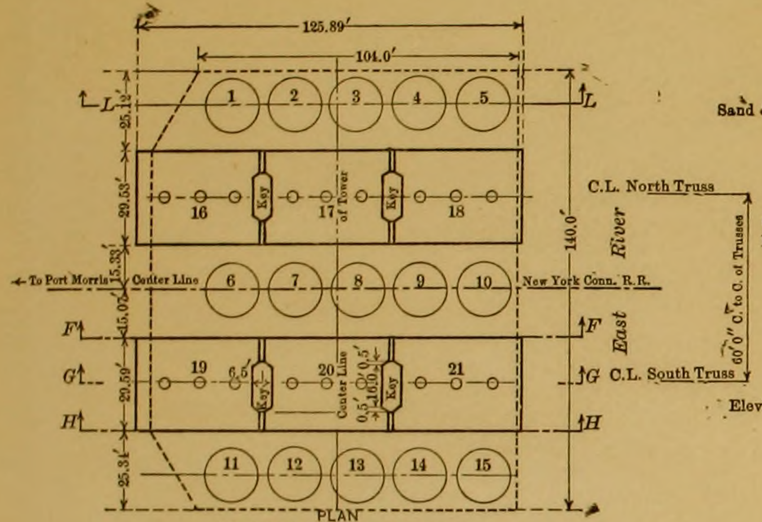
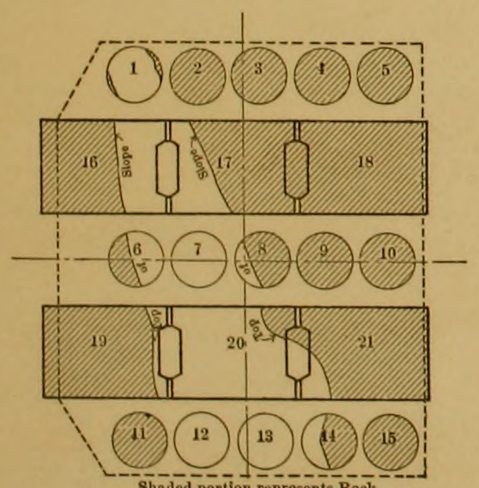


FIG. 10.—LONG ISLAND TOWER OF HELL GATE BRIDGE.

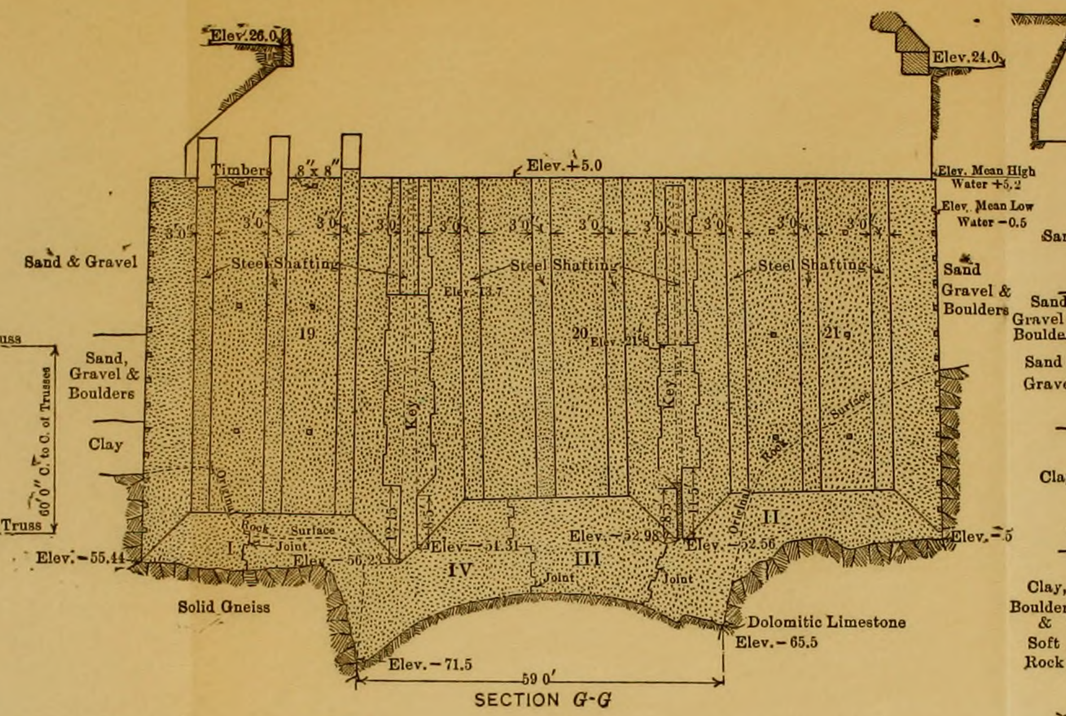




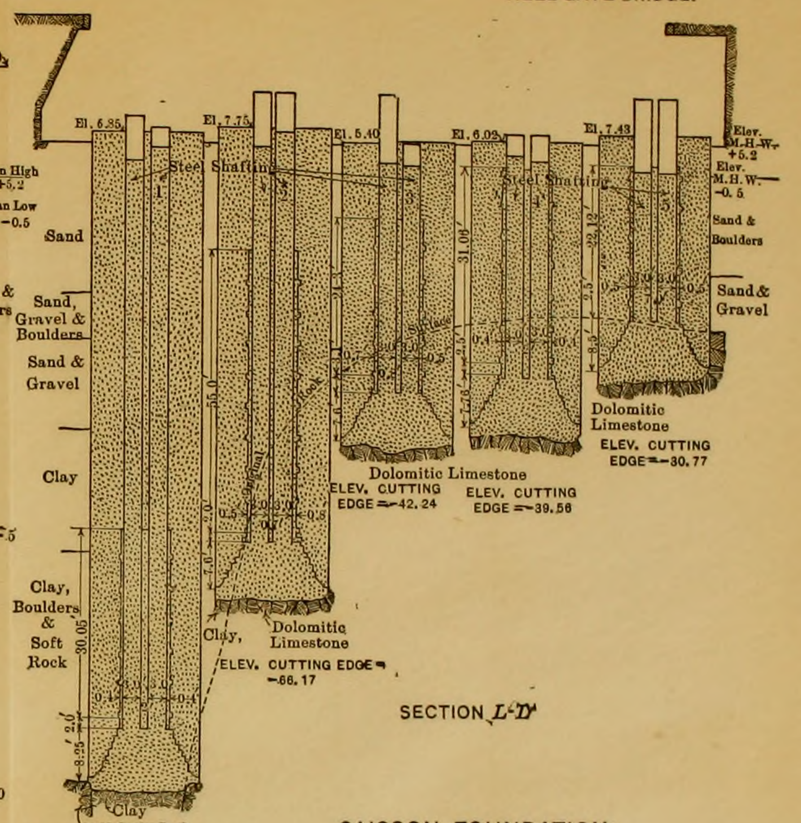
PLAN
SHOWING DIMENSIONS AND
LOCATION OF CAISSONS AT ELEVATION +5.0



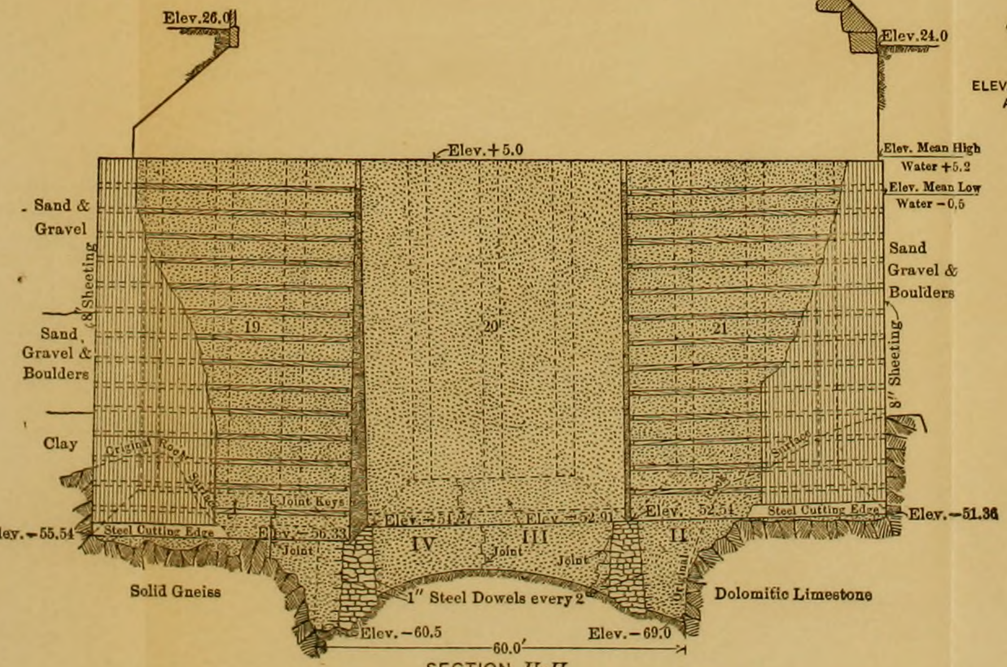
PLAN SHOWING LOCATION OF ROCK
AT ELEVATION OF CUTTING EDGES



SECTION G-G



SECTION L-D



SECTION H-H

CAISSON FOUNDATION OF WARDS ISLAND TOWER

NOTES:

Caissons shown on this plate, were sunk in the following order: 20 19-21 Rubble masonry piers built under E. & W. Cutting edges of Caissons 19 & 20 at N. & S. sides and under 20 & 21 at S. side only. These piers were constructed to support caissons during excavation for Arch, and were left in place.

Order of procedure in excavating and concreting working chamber.

1st- Part I excavated and concreted continuously
2nd- " II " " " "
3rd- " III " " " "
4th " IV " " " "

Keyways between caissons 19 & 20 and 20 & 21 excavated and sealed under air pressure.

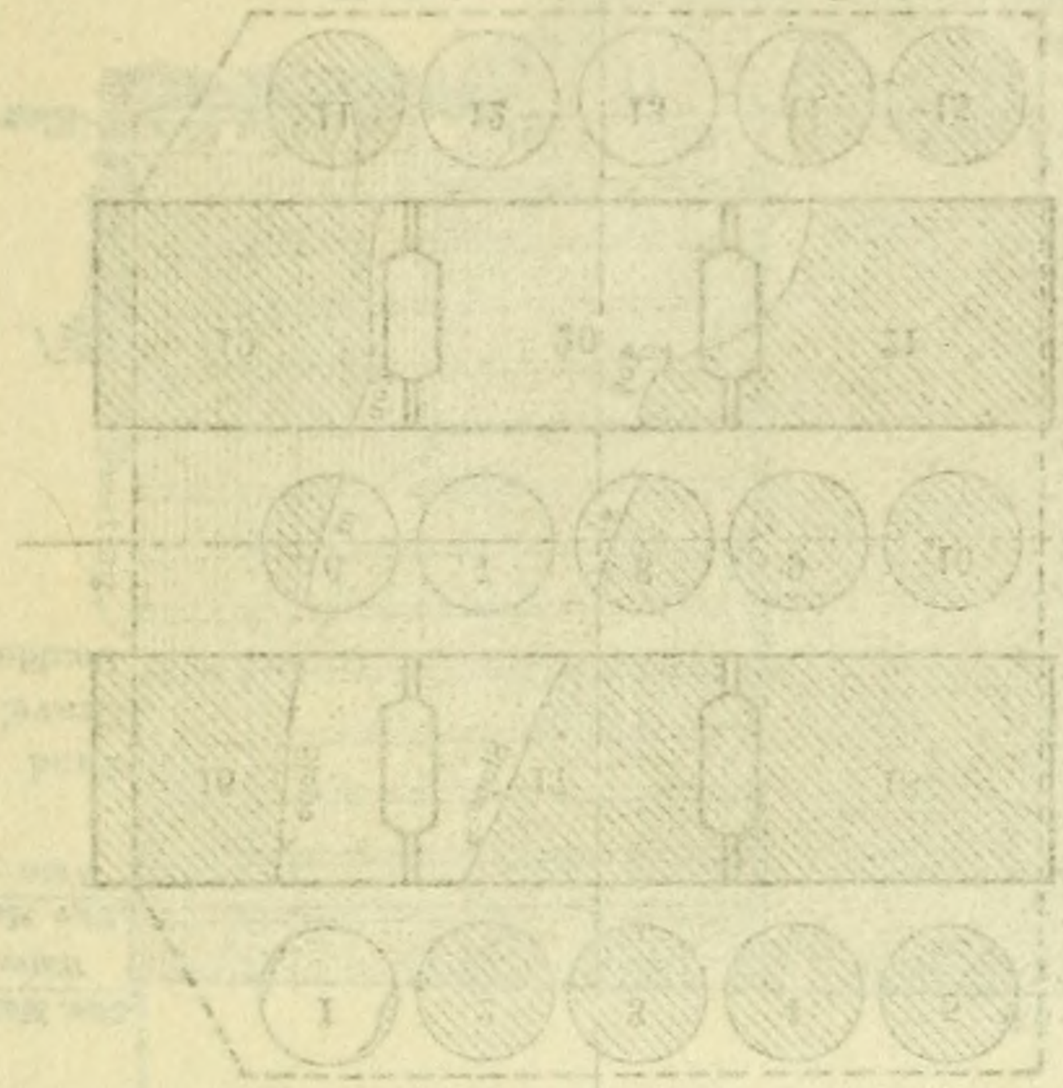
Total linear feet of sinkage = 171'

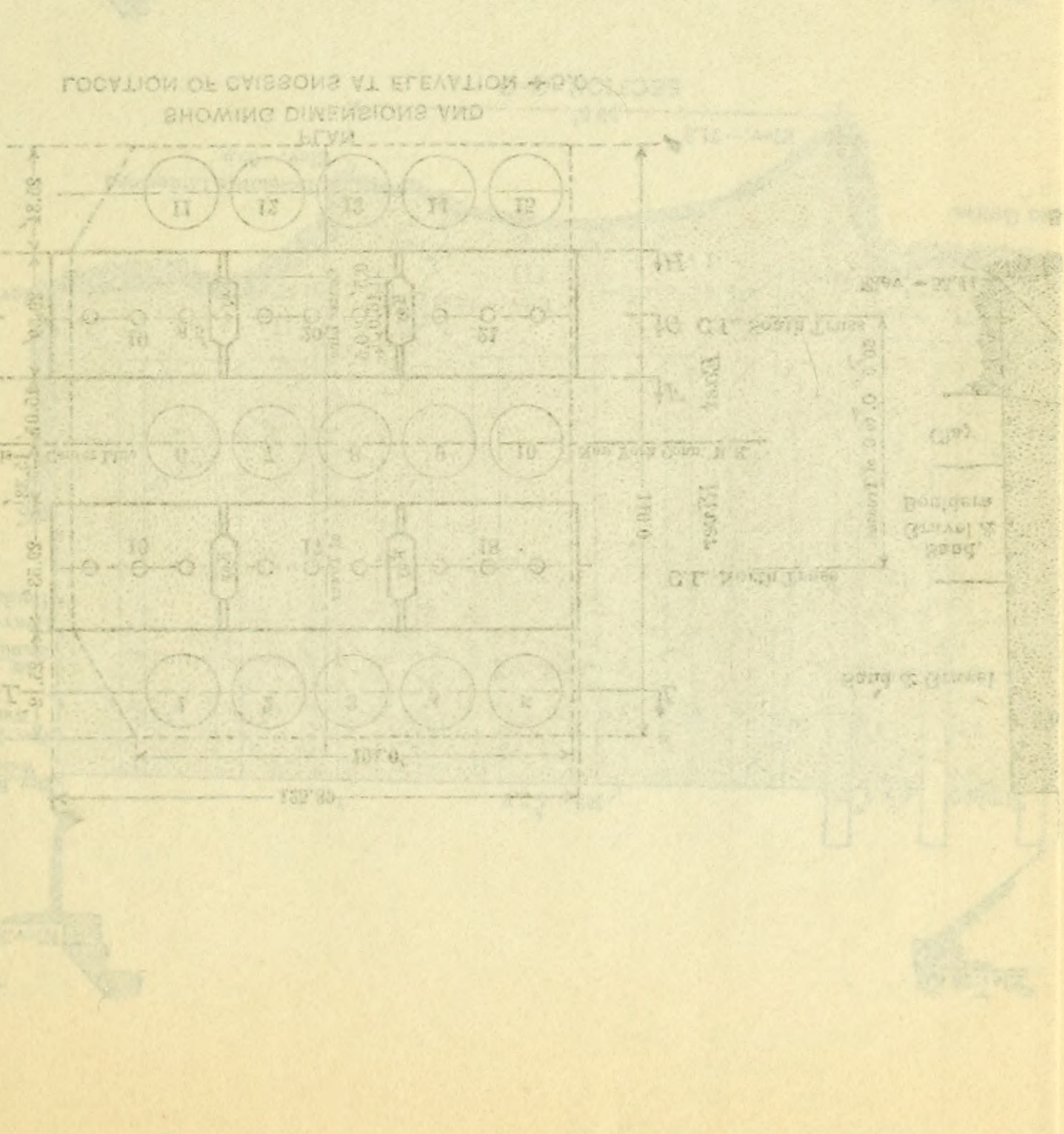
Steel shafting left in place.

Average air pressure during arch construction = 30 lb.
" " " " sinking = 30 lb.

Concrete in all Caissons: 1 part Portland Cement, 3 parts Sand, 4 parts Broken Stone. (Large Rubble Stone embedded in concrete)

Datum: All elevations given, refer to datum of New York Connecting R.R., which is equal to Mean Low Water at Battery = Elev. 0.0





bridged over by a concrete arch which was built at considerable risk below the cutting edge of the caissons. For this purpose, the caissons were supported temporarily by pilasters of rubble masonry built down to the intrados of the arch previous to the excavation for the latter (Plate XXVII). All caissons are of 1:2:4 concrete, well reinforced horizontally and vertically with steel rods. Details of the working chambers and cutting edge are shown in Fig. 11.

Owing to the unusual and difficult character of the Wards Island foundation, the Company decided to do the work with its own forces, under the direction of the Chief Engineer. P. G. Brown, M. Am. Soc. C. E., as Managing Engineer, was in immediate charge of this work.

Quantities.—The two towers contain approximately 110 000 cu. yd. of masonry, of which 28 000 cu. yd. are in the Wards Island foundation. The principal quantities are:

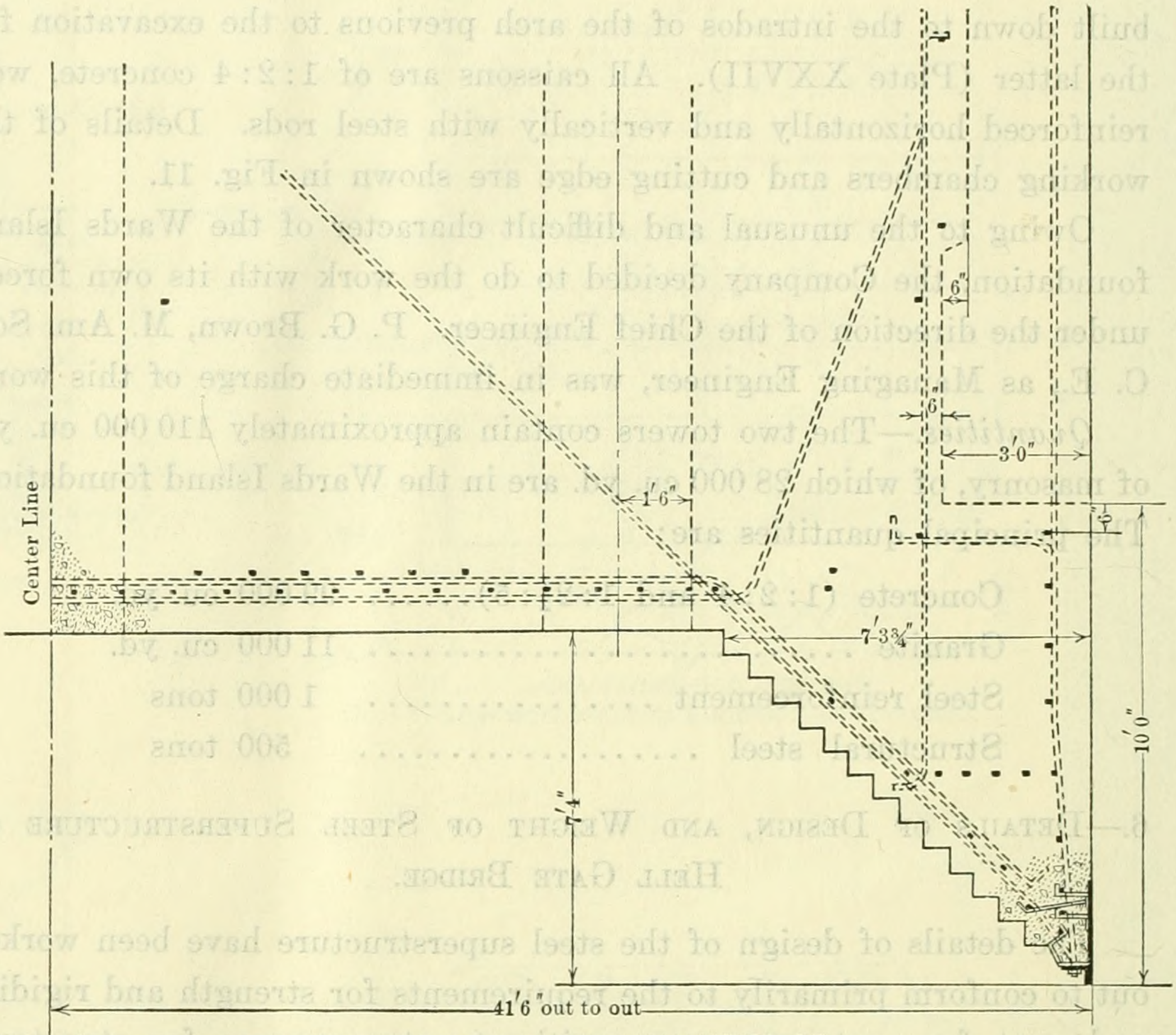
Concrete (1:2:4 and 1:2½:5)	99 000	cu. yd.
Granite	11 000	cu. yd.
Steel reinforcement	1 000	tons
Structural steel	500	tons

6.—Details of Design, and Weight of Steel Superstructure of Hell Gate Bridge.

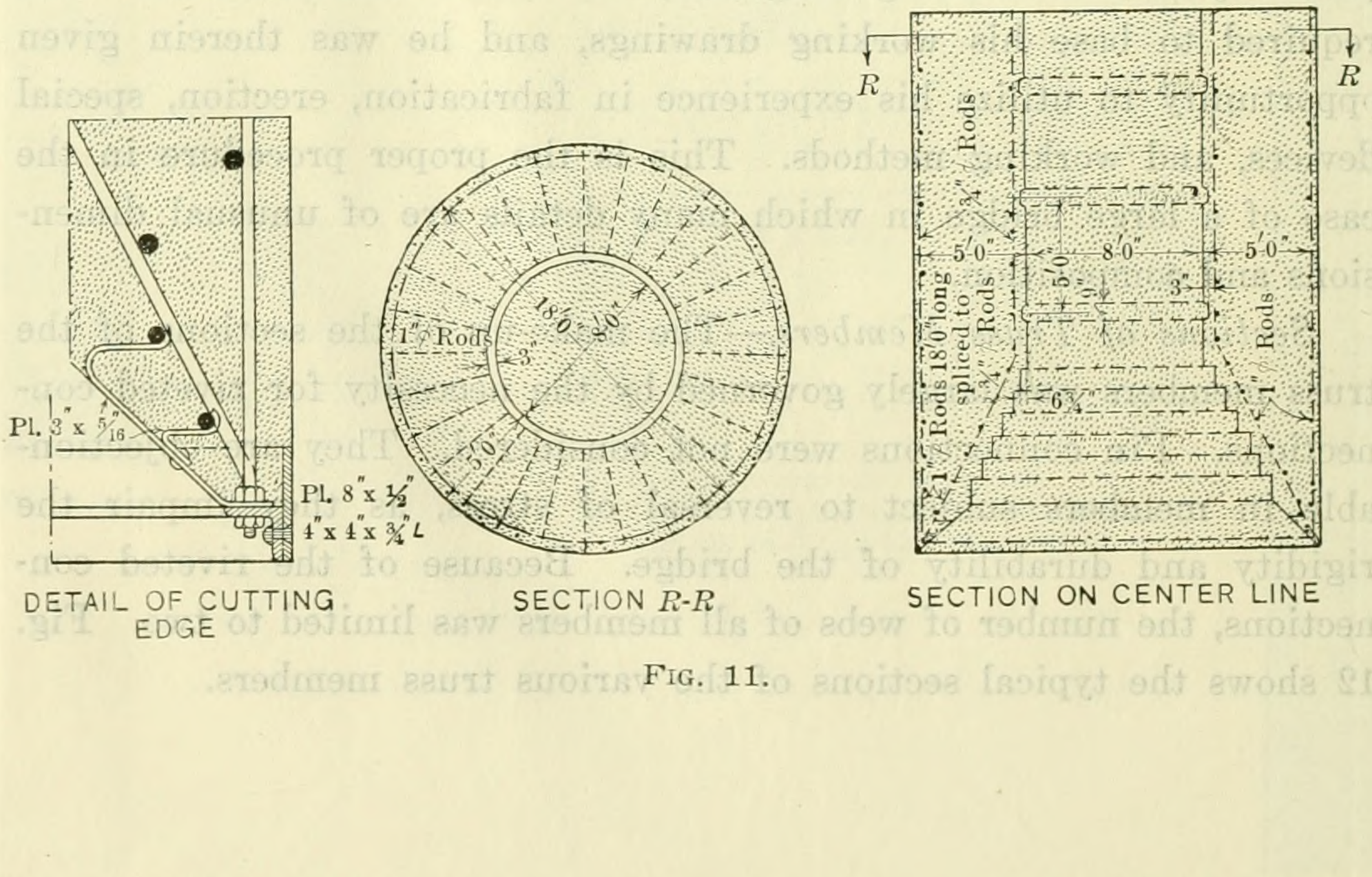
The details of design of the steel superstructure have been worked out to conform primarily to the requirements for strength and rigidity and next for neat appearance, without extra expense for structural ornamentation. Stress sheets and complete detailed plans were prepared by the Consulting Engineer, on which the Contractor was required to base his working drawings, and he was therein given opportunity to utilize his experience in fabrication, erection, special devices, and working methods. This is the proper procedure in the case of a large bridge in which many details are of unusual dimensions and composition.

Sections of Truss Members.—The make-up of the sections of the truss members was largely governed by the necessity for riveted connections. Pin connections were not considered. They are objectionable in members subject to reversal of stress, as they impair the rigidity and durability of the bridge. Because of the riveted connections, the number of webs of all members was limited to two. Fig. 12 shows the typical sections of the various truss members.

DETAILS OF WORKING CHAMBER OF
RECTANGULAR CAISSONS
WARDS ISLAND TOWER FOUNDATION.



DETAILS OF WORKING CHAMBER OF
CYLINDRICAL CAISSONS
WARDS ISLAND TOWER FOUNDATION.



DETAIL OF CUTTING EDGE

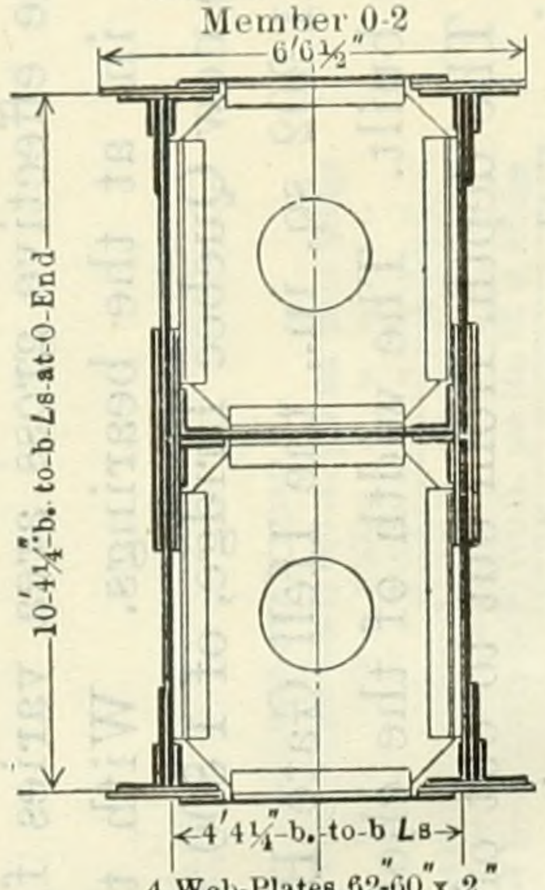
SECTION *R-R*

SECTION ON CENTER LINE

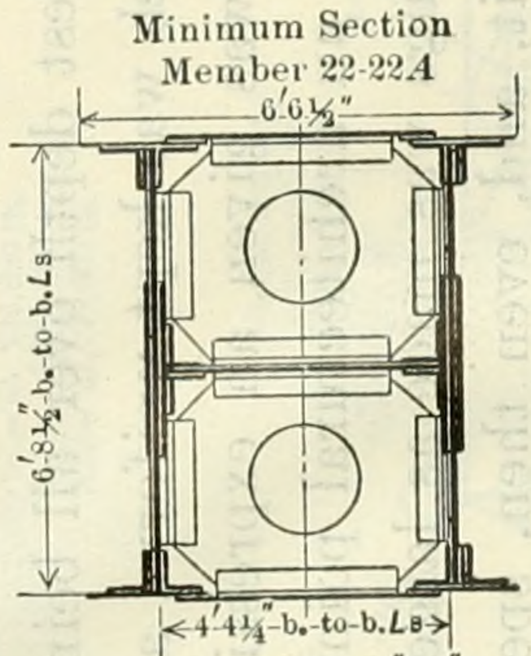
FIG. 11.

SECTIONS OF TRUSS MEMBERS

BOTTOM CHORD

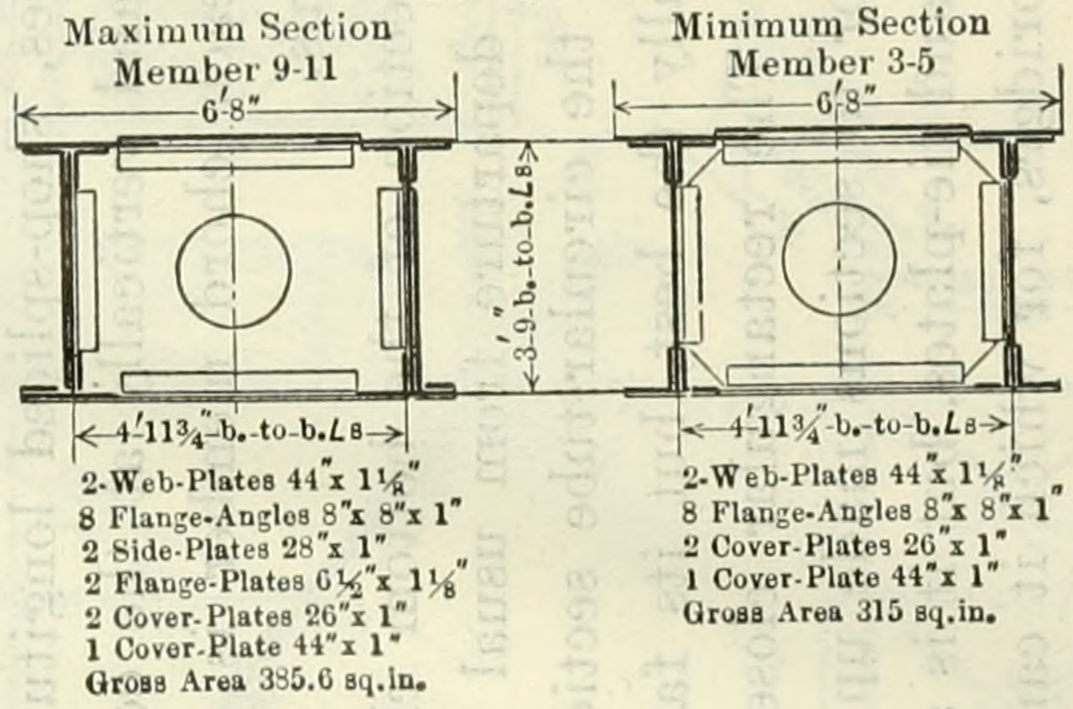


4-Web-Plates 62-60″ x 2″
12 Angles 8″x 8″x 1″
4 Plates, inside 40″x 13/16″
4 Plates, outside 40″x 13/16″
4 Flange-Plates 18″x 1⅝″
8 Flange-Plates 22″x 13/16″
2 Cover-Plates 50″x 1″
1 Center Web 48″x 2″
Max. Gross Area 1392 sq.in.



4-Web-Plates 40″x 2″
12 Angles 8″x 8″x 1″
2 Plates, inside 32″x 1½″
2 Plates, outside 32″x 1″
4 Flange-Plates 22″x 7/8″
4 Flange-Plates 8″x 1″
2 Cover-Plates 50″x 1″
1 Center Web 48″x 1¼″
Gross Area 929 sq.in.

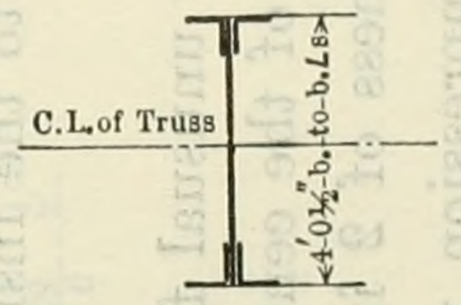
TOP CHORD



2-Web-Plates 44″x 1⅛″
8 Flange-Angles 8″x 8″x 1″
2 Side-Plates 28″x 1″
2 Flange-Plates 6½″x 1⅛″
2 Cover-Plates 26″x 1″
1 Cover-Plate 44″x 1″
Gross Area 385.6 sq.in.

2-Web-Plates 44″x 1⅛″
8 Flange-Angles 8″x 8″x 1″
2 Cover-Plates 26″x 1″
1 Cover-Plate 44″x 1″
Gross Area 315 sq.in.

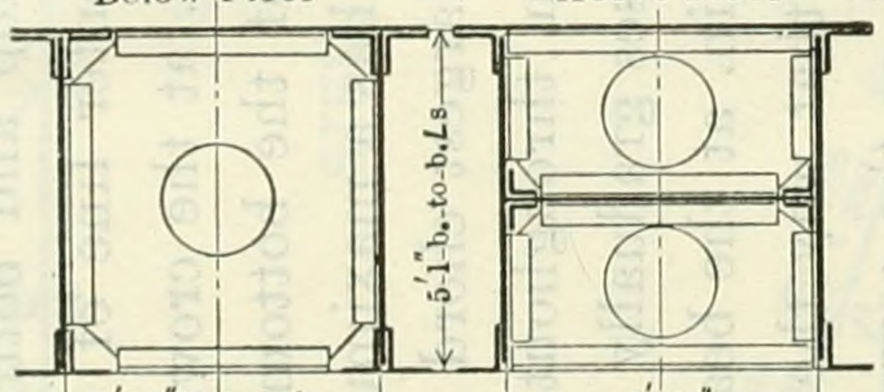
SUSPENDERS



1 Web-Plate 48″x 1″
4 Angles 8″x 8″x 1″
Gross Area 108 sq.in.

WEB MEMBERS

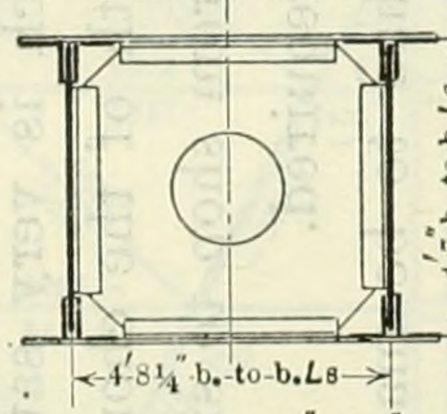
End Vertical 0-1
Below Floor / Above Floor



2 Web-Plates 60″x 1⅝″
8 Flange-Angles 8″x 8″x 1″
1 Cover-Plate 76″x ½″
Gross Area (excl. of Cov. Pl.) 315 sq.in.

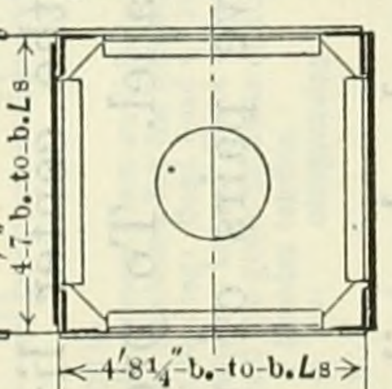
2 Web-Plates 60″x 1⅝″
4 Flange-Angles 8″x 8″x 1″
1 Cover-Plate 76″x ½″
1 Center Web 56″x ½″
4 Angles 6″x 6″x ½″
Gross Area (excl. of Cov. Pl.) 306 sq.in.

Verticals 2-3 & 4-5
Below Floor



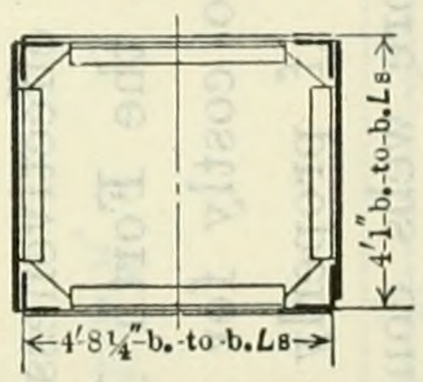
2 Web-Plates 54″x 1⅝″
8 Flange-Angles 8″x 8″x ¾″ & ⅞″
Gross Area, max. 281.3 sq.in.
Gross Area, min. 267.0 sq.in.

Verticals 2-3 & 4-5
Above Floor
Diagonals 1-2 & 3-4



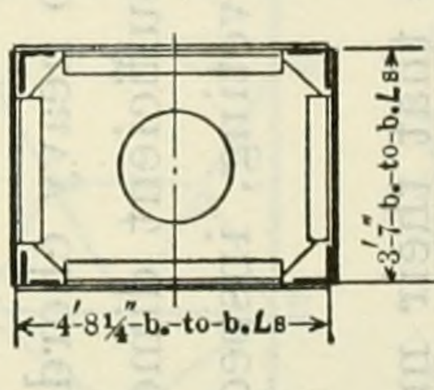
2 Web-Plates 54″x 1⅝″
4 Flange-Angles 8″x 8″x ¾″ to 1″
Gross Area, max. 235.0 sq.in.
Gross Area, min. 221.3 sq.in.

Diagonal 5-6



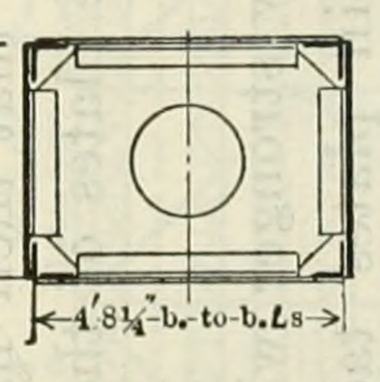
2 Web-Plates 48″x 1⅝″
4 Flange-Angles 8″x 8″x ¾″
Gross Area, 201 8 sq.in.

Verticals 6-7 to 22-23
Diagonals 7-8 to 13-14



2 Web-Plates 42″x 1″
4 Flange-Angles 8″x 8″x 11/16″ to 1⅛″
Gross Area, max. 151 sq.in.
Gross Area, min. 126.1 sq.in.

Diagonals
15-16 to 21-22



2 Web-Plates 42″x 1″
4 Flange-Angles 8″x 8″x ¾″ to 1″
2 Side-Plates 26″x ¾″ to 1″
Gross Area, max. 196 sq. in.
Gross Area, min. 168.6 sq.in.

FIG. 12.

Bottom Chord Section.—The bottom chord has a closed double-box section, consisting of two vertical webs, top and bottom covers, and a solid horizontal diaphragm along the center line of the chord. The effective gross area varies from 929 sq. in. at the crown to 1 392 sq. in. at the bearings. With the exception of the bottom chord of the new Quebec Bridge, of 1 800 ft. span, which has a maximum section of 1 902 sq. in., the Hell Gate Bridge has the largest chord section so far built. The width of the chord is 6 ft. $6\frac{1}{2}$ in. throughout.

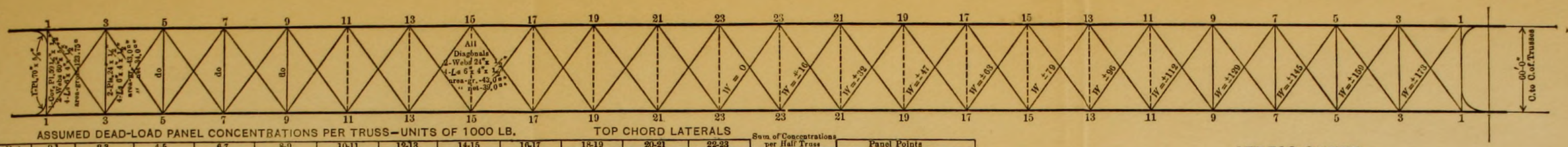
The depth, from out to out of covers, increases gradually from 7 ft. $0\frac{1}{4}$ in. at the crown to a maximum of 10 ft. $9\frac{3}{4}$ in. at the bearings, the greatest depth over all being 11 ft. $4\frac{1}{4}$ in. In that way the thickness of web was kept uniform, and the bottom chord, as the carrying member, was given an expression of strength which is very satisfactory from an architectural point of view. The depth of the chord at the bearings was made as large as transportation from shop to site would permit, and, even then, special low cars were required.

Each web-plate between two panel points had to be made up of four pieces, shop-spliced longitudinally along the center line of the member and vertically at the center of the panel. To prevent distortion, each chord member is stiffened by five pairs of transverse diaphragms.

The section of the bottom chord, as previously described, marks a radical departure from usual practice. For effectiveness to resist buckling, the circular-tube section, as used in the Forth Bridge, is theoretically the best, but its fabrication is too costly for American practice. The rectangular closed-box section, if properly stiffened, is superior to sections made up of two or more webs connected by latticing and tie-plates, but it is adapted only to heavy chords or posts of large bridges, for which it can be made of sufficient dimensions to allow access to the inside for the purpose of riveting, inspection, and painting.

A further unusual feature of these chords is that their main webs, and the web of the center diaphragm, are single plates of the extraordinary thickness of 2 in.

Large compression members are undoubtedly stronger when made up of single thick plates than of several thin plates tack-riveted together; besides, many rivets are saved by using single thick plates. It is frequently maintained that better material is obtained in the



ASSUMED DEAD-LOAD PANEL CONCENTRATIONS PER TRUSS—UNITS OF 1000 LB.

Abut.	0-1	2-3	4-5	6-7	8-9	10-11	12-13	14-15	16-17	18-19	20-21	22-23	Sum of Concentrations per Half Truss	Panel Points
17	188	187	174	157	160	155	135	140	140	140	137	130	1860	Steel Work at Upper Chord
65	308	281	234										838	Steel Work at Floor Height
	324	538	484	638	588	583	537	516	515	505	498	510	6236	Steel Work at Bottom Chord
	496												496	Bearings
52	820	988	892	795	748	738	672	656	655	645	635	640	8934	Total Steel Work except Bearing
65	160	317	317										859	Flooring at Floor Height
				317	317	317	317	317	317	317	317	317	2853	Flooring at Bottom Chord
117	980	1303	1209	1112	1065	1055	989	973	972	962	952	957	12616	Total Concentration St. & Fl.

TOP CHORD LATERALS

BOTTOM CHORD LATERALS

FLOOR LATERALS

FLOOR SYSTEM

STRESS SHEET OF STEEL SUPERSTRUCTURE

ASSUMED DEAD LOAD

4-Railroad tracks @ 2 900 lb. per ft. = 11 600 lb. per lin. ft. of bridge
2-Sidewalk floors @ 650 " " " = 1 300 " " " " "
2-Electric Conduits @ 1 000 " " " = 2 000 " " " " "

Total Flooring & Conduits = 14 900 lb. per lin. ft. of bridge
Steelwork average (excl. Bearings) = 35 800 " " " " "
Total Dead Load (average) = 50 700 lb. per lin. ft. of bridge

TOTAL WEIGHT OF STEEL WORK

Structural Steel 4 x 8,934,000 lb. = 35, 736,000 lb.
Cast Steel Bearings 4 x 496 lb. = 1,984,000
Total = 37,720,000 lb.

NOTE:-
All stresses given on this half of truss are maximum for the following conditions of loading: Dead Load + Live Load + Impact + Lateral Force + Excess Load, and are given in units of 1000 lb.
+ denotes tension
− " compression

SECTION 2-3

SECTION 8-9

SECTION 10-11

SECTION 6-7

SECTION 4-5

EAST VIEW

REACTION ON SKEWBACK OF PIER

thinner than in the thicker plates. This was not substantiated by the great number of specimen tests made for the Hell Gate Bridge from material varying from ½ in. to 2 in. in thickness. In general, the thick material showed as high elastic properties and ultimate strength as the thinner material rolled from the same heat.

The tests* made by the Society's Special Committee on Steel Columns and Struts show a marked falling off in the compressive strength of heavy columns in comparison with light columns of approximately the same outside dimensions. It would be wrong, however, to conclude that this is due to the thicker metal in the heavy columns. It would seem to be due rather to the less efficient distribution of metal in the heavier columns. If the heavy columns had been built up of several thin plates, tack-riveted together, their compressive strength would probably have been even less. Comparative tests, to throw light on this question, would be highly desirable.

Top Chord and Web Members.—The top chord and web members have a rectangular box section with two solid webs parallel to the plane of the truss. The top chord and end posts are properly provided also with a solid cover. All open sides have stiff-angle latticing (Plates XXIX and XXX). The section of the top chord ranges from 315 to 386 sq. in., and has a uniform depth of 4 ft., except in the end panel, where, for better appearance, and to allow sufficient height for entrance at the tower, the chord tapers to 5½ ft. From the end a stair leads through the chord to its top, along which two light handrails are provided.

The sections of the web members increase from 126 sq. in. at the crown to 315 sq. in. at the end, and the depth increases correspondingly from 42 to 60 in. A good section for compression members of moderate area is that of the two vertical posts, 2-3 and 4-5, below the floor, each flange consisting of two angles. The latticing connects only to the inside angle, but its stresses are transmitted to both angles through short tie-plates placed across the two angles (Plate XXX).

Floor Suspenders.—The suspenders, which carry the floor between its intersections with the trusses, have an I-section with a single web, 48 in. wide, placed at right angles to the plane of the truss. For appearance, and to prevent large bending stresses in the suspenders, due to the longitudinal expansion and contraction of the floor, they

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* *Proceedings,* Am. Soc. C. E., for December, 1917.

were purposely made slender in elevation. To prevent bending stresses in the suspenders, due to the vertical deflection of the floor-beams and the horizontal deflection of the floor-lateral truss, the suspenders are connected to the floor-beams at the bottom and to the trusses at the top with 16-in. pins placed parallel to the plane of the truss (Plate XXX).

Latticing of Truss Members.—Since the failure of the Quebec Bridge in 1907, increased attention has been given to the latticing of compression members, which has led to marked improvement in this respect. Numerous tests, and a number of failures that have occurred since, have given further proof that inadequate latticing greatly reduces the buckling strength of compression members. It is now generally recognized that the latticing has a distinct static function, and bears a certain relation to the dimensions, shape, and area of section of the member.

The following simple and easily remembered rule was applied in proportioning the latticing in the Hell Gate Bridge and Approaches:

"The latticing and its connections shall be designed to resist at any section at right angles to the axis of the member a shearing force, in pounds, at least equal to 300 times the gross area of the member, in square inches."

Stiff-angle latticing was used throughout, with a minimum thickness of $\frac{1}{2}$ in., and at least two rivets for each angle. Flat lattice bars in heavy compression members are objectionable, as they have little resistance against compression. They are easily bent in handling the members, and, even if subsequently straightened, as is a too common practice, they constitute a permanent defect in a bridge.

Special attention was given to uniformity and the neat appearance of the latticing. Main truss members, as well as laterals, have double lattice angles, with an inclination of about 45° to the axis of the member. One of each pair of lattice angles is spliced at the intersection by a square plate (Plates XXIX and XXX). All latticing is placed inside the flange angles. The tie-plates are as near as practicable to the end of the member, in any case, well within the edge of the gusset-plate, and all cover-plates of the bottom chord are continuous across the panel point, being notched out for the gusset-plates. This is an important improvement over the very common practice of stop-

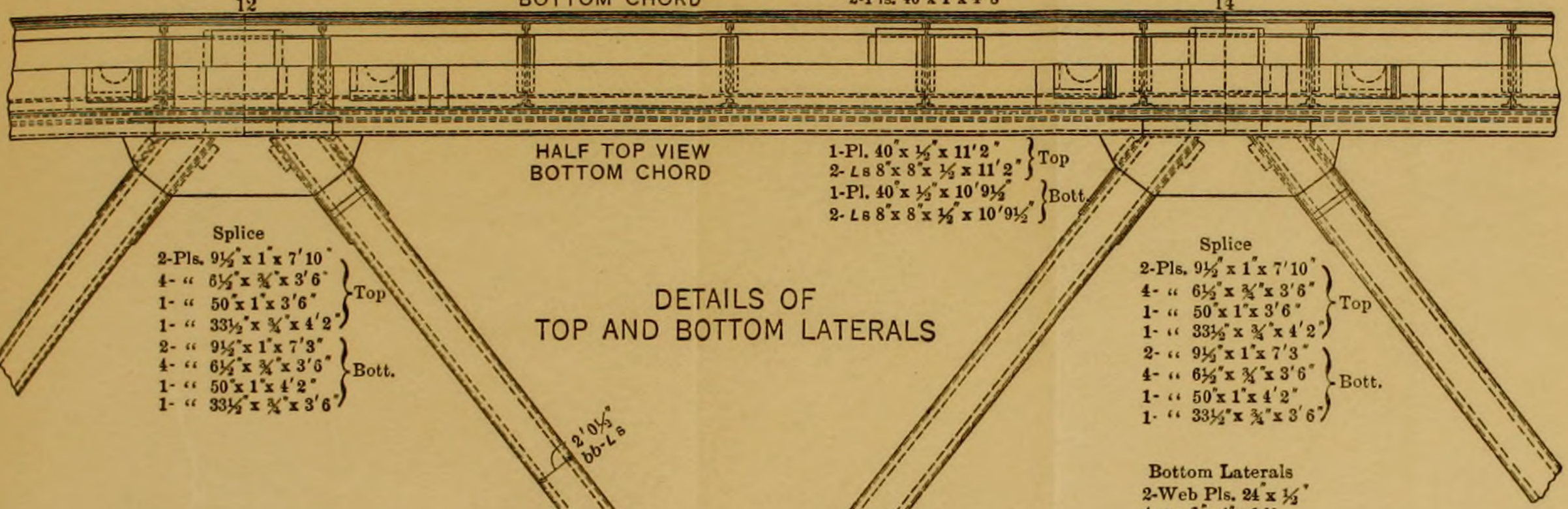
Manhole Reinforcement Material
1-Pl. 50" x 1" x 4'3"
2-Bars 4" x ½" x 1'8"
2-Bars 4" x ½" x 2'4"

Splice
1-Pl. 40" x 1" x 2'11½"
1-Pl. 40" x 1" x 3'7½"
12

HALF SECTiON-CENTER WEB
BOTTOM CHORD

Splice
2-Pls. 32" x 1" x 5'4"
2-Pls. 40" x 1" x 4'8"

Splice
1-Pl. 40" x 1" x 2'11½"
1-Pl. 40" x 1" x 3'7½"
14



HALF TOP VIEW
BOTTOM CHORD

1-Pl. 40" x ½" x 11'2" } Top
2-Ls 8" x 8" x ½ x 11'2" }
1-Pl. 40" x ½" x 10'9½" } Bott.
2-Ls 8" x 8" x ½" x 10'9½" }

DETAILS OF
TOP AND BOTTOM LATERALS

Splice
2-Pls. 9½" x 1" x 7'10"
4- " 6½" x ¾" x 3'6" } Top
1- " 50" x 1" x 3'6"
1- " 33½" x ¾" x 4'2"
2- " 9½" x 1" x 7'3"
4- " 6½" x ¾" x 3'6" } Bott.
1- " 50" x 1" x 4'2"
1- " 33½" x ¾" x 3'6"

Splice
2-Pls. 9½" x 1" x 7'10"
4- " 6½" x ¾" x 3'6" } Top
1- " 50" x 1" x 3'6"
1- " 33½" x ¾" x 4'2"
2- " 9½" x 1" x 7'3"
4- " 6½" x ¾" x 3'6" } Bott.
1- " 50" x 1" x 4'2"
1- " 33½" x ¾" x 3'6"

Bottom Laterals
2-Web Pls. 24" x ½"
4-Ls 6" x 4" x ½"
Tie Pls. ½" thk.
Latt. L s 3½" x 3½" x ⅜"

2'0½" bb-Ls

2-Pls. 68" x ⅝" x 6'8"

TYPICAL CROSS-SECTION & LATTICING
OF ALL TOP & BOTT. LATERALS FROM 10 TO 20

3'7½" bb-Ls

2'0½" bb-Ls

Top Laterals
2-Web Pls. 24" x ½"
4-Ls 6" x 4" x ½"
Tie Pls. ½" thk.
Latt. L s 3½" x 2½" x ⅜"

2'0½" bb-Ls

HALF TOP VIEW
TOP CHORD

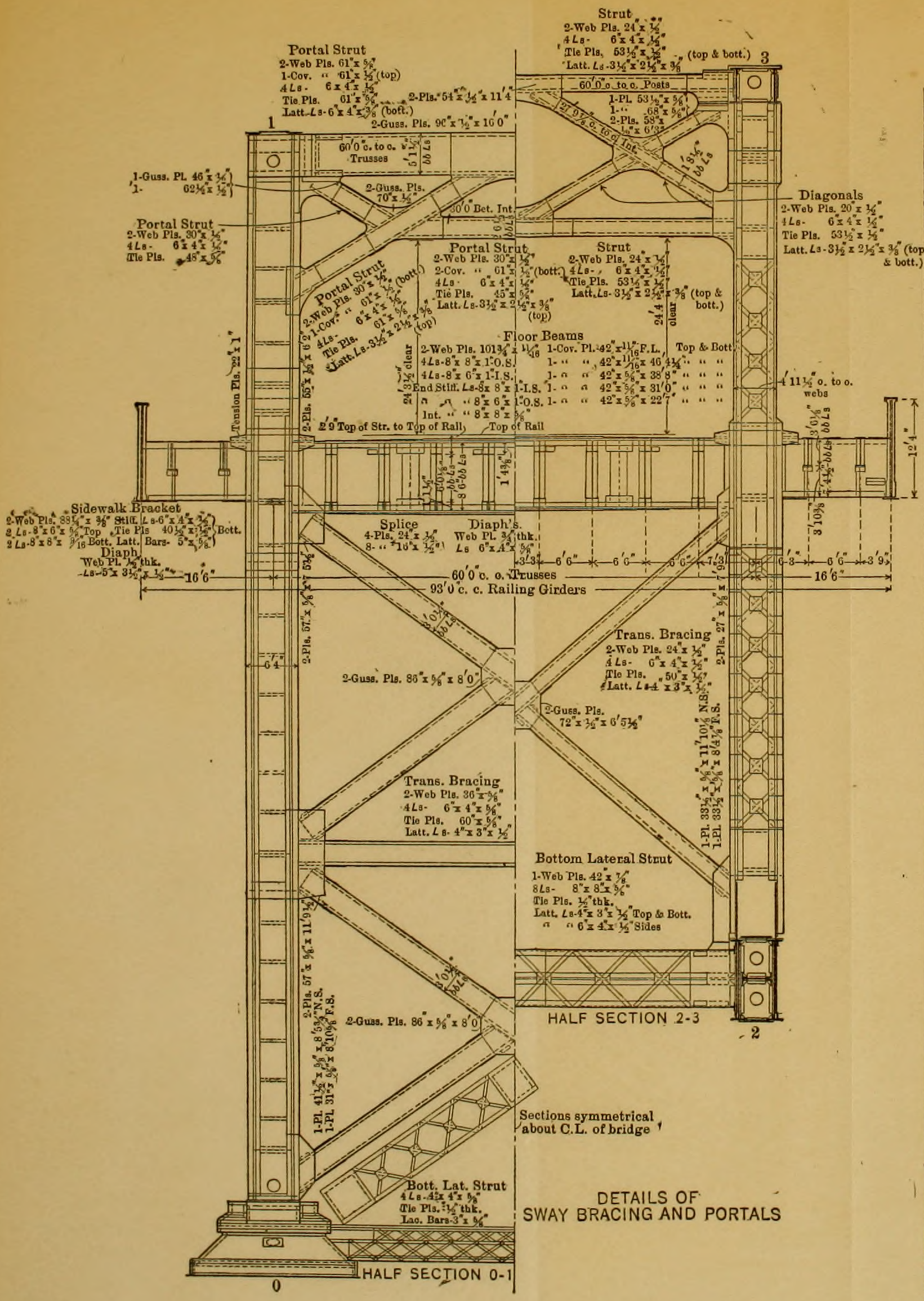
2-Pls. 36" x 1" x 13'2"

13

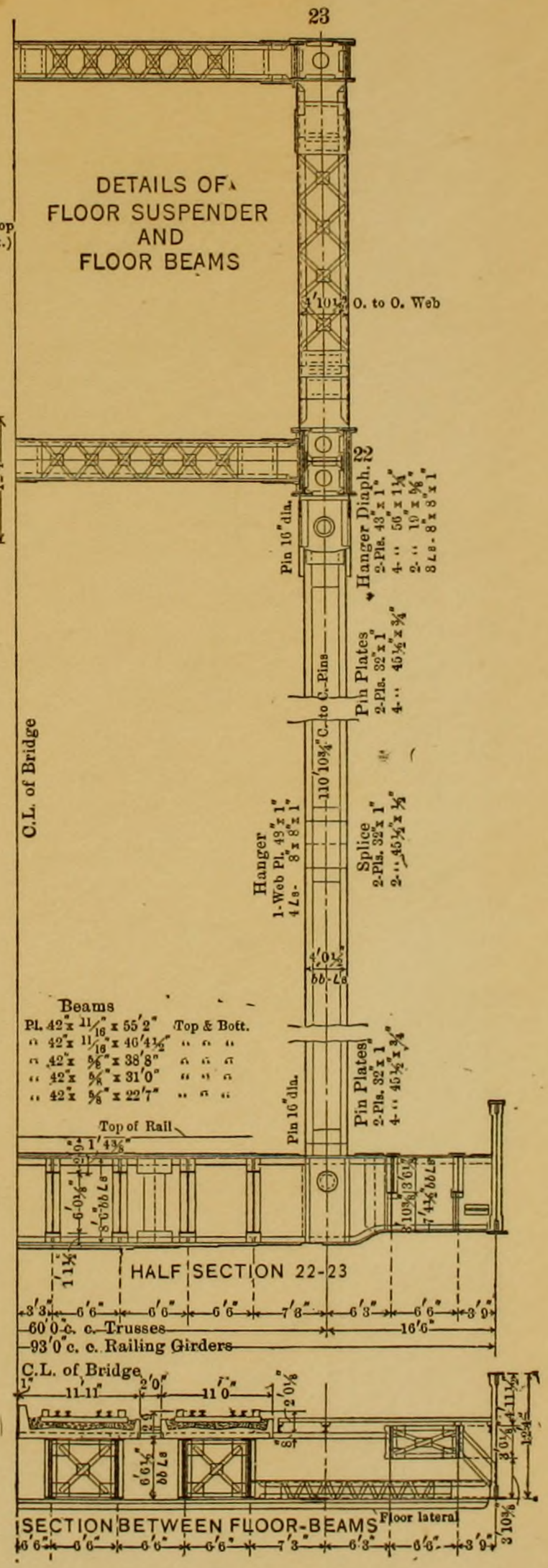
HALF BOTTOM VIEW TOP CHORD

15

Bolts
Top & Bott.
Splice
Bolts
Splice
Bolts
Top & Bott.
Splice
Bolts
Top & Bott.
Splice
Bolts
Top & Bott.



DETAILS OF
SWAY BRACING AND PORTALS



ping tie-plates or cover-plates outside of the gusset, thus leaving the flange of the member unsupported for a considerable length.

Riveted Connections and Splices of Truss Members.—As yet, little is known about the correct distribution of stresses in a riveted connection. In proportioning, the favorable assumption of uniform distribution of stresses among all rivets of a connection is made, and the secondary stresses in the rivets caused by the stiffness of the connections is generally disregarded, on the assumption that they are fully covered by the margin of safety of the main stresses. In view of the unusually large connections and splices in the main trusses of the Hell Gate Bridge, it was important to guard against local over-stressing.

The following requirements, as quoted from the "Rules of Design," governed the detailing of the connections and splices:

"The strength of connections shall be sufficient to develop the full strength of the member, even though the computed stress is less, the kind of stress to which the member is subjected being considered.

"Truss members with alternating axial stresses caused by live load (including impact) shall be proportioned for the stress requiring the larger sections. The connections and splices shall be proportioned for the larger stress plus 50% of the smaller stress of opposite sign.

"All joints in riveted work, whether in tension or compression, shall be fully spliced, except when differently noted on drawings.

"Rivets carrying calculated stress, and having a grip which exceeds four diameters, shall be increased in number 1% for each additional $\frac{1}{8}$ in. of grip.

"Where splice-plates are not in direct contact with the parts which they connect, the number of rivets on each side of the joint shall be increased over the number theoretically required to the extent of one-third of the number for each intervening plate.

"Rivets carrying strain and passing through fillers shall be increased 100% in number, if the thickness of the filler is equal to the diameter of the rivet, the increase in number being proportionately more or less as the filler shall be more or less thick than the diameter of the rivet."

The further rule was observed to connect, or splice, each individual part of the section of a member as directly as possible, so as to avoid the transfer of stresses through other parts. Typical connections are shown in Figs. 13, 14, 15, and 16.

The connections at each panel point are made with two gusset-plates, 1 in. thick in the middle panels and $1\frac{5}{8}$ in. in the panels nearer the ends. The largest of these are 120 by $1\frac{5}{8}$ in. by 17 ft. 6 in. and

126 by $1\frac{5}{8}$ in. by 14 ft. 6 in., and mark the limits of present rolling mill capacities.

The general scheme in detailing the connections of the web members to the gussets was as follows: The web-plate of the member is in the same plane and has the same thickness as the gusset-plate. It is cut off at the edge of the gusset and spliced to the latter by a splice-plate on each side. The flange angles, however, extend as far as possible over the gusset, their outstanding flanges being connected by lug-

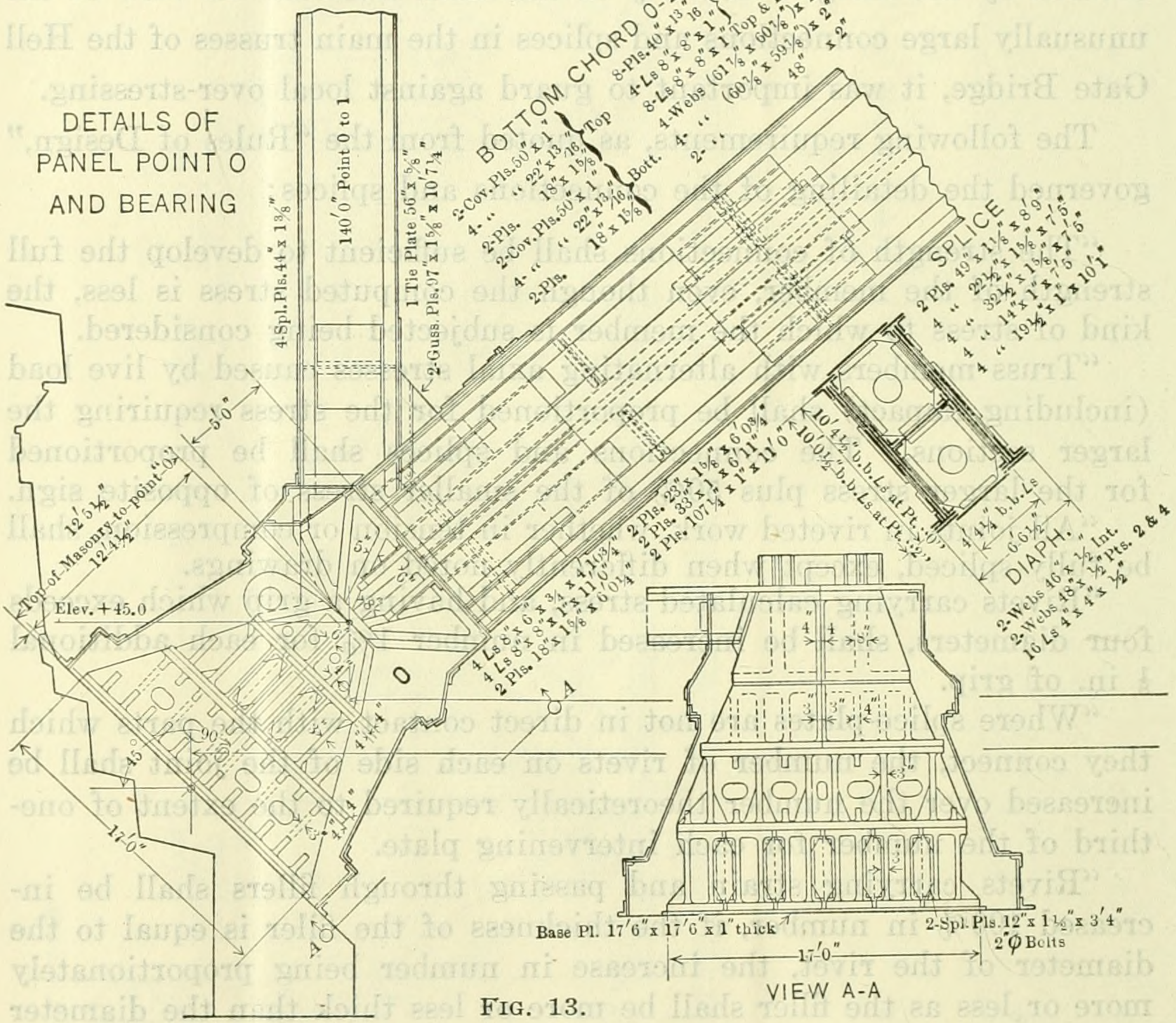


FIG. 13.

angles. This type of connection is efficient and economical, as most of the rivets are in double shear, and the size of the gusset is reduced to a minimum. On account of the re-entrant joints, this connection caused some inconvenience in erection, but no serious difficulties were encountered. The splice-plates were shop-riveted to the member, but, to allow them to spring slightly in entering the member, the two rows of rivets nearest the edge of the gusset-plate were left for field driving.

The top and bottom chords have a butt joint at every panel point on the intersection of the axes of the truss members. The connection

DETAILS OF
PANEL POINTS
6 AND 7
WITH FLOOR SUSPENDED

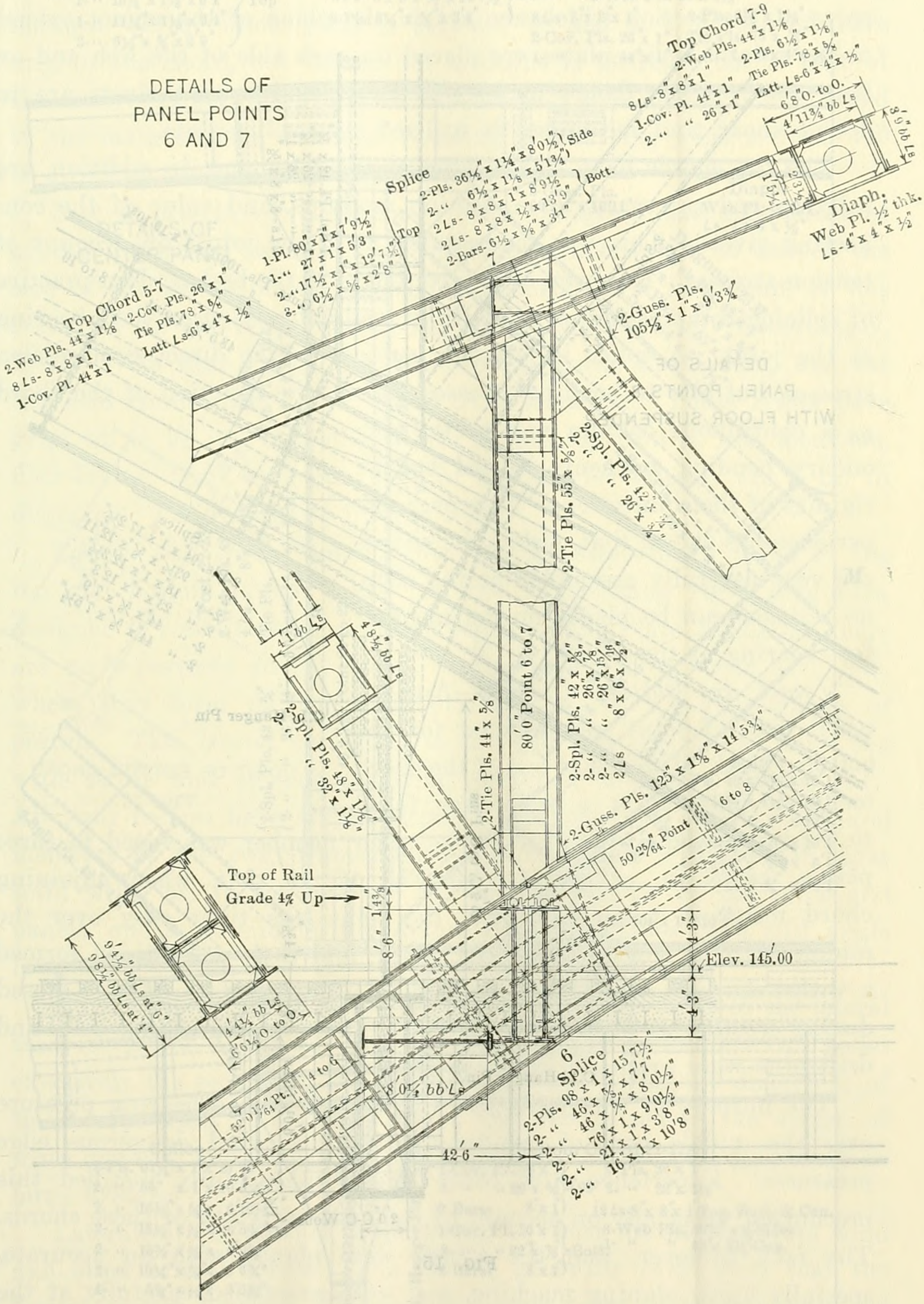


FIG. 14.

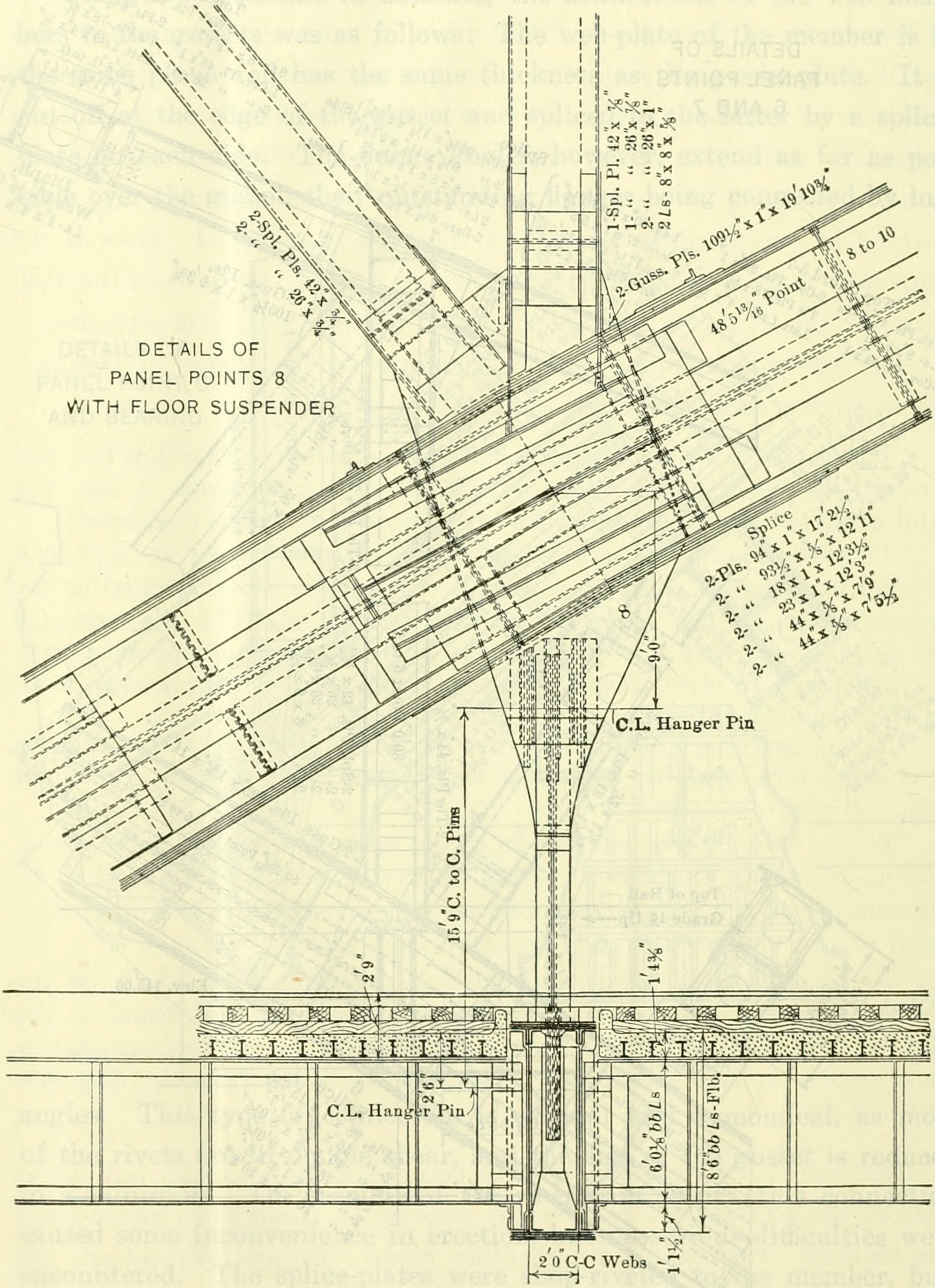

DETAILS OF
PANEL POINTS 8
WITH FLOOR SUSPENDER
2-Spl. Pls. 42" x 3/4"
2- " " 26" x 3/4"
1-Spl. Pl. 42" x 7/8"
1- " " 26" x 7/8"
2- " " 26" x 1"
2 Ls- 8" x 8" x 5/8"
2-Guss. Pls. 109 1/2" x 1" x 19' 10 3/4"
8 to 10
48' 5 13/16" Point
Splice
2-Pls. 94" x 1" x 17' 2 1/2"
2- " 93 1/2" x 5/8" x 12' 11"
2- " 18" x 1" x 12' 3 1/2"
2- " 23" x 1" x 12' 3"
2- " 44" x 5/8" x 7' 9"
2- " 44" x 5/8" x 7' 5 1/2"
8
9'-0"
C.L. Hanger Pin
15' 9" C. to C. Pins
2' 9"
1' 4 3/8"
2' 6"
C.L. Hanger Pin
6' 0 1/8" bb Ls
8' 6" bb Ls-Flb.
1' 1 1/2"
2' 0" C-C Webs

FIG. 15.

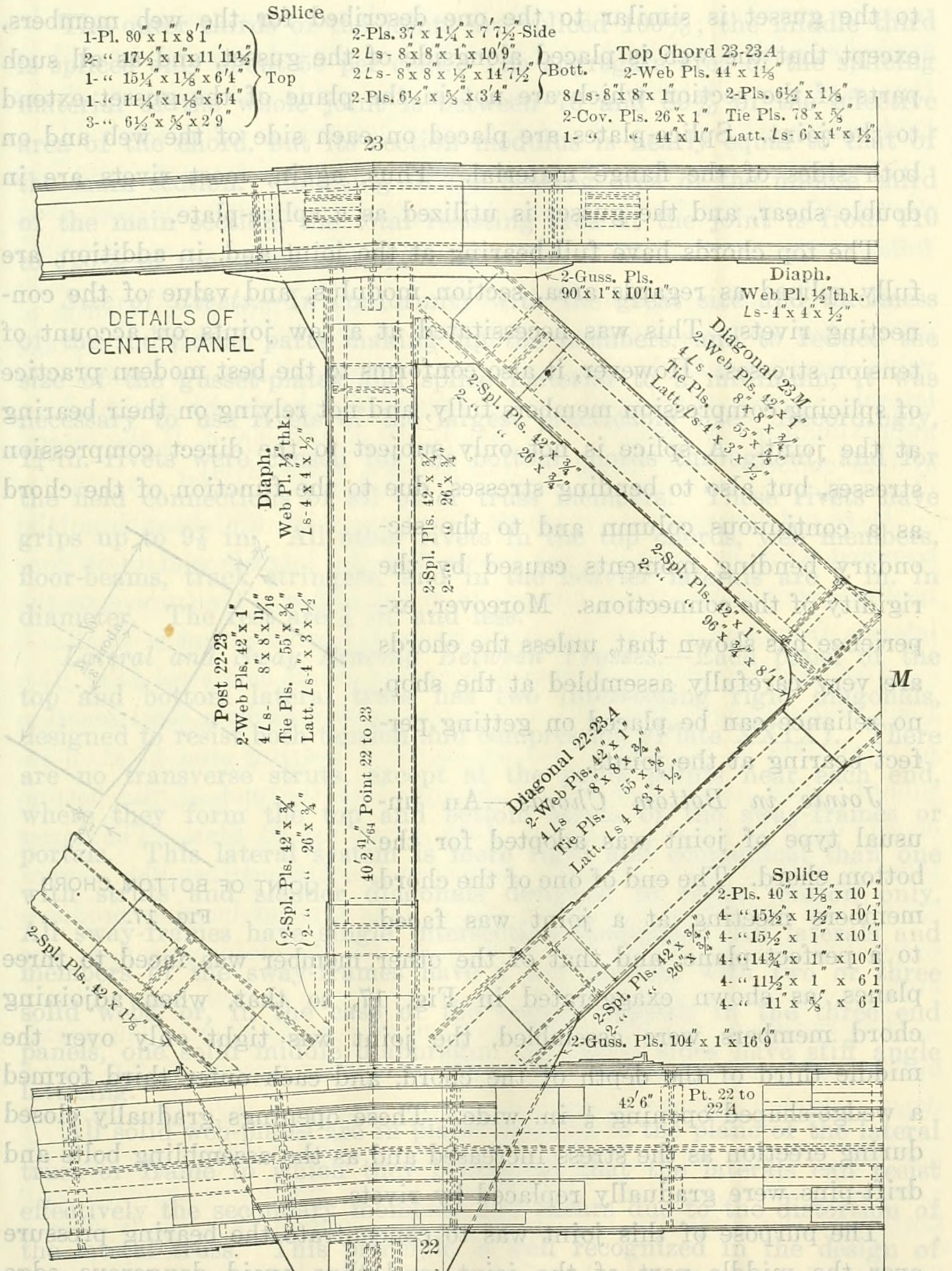
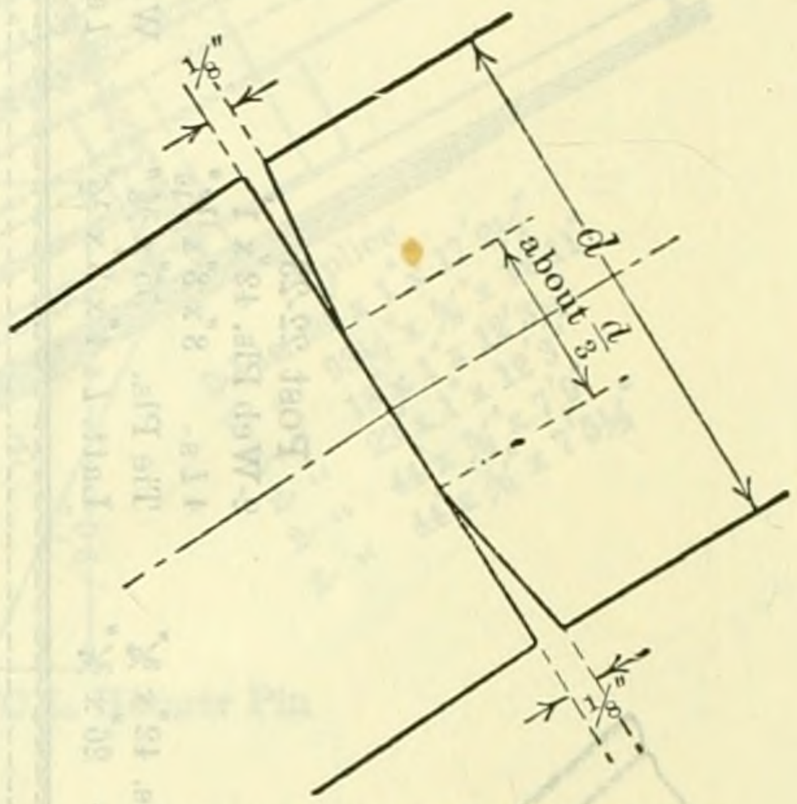

DETAILS OF CENTER PANEL
Splice
1-Pl. 80" x 1" x 8' 1"
2-" 17½" x 1" x 11' 11½"
1-" 15¼" x 1⅛" x 6' 4"
1-" 11¼" x 1⅛" x 6' 4"
3-" 6½" x ⅝" x 2' 9"
Top
2-Pls. 37" x 1¼" x 7' 7½"-Side
2 Ls - 8" x 8" x 1" x 10' 9"
2 Ls - 8" x 8" x ½" x 14' 7½"
2-Pls. 6½" x ⅝" x 3' 4"
Bott.
23
Top Chord 23-23A
2-Web Pls. 44" x 1⅛"
8 Ls - 8" x 8" x 1"
2-Pls. 6½" x 1⅛"
2-Cov. Pls. 26" x 1" Tie Pls. 78" x ⅝"
1-" " 44" x 1" Latt. Ls - 6" x 4" x ½"
2-Guss. Pls. 90" x 1" x 10' 11"
Diaph. Web Pl. ½" thk. Ls - 4" x 4" x ½"
Diagonal 23M
2-Web Pls. 42" x 1"
4 Ls - 8" x 8" x ¾"
Tie Pls. 55" x ⅜"
Latt. Ls - 4" x 3" x ½"
2-Spl. Pls. 42" x ¾"
2-" " 26" x ¾"
2-Spl. Pls. 42" x 1"
2-" " 96" x ¾" x 8' 1"
M
Post 22-23
2-Web Pls. 42" x 1"
4 Ls - 8" x 8" x 11/16"
Tie Pls. 55" x ⅜"
Latt. Ls - 4" x 3" x ½"
Diaph. Web Pl. ½" thk. Ls - 4 x 4 x ½"
2-Spl. Pls. 42" x ¾"
2-" " 26" x ¾"
40' 2 41/64" Point 22 to 23
Diagonal 22-23A
2-Web Pls. 42" x 1"
4 Ls 8" x 8" x ¾"
Tie Pls. 55" x ⅜"
Latt. Ls 4 x 3" x ½"
2-Spl. Pls. 42" x 1⅛"
2-Spl. Pls. 42" x ¾"
2-" " 26" x ¾"
Splice
2-Pls. 40" x 1⅛" x 10' 1"
4-" 15½" x 1½" x 10' 1"
4-" 15½" x 1" x 10' 1"
4-" 14¾" x 1" x 10' 1"
4-" 11½" x 1" x 6' 1"
4-" 11" x ⅝" x 6' 1"
2-Spl. Pls. 42" x ¾"
2-" " 26" x ¾"
2-Guss. Pls. 104" x 1" x 16' 9"
42' 6"
Pt. 22 to 22A
22
C.L. Hanger Pin
8' 6"
Splice
2-Pls. 67¼" x 1" x 15' 10"
2-" 66" x 1" x 10' 6½"
2-" 16½" x ⅝" x 6' 10½"
2-" 16½" x ⅝" x 6' 5¾"
2-" 15⅜" x ⅝" x 6' 9¼"
2-" 15¼" x ⅝" x 6' 6¾"
4-" 6¾" x ½" x 3' 6½"
Bottom Chord 22-22A
1-Cov. Pl. 50" x 1"
2-" " 22" x ⅞" Top
2-Bars- 8" x 1"
1-Cov. Pl. 50" x 1"
2-" " 22" x ⅞" Bott.
2-Bars- 8" x 1"
2-Pls. 32" x 1"
2-" 32" x 1½"
12 Ls - 8" x 8" x 1" Top, Bott. & Cen.
8-Web Pls. 39⅛" x 2" Sides
1-" " " 48" x 1¼" Cen.

FIG. 16.

to the gusset is similar to the one described for the web members, except that the web is placed alongside of the gusset, and as all such parts of the section which are not in the plane of the gusset extend to the joint. Splice-plates are placed on each side of the web and on both sides of the flange material. Thus, again, most rivets are in double shear, and the gusset is utilized as a splice-plate.

The top chords have full bearing at the joint and, in addition, are fully spliced as regards area, section modulus, and value of the connecting rivets. This was necessitated at a few joints on account of tension stresses. However, it also conforms to the best modern practice of splicing compression members fully, and not relying on their bearing at the joint. A splice is not only subject to the direct compression stresses, but also to bending stresses, due to the function of the chord as a continuous column and to the secondary bending moments caused by the rigidity of the connections. Moreover, experience has shown that, unless the chords are very carefully assembled at the shop, no reliance can be placed on getting perfect bearing at the joints.



JOINT OF BOTTOM CHORD
FIG. 17.

Joints in Bottom Chords.—An unusual type of joint was adopted for the bottom chord. The end of one of the chord members meeting at a joint was faced to a perfect plane, and that of the other member was faced to three planes, as shown exaggerated in Fig. 17, so that, when adjoining chord members were assembled, the joint was tight only over the middle third of the depth of the chord, and each outer third formed a wedge-shaped opening $\frac{1}{8}$ in. wide. These openings gradually closed during erection as the stress increased and as the assembling bolts and drift-pins were gradually replaced by rivets.

The purpose of this joint was to concentrate the bearing pressure over the middle part of the joint, and thus avoid dangerous edge pressures. Experience with other large bridges fully justified this precaution, especially on account of the unusual width of these chords. The fact, however, that the chords were planed with an accurate, specially built, planing machine, and were assembled carefully at the shop, accounts for the accuracy with which they fitted together in the field.

The outer thirds of the joints are spliced 100%; the middle third is spliced only about 50 per cent. The aggregate area of the splicing material of the whole joint is between 70 and 80% of the effective area of the chord, but its section modulus is nearly equal to that of the main section. Counting in the bearing area of the middle third of the main section, the total resisting area at the joint is from 110 to 120% of the chord section.

Size of Rivets.—In conformity with the great size and thickness of the individual parts making up the members, and to reduce the size of the gusset-plates and splice material to a minimum, it was necessary to use rivets of the largest practicable size. Accordingly, $1\frac{1}{4}$-in. rivets were chosen for the bottom chords throughout, and for the field connections of all other truss members. These rivets have grips up to $9\frac{7}{8}$ in. All other rivets in the top chords, web members, floor-beams, track stringers, and in the heavier laterals are 1 in. in diameter. The rest are $\frac{7}{8}$ in. and less.

Lateral and Sway Bracing Between Trusses.—Each panel of the top and bottom lateral truss has two intersecting rigid diagonals, designed to resist both tension and compression (Plate XXIX). There are no transverse struts, except at the panel points near each end, where they form the top and bottom struts of the sway-frames or portals. This lateral system is more rigid and economical than one with struts and slender diagonals designed to resist tension only. All sway-frames have single intersection diagonals. All laterals and members of the sway-frames have a box section, with two or three solid webs, or, in the case of the bottom laterals in the three end panels, one solid middle diaphragm. All open sides have stiff angle latticing.

All solid web-plates are in planes parallel to the plane of the lateral truss or frame to which they belong, so that the laterals can resist effectively the secondary moments and shears due to the distortion of the lateral truss. This principle is well recognized in the design of main trusses, and is equally applicable to the design of bracing in large bridges.

Laterals are commonly proportioned to resist the stresses from wind and other lateral forces. The fact is generally overlooked that the lateral system between compression chords bears to these chords the same relation as the latticing to the different ribs making up a com-

pression member. It forms with the chords a column, which must be strong enough to resist lateral buckling as a whole, and it is evident that inadequate laterals may impair considerably the strength of the bridge as a whole.

A simple approximate rule, similar to that mentioned for the proportioning of the latticing, can be applied to the lateral system, as follows:

If a is the aggregate gross section of the compression chords of all trusses, in square inches (if the chord section varies, an average value may be taken), the transverse shearing force for which the laterals in any panel and their connections should be designed is, approximately, in pounds, $S = 400\ a$, $330\ a$, and $300\ a$, if the lateral system connects two, three, or four trusses, respectively.

It is not necessary to combine this force with the shear from the assumed wind or other lateral forces, but the laterals and their connections should be strong enough to resist either. This prevents the laterals from being made too light where the wind stress is small.

Arch Bearings.—The four arch bearings are of cast steel (Fig. 13). Each bearing has to transmit to the granite skewbacks a total reaction of 30 262 000 lb., or 700 lb. per sq. in. on a bearing area 17 ft. 6 in. square. The upper shoe, which is bolted to the end of the bottom chord, consists of two castings, each weighing 30 tons. The lower face of this shoe is perfectly plane, and bears against the convex cylindrical surface of the lower shoe. This type of bearing produces a rocking motion with little friction under the deformation of the arch.

The radius of the cylindrical surface is $r = 1\,150$ in. The maximum angular motion under live load is approximately 1° 30′ up or down, which produces an eccentricity of only 2.5 in. The pressure per linear inch of line of contact is $p = \frac{30\ 274\ 000}{116} = 261\ 000$ lb. Owing to the elasticity of the metal, the contact is actually over a rectangular area, the width of which is approximately $b = \sqrt{\frac{p\ r}{E}} = 9.5$ in., wherein E is the modulus of elasticity of the material.

The pressure per square inch increases from zero, at the edge of this area, to a maximum of $s = 0.42\sqrt{\frac{p\ E}{r}} = 34\ 500$ lb. per sq. in.,

at the center of the bearing. The average pressure is 27 500 lb. per sq. in., which is safe.

The maximum tangential force is 3 570 000 lb., or 12% of the normal pressure, and is easily resisted by the friction. However, to prevent displacement of the upper shoe, four steel dowels, 5½ in. in diameter, are set into the lower shoe and engage holes in the upper shoe.

The lower shoe consists of eleven castings, arranged in three tiers, in which the joints between the individual castings are placed alternately parallel and at right angles to the plane of the truss, so as to insure proper distribution of the pressure. A 1-in. steel plate is placed between the lower shoe and the masonry. Sixteen anchor-bolts, 2½ in. in diameter and 10 ft. long, secure the lower shoe to the masonry.

The total weight of one complete bearing is 248 tons. For better appearance, the whole bearing is enclosed in a steel hood, which produces the effect of a massive pedestal. No mortar or other bed was used between the base plate and the masonry. The skewbacks were carefully dressed to a perfect plane on which dry cement powder was evenly distributed, before the base plate was placed, so as to fill out any unevenness. The holes for the anchor-bolts were drilled to a template after the exact location of the bearings had been fixed.

The failure of the suspended span of the Quebec Bridge, during its erection in 1916, which is ascribed to the breaking of one of the cast-steel bearings, has brought to the foreground the question as to whether cast steel is a suitable material for bridge construction.

Cast steel for bearings has the advantage that it can be built into forms and thicknesses for which riveted work is impracticable. Large steel castings, however, are difficult to make. Experience has shown that unless great care is taken in casting the metal, internal defects may develop, which are difficult to detect. Further, unless properly annealed, as soon as the cores are removed, or after any subsequent local heating, internal stresses will remain in the casting, and may be large enough to cause breakage under slight shock. This applies particularly to complicated castings, or castings in which the metal varies considerably in thickness and, therefore, does not cool uniformly.

Simplicity and uniform thickness, therefore, should be the aim in designing the castings. Provision should also be made to insure quick removal of the cores; otherwise, their resistance to the free shrinkage

of the metal, while it cools, may cause serious stresses in the latter and permanent defects which may not be removed by the subsequent annealing. Bearings which have to distribute a concentrated load over a certain area should have a height of at least one-half the width of the bearing area and be safely proportioned for the bending and shearing stresses. If these precautions are taken in the design and manufacture, and scrupulous inspection is exercised, there need not be any apprehension concerning the use of cast steel for bearings.

Stringers and Floor-Beams.—Typical details of the floor system are shown in Plates XXX and XXXI. The railroad stringers are 6 ft. deep, and of the ordinary make-up. They are framed into the floor-beams. The connection was designed to allow the stringers to be swung into place after the floor-beams were erected. The stringers are connected in pairs by top and bottom lateral bracing and two sway-frames in each panel.

The floor-beams are heavy box-girders, with two webs, $8\frac{1}{2}$ ft. deep. The webs are joined by top and bottom cover-plates, 42 in. wide, and by vertical diaphragms in the lines of the stringers and horizontal diaphragms opposite the floor-lateral connections. On the suspended portion of the floor the floor-beams are continuous between railing girders and connected to the floor suspenders by 16-in. pins. At the four end panel points, the floor-beams are framed into the truss verticals, and separate cantilever brackets are provided on the outside. One of the floor-beams weighs 86 tons.

Floor-Lateral Truss and Braking Girders.—The floor-lateral truss is in the plane of the bottom flanges of the railroad stringers, the diagonals being riveted to the stringers at intersection points and also to the floor-beams. The diagonals take tension and compression. Each diagonal consists of a horizontal web-plate with two or four angles riveted to its lower side (Plate XXXI). On account of the great unsupported length between the outer stringer and the wind chord, the diagonals are stiffened in that portion by a latticed strut parallel with and 2 ft. above the diagonal, and connected to the latter by lattice angles.

At the expansion joint of the floor, the two diagonals, on the sliding side, are connected to a horizontal gusset-plate which slides longitudinally, but transmits the lateral reaction to a bracket attached to the center of the floor-beam (Plate XXXI). The chords of the

SECTION SHOWING BOTTOM FLANGE OF RAILING GIRDER

42'6" 42'6" 42'6"

1-Pl. .52" x 3/8" x 6'10½"

BOTTOM VIEW - SIDEWALK STRINGERS
Top Lats. 1 L-6" x 4" x 3/8"
Bott. Lats. 1 L-3½" x 3½" x 3/8"
Cross Frame L s-4" x 4" x 3/8"

Stringer Material:
1 Web 42" x 3/8" x 40'4½"
4 Ls 6" x 4" x ½" x 40'4½"
Stiffeners 5" x 3½" x 3/8" x 3'5⅛"

1 Web 72" x ½" x 39'0"
4 Ls 8" x 8" x ½" x 39'0"
Stiffeners 6" x 4" x ½" x 5'10¼"

12 14 16

Center Line of Truss

TOP VIEW

BOTTOM VIEW R.R. STRINGERS

Center Line

BOTTOM VIEW

1-Pl. 50" x ¾" x 13'5"

1-Pl. 46" x ¾" x 13'5"

1-Pl. 21" x 1" x 4'11"
1-" 46" x ¾" x 11'1"
1-" 46" x ¾" x 4'10½"
2-" 21" x 1¼" x 2'8"
4-" 18¾" x 1¼" x 4'0"
2-" 34" x ⅝" x 4'10¼"
2-" 11½" x 1⅛" x 1'6½"

Top Lats. 1 L-6" x 4" x ½"
Bott. Lats. 1 L-3½" x 3½" x ¾"
Cross-Frame L s-4" x 4" x ½"

1-Pl. 33" x ⅝" x 13'5"

1-Pl. 33" x ⅝" x 13'5"

1-Pl. 28" x 1⅛"
4 Ls-6" x 4" x ⅝"

1-Pl. 18" x 1⅛"
2 Ls-6" x 4" x ½"

1-Pl. 18" x ½"
2 Ls-6" x 4" x ½"

2 Ls-6" x 4" x ½"
Lac. Bars 3" x ½" Top
Lac. Ls-3" x 3½" x ½" Sides

Braking Girder
1-Web Pl. 90" x ⅜"
4 Ls-8" x 8" x ½"
2-Cov. Pls. 20" x ⅝" P. L. Top & Bott.
2 " " 20" x ⅝" x 46'3½" Top & Bott.
2 " " 20" x ⅝" x 38'4½" " "
2 " " 20" x ⅝" x 32'4½" " "
2nd Stiffs 32" x L Bear. Pls.
4 Ls-8" x 6" x ½"
1-Pl. 16" x ¾" Stringer Conn.

93'0" c. to c. Railing Girders
60'0" c. to c. Trusses
16'6"

Center Line of Truss

8'0" c. to c. Ls

Center Line of Stringer

Center Line of Stringer

A A

Top of rail

5'4½"
6'0½"
3'0"
1'6"

C.L. Bott. Chord 6-8

Panel Point No. 6

C.L. Bott. Chord 4-6

SECTION A-A
Showing connection of stringer & bottom strut to braking girder and floor beam.

Center Line of Truss

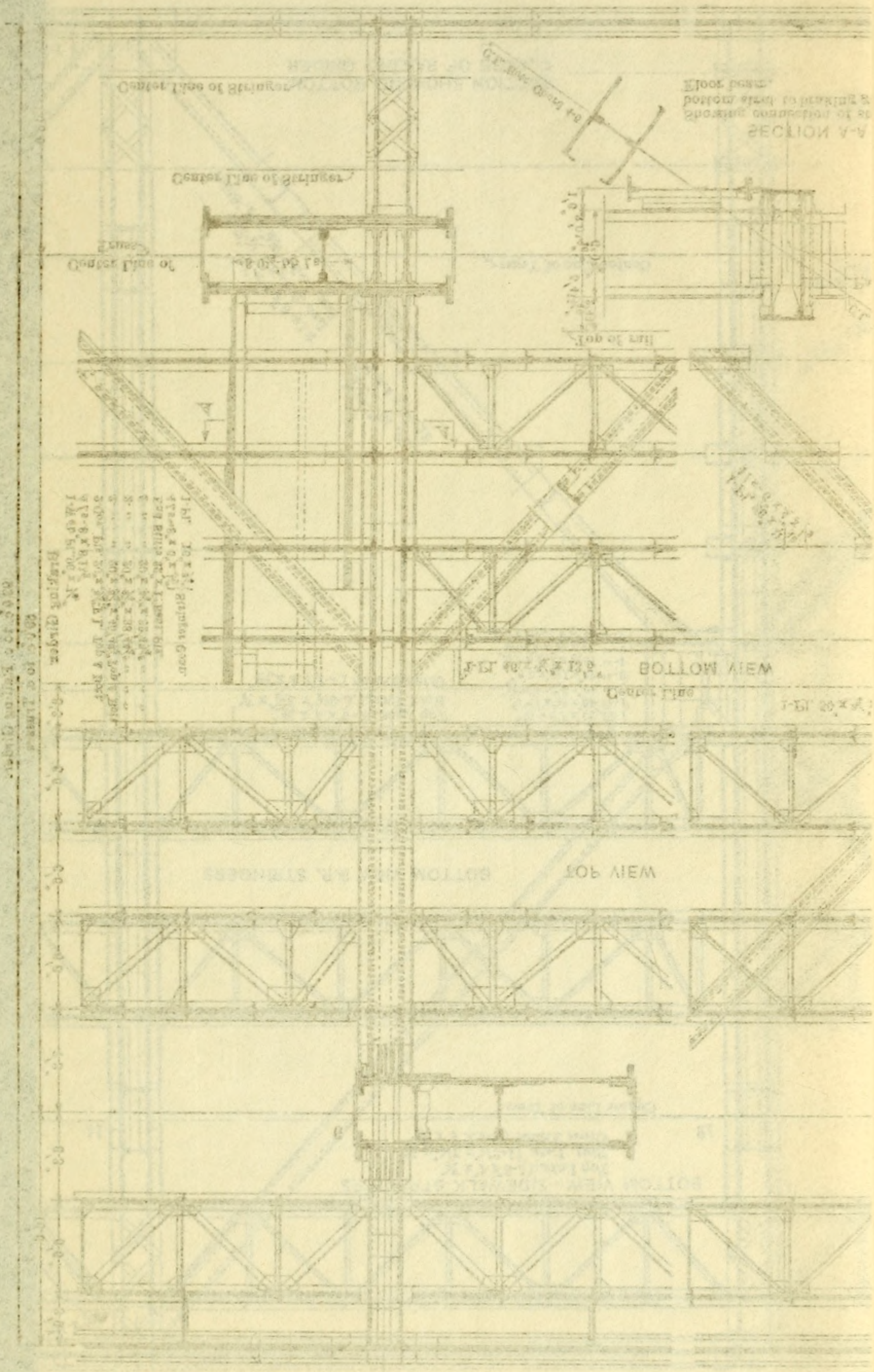
Center Line of Truss

Center Line of Truss

SECTION SHOWING BOTTOM FLANGE OF RAILING GIRDER

1-Pl. 52½" x ⅝" x 6'10½"

DETAILS OF FLOOR SYSTEM AND BRAKING GIRDERS



floor-lateral truss which form the bottom chords of the railing girders have the shape of an inverted T, the vertical web-plate forming the gusset for the web members of the railing girder. To reduce the unsupported length of this chord between floor-beams, the railing girder is connected with the highway stringers by intermediate brackets, one opposite each stringer cross-frame (Plate XXX).

The two braking girders at the intersections of the floor with the trusses are horizontal, single-web plate girders, 8 ft. wide, and placed immediately below the floor laterals (Plate XXXI). These girders are rigidly framed into the bottom chords of the arch trusses (Fig. 14) to which they deliver their reactions. Each railroad stringer is connected to the braking girder by a vertical diaphragm.

Steel Weights.—The weight of the steel superstructure of the Hell Gate Bridge is made up as follows:

Stringers	4 010 000	lb.
Floor-beams	3 870 000	"
Railing girders with wind chords	635 000	"
Floor laterals and braking girders	575 000	"
Total floor system	9 090 000	lb.
Arch trusses	21 640 000	"
Floor suspenders	1 380 000	"
Arch bracing	3 670 000	"
Bearings	1 990 000	"
Total steel weight	37 770 000	lb.

or an average of 37 000 lb. per lin. ft. of bridge. This does not include 470 tons of I-beams in the concrete floor-slabs.

7.—Camber and Deformation of Arch Trusses.

For a large bridge of unusual design, an investigation of the elastic behavior of its trusses under load is essential for a proper judgment of its merits. The elastic line gives at once an indication of the comparative rigidity of a bridge, and as to whether and where large secondary stresses may occur. Further, the calculation of the greatest deflection is necessary for determining the proper camber, in order to satisfy clearance conditions under the bridge, and to establish the desired grade of the floor.

Camber.—The arch trusses of the Hell Gate Bridge were cambered for dead load only by increasing or decreasing the "geometric length" of each member, including the floor suspenders, by an amount equal—but opposite—to its change in length from its dead-load stress, so that, under full dead load, the trusses would assume their true "geometric form", for which the stresses are calculated. This is the proper method of cambering large bridges. There was no need for cambering the trusses for live load, as the deflections have only a negligible influence on the direct stresses in the truss members. The only object was to prevent the top of rail from sagging below a horizontal line under full live load and extreme low temperature, and this was accomplished by establishing an initial vertical curve for the top of rail.

Deflections.—Plate XXXII shows the assumed initial vertical curve, and gives a summary of the vertical deflections for the points along the top of rail. In the calculation for the deflections, the "gross area" of all members was assumed, and the influence of the details, and of the rigidity of the connections, was neglected. The modulus of elasticity was assumed to be 30 000 000 lb. per sq. in., and the linear temperature expansion as 0.0000065. The effect of the elastic deformation of the web members on the deflections of an arch of this type is very considerable, especially for partial load, and cannot be neglected.

Dead-Load Deflections.—The maximum vertical deflection of the floor, due to the total dead load (average 51 000 lb. per ft. of bridge), from the theoretical cambered position, is 8.35 in. at Panel Point 22 (Wards Island side) and 7.13 in. at Panel Point 22 (Long Island side). This difference for the two sides, and the kink at Panel Point 22, Wards Island side, in the cambered position, is due to the fact that, in its cambered position, the truss forms an unsymmetrical, three-hinged arch, and was erected as such, the hinge being at Panel Point 22, Wards Island side, whereas, in the final position, the arch was to assume its true geometric form, which is symmetrical about the center.

Live-Load Deflections.—Plate XXXII (*b*) shows the elastic lines of the floor for a live load of 12 000 lb. per ft. of truss covering one half span and the whole span, respectively, and also the lines connecting the extreme positions of the panel points under most unfavorable positions of the load. The greatest downward deflection is 5.19 in., or $\frac{1}{2\,300}$ of the span length, and occurs at about the quarter point of the

DEFLECTIONS OF ARCH TRUSSES

Note:- All Deflections in these Diagrams are those of Floor Level, and are given in Inches

\+ Denotes Downward Deflections, — Upward Deflections.

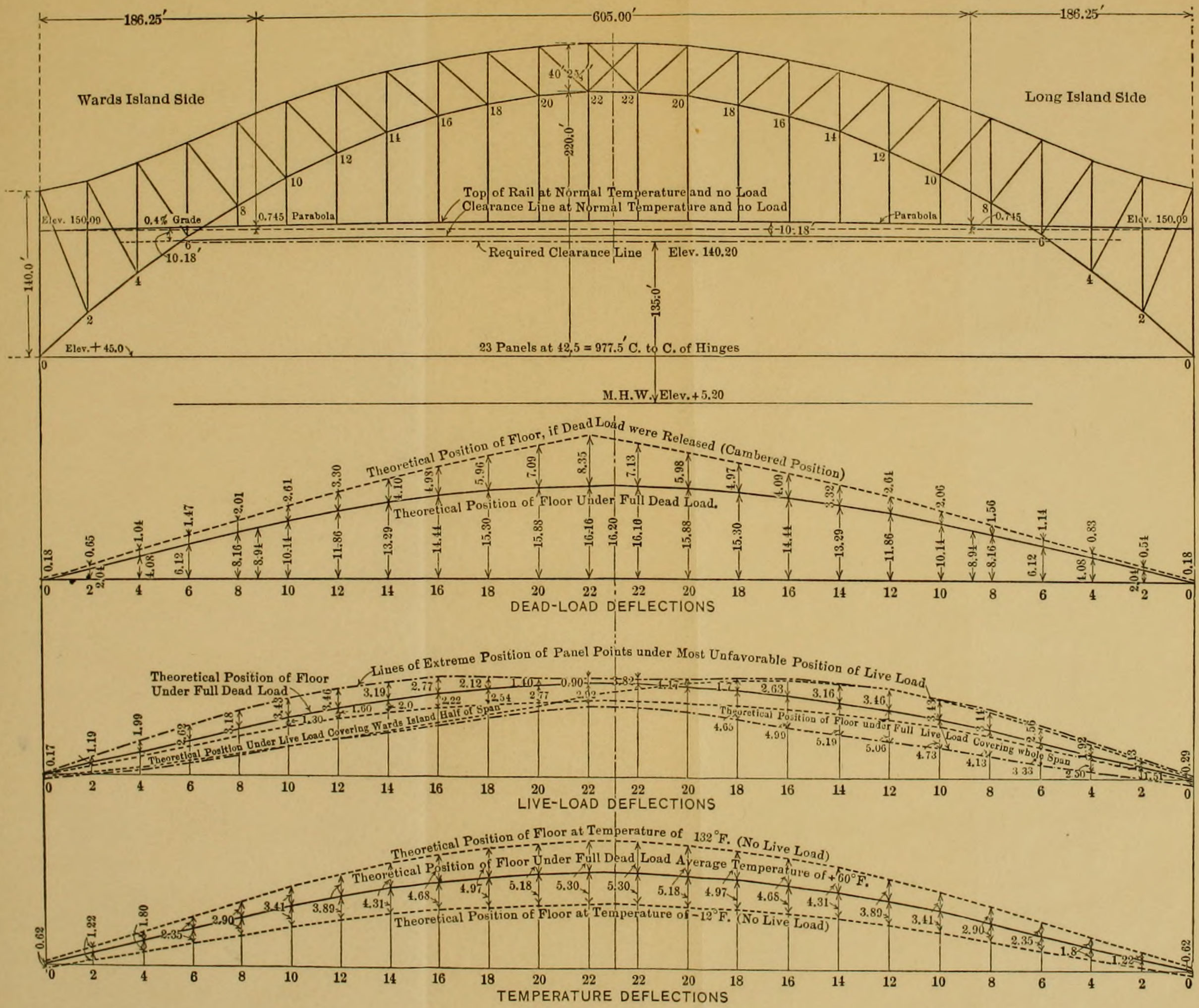
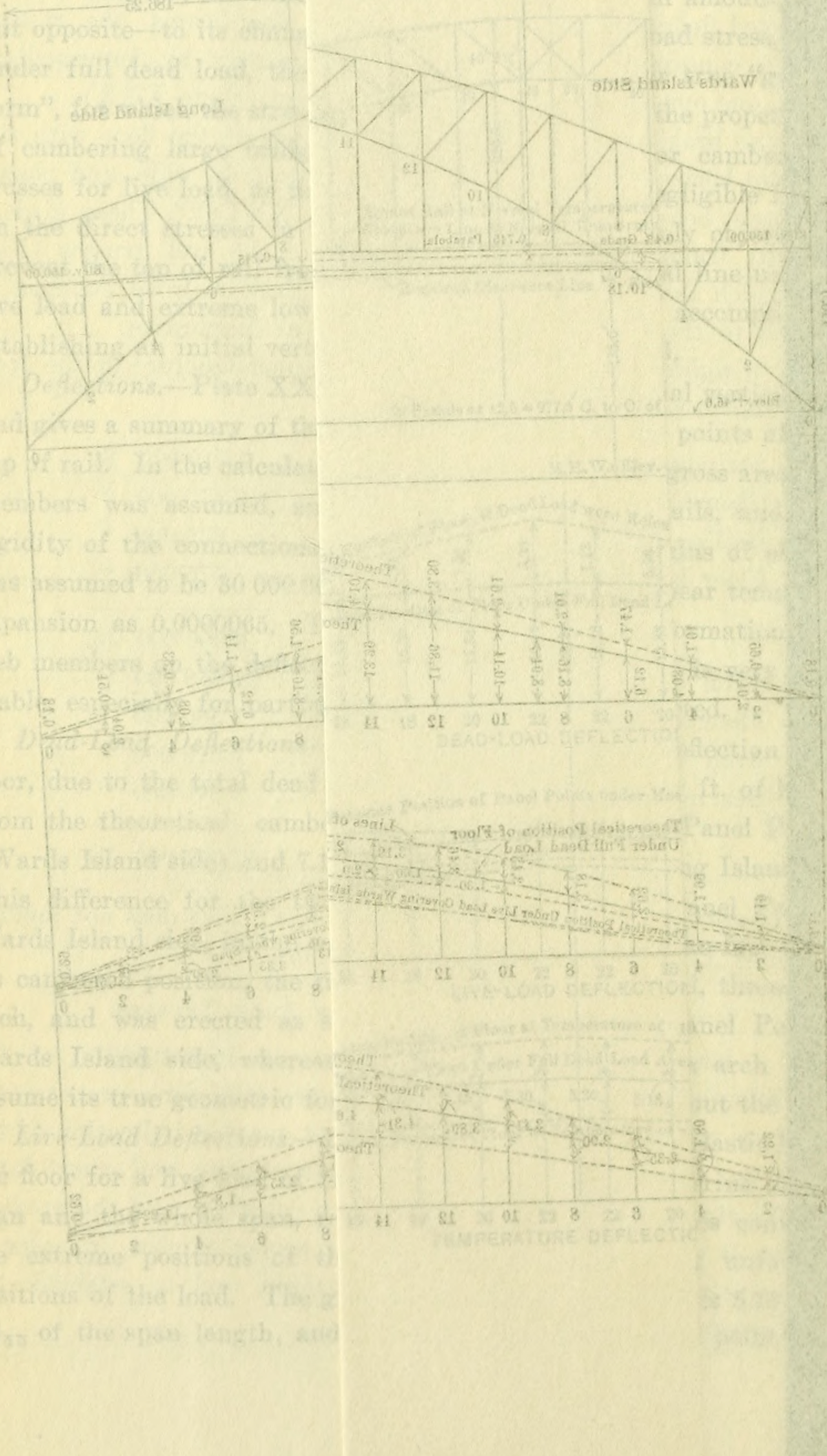


PLATE XXXII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LXXXII, No. 1417.
AMMANN ON
HELL GATE BRIDGE.



span under a load covering about one-half of the span. The quarter point on the other half of the span rises 3.46 in. under the same loading condition. The greatest deflection at the center is 3.82 in., or $\frac{1}{3\,100}$ of the span length, and occurs under a load covering approximately the middle half of the span. The maximum live-load deflection at the center of a cantilever bridge of the same span and loading would be about 8 in., and that of a simple span about 5 in.

The elastic lines are smooth curves, or rather polygons, free from local kinks, such as occur at the hinges of cantilevers or three-hinged arches. They are also free from sharp corners, such as are produced over the intermediate supports of cantilevers, or at every panel point of trusses with subdivided panels. From this it may be concluded that the secondary stresses in the arch trusses are comparatively small, which fact is substantiated by the calculations of these stresses.

Temperature Deflections.—A change in temperature from the normal (60° Fahr.) produces an elastic line of approximately parabolic shape with greatest ordinate at the center of the span (Plate XXXII (*c*)). The latter rises or falls, respectively, by 0.74 in. for each degree of rise or fall in temperature, the total deflection for a change of 72° Fahr. being ± 5.3 in., or $\frac{1}{2\,250}$ of the span length. This additional deflection from temperature change is not objectionable, as it has no bearing on the rigidity of the bridge. The temperature deformation is the result of the change in length of each member, first, due to unconstrained expansion or contraction, and, second, due to the elongation or compression from temperature stress. The former effect raises or lowers each point in proportion to its height above the bearings.

8.—Material.

Quality of Steel.—All material in the steel superstructure of the Hell Gate Bridge and Approaches is rolled, forged, or cast steel, made by the open-hearth process.

The following four grades of steel were used:

a.—Hard steel for all rolled parts and pins of the Hell Gate Arch Bridge;

b.—Structural steel for all rolled parts and pins of the Approaches;

c.—Rivet steel for all rivets;

d.—Cast steel for all castings for bearings, etc.

These different grades had to conform to the chemical and physical requirements given in Table 1.

TABLE 1.—Chemical and Physical Requirements for Steel.

	Hard steel.	Structural steel.	Rivet steel.	Cast steel.
Phosphorus, max. { basic	0.04	0.04	0.04	0.05
Phosphorus, max. { acid	0.06	0.06	0.04	0.08
Sulphur, max	0.05	0.05	0.04	0.05
Ultimate tensile strength, in pounds per square inch { max	76 000	70 000	58 000	
Ultimate tensile strength, in pounds per square inch { desired	71 000	66 000		
Ultimate tensile strength, in pounds per square inch { min	66 000	62 000	50 000	65 000
Yield point, min	38 000	35 000	28 000	33 000
Elongation, min. { in 2 in. for cast steel.. in 8 in. for other steel.	*	22%	28%	20%
Character of fracture	Silky.	Silky.	Fine, silky.	Silky or fine granular.
Cold bend without fracture	*	180° around pin of thickness of test piece.	180° flat.	90° around pin of thickness of test piece.

* Minimum elongation for "hard steel": 1 400 000 divided by ultimate strength for thicknesses up to and including ¾ in.; 1% less for each additional ⅛ in. in thickness, with a limit of 16% for thicknesses up to and including 2 in. and 15% for thicknesses greater than 2 in.

Cold-bend test for "hard steel": 180° around a pin of double the thickness of the test piece for material up to and including ¾ in., 180° around a pin three times that thickness for material of greater thickness than ¾ in.

The chemical analysis was made from test ingots during the casting of the melt, and included the determination of carbon, manganese, and silicon, in addition to phosphorus and sulphur. The carbon ranged between 0.27 and 0.34% in the hard steel and between 0.23 and 0.28% in the structural steel, and manganese from 0.52 to 0.64 and 0.36 to 0.61%, respectively. Special stress was laid on sufficient discard being made from the ingot to insure sound material free from piping and excessive segregation.

The physical tests were made on standard test specimens cut from the rolled, forged, or cast piece, in the latter two cases after annealing. The specifications required that at least two tensile and two bending tests be made from each heat of 25 tons or less, at least three tests from each heat up to 40 tons, and four tests for heats exceeding 40 tons. If the material rolled from the same heat varied in thickness ⅜ in., or more, the foregoing number of tests were to be made from the thickest as well as from the thinnest pieces. In all about 7 000 tests, covering

accepted heats only, were made, or an average of eight tests per 100 tons.

Full-Sized Eye-Bar Tests.—About 1 900 tons of structural steel eye-bars, 16 in. wide and from $1\frac{7}{8}$ to $2\frac{3}{8}$ in. thick, with forged heads $37\frac{1}{2}$ in. in diameter, and 16-in. pin-holes were used in the Little Hell Gate Bridge. Twenty-one full-sized bars were tested, after annealing, and had to show the following results: Ultimate strength: 56 000 to 68 000 lb. per sq. in.; elastic limit: minimum, 33 000, and maximum, 38 000 lb. per sq. in.; minimum elongation in 10 ft.: 12 per cent.

Table 2 gives a summary of the full-sized tests, together with the corresponding unannealed specimen tests.

TABLE 2.—Results of Twenty-One Full-Sized Eye-Bar Tests.

	Annealed Full-Sized Bars.			Unannealed Specimen.			Difference between average full size and specimen.
	Min.	Average.	Max.	Min.	Average.	Max.	
Ultimate strength, in pounds per square inch	58 500	61 800	64 700	64 300	67 800	72 600	6 000
Elastic limit, in pounds per square inch	32 900	34 600	36 200	35 700	38 800	41 000	4 200
Elongation: percentage in 10 in	17.8	22.8	29.8	.....	.....	.....	.....
Elongation: percentage in 12 in	31.6	39.3	49.3	in 8 in., 21.2	26.4	30.0	.....
Reduction: percentage	29.1	36.0	43.2	30.0	36.9	43.0	.....

All specimens broke in the body of the bar.

Selection of High-Carbon Steel for Arch Bridge.—A high-grade carbon steel was adopted for the Hell Gate Bridge, principally because it permitted higher unit stresses, which resulted in a considerable saving in steel. The consequent saving in cost was offset only to a small extent by the slightly greater unit price of hard steel over ordinary structural steel. This slightly greater price is not due so much to greater cost of manufacture at the mill and shop as to the fact that, for any special steel, the quantity to be scrapped, on account of not fulfilling the requirements, is greater than for ordinary material. The contractor was given the option of furnishing, at the same price, either hard or structural steel for the floor system and suspenders, which parts have been designed with the unit stresses allowed for structural steel. He preferred, however, to use the same material, that is, hard steel, for the whole bridge. Further, with ordinary steel, the sections

of truss members, gusset-plates, splices, and size of rivets would have become much larger and, in some cases, excessive. Even with the hard steel as used the dimensions of some of these details are up to the practicable limits.

The hard steel showed an elongation of generally more than 20%—in the average about 25%—and an average reduction of 45%, and no difficulties were experienced in complying with the bending test. This proves that the material is tough and ductile. It required specially hard tools for drilling and reaming, and these operations were somewhat slower than with ordinary carbon steel, but otherwise no serious difficulties were experienced in the fabrication.

Adaptability of Alloy Steel.—The use of nickel steel for the arch trusses was also taken into consideration, but it was found that, with permissible unit stresses 50% higher than for structural steel, there would have been no saving in cost. The unit price for nickel steel obtaining at that time was about $40 per ton higher than for carbon steel. Moreover, unless nickel steel, or any other alloy steel, would have resulted in an appreciable saving, or other important advantages, carbon steel would still have been given the preference. Carbon-steel trusses, on account of their greater weight or inertia and smaller elastic deformations, are less subject to vibrations from live load, and, consequently, insure greater stiffness and permanency. Moreover, as the proportion of their dead weight to live load is greater, they will sustain safely a comparatively greater future increase in live load than alloy steel.

9.—Loads and Unit Stresses.

The safety and useful life of a bridge depend essentially on the proper assumption and co-ordination of the various loads and permissible unit stresses. In the design of bridges of ordinary span and capacity, the assumption of loads and unit stresses is a matter of well-established routine, whereas, for a large bridge, for which there are few or no precedents, it is a complex problem, which must be solved largely by judgment. The more accurate and complete the load assumptions, the higher can be the permissible unit stresses. It is safer and more in conformity with true economy to make sure that under the most unfavorable, but possible, conditions and combinations of loading the elastic limit of the material or a comparatively high proportion thereof will not be exceeded, than to trust to ordinary

loading conditions and low unit stresses and allow a large, but uncertain, margin of safety. This principle has been followed in the design of the Hell Gate Bridge. All possible forces have been taken into consideration, namely, dead load, live load, vertical impact, lateral force or impact, longitudinal force from traction and braking of trains, wind pressure, and forces due to change in temperature, and comparatively high unit stresses (from five-eighths to three-fourths of the minimum elastic limit) have been allowed for the combination of these forces.

Dead Load.—Before determining on the final sections of the truss members, the dead load was carefully calculated and checked. From the preliminary designs and detailed drawings the panel concentrations were ascertained very closely. After the work had been let a recalculation was made from drawings worked out by the contractor sufficiently in detail to allow the ordering of material therefrom.

The dead load, in pounds per linear foot of bridge, is made up as follows:

Tracks and ballast	4 900	lb.
Steel concrete and timber flooring	8 100	"
Conduits, cables, wires, etc	1 000	"
Total tracks and flooring, etc	14 000	lb.
Steelwork, average	37 000	"
Total average dead load per foot of bridge	51 000	lb.

The dead load assumed for the final calculation is 51 900 lb., average, and varies from 45 000 lb. at the center to 62 000 lb. at the ends. The excess over the actual dead load is due to the excess of the assumed over the actual weight of the conduits. For the weight of steelwork (exclusive of steel in floor slabs), the stresses have been calculated by assuming the arch trusses to be three-hinged, under which condition they were erected (middle hinge at bottom chord, Point 22, Wards Island side, Plate XXVIII). The stresses for the remainder of the dead load, as well as for all other forces, have been calculated for the final two-hinged condition.

Live Load.—The arch bridge and approaches are designed for the following live load:

1.—All bridges and viaducts of the approaches, and the floor system and floor suspenders of the arch bridge, for Cooper's E-60 on each of the four tracks, or an alternative three-axle load of 70 000 lb. on each axle wherever this causes greater stresses (Fig. 18).

2.—The arch trusses for a uniform load of 6 000 lb. per ft. of track, or 24 000 lb. per ft. of bridge, placed in the most unfavorable position in either a single stretch or in two separated stretches, when the latter condition gives a greater stress.

The assumption of a uniform train load for the arch trusses, instead of the engine concentrations, was justified in view of the highly improbable, and almost impracticable, condition of maximum engine and car loads, in the most unfavorable position simultaneously on all four tracks. This hypothetical condition would have required addi-

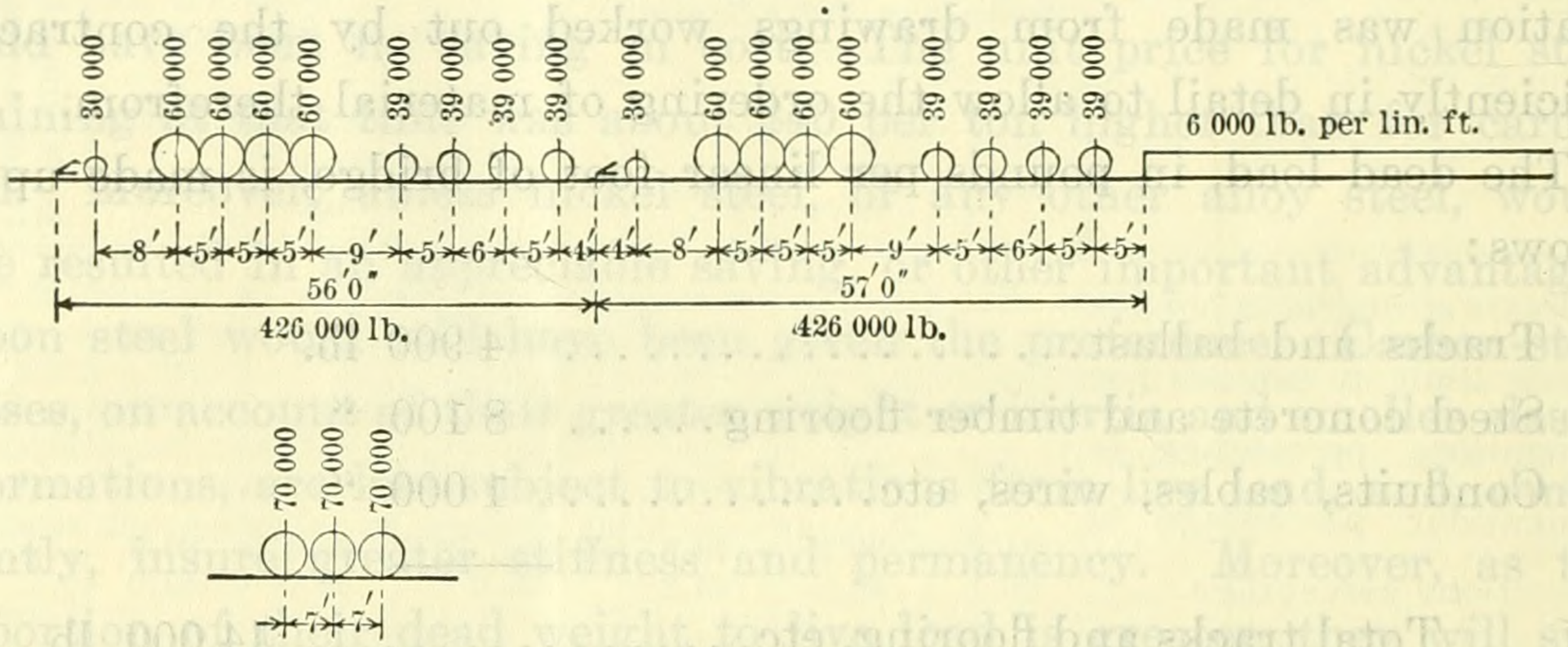


DIAGRAM OF ASSUMED LIVE LOAD

FIG. 18.

tions up to about 5% to the sections of some of the truss members. An E-75 loading on each track in the most unfavorable position, but without impact, would have increased the sections of a few of the heaviest bottom chord members by not more than 3½%, the sections of all the other members being ample. Such a heavy load, if at all possible, with present limits of gauge and clearance, represents probably the heaviest future freight trains, and as these will move comparatively slowly, the impact effect of which on the truss members will be negligible, it is considered, therefore, that the bridge will carry safely any possible future moving load.

Impact.—The impact stresses, or vertical dynamic effects of the locomotives and cars, have been determined according to Lindenthal's formula, as published and fully explained in an article* by its author.

* *Engineering News*, August 1st, 1912.

Since the appearance of that article, the formula has been modified slightly so as to be applicable automatically to bridges having two or more tracks.

The formula in its final form, applied to E-60 loading, is:

$$I = \frac{L^2}{D + L} \times \frac{1\,200 + \frac{a}{n}}{600 + 4a}$$

wherein, D = stress from dead load, in pounds,
L = stress from live load, in pounds.
a = length of train behind locomotive tender for position of maximum stress, in feet,
n = number of tracks loaded for maximum stress.

The modification of the formula consists of the introduction of the value, n, and is based on the assumption of full impact from the locomotives on all tracks, but impact from the cars on one track only. The principal object sought in deriving this formula was that it should be applicable equally to all kinds of bridges, of the shortest as well as the longest spans, to bridges with open or solid ballasted floors, and to steel or concrete bridges; whereas most of the existing formulas have only limited usefulness, particularly the widely adopted formula of the American Railway Engineering Association. The latter formula gives excessive results for long spans and heavy ballasted bridges, and insufficient values for very short stringers, rails, ties, etc., on which the dynamic effect of the driving wheels is unquestionably more than 100 per cent. Lindenthal's formula, in combination with the comparatively high permissible unit stresses, secures economical and well-proportioned structures. Seemingly, it is complicated, but, by tabulation or graphical diagram, its application is very simple.

Lateral Force from Live Load.—The lateral force or lateral impact, due to the swaying motion of fast-moving trains on tangents, or the centrifugal force on curves up to 2°, has been assumed at 600 lb. per lin. ft. of single track. For curves sharper than 2°, 300 lb. were added for each additional degree up to 6 degrees. These forces were increased by 50% for each additional track. This gave, for a four-track bridge on tangent, a total lateral force of 1 500 lb. per ft.

The lateral force occurs simultaneously with the live load, and may act at the same time as the wind pressure, and it is proper, therefore, to provide for it in the proportioning of the laterals as well as the

main truss members. The foregoing value of the lateral force, for a four-track structure, is probably higher than any which may ever actually occur, but it tends to secure a strong and rigid lateral system along the floor. As far as the writer knows, this is the first attempt to make adequate provision for the lateral force from trains on tangents, and to separate it clearly from the wind pressure. The American Railway Engineering Association Specifications of 1914, in this case, would call for a total combined lateral and wind force of only 800 lb. per lin. ft. of floor.

Wind Pressure.—Wind pressure was assumed as a moving load of 500 lb. per lin. ft. of bridge at track level, irrespective of the number of tracks, plus a static load of 30 lb. per sq. ft. on all such vertical surfaces of the unloaded bridge as are exposed at an angle of between 20° above and 20° below the horizontal, or at an angle of 45° from the axis of the bridge. This resulted, for the Hell Gate Arch, in an average wind pressure of 600 lb. per lin. ft. of horizontal projection of top chord, 1 000 lb. per lin. ft. of horizontal projection of bottom chord, and 1 500 lb. per lin. ft. of floor (inclusive of the 500 lb. moving wind pressure), or a total of 3 100 lb. per lin. ft. of bridge. Adding to this the lateral force of 1 500 lb., it is seen that, with a total force of 4 600 lb. per ft. of bridge, ample lateral strength, rigidity, and stability are secured.

Longitudinal Force from Traction and Braking.—The longitudinal force, acting along the rail, from traction and braking has been assumed either at 15 000 lb. for each of the eight driving axles of the two locomotives (25% of the load on each driving axle), or at 1 000 lb. per lin. ft. of train (approximately 15% of the average weight of the train), whichever gave the greater results. For the four-track bridges this force was assumed as acting on two tracks only, in view of the practical impossibility of its acting simultaneously on all four tracks in the same direction.

Recent tests with improved air-brake equipment, yielding a considerably increased brake power, have led to suggestions that structures be designed for a higher braking force than has been the practice heretofore. The effect of the more powerful brakes, that is, the greater force exerted by the brakes to the wheels, is to reduce the time in which a stop can be made without causing dangerous slipping of the wheels on the rails; in other words, the braking force between wheel

and rail attains its maximum value in a shorter time. Its amount, which depends solely on the friction between rail and wheel, is not increased. For reasons of safety, the action of the brakes will never be so sudden that the braking force at the rail acts with impact, that is, the braking force can always be regarded as a static force which increases gradually to its maximum value.

The friction between wheels and rails has been determined variously at from 15 to 30% of the total vertical load on the wheels to which brakes are applied, the percentage depending on the smoothness of the rail surface under varying climatic conditions. As the greatest possible braking force of the two heaviest trains will rarely, if ever, be applied to the bridge, the assumed longitudinal force appears to be ample, no matter how efficient the brake equipment may be made in the future.

Temperature Stresses.—The stresses from temperature have been determined for a variation of $\pm$ 72° Fahr. from the normal temperature of 60° Fahr. The temperature stresses are greatest in the center panels of the top chord of the arch trusses, where they attain values of 4 000 lb. per sq. in., or about 40% of the live-load stresses.

Total Stresses.—All members were proportioned for the following combination of stresses:

1.—For members which carry dead and live load, the "total stress" was obtained by adding the stresses from dead load (D), live load (L), impact (I), lateral force (Lat.), and the so-called "excess stress" (Exc.). This excess stress is the sum of the stresses from wind pressure (W), braking force (B), and temperature (T), less 20% of the sum ($D + L + I +$ Lat.). This is equivalent to allowing up to 25% higher unit stresses for the sum ($D + L + I +$ Lat. $+ W + B + T$) than for the previously mentioned "total stress", the percentage decreasing with increasing value of ($W + B + T$).

2.—For members which carry no dead and live load, the "total stress" was made equal to ($W + B + T +$ Lat.).

The "total stress," divided by the permissible unit stress, gave the minimum required area.

Permissible Unit Stresses.—The permissible unit stresses assumed are given in Table 3.

The stresses in Table 3 are higher than those allowed by most specifications, notably those of the American Railway Engineering

TABLE 3.—PERMISSIBLE UNIT STRESSES ASSUMED.

		For Trusses and Bracing of Hell Gate Bridge. (Hard steel). In pounds per square inch.	For Approach Spans and Floor System and Suspenders of Hell Gate Bridge. (Structural steel). In pounds per square inch.
Axial tension, net section		24 000	20 000
Bending on extreme fiber of beams, girders, and steel castings, net section		.....	20 000
Axial compression, net section :			
(*a*) Closed section, or section with two diaphragms, or one diaphragm and two planes of latticing	$\frac{l}{r}$ = 20	24 000	20 000
	40	23 000	19 000
	60	22 000	17 000
	80	20 000	15 000
	100	18 000	14 000
	120	15 000	12 000
(*b*) Half-open section, with one cover and one latticing, or with one diaphragm without latticing	$\frac{l}{r}$ = 20	23 000	20 000
	40	22 000	18 000
	60	20 000	16 000
	80	18 000	14 000
	100	16 000	13 000
	120	14 000	11 000
(*c*) Open section, with two or more planes of latticing	$\frac{l}{r}$ = 20	22 000	19 000
	40	20 000	17 000
	60	18 000	15 000
	80	16 000	13 000
	100	14 000	12 000
	120	12 000	10 000
Shearing stress :			
On plate girders, net section		.....	15 000
Shop rivets and pins		15 000	
Field rivets and turned bolts		12 000	
Bearing stress :			
Pins		24 000	20 000
Shop rivets		30 000	
Field rivets and turned bolts		24 000	
Pressure on :			
Expansion rollers per linear inch		Diameter of roller, in inches, × 1 000	
Granite masonry		800	800
Concrete masonry		600	800

Association. In combination with Lindenthal's impact formula, however, they result in heavier bridges for spans up to about 250 ft. and lighter ones for longer spans. The assumption of the permissible compression stress per unit of net area, that is, with rivet holes deducted, is unusual. Most specifications base the compression stress on the gross section, the assumption being generally made that the rivet shanks replace the compression value of the metal cut out for the holes. This assumption is proper, provided the rivet fills

the hole perfectly, and its metal has the same elastic properties as that of the member. To a certain extent, also, the friction of the rivet heads on the member makes up for loss in compression strength due to the holes.

These conditions, however, are not always fulfilled, and, moreover, compression members are subject to shearing stresses and, near the point of failure, possibly even to tension stresses, both of which cannot be resisted by the rivets, or only to a limited extent. It is evident, therefore, that rivet holes, even after riveting, decrease the strength of a compression member. This decrease may not be appreciable in ordinary compression members in which the rivets have short grips, generally fill the holes well, and develop a comparatively high friction under their head. It is to be assumed, however, that in heavier members the decrease in strength, due to the holes, is greater on account of the longer grip of the rivets and the probability of less perfect rivets.

The extent to which compression members are weakened by the holes can only be determined by a systematic series of comparative tests, embracing both light and heavy sections. In the absence of sufficient experimental data, it is advisable to remain on the safe side.

Another unusual feature of these specifications is the diversity of permissible stresses for different types of compression members, greater stresses being allowed for members having solid diaphragms or cover-plates than for those having flanges which are connected by latticing. The usual compression formulas do not discriminate in this respect, the only provision, in most specifications, is that the portion of the flange between connections of the latticing shall be as strong as the member as a whole. This is usually interpreted to mean that the ratio, $\frac{l}{r}$, of that portion of the flange shall not be less than for the member as a whole, and implies that the strength of the member is the same for any value, $\frac{l}{r}$, of the flange, within the value, $\frac{l}{r}$, of the member as a whole. This assumption is fallacious. The flange of a member, subject to buckling, should have an unreduced compressive value, the same as the flange of a beam subject to bending. If it has not, that is, if the flange itself has a reduced buckling strength, then the strength of the compression member as a whole is doubly reduced.

There are other weaknesses of integral parts, such as insufficient thickness of webs, cover-plates, outstanding flanges of angles, excessive rivet pitch, etc., all of which tend to reduce the strength of a compression member as a whole. It is evidently impossible to cover all these influences by cut-and-dried rules or formulas, particularly in view of the lack of sufficient comparative experimental data, but it is obvious that some distinction should be made in the permissible unit stresses for different types of sections, types of latticing, etc., and not merely for different values of the ratio, $\frac{l}{r}$.

Secondary Stresses.—Care was taken in designing and detailing all the bridges to avoid large secondary stresses, and, in a few cases, where this was not possible, additions to the sections were made.

In the Hell Gate Arch the maximum calculated secondary stresses which occur simultaneously with the maximum primary stresses are as follows, in pounds per square inch:

	Dead load.	Live load.	Combined dead and live load.
Bottom chord.....	± 1 300	± 1 300	± 2 100
Top chord.....	+ 2 100	± 3 100	+ 5 200
	— 2 500		— 5 600
Diagonals	± 5 000	± 2 800	± 6 300
Verticals	± 2 200	± 4 400	± 4 800

These, being extreme fiber stresses, can safely be assumed to be covered by the margin of safety of the primary or axial stresses, especially as the greatest secondary stresses occur in members which have considerable excess of section. Moreover, stress measurements made during and after erection indicate that the actual secondary stresses are below the calculated ones.

No attempt, therefore, was made in the fabrication and erection of the trusses to eliminate or even reduce the secondary stresses. In fact, this would have been a very difficult problem on account of the rigidity of the members, particularly of the bottom chord. This question is discussed more fully under "Workmanship and Fabrication."

Erection Stresses.—The greatest stresses in the arch trusses during erection were: + 18 600 and — 16 600 lb. per sq. in. $\left(\frac{l}{r} = 46\right)$ from dead load and erection traveler without wind, and + 20 400 and — 19 700

lb. per sq. in. $\left(\frac{l}{r} = 46\right)$ from the dead load, traveler, and an assumed wind pressure of 30 lb. per sq. ft. of exposed surface.

These stresses were well within the safe limits allowed for the total stresses in the completed bridge. Only in the end panel of the top chord was the erection stress greater than the total stress in the permanent bridge, but these members required a certain minimum section which was sufficient for the erection stress. No extra metal was required, therefore, in the arch trusses for erection purposes. The maximum values allowed for the erection stresses without wind in the temporary back-stays (structural steel) were the same as those allowed for the total stresses in the finished structures (20 000 lb. per sq. in. tension) and 25% more with a wind pressure of 30 lb. per sq. ft. of exposed surface (25 000 lb. per sq. ft. tension).

10.—Workmanship and Fabrication.

The specifications were prepared by the Consulting Engineer with a view to securing the best class of workmanship which current practice and possible improvement therein would admit. The steelwork for the arch bridge was manufactured at the Ambridge Plant of the American Bridge Company, whose shop management gave careful study to new shop methods and the use of more efficient tools and machinery, as necessitated for the unprecedented work.

Punching, Reaming, and Drilling.—The punching, reaming, and drilling of rivet holes was governed by the following clauses in the specifications:

"Steel up to and including a thickness of $\frac{1}{2}$ in. may be punched without subsequent reaming, unless the same shall be necessary to insure smooth holes of the assembled parts to be riveted together.

"Steel up to and including $\frac{5}{8}$ in. in thickness shall be punched with holes $\frac{1}{8}$ in. smaller and then reamed after assembling $\frac{1}{16}$ in. larger than the nominal size of the rivets shown on the drawings.

"Structural steel up to and including $\frac{3}{4}$ in. in thickness may be punched and reamed, provided that in no instance shall the punched rivet hole be more than $\frac{1}{16}$ in. in diameter in steel $\frac{3}{4}$ in. thick. This applies to any diameter of rivet, whether $\frac{7}{8}$ in. or larger. After assembling the holes shall be reamed out for the removal of the bruised metal to a diameter $\frac{1}{16}$ in. larger than the nominal size of the rivet. A misfit or overlapping of the punched holes in the assembled pieces greater than $\frac{1}{16}$ in. will not be allowed, and when the Engineer is satisfied, after reasonable trial, that the punching is not within that limit

of accuracy then punching must be discontinued and the holes drilled as per paragraph 55.

"Rivet holes of more than $\frac{5}{8}$ in. in thickness shall be either drilled from the solid $\frac{1}{16}$ in. smaller and then reamed after assembling $\frac{1}{16}$ in. larger than the nominal size of the rivets shown on the drawings, or the rivet holes shall be drilled from the solid after assembling $\frac{1}{16}$ in. larger than the nominal size of the rivet.

"Rivet holes in members composed of material of two or more thicknesses, any one of which is greater than $\frac{5}{8}$ in., shall be punched or drilled, respectively, as specified in paragraphs 54 and 55, and after assembling reamed to proper size, or the rivet holes shall be drilled from the solid after assembling $\frac{1}{16}$ in. larger than the nominal size of the rivet.

"All reaming or drilling of holes in assembled parts shall be done with the various pieces bolted together in their respective relative and exact positions. After reaming, every hole shall be entirely smooth, showing that the reaming tool has everywhere touched the metal.

"All drilling and reaming of holes shall be done dry with self-hardening tool steel.

"After the drilling or reaming is completed, a special reamer shall be run over both edges of every rivet hole to remove the sharp edges and burrs and make a fillet $\frac{1}{16}$ in. deep in each rivet head. When necessary the work shall be taken apart and any shavings between pieces carefully removed."

On account of the thick material composing the truss members of the arch bridge, nearly all the holes in these members had to be drilled. The general procedure was as follows: After the holes had been laid out with wooden templates and punch-marked, from 10 to 15% of all of them were drilled from solid to a diameter of $\frac{11}{16}$ in. The parts were then assembled and bolted up with $\frac{5}{8}$-in. tack-bolts. All the other holes were then drilled from the solid to full size, except those for the field rivets, which were also drilled to a diameter of $\frac{11}{16}$ in. The holes of the $\frac{5}{8}$-in. tack-bolts were then reamed to full size, the bolts being gradually replaced by larger ones, and finally the sharp edges of the holes were reamed off. The member was then riveted.

For the bottom chord members of the arch, the two webs with their flanges had first to be assembled and riveted separately and then joined and riveted to the middle diaphragm and finally the top and bottom cover-plates were added. All reaming and drilling of

the hard steel was done with tools of special high-speed steel. Lubricating with oil was not permitted, but it was found necessary to use occasionally a few drops of soap water to reduce the friction, particularly in drilling through thick material.

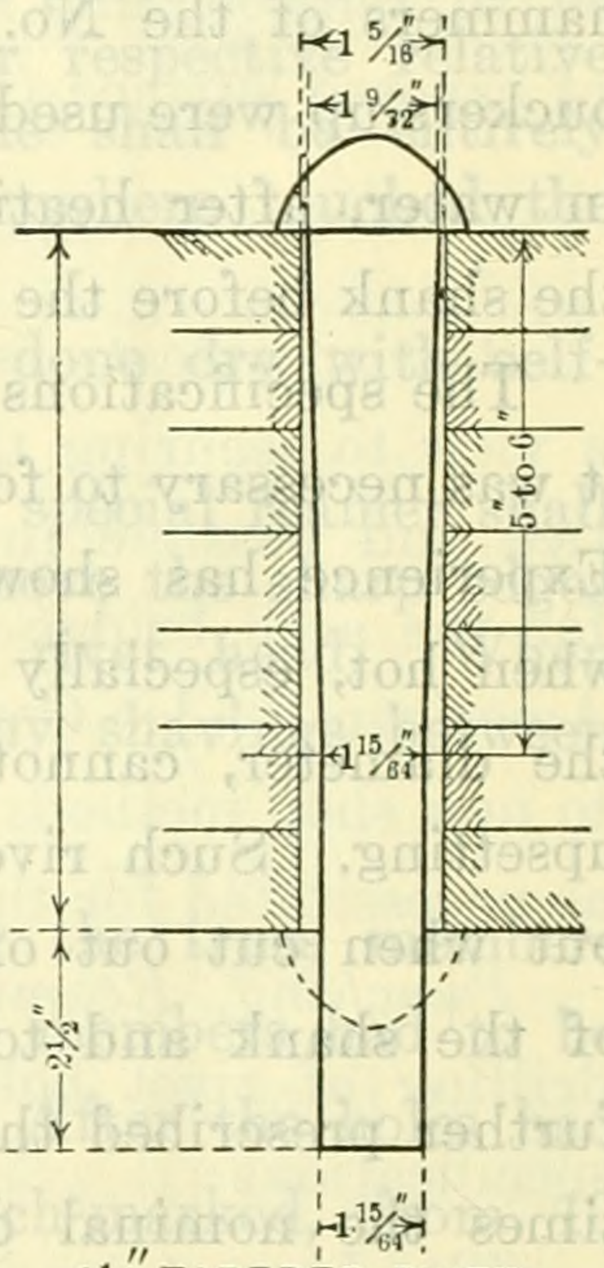
Rivets and Riveting.—There are, in the whole arch bridge, about 840 000 shop rivets and 334 000 field rivets, of which 400 000 are $1\frac{1}{4}$ in. in diameter. Most of the 1- and $1\frac{1}{4}$-in. shop rivets were driven with hydraulic riveting machines having a pressure capacity of 100 tons (100 lb. per sq. in.). The output of one of these machines is about 3 500 rivets per day. For all heavy field rivets and such shop rivets as could not be driven with pressure machines, pneumatic riveting hammers of the No. 90 Boyer type with 9-in. stroke and pneumatic buckers-up were used. The points of the long rivets had to be dipped in water, after heating, so as to insure more complete upsetting of the shank before the head was formed.

The specifications required the rivets to be of such a diameter that it was necessary to force them into the holes with a hammer when hot. Experience has shown that rivets which drop easily into the holes when hot, especially those of long grip exceeding about three times the diameter, cannot be relied on to fill the holes completely after upsetting. Such rivets may seem tight when tested with a hammer, but when cut out often show incomplete upsetting near the middle of the shank and toward the shop-formed head. The specifications further prescribed that rivets with a grip equal to, or exceeding, four times the nominal diameter should be tapered so that the base of the rivet should be $\frac{3}{64}$ in. larger and the point $\frac{1}{64}$ in. smaller than the nominal diameter.

Experiments showed that such rivets, when smooth and of perfect size, and the holes exactly $\frac{1}{16}$ in. larger than the nominal diameter of the rivets or $\frac{1}{64}$ in. larger than the actual diameter at the base, can be driven, without scraping the hot metal of the shank and forming a film under the rivet head, and fill the holes more perfectly than ordinary cylindrical rivets. It was found, however, that allowance had to be made for the irregularity in diameter of rivets and holes due to the rapid wear of the rivet dies and the drills. After extended experiments, the following size and shape was finally adopted for the $1\frac{1}{4}$-in. rivets (Fig. 19): Minimum diameter under the head, $1\frac{9}{32}$ in., tapered

down to $1\frac{15}{64}$ in. in a distance of from 5 to 6 in. from the head, depending on the length of the rivets, the rest of the shank being cylindrical.

The rivets were required to be perfectly round, and free from loose scale and projecting fins, and before they were entered into the holes, the scale, formed in heating, had to be carefully scraped off. To produce perfectly round and smooth rivets, it was found necessary to deviate from the ordinary practice of upsetting the rivet between dies by a single stroke of the machine and cutting it off at the same time from the feeding rod. Pieces of the proper length required for the various rivets were cut cold from the rivet rods and, after heating, were placed individually into the upsetting machine. Each rivet was given at least two strokes, and was turned after each stroke, so as to make it perfectly round and press down the projecting fins which form in the first stroke at the joint between the dies. Further, by more careful individual handling of the rivets in the heating furnace, it was possible to avoid the scale which often forms on the rivets due to improper heating when handled in bulk. Whatever slight scale was left was removed by running the finished rivets through a rumbling process and, where necessary, projections were removed by grinding.



1¼" TAPERED RIVET

FIG. 19.

Planing Ends of Chord Members.—Great care was taken to secure perfect planing of the ends of the chords. The planing was done by a Bermet Miles horizontal and vertical planer which was especially erected for this work. It is mounted on a rotary platform, and has a cutting range of 12½ ft. horizontally and 10½ ft. vertically.

The three-faced joint of the bottom chord members was obtained by facing the member first to a single plane for the full width. In this condition the chord member was assembled in the truss, and after the truss had again been taken apart the two outer thirds of the face of the chord were planed to the required bevel. About 1 in. of metal was cut away at each face, first by a roughing cut parallel to the vertical webs, and then by a horizontal finishing cut. The length

of the members had to be accurate within $\frac{1}{64}$ in. and the bevels of the faces within 30 sec. ($= \frac{1}{64}$ in. in 10 ft.).

Assembling Arch Trusses.—Complete or partial assembling of riveted trusses, or of continuous chords of pin-connected trusses, is gradually becoming the ordinary method in American practice, at least for important work. With improved facilities for assembling large trusses at the shop, the reaming and drilling of the field connections can be done more cheaply than by the formerly prevailing practice of reaming or drilling to iron templates. The assembling at the shop, moreover, insures greater accuracy and decreases the chances for errors or unforeseen difficulties, with the resulting delays and added expenses, in the field.

As the complete assembling of the arch trusses would have required a very large space, and corresponding facilities for handling the members, the Bridge Company was permitted to assemble the truss in sections of four panels, the last panel of each section being again assembled with the following three panels.

Each truss section was laid out carefully in the bridge shop yard to the correct cambered form with a transit. The members were supported by timber grillages of sufficient bearing area to prevent excessive settlements. Levels were taken each morning before any work was done to make sure that the truss section was in a perfect plane during the drilling of the holes. The members were handled with a gantry crane of 130 ft. span and 150 tons lifting capacity (Fig. 20). This crane was erected specially for this work.

As previously mentioned, all holes for the field connections were drilled to a diameter of $\frac{11}{16}$ in. from the solid, and $\frac{5}{8}$-in. bolts were used for assembling. After a truss section was assembled, the holes were drilled to full size with long high-speed drills which reached through both webs. The $\frac{5}{8}$-in. bolts were gradually replaced by larger ones as the drilling progressed, so as to keep the members and gussets always firmly tied together. Before the members were taken apart, the field connections were match-marked carefully and the marks were recorded on a chart for the use of the field force. Fig. 21 shows the special lifting device used for handling the heavy chords.

Only the best class of labor was employed on this work, and the bridge shop deserves full credit for its excellent work, which manifested itself in the accuracy and expediency with which the members

were erected in the field. The 250-ton cast-steel bearings (Fig. 22) were completely fitted together at the shop.

Angles Between Truss Members and Bevels of Joints.—In the fabrication of large riveted trusses, careful consideration must be given to the question as to whether the angles between members and the bevels of faced ends of chord members shall be made to conform to the "cambered", or to the "geometric", or some other form of the truss. The method to be used depends essentially on the desirability of reducing the secondary stresses, and therefore on the kind and system of truss.

If the angles are laid out to the geometric form of truss, that is, the form which the truss is expected to assume after completion, it is possible to reduce the secondary stresses considerably. This method, therefore, is advisable for trusses with high secondary stresses, such as cantilevers or continuous trusses.

If the angles are made to conform to the cambered form of truss, that is, the form which the truss would assume when entirely relieved of stress, the secondary stresses are, theoretically, fully developed. This method was used for the Hell Gate Bridge, as the secondary stresses are negligible, and because this method has the following important advantages:

First, the whole truss or any number of panels can be completely put together at the shop, with tight joints, and the holes for the connections can be reamed or drilled while the truss is thus assembled, with the greatest possible accuracy and least chance of errors.

Second, when the truss members are erected in the field, the holes of the connections should match perfectly, and the joints should be tight, without initial bending of the members. The riveting of the connections can start at once, if desired, or the holes can be filled temporarily with tight-fitting drift-pins and bolts to prevent motion of the ends of the member during the deformation of the truss. This secures the greatest safety and expediency in erection.

11.—Erection of Hell Gate Bridge.

Method of Erection.—As is necessary in the case of a large bridge which has few or no precedents, the question of erection was given thorough consideration by the Consulting Engineer, who prepared general erection plans and stress sheets before the design was finally

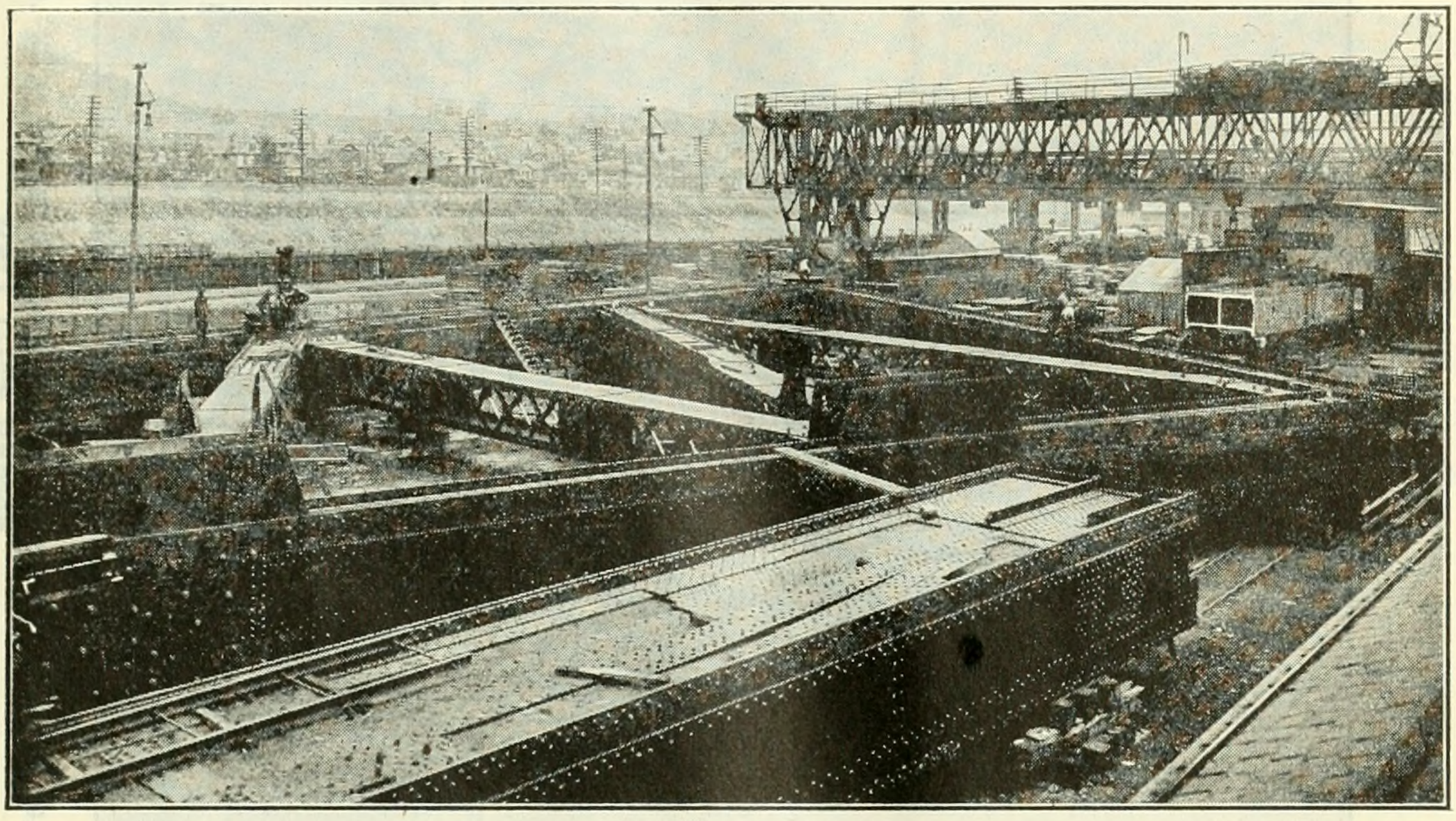


FIG. 20.—ASSEMBLING ARCH TRUSSES AT SHOP, FOR HELL GATE BRIDGE.

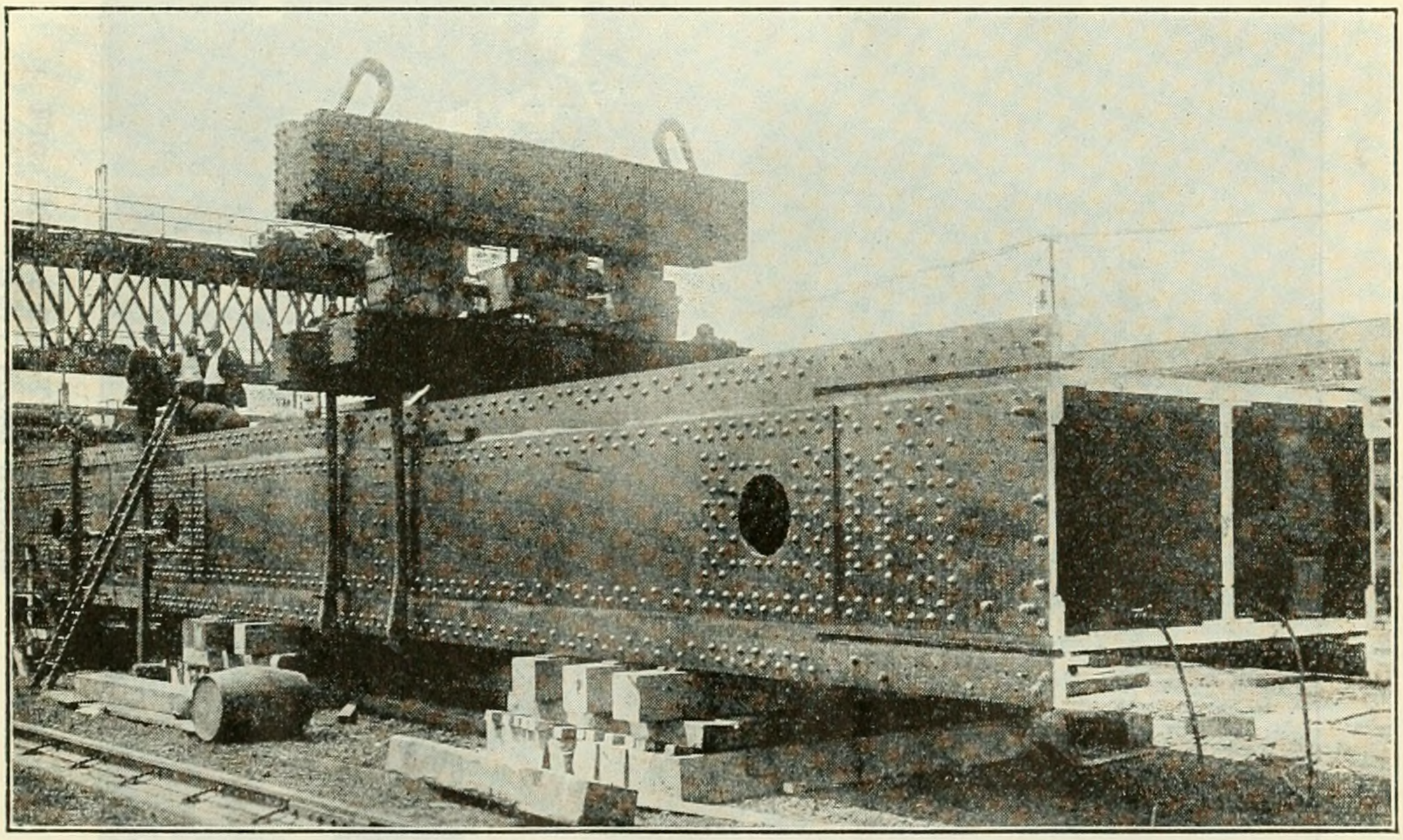


FIG. 21.—BOTTOM CHORD SECTION WITH LIFTING DEVICE, HELL GATE BRIDGE.

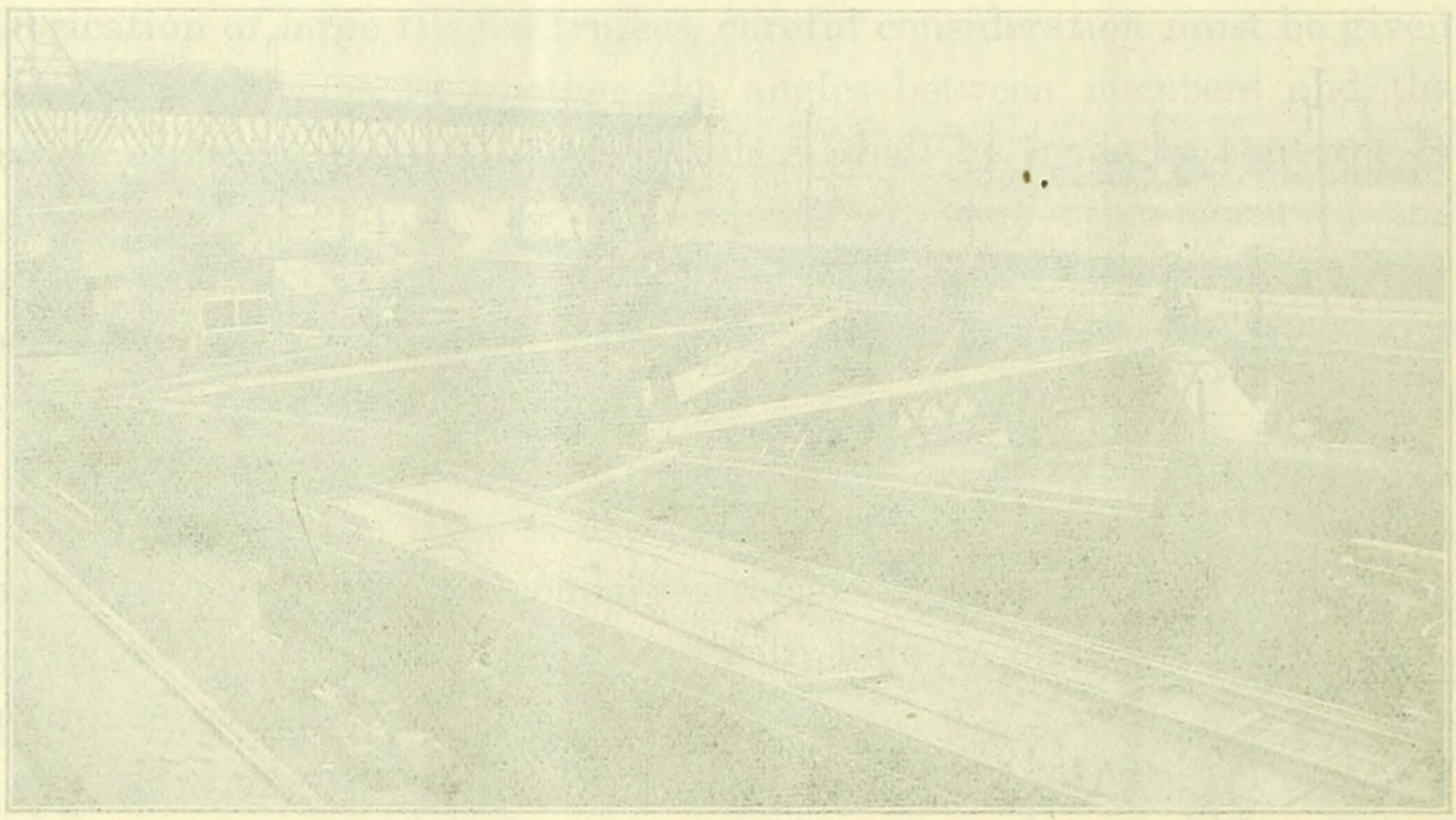


FIG. 20.—ASSEMBLING ARCH TRUSSES AT SHOP, FOR HELL GATE BRIDGE.

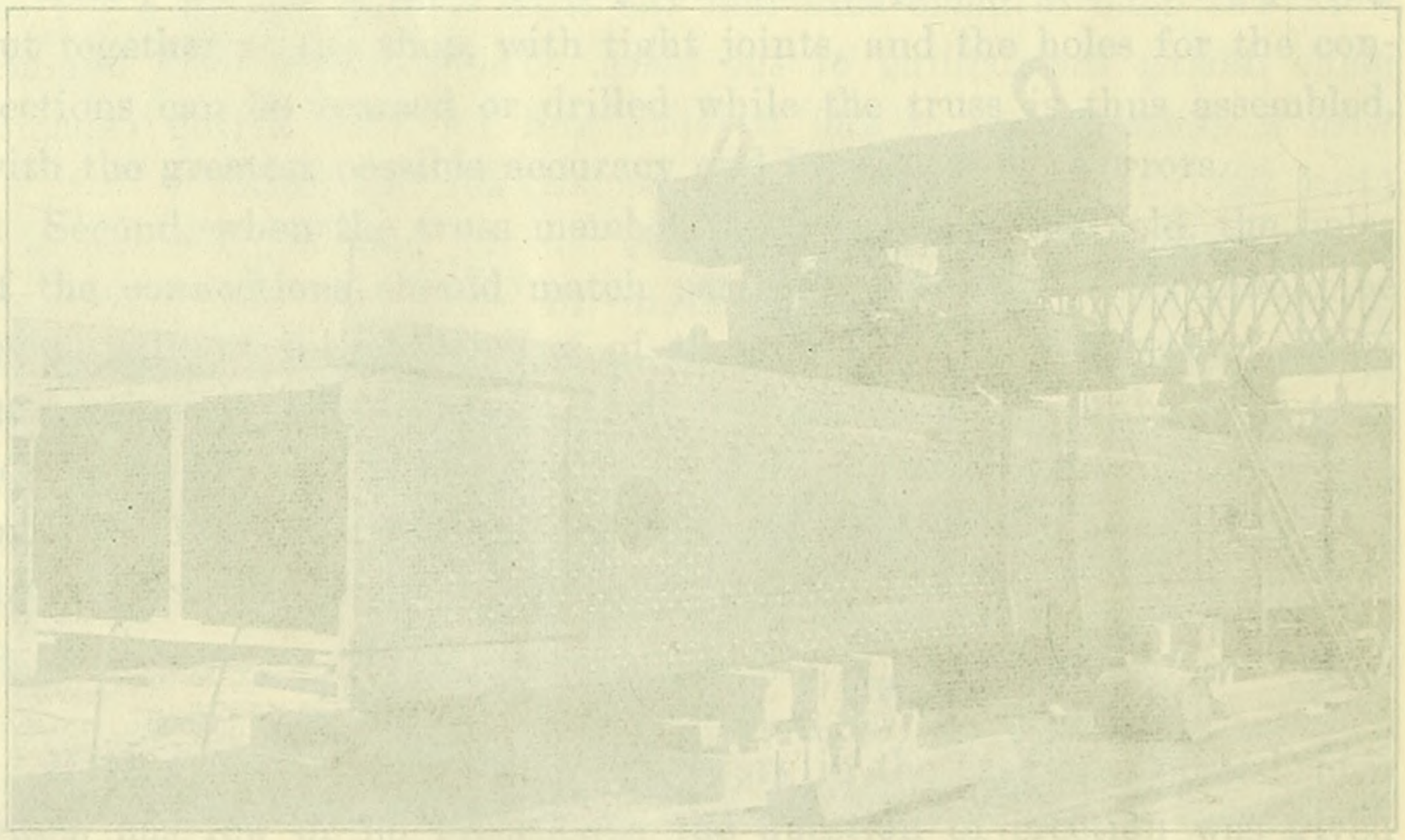


FIG. 21.—BOTTOM CHORD SECTION WITH LIFTING DEVICE, HELL GATE BRIDGE.

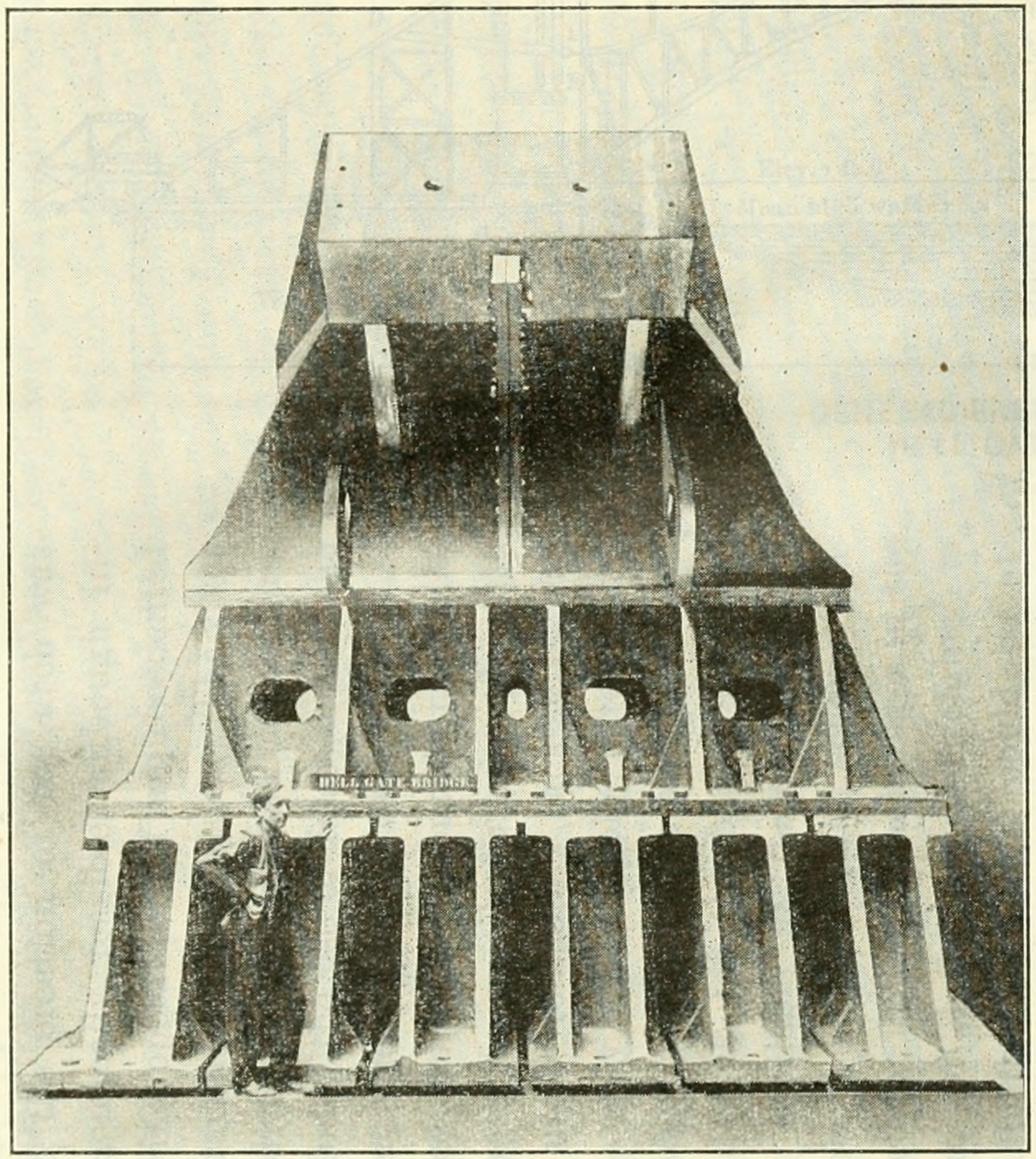


FIG. 22.—ASSEMBLING OF 250-TON ARCH BEARING, HELL GATE BRIDGE.

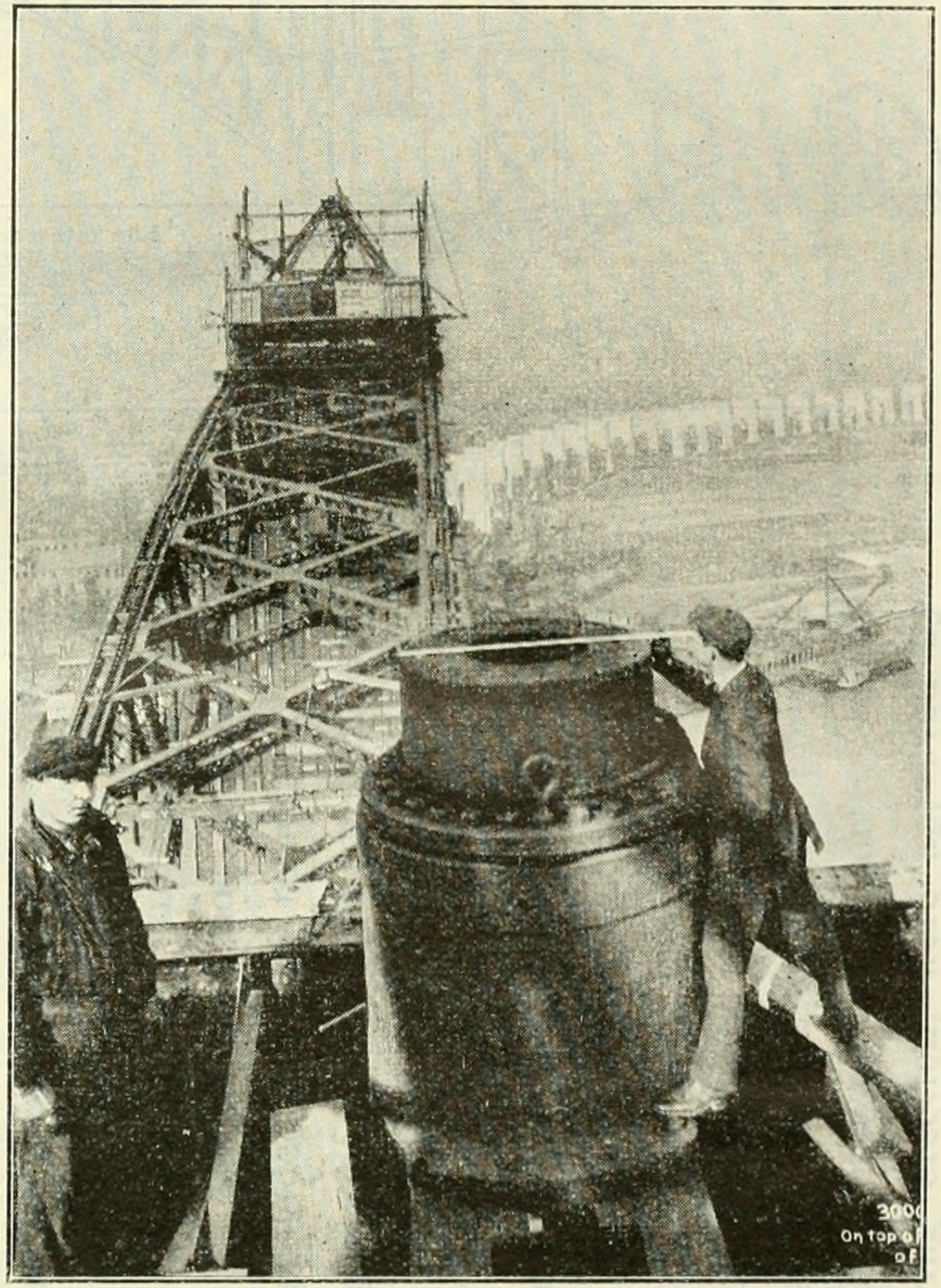


FIG. 23.—3 000-TON HYDRAULIC JACK, HELL GATE BRIDGE.

FIG. 53.—3,000-TON HYDRAULIC JACK.

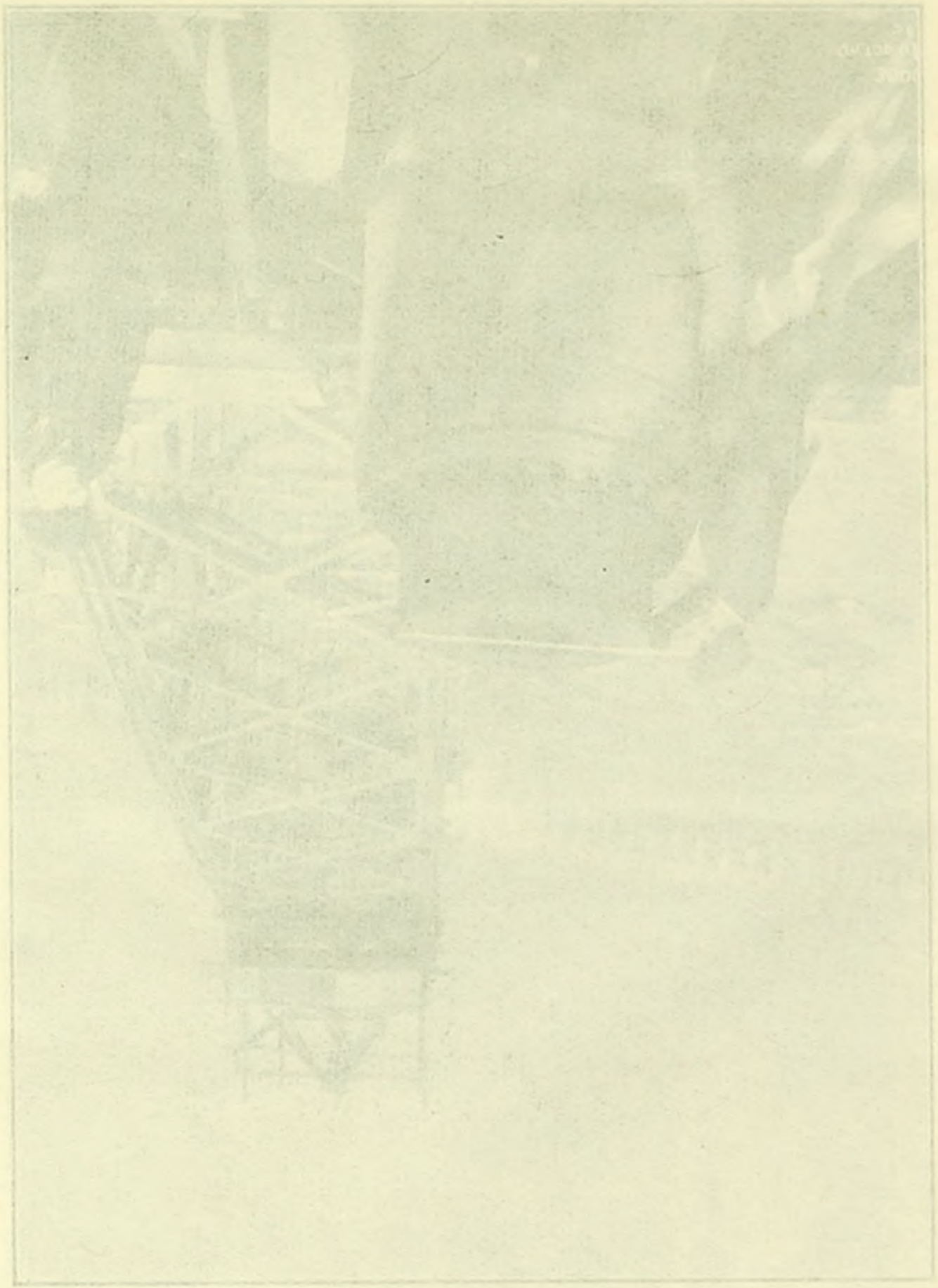
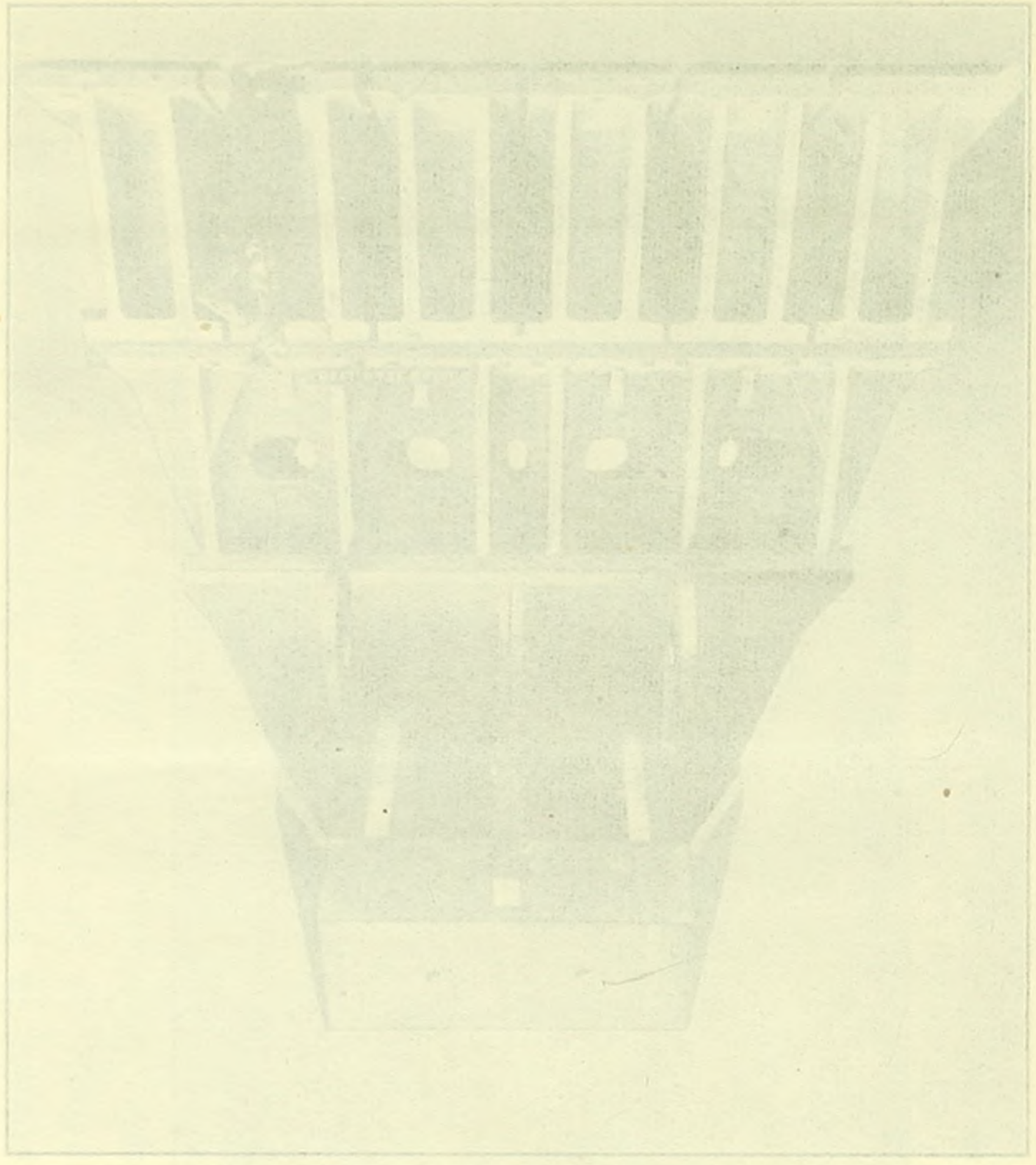
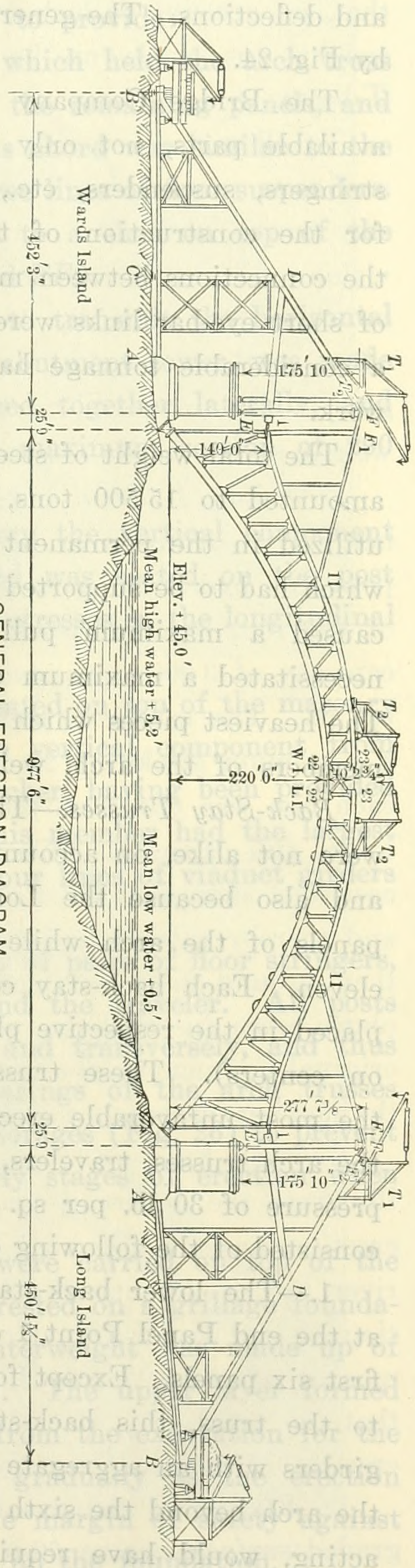


FIG. 53.—ASSEMBLING OF 520-TON ARCH BEARING.



decided upon. These plans indicated that the erection of the arch was economically feasible, although involving operations of unprecedented character and magnitude. The river conditions, as outlined heretofore, excluded the use of falsework in the river, except for a very short distance from each abutment. Erection on the cantilever principle, with the use of temporary back-stays, was the only practicable alternative. However, as natural anchorages in solid rock for the back-stays were not available at reasonable depth below the surface, the scheme provided for artificial anchorages in the form of huge counterweights to which the back-stays could be attached. To transmit the horizontal pull of the back-stays to the abutment towers, where it could find resistance, struts had to be provided along the ground surface between the counterweights and the towers. To render this method economical, it provided for the temporary use of parts of the permanent bridge in the back-stays.



GENERAL ERECTION DIAGRAM
HELL GATE BRIDGE

FIG. 24.

The general features of this scheme were adopted by the erectors, the American Bridge Company, with whom the responsibility for the erection properly rested. The erectors' plans, which were prepared with a view to secure perfect safety during erection, were approved by the Consulting Engineer after thor-

ough examination and independent calculation of all erection stresses and deflections. The general scheme of erection, as used, is illustrated by Fig. 24.

The Bridge Company resorted to a very skillful utilization of available parts, not only of the arch bridge proper, such as floor stringers, suspenders, etc., but also of the plate-girder approaches, for the construction of the back-stays and counterweights. Only the connections between members, some light bracing, and a number of short eye-bar links were made of extra material, and even of this a considerable tonnage had been used previously on other erection work.

The total weight of steel in the back-stays, for both sides together, amounted to 15 500 tons, of which only about 2 300 tons are not utilized in the permanent structure. The total weight of the arch which had to be supported by the back-stays amounted to 14 500 tons, caused a maximum pull of 6 500 tons in each back-stay, and necessitated a maximum counterweight of 5 300 tons on each side. The heaviest pieces which had to be lifted as units, the bottom chord members of the arch, weighed 180 tons.

Back-Stay Trusses.—The back-stays on the two sides of the river were not alike, on account of different configuration of the surface, and also because the Long Island back-stay had to carry twelve panels of the arch while the Wards Island back-stay carried only eleven. Each back-stay consisted of two separate back-stay trusses placed in the respective planes of the two arch trusses (60 ft. apart on centers). These trusses were designed to resist safely, under the most unfavorable erection conditions, their own weight, that of the arch trusses, travelers, and other erection equipment, and a wind pressure of 30 lb. per sq. ft. of exposed area. Each back-stay truss consisted of the following essential parts (Fig. 24):

1.—The lower back-stay chord, *BD1,* which held the arch truss at the end Panel Point 1 of the top chord during the erection of the first six panels. Except for the short eye-bar links at the connection to the truss, this back-stay chord consisted of two lines of plate girders with an aggregate net section of 316 sq. in. The erection of the arch beyond the sixth panel, with only the lower back-stay chord acting, would have required an excessively large section for this

back-stay and considerable extra material in the top chord of the arch trusses. It was necessary, therefore, to provide:

2.—An upper back-stay chord, *DF*11, which held the arch truss at Panel Point 11 during the erection of the remaining panels, and the closure of the arch. Part, *DF,* of this chord was similar to the lower chord, and part, *F*11, consisted of three lines of floor suspenders. Both parts of this chord were connected to a shoe on top of the column, *EF,* by short eye-bar links, *FF*1 and *FF*2.

3.—The bottom strut, *AB,* which had to transmit the horizontal component of the back-stay chord to the abutment tower, was made up of four lines of floor stringers, braced together laterally, and supported by timber grillages. It had a maximum section of 290 sq. in.

4.—The vertical post, *CD,* had to carry the vertical component from the lower tension chord. The chord was seated on the post by a roller nest in order to prevent serious stresses in the longitudinal bracing between posts.

5.—The vertical post, *EF,* which was seated on top of the masonry tower at track level had to transmit the vertical component from the upper-tension chord. It acted as a rocker, having been provided with pin bearings at top and bottom. This member had the largest section, 348 sq. in., and was made up of four lines of viaduct girders with their bracing.

All other vertical posts, made up mostly of pairs of floor stringers, carried only the weight of the chords and the traveler. All posts were well braced in pairs longitudinally and transversely, and thus formed rigid towers and bents. The bearings of the arch trusses were provided with temporary eye-bar anchorages (Fig. 26) to prevent sliding on the skewbacks during the early stages of erection when the reaction had the steepest inclination.

Counterweights.—The counterweights were carried on top of the rear ends of the back-stay trusses, which rested on a grillage foundation. On the Long Island side the counterweight was made up of three layers of viaduct girders (Fig. 25). The upper layer formed a box which was filled with earth taken from the excavation for the bottom strut. The earth was filled in gradually as the erection proceeded so that there was always ample margin of safety against uplift and at the same time safe pressure on the foundation.

The counterweight on the Wards Island side was similar, except that, on account of lack of sufficient earth excavation, the necessary weight was made up by additional viaduct girders and flooring I-beams.

To allow free expansion and contraction of the back-stay trusses, and thus prevent large temperature stresses, the ends of the back-stay trusses under the counterweights rested on roller bearings. Provision was also made to raise the counterweights to proper height by eight 500-ton hydraulic jacks (four under each truss) so as to offset settlements of the foundations. These jacks, having been gauged for pressure, served at the same time as a check for the weight of the earth fill.

3 000-Ton Hydraulic Jacks.—Adjustment of the arch trusses in height was required at various erection stages. For this purpose a powerful hydraulic jack (Fig. 23) was placed at the top of each of the four erection posts, *EF* (Fig. 24). Each jack had a lifting capacity of 3 000 tons under a water pressure of about 5 000 lb. per sq. in. The cast-iron jack plungers had a diameter of 39 in. and a maximum stroke of 26 in. The cylinders were of cast steel, and each one was tested at the United States Government testing plant to its full capacity. When operated, the plunger acted against the cast-steel shoe on top of the post, raising or lowering it as desired, and thereby raising or lowering the arch trusses or rather revolving them around their bearings. As the shoe was raised or lowered, shim plates were inserted or removed from between the shoes and their original bearings on the post, so that the jacks could be relieved after the jacking operation was completed.

General Erection Procedure.—The diagrams on Plate XXXIII illustrate the general procedure and the progress of erection. The erection was to be carried out simultaneously from both sides of the river, but, on account of delays in the construction of the deep foundation of the Wards Island tower, erection on that side was started 4½ months later than on the Long Island side. It was timed, however, so that the two halves of the arch were completed simultaneously. The erection of each half span proceeded in the following order (Fig. 24):

1.—Placing of compression chords of back-stay, on the previously graded surface, with a 60-ton locomotive crane.

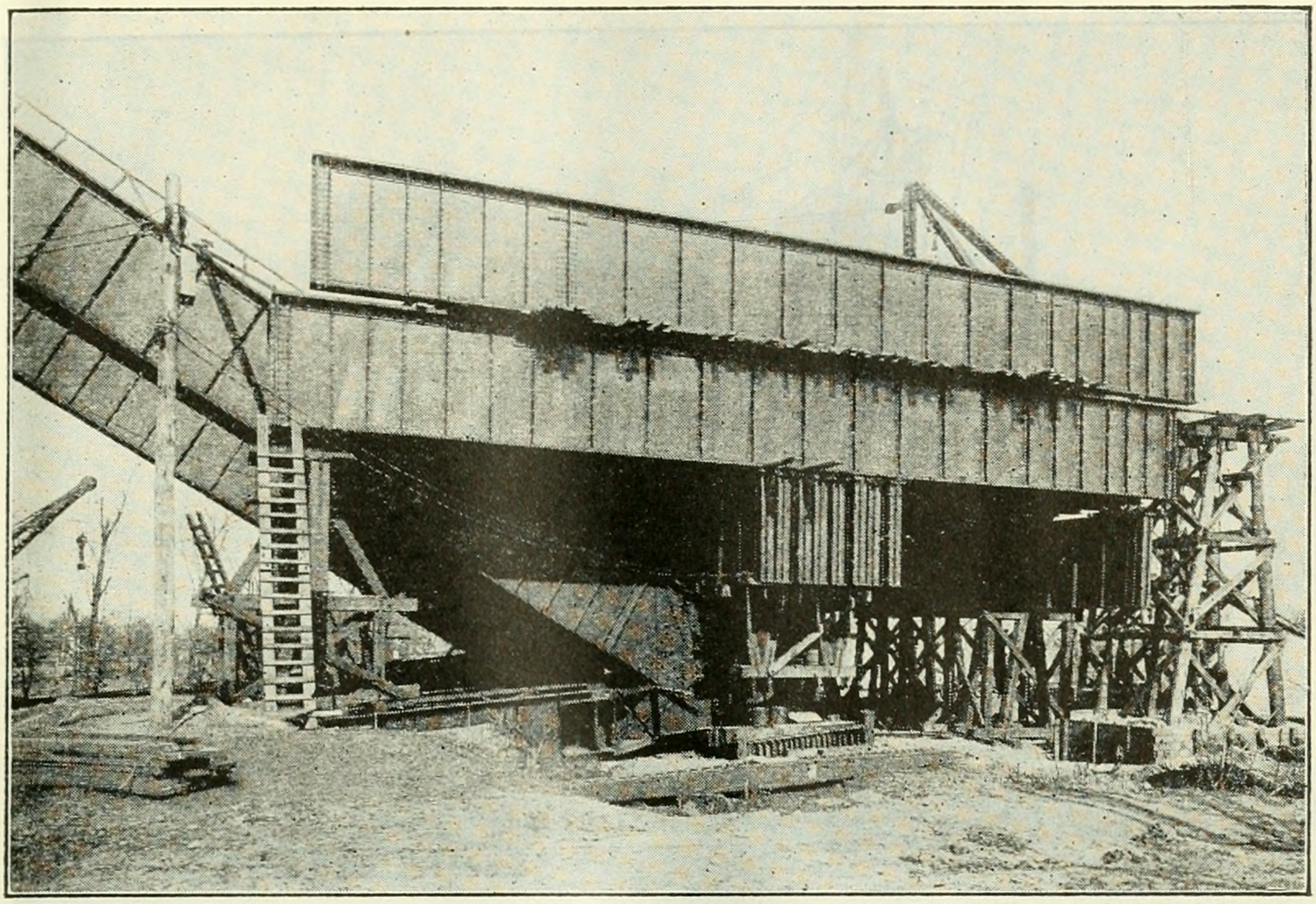


FIG. 25.—LONG ISLAND BACK-STAY COUNTERWEIGHT, FOR HELL GATE BRIDGE.

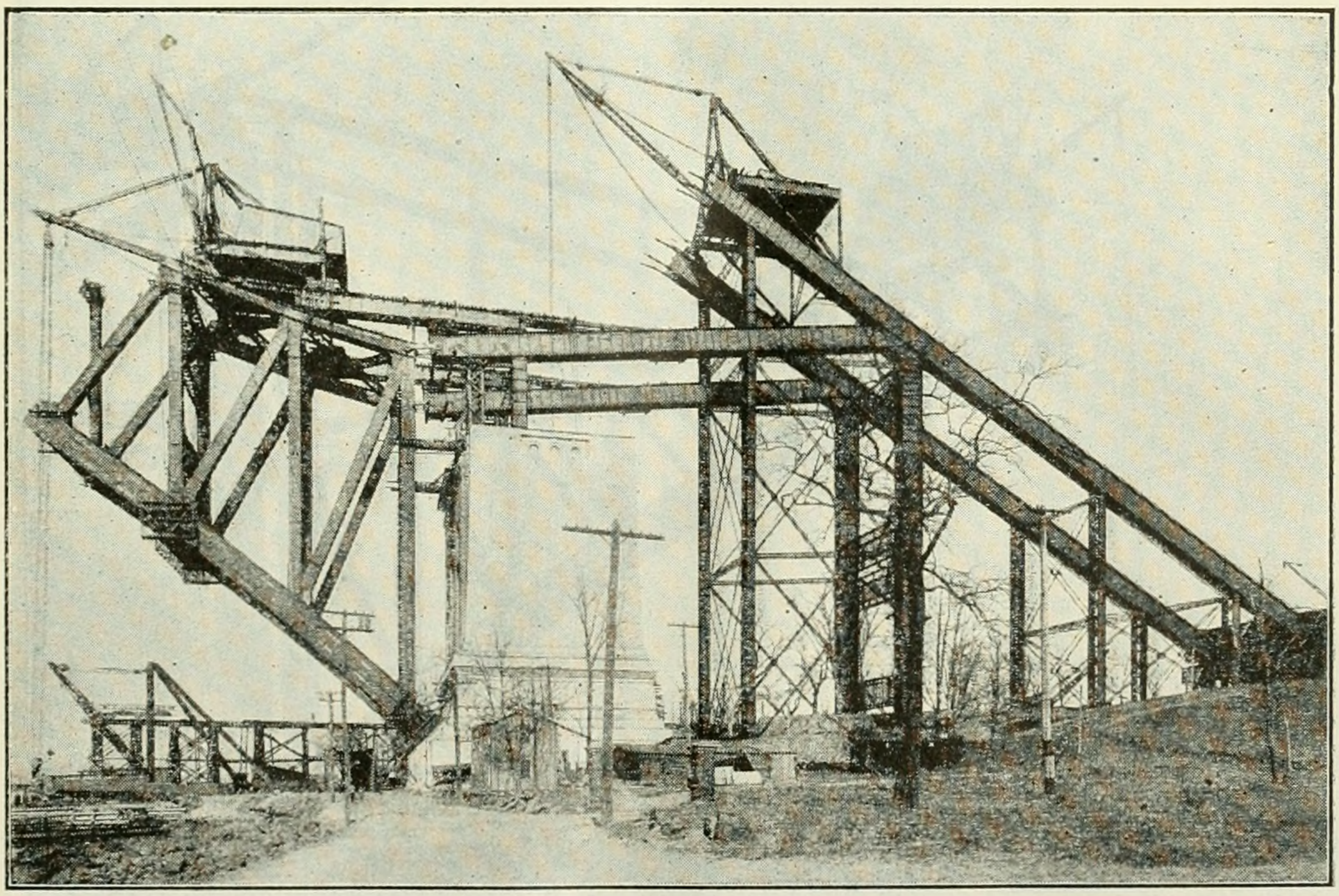


FIG. 26.—HELL GATE BRIDGE: ERECTION STAGE, APRIL 1ST, 1915, LONG ISLAND SIDE.

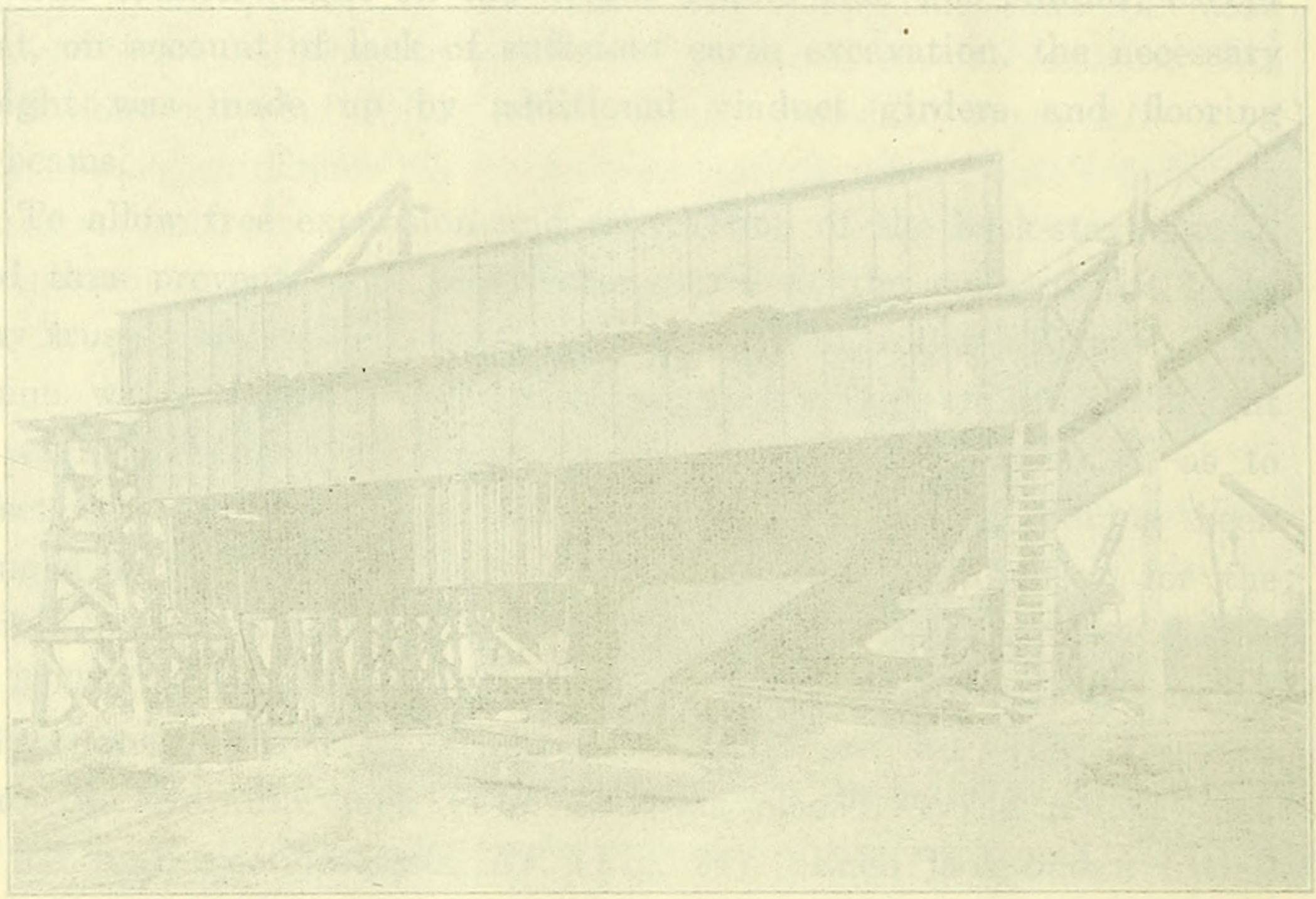


FIG. 25.—LONG ISLAND BACK-STAY COUNTERWEIGHT, FOR HELL GATE BRIDGE.

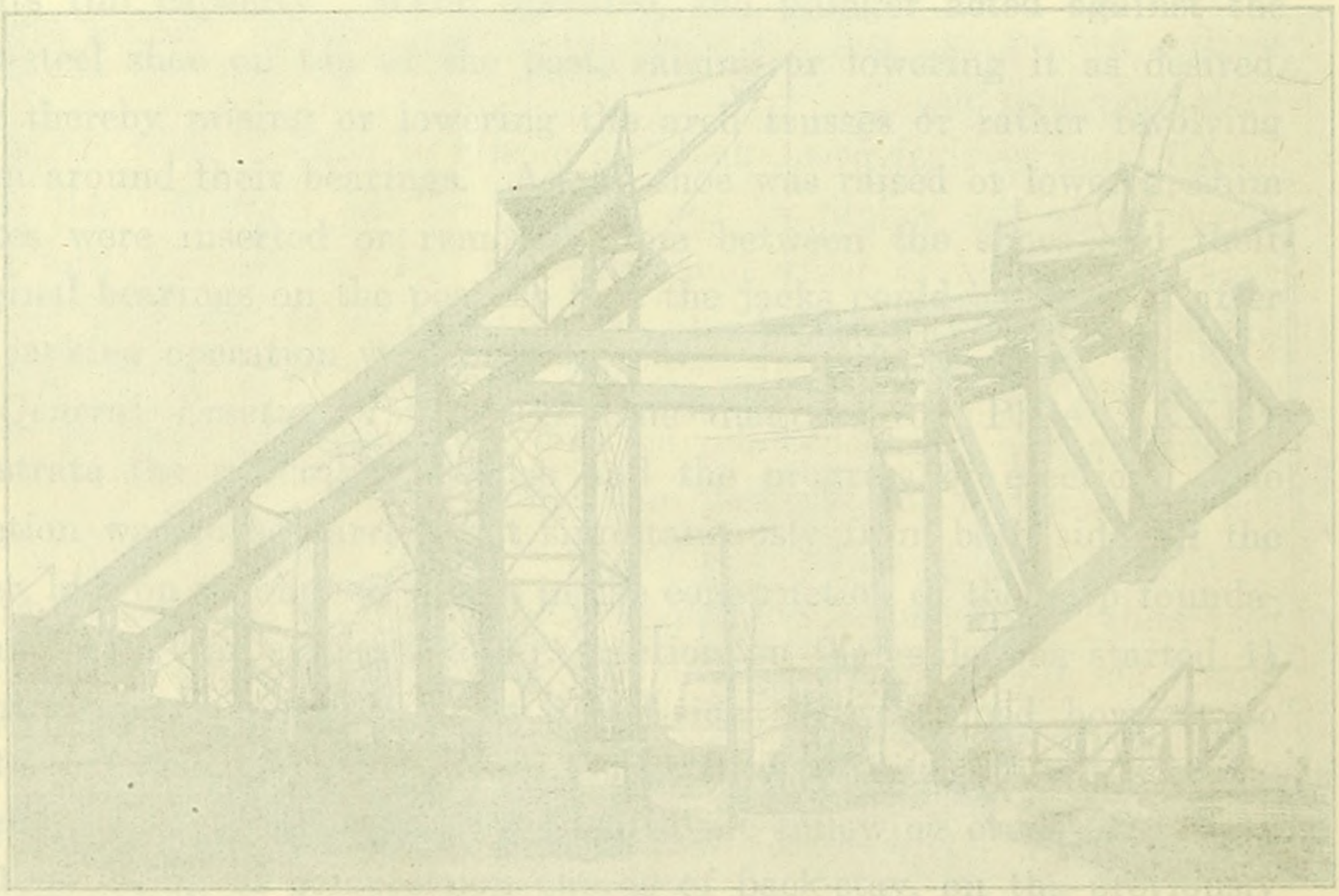


FIG. 26.—HELL GATE BRIDGE: ERECTION STAGE, APRIL 1ST, 1915, LONG ISLAND SHORE.

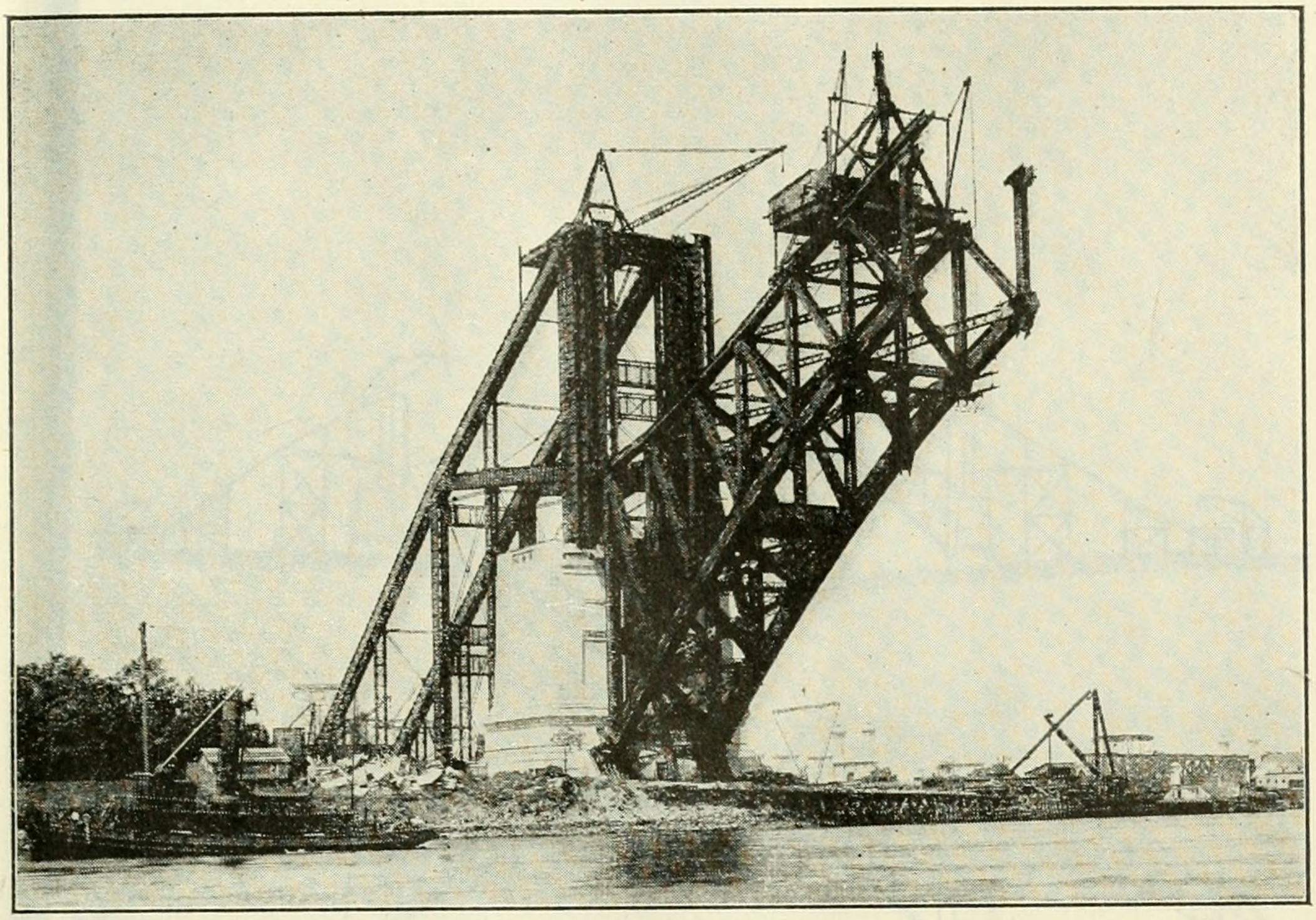


FIG. 27.—HELL GATE BRIDGE: ERECTION STAGE, AUGUST 18TH, 1915, WARDS ISLAND SIDE.

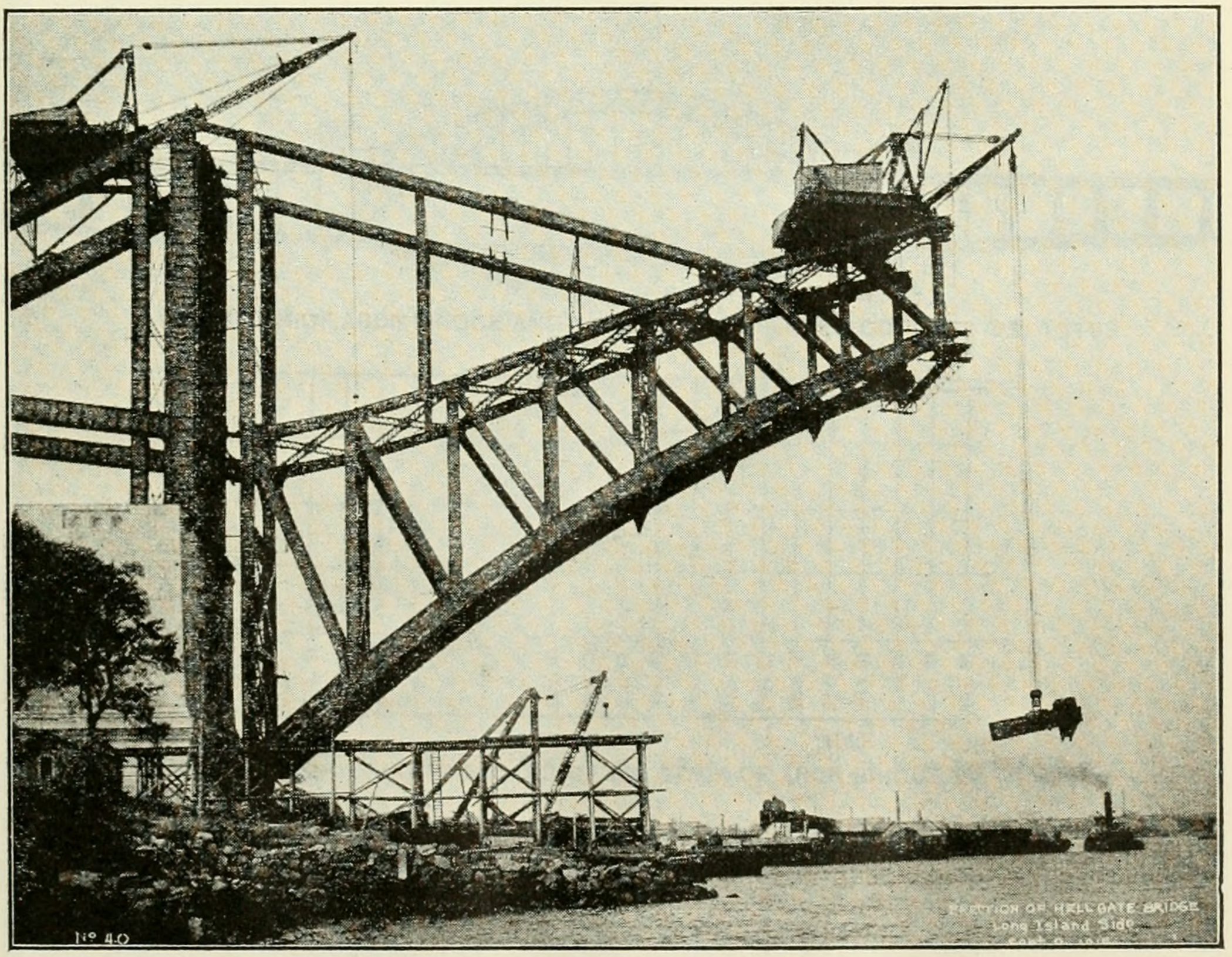


FIG. 28.—HELL GATE BRIDGE: ERECTION STAGE, SEPTEMBER 2D, 1915, LONG ISLAND SIDE.

FIG. 50.—HELL GATE BRIDGE: ERECTION STAGE, SEPTEMBER 3D, 1915.
LONG ISLAND SIDE.

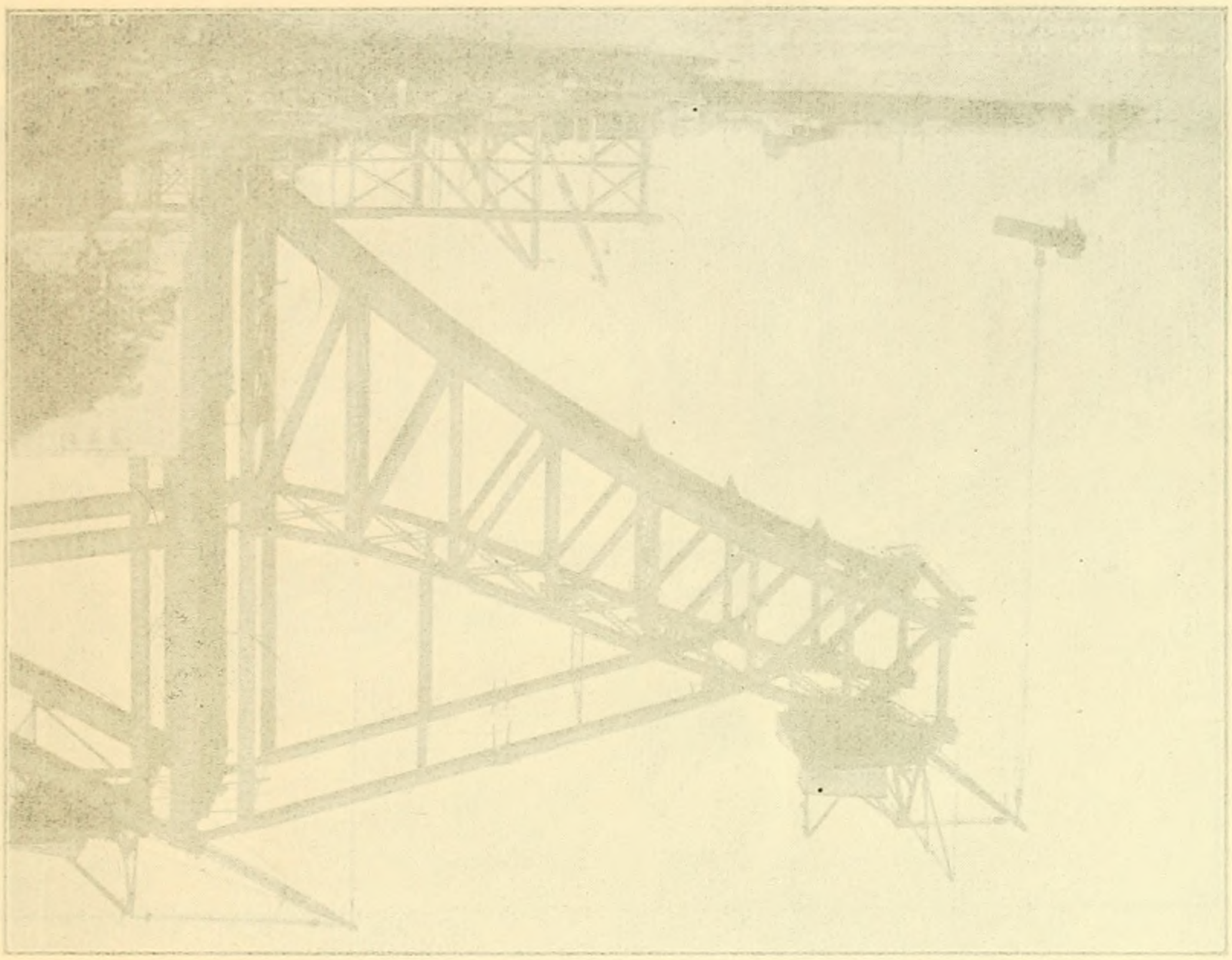
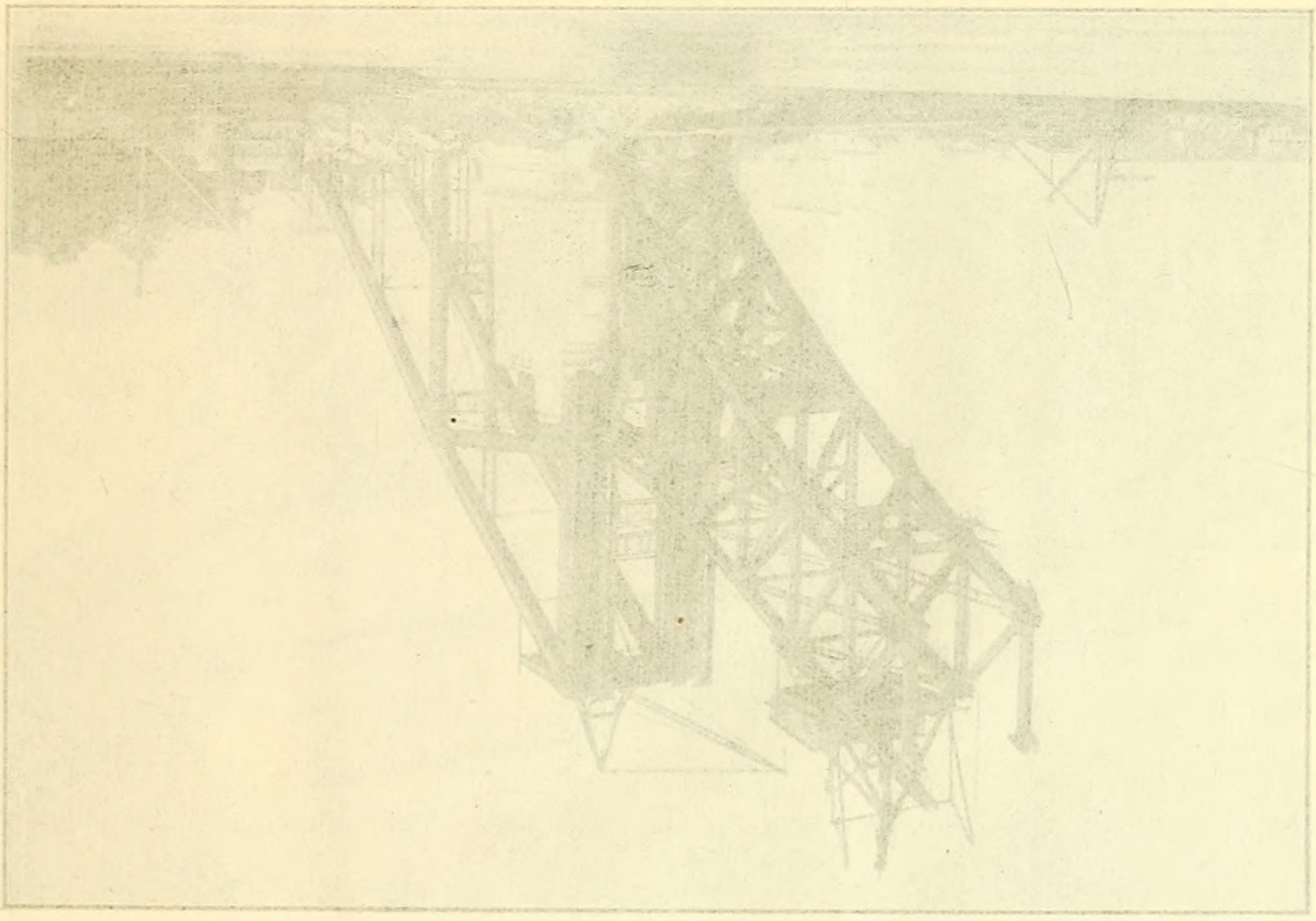


FIG. 51.—HELL GATE BRIDGE: ERECTION STAGE, AUGUST 18TH, 1915.
MANHATTAN ISLAND SIDE.



PROGRESS DIAGRAM OF ERECTION OF HELL GATE BRIDGE

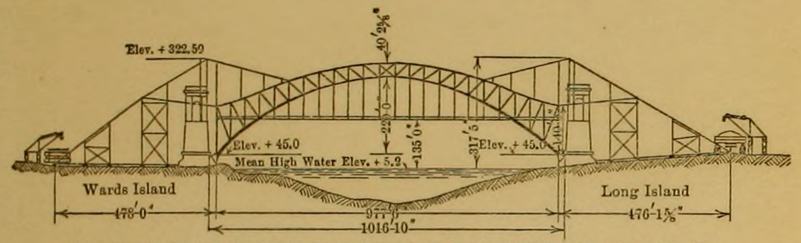


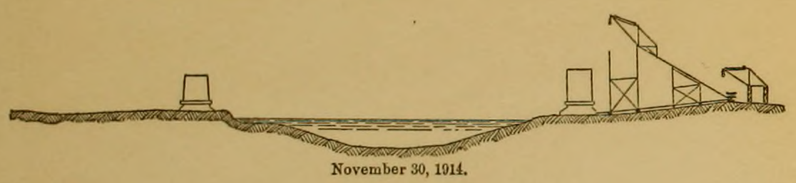
DIAGRAM SHOWING GENERAL DIMENSIONS OF ARCH BRIDGE AND BACK-STAYS



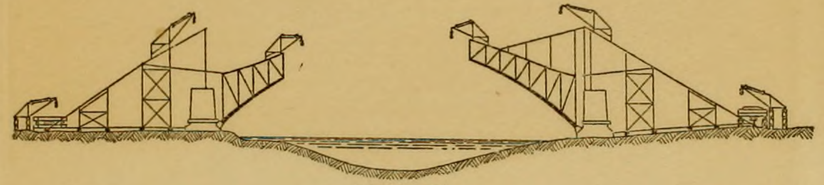
May 31, 1915.



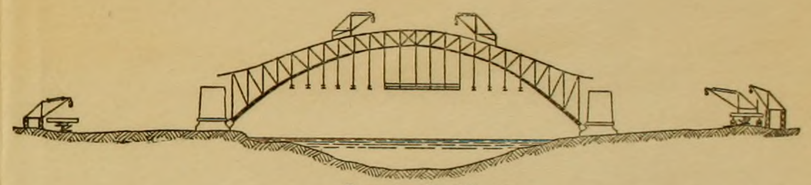
October 31, 1915.



November 30, 1914.



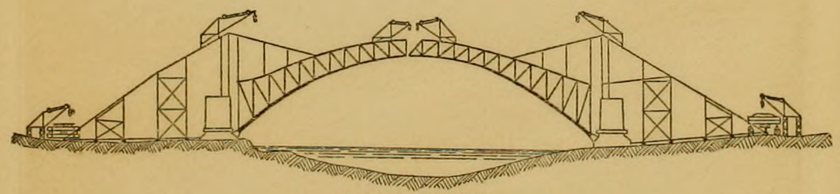
July 31, 1915.



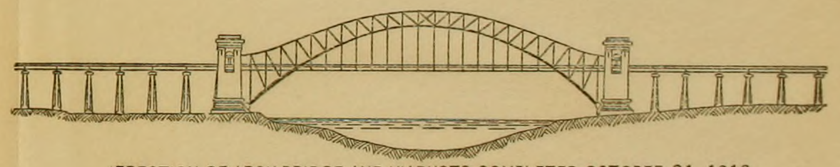
December 31, 1915.



January 31, 1915.



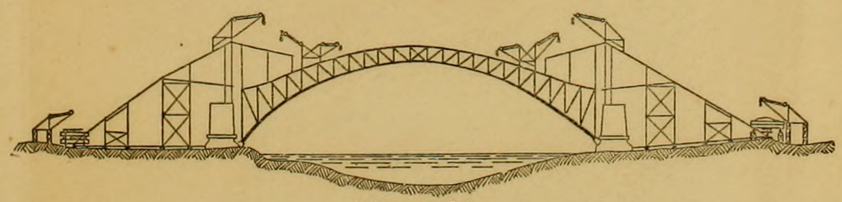
September 30, 1915.



ERECTION OF ARCH BRIDGE AND VIADUCTS COMPLETED OCTOBER 31, 1916.



March 31, 1915.



October 15, 1915.

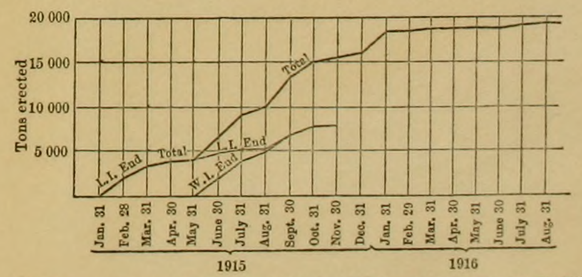
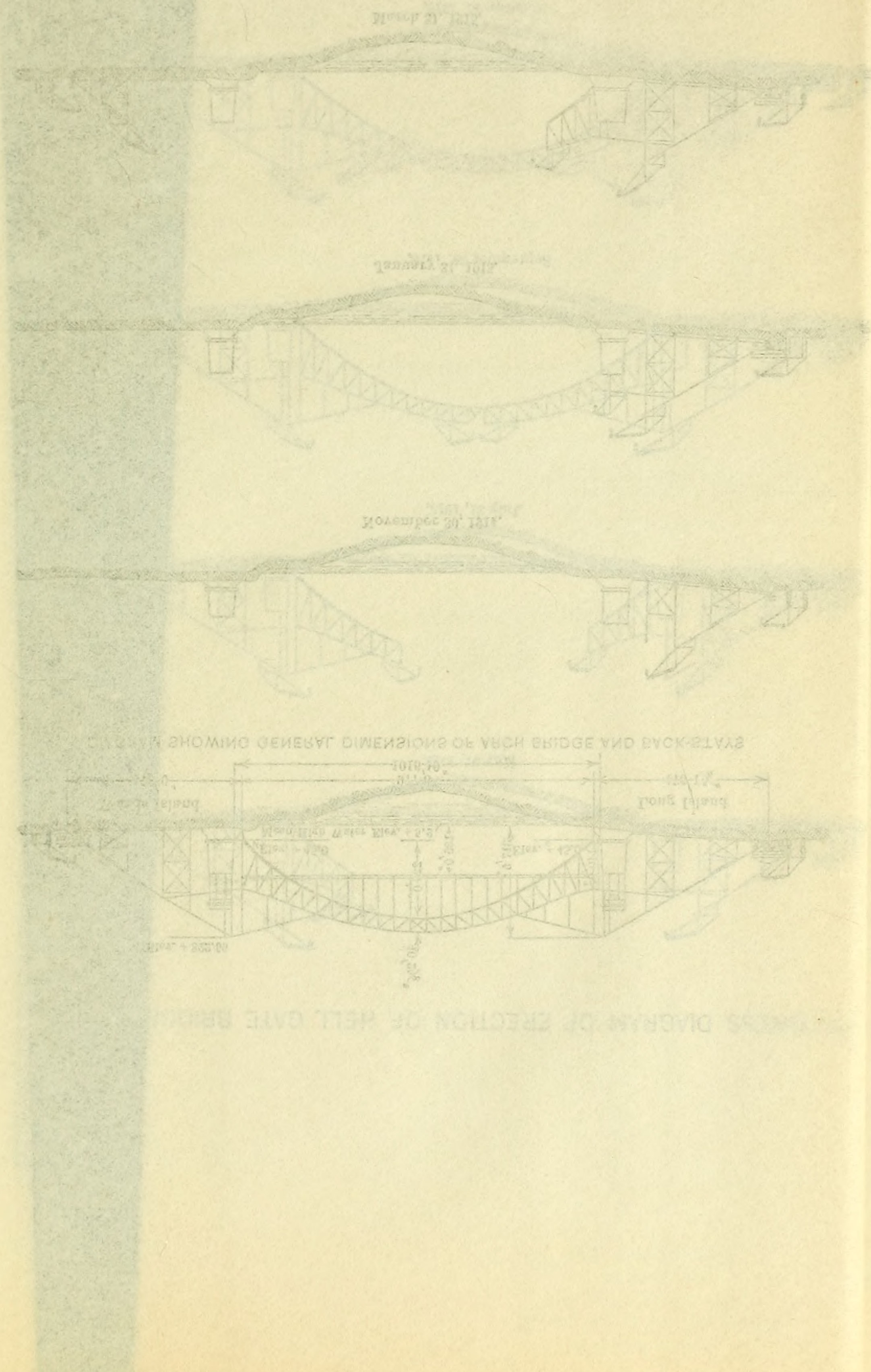


DIAGRAM SHOWING PROGRESS OF ERECTION OF ARCH BRIDGE BY TONNAGE



2.—Erecting bottom layer of counterweight girders and on top of these the back-stay traveler, T_1, with a 30-ton stiff-leg derrick placed back of the counterweight.

3.—Erecting lower part, $BD1$, of back-stay with the back-stay traveler, T_1, which moved along the top of the back-stay.

4.—Erecting arch traveler, T_2, on top of lower tension chord of back-stay by the traveler, T_1.

5.—Setting of bearings for arch bridge, and erecting first six panels of arch trusses, bracing, and floor-beams, by the traveler, T_2, which moved along the top chord of the arch. At the same time the traveler, T_1, completed the erection of the upper part, DF, of the back-stay, and portion of the tie, F-11, by advancing on top of the back-stay as far as the point, F. The front portion of the tie, F-11, was erected by a derrick mounted on the rear end of the traveler, T_2.

6.—Connecting the upper chord, F-11, to Panel Point 11 of the arch, and raising the point, F, with the hydraulic jacks until the lower chord, D-1, was relieved of its stress and disconnected.

7.—Continuing erection of arch trusses and bracing to the center (eleven panels on Wards Island side and twelve panels on Long Island side) leaving a small gap between the two ends (at bottom chord point 22 WI).

8.—Lowering Points F with the hydraulic jacks until the trusses became self-supporting three-hinged arches.

9.—Moving the travelers, T_2, back to Panel Point 11, where they started the removal of the forward stay and the erection of the floor suspenders and floor-beams, proceeding again toward the center. (The stringers could not be erected in this operation because they formed the compression chords of the back-stays and were at that time not yet dismantled.)

10.—Dismantling of back-stays and back-stay travelers in the reverse order in which they had been erected.

11.—Erecting stringers, railings, and floor bracing of arch by the traveler, T_2, from the center toward the ends. The end panels of the floor were erected, and the travelers, T_2, were dismantled later by a derrick set up at the end of the top chord.

12.—Connecting up of top chord and diagonals in the center panel so as to transform the trusses into two-hinged arches.

13.—Placing of concrete flooring, ballast, and tracks, after the riveting of the arch trusses had been completed.

Erection of Truss Members.—The simplicity and uniformity in the design of the various truss panels greatly facilitated and expedited the erection. The typical procedure in erecting a panel of the arch was as follows: The two bottom chords were raised into position, one at a time, the arch traveler standing with its front truck at the end of the previously completed panel. As soon as a chord was in place, the connection at the rear end was made with bolts and drift-pins, and then the falls were released and the chord was allowed to cantilever out. The gusset and splice-plates at the front end of the chord had previously been bolted to the chord on the ground.

Next, the diagonals were raised and connected at both ends, then followed the laterals between bottom chords, the vertical posts, top chord members, and last the sway-bracing between the posts and the top laterals, only one member being raised at a time. On account of the great weight of the chord members it was considered that better progress could be made by raising them individually, instead of in pairs, and therefore the traveler had been designed with a single central boom.

After the completion of a panel, the traveler was moved forward to the next panel point and blocked. The time consumed for the erection of a panel decreased as the erection advanced toward the center, as the erection gangs became more experienced and the members decreased in weight. It took about 3 weeks to erect the end panel on the Long Island side, and the tenth panel on the Wards Island side was completely put up in 7½ hours. The members of the center panel were raised jointly by both arch travelers, the latter standing with their front trucks at Points 21 (Fig. 29). Owing to the accuracy of the shop work, and the careful assembling of the trusses at the shop, no serious difficulties or delays were experienced in making the connections in the field, notwithstanding the unusual number and dimensions of gusset and splice-plates and the fact that no clearance had been allowed for entering parts.

Deflections and Adjustments of Arch Trusses During Erection.—During their erection, the arch trusses passed through the following four principal and distinct static conditions:

1.—Cantilever condition. During erection of first six panels, truss held at end of top chord by lower back-stay (Figs. 26 and 27).

2.—Cantilever condition. During erection of remaining panels, truss held at top chord Point 11 by upper back-stay (Figs. 28 and 29).

3.—Three-hinged arch condition. Back-stays released and trusses connected at bottom chord Point 22 Wards Island side, which acted as hinge, top chord 23-23, and diagonal 23 Wards Island side—22 Long Island side not connected at 23 Wards Island side (Fig. 30). Arch left in this condition until all steelwork had been erected.

4.—Final or two-hinged arch condition. All steelwork erected and all members of the center panel fully connected.

Each transformation from one to the next of these principal static conditions marked a critical erection operation. The first two operations required adjustments of the arch trusses, and to determine the amounts of these adjustments it was necessary to calculate the deflections of certain points of the trusses during erection.

To obtain a clear illustration of the elastic deformation of the trusses during erection, and for the purpose of comparison with observations in the field, complete deflection diagrams for the various erection stages were prepared. The diagrams for the principal stages are shown on Plate XXXIV. The deflections of the arch trusses during the cantilever conditions were due, first, to the elastic deformation of the trusses themselves from dead load and weight of traveler, etc.; second, to the elastic deformation of the back-stays from the same loads; and, third, to the change in length of the members from temperature changes. These deflections were very considerable. The total deflection of the bottom chord Point 22, Wards Island side, for instance, under the extreme cantilever condition and at normal temperature (60° Fahr.), was theoretically as follows:

	Wards Island side.	Long Island side.
Vertical downward deflection....	20.6 in.	26.6 in.
Horizontal outward deflection....	7.6 in.	8.1 in.

At a temperature of 30° above normal, these points would have moved 1.6 in. farther out, which would have resulted in an overlap of the two ends of about 19 in. In order that the ends of the two arms should clear each other and that connection at the center could be made by lowering the trusses, it was necessary, therefore, to erect each half in a raised position (revolved around its end bearing).

This position was determined so that, at a temperature of 30° Fahr. above normal, which was considered the extreme high temperature at which the connection might have to be made, there would still be an opening between the ends of the two arms of about $1\frac{1}{2}$ in., to allow for discrepancy between theoretical and actual deflections. That corresponded to an opening of 4.8 in. at normal temperature and 8.1 in. at 30° below normal. The raised position of the trusses was established by making the eye-bar links of the back-stays at Points *F* shorter than their geometric lengths. Link *FF*1 was shortened by $12\frac{9}{16}$ in. on the Long Island side and by $11\frac{1}{16}$ in. on the Wards Island side, and *FF*2 by $1\frac{3}{8}$ in. on both sides. All other members of the back-stays were made to their true geometric length, and the pin at Point *F* was assumed to be raised to its normal or geometric elevation. The position for that extreme cantilever condition (Stages 12 and 12*a*) and for normal temperature is shown (by full lines) in the third diagram on Plate XXXIV. The positions of the various panel points are given with reference to the geometric position of the arch trusses (shown by dotted lines).

To determine the required range of adjustment by the hydraulic jacks at Points *F*, that is, the amount by which Points *F* had to be lowered by the jacks to close the opening at the center and transform the trusses into a self-supporting three-hinged arch, it was necessary to calculate the position of the trusses for the latter condition. That position is shown in the fourth diagram (Stage 13) on Plate XXXIV. This diagram shows also the intermediate position (dash and dotted line) which the trusses assumed when the ends of the two arms just came into contact, without transmitting any stress. From the commencement of lowering to this intermediate stage the movement of the trusses was only a downward rotation around their bearings; during the remainder of the movement the trusses changed their elastic form, rising in the center about $1\frac{3}{4}$ in. and sagging at the quarter points about 3 in.

To determine the movements taking place during the operation of releasing the lower back-stay, it was necessary to calculate the positions of the trusses immediately before and after this operation. These positions are shown in the first and second diagrams on Plate XXXIV (Stages 6 and 7). The position at Stage 7 is obtained from the position at Stage 6 by raising Point *F* from an initial low position

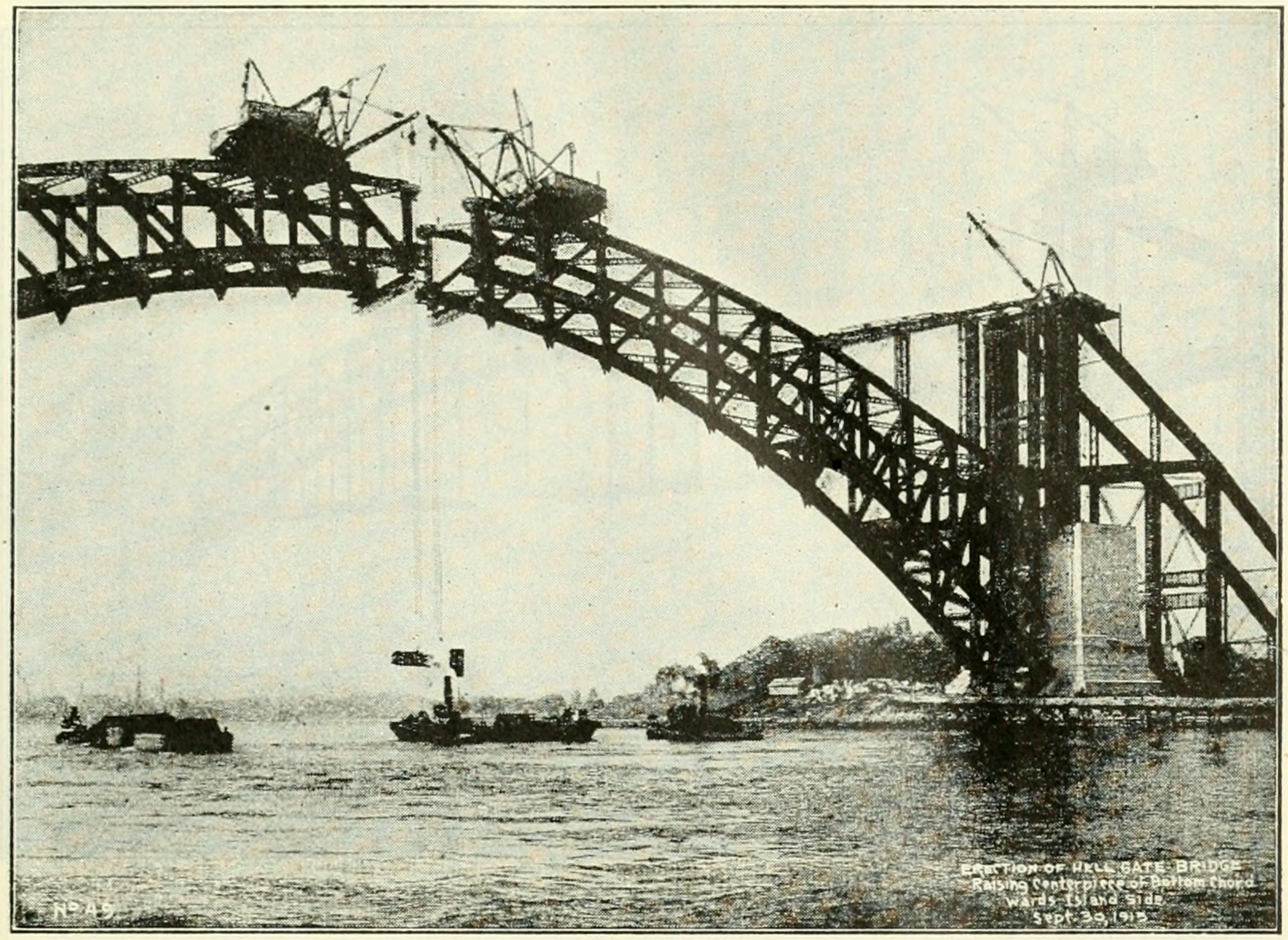


FIG. 29.—ERECTION STAGE, SEPTEMBER 30TH, 1915. RAISING CENTER PIECE OF BOTTOM CHORD, HELL GATE BRIDGE.

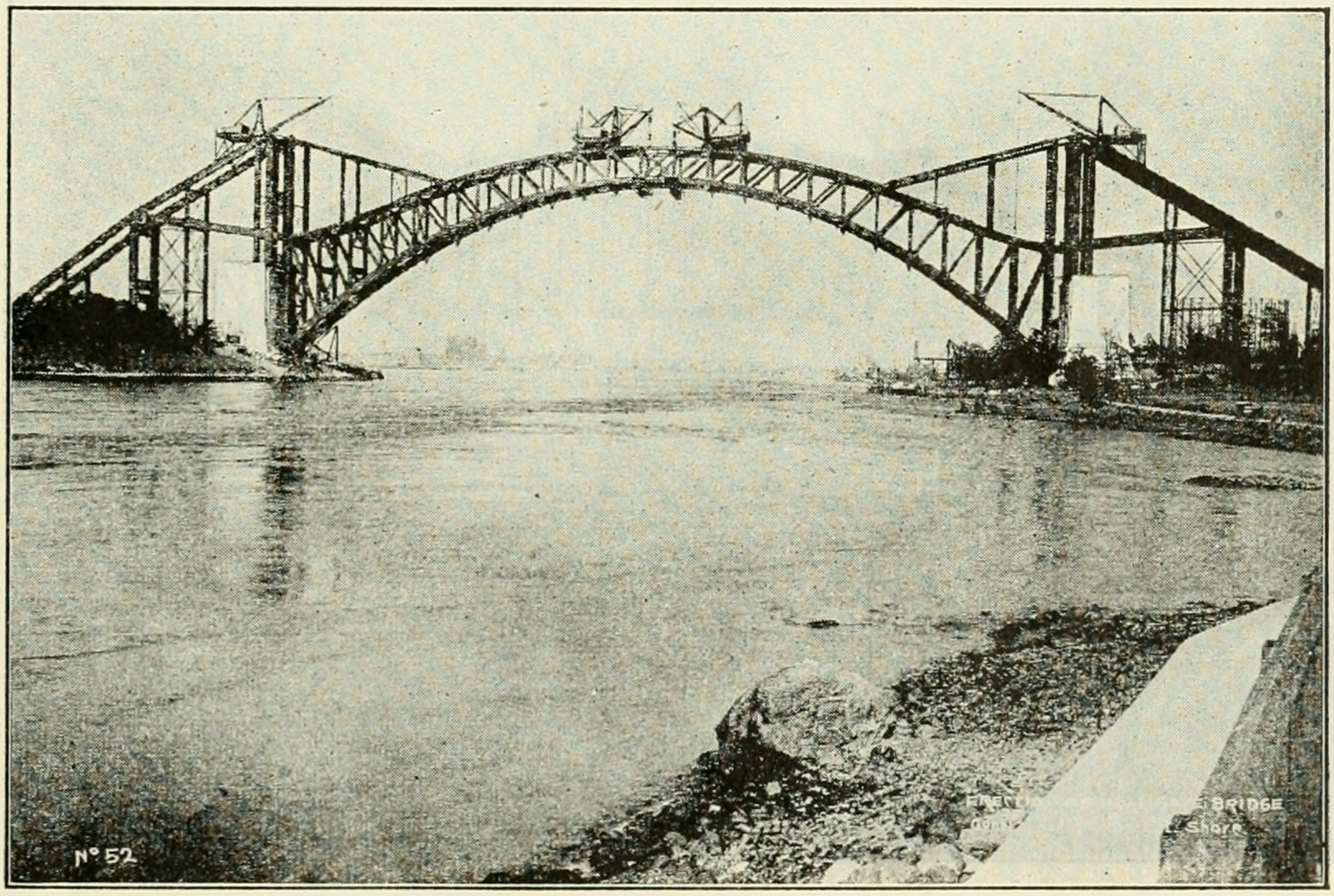
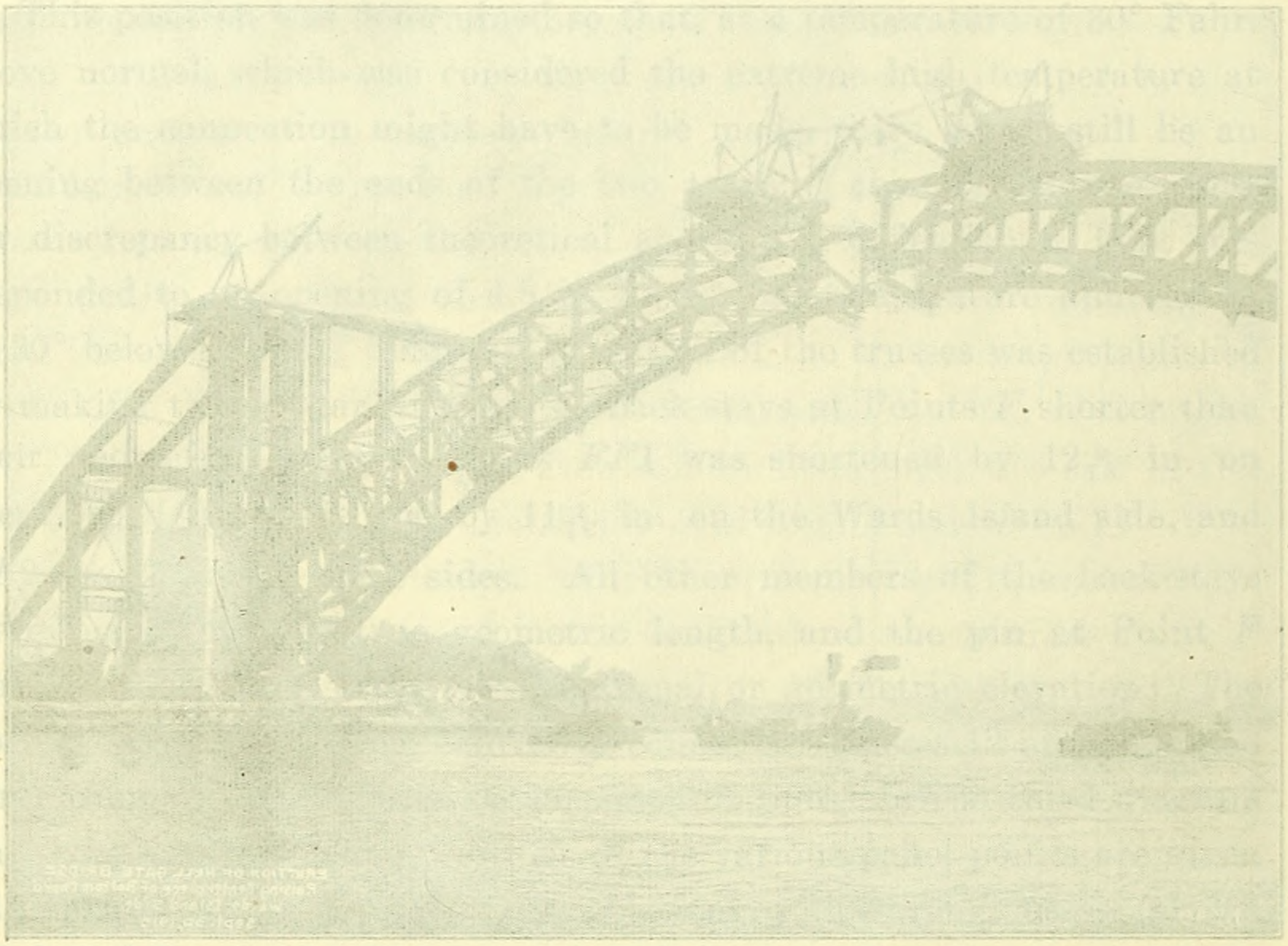


FIG. 30.—HELL GATE BRIDGE: ERECTION STAGE, OCTOBER 4TH, 1915.



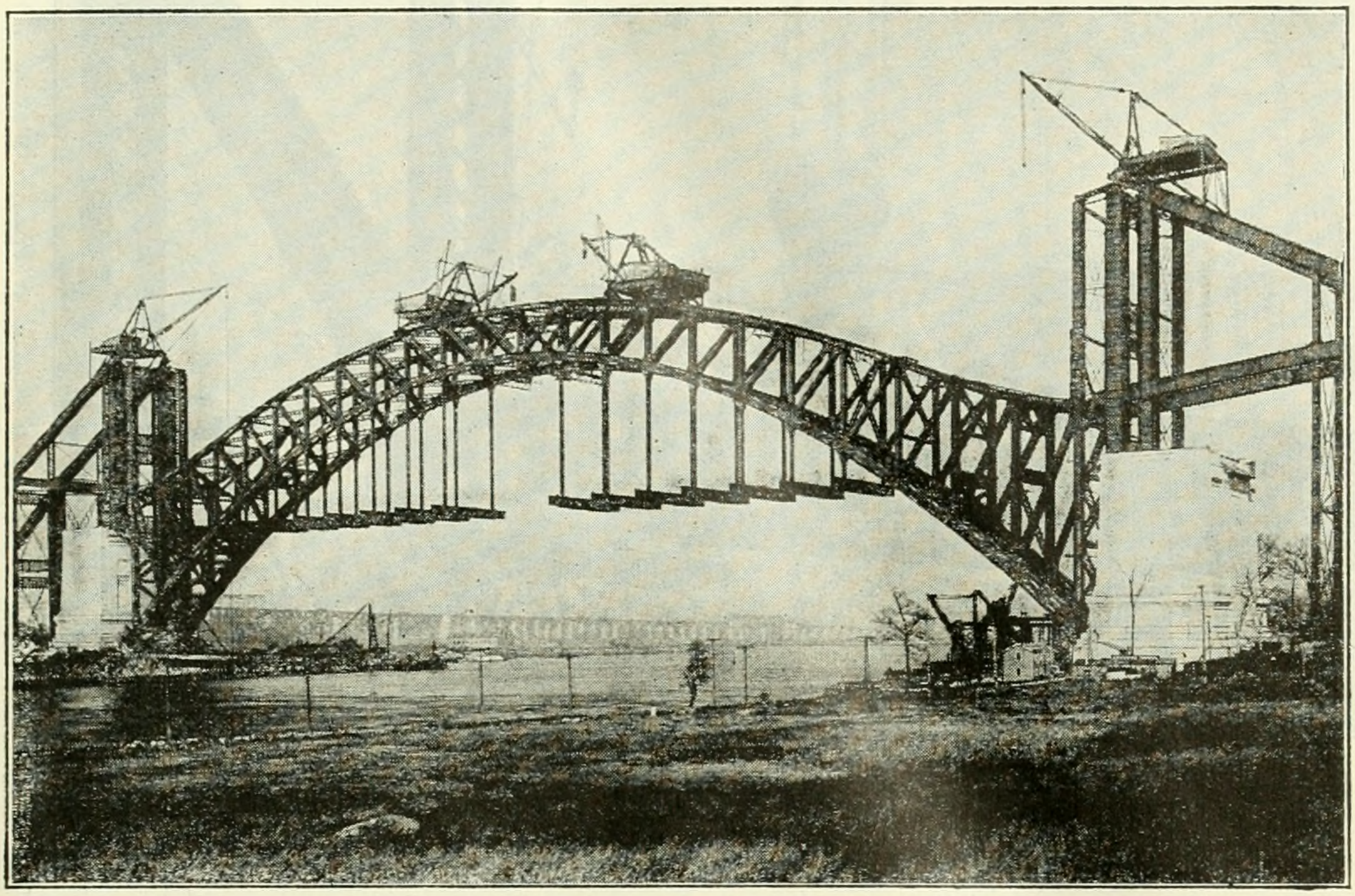


FIG. 31.—HELL GATE BRIDGE: ERECTION STAGE, NOVEMBER 1ST, 1915.

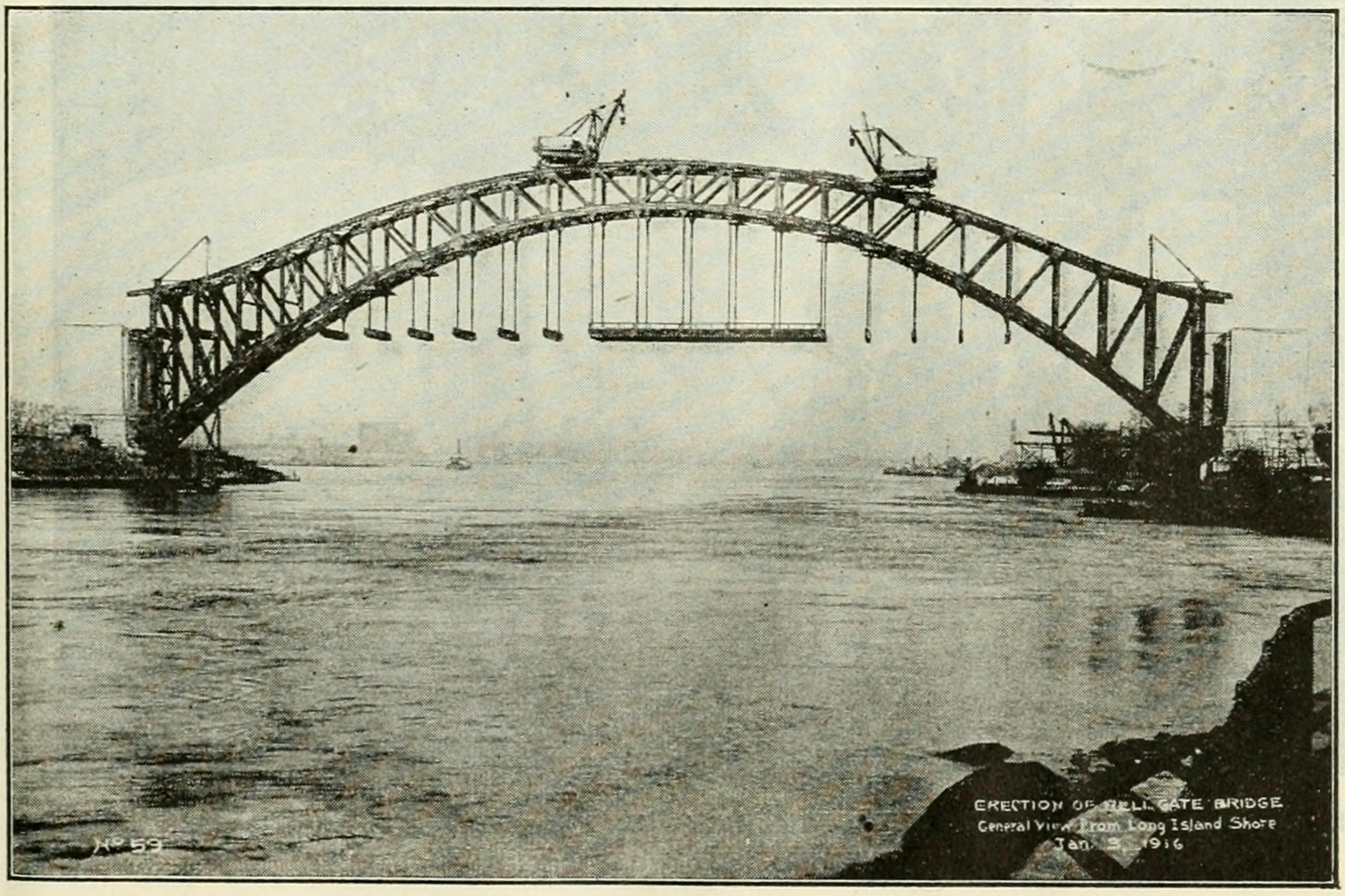


FIG. 32.—HELL GATE BRIDGE: ERECTION STAGE, JANUARY 3D, 1916.



FIG. 31.—HELL GATE BRIDGE: ERECTION STAGE, NOVEMBER 1ST, 1915.

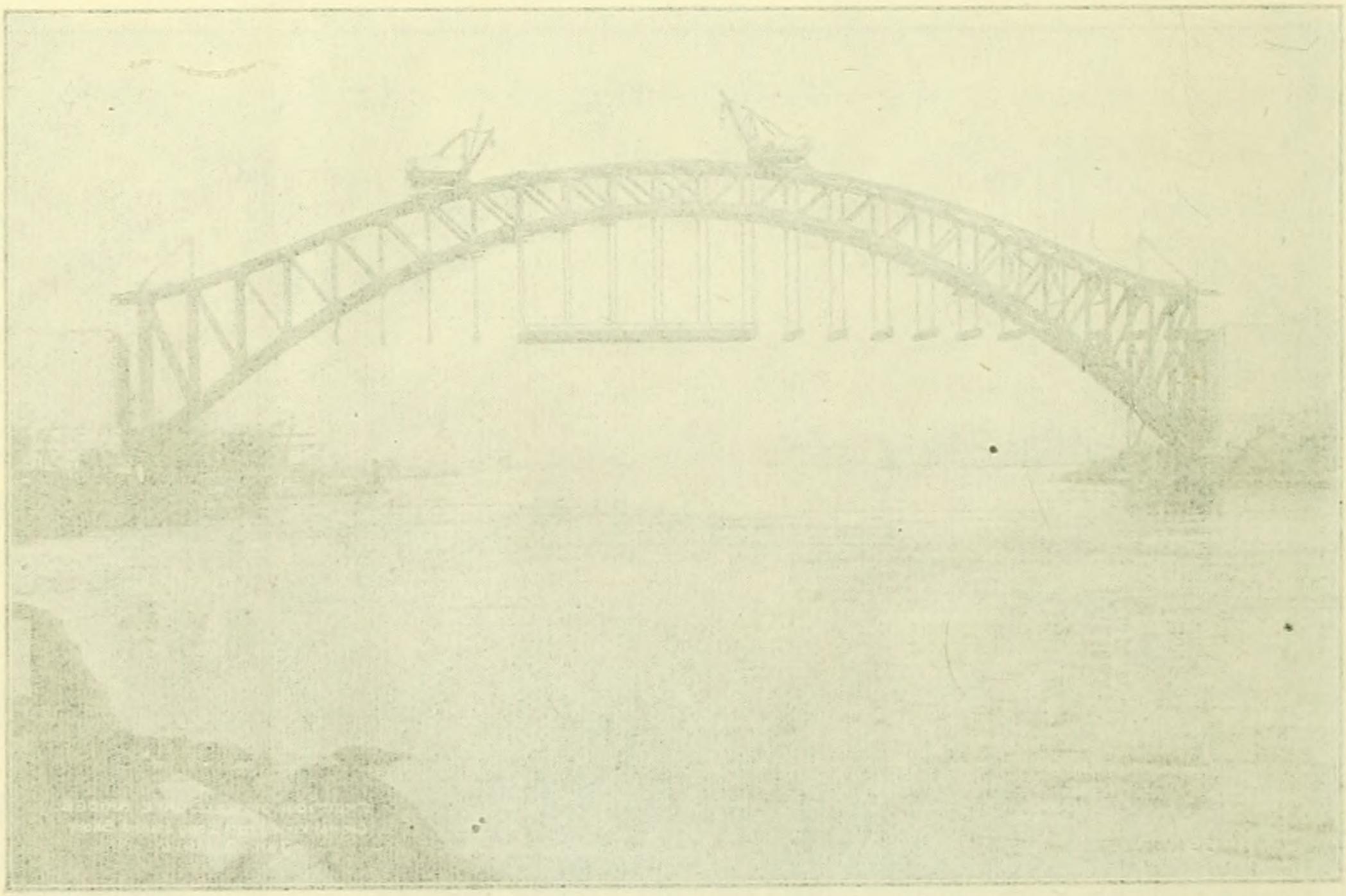


FIG. 32.—HELL GATE BRIDGE: ERECTION STAGE, JANUARY 3D, 1916.

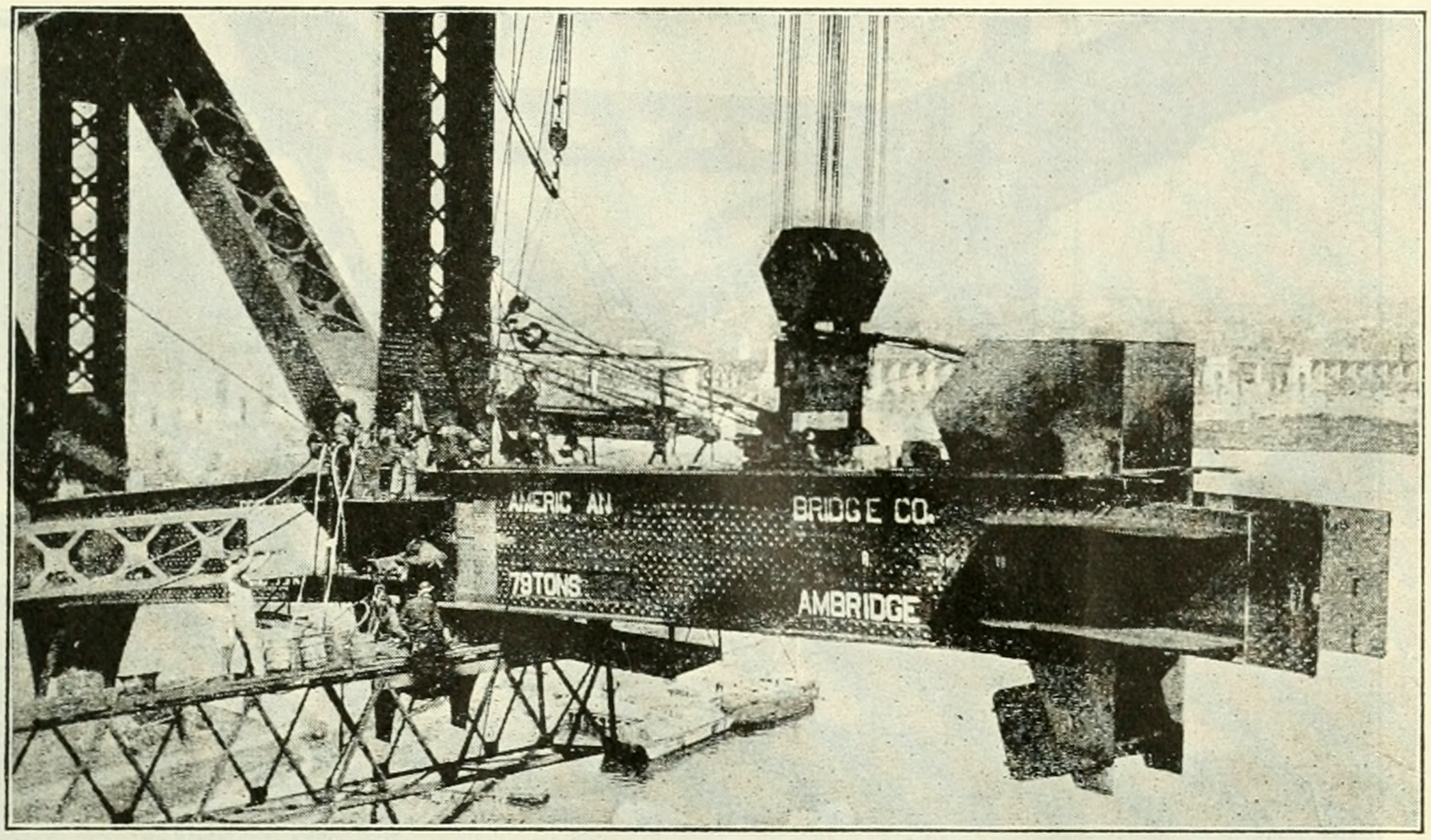


FIG. 33.—CONNECTING BOTTOM CHORD MEMBER, HELL GATE BRIDGE.

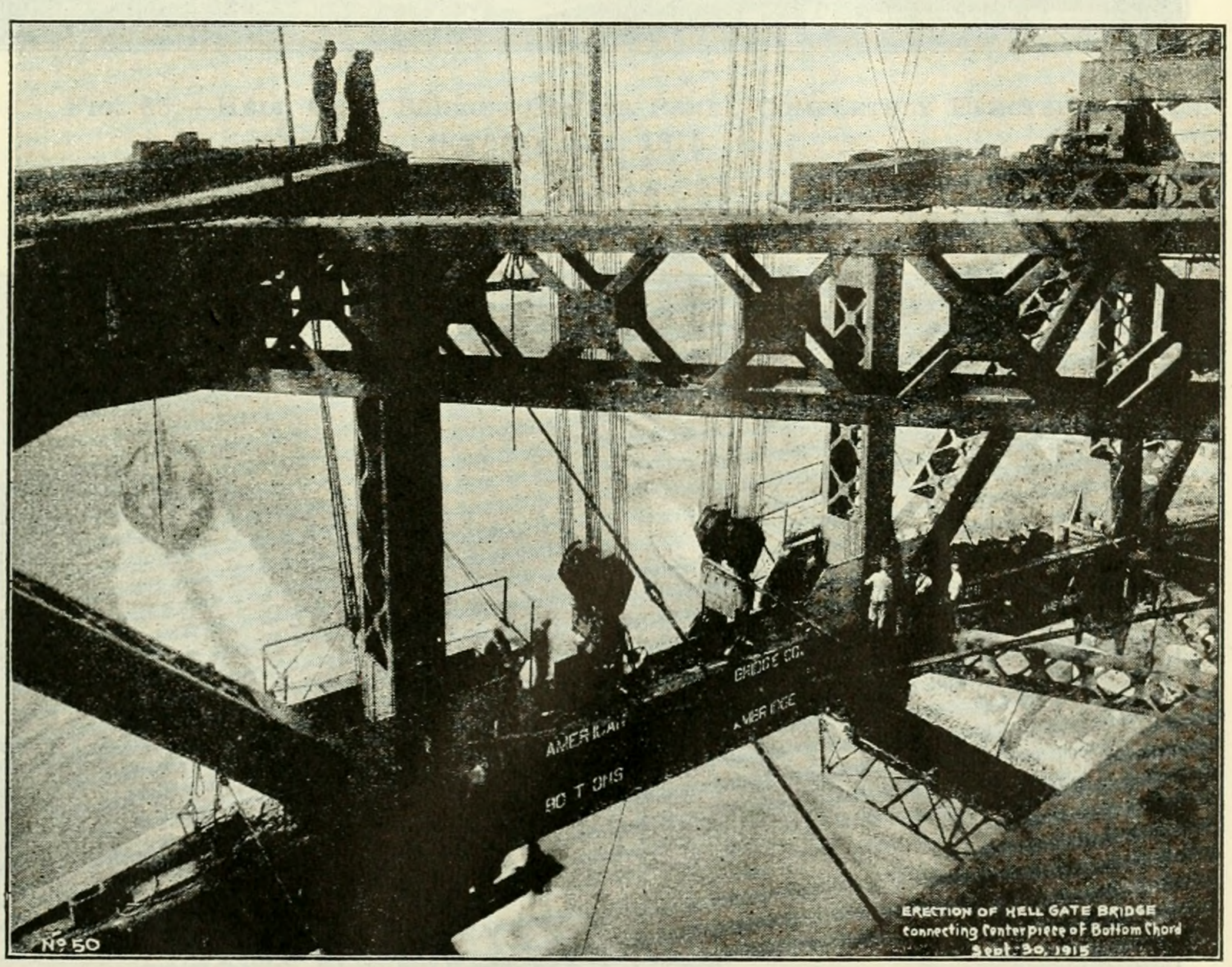


FIG. 34.—CONNECTING CENTER PANEL OF BOTTOM CHORD, SEPTEMBER 30TH, 1915, HELL GATE BRIDGE.

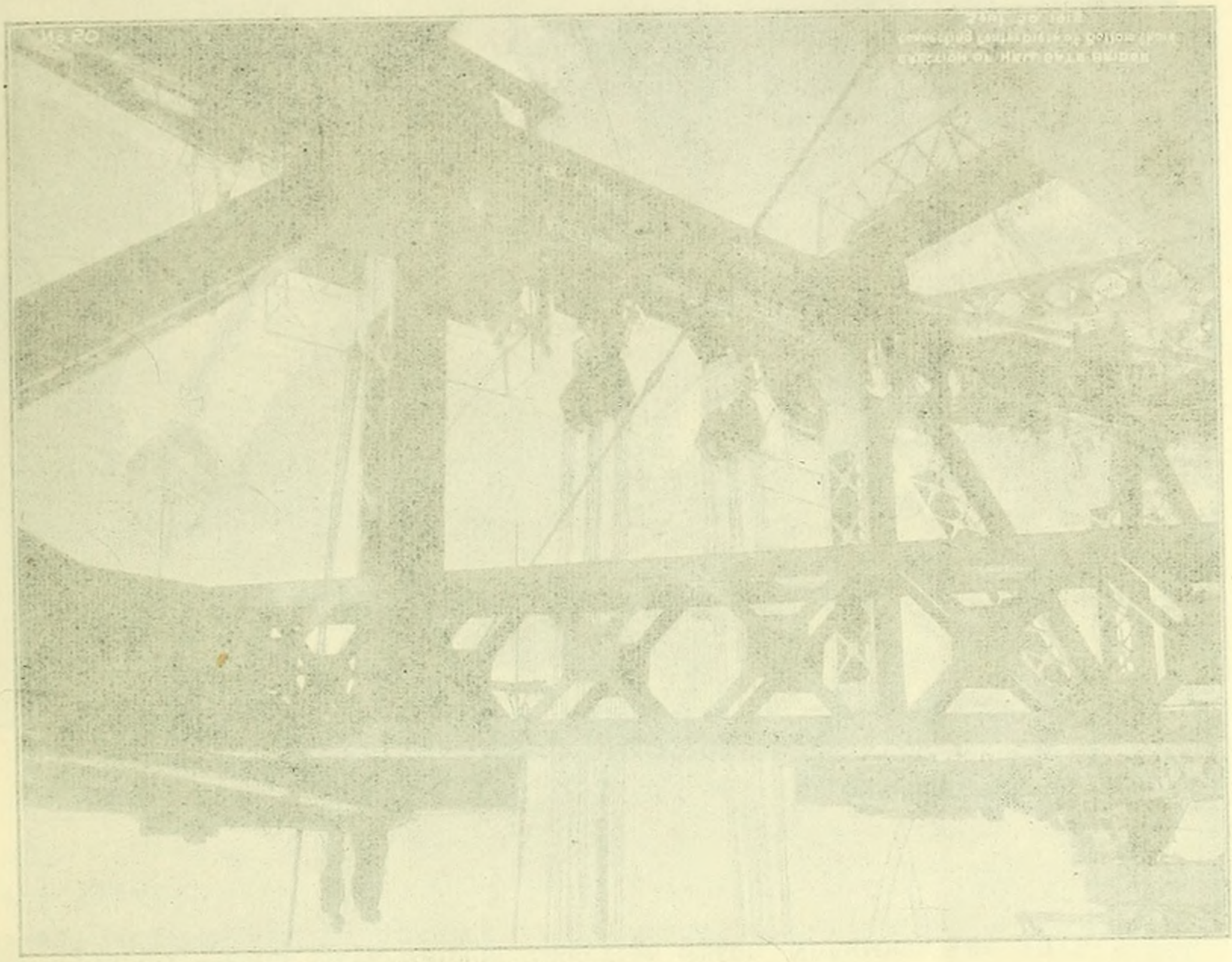


FIG. 32.—CONNECTING CENTER PANEL OF BOTTOM CHORD, SEPTEMBER 30TH, 1915.
HELL GATE BRIDGE.

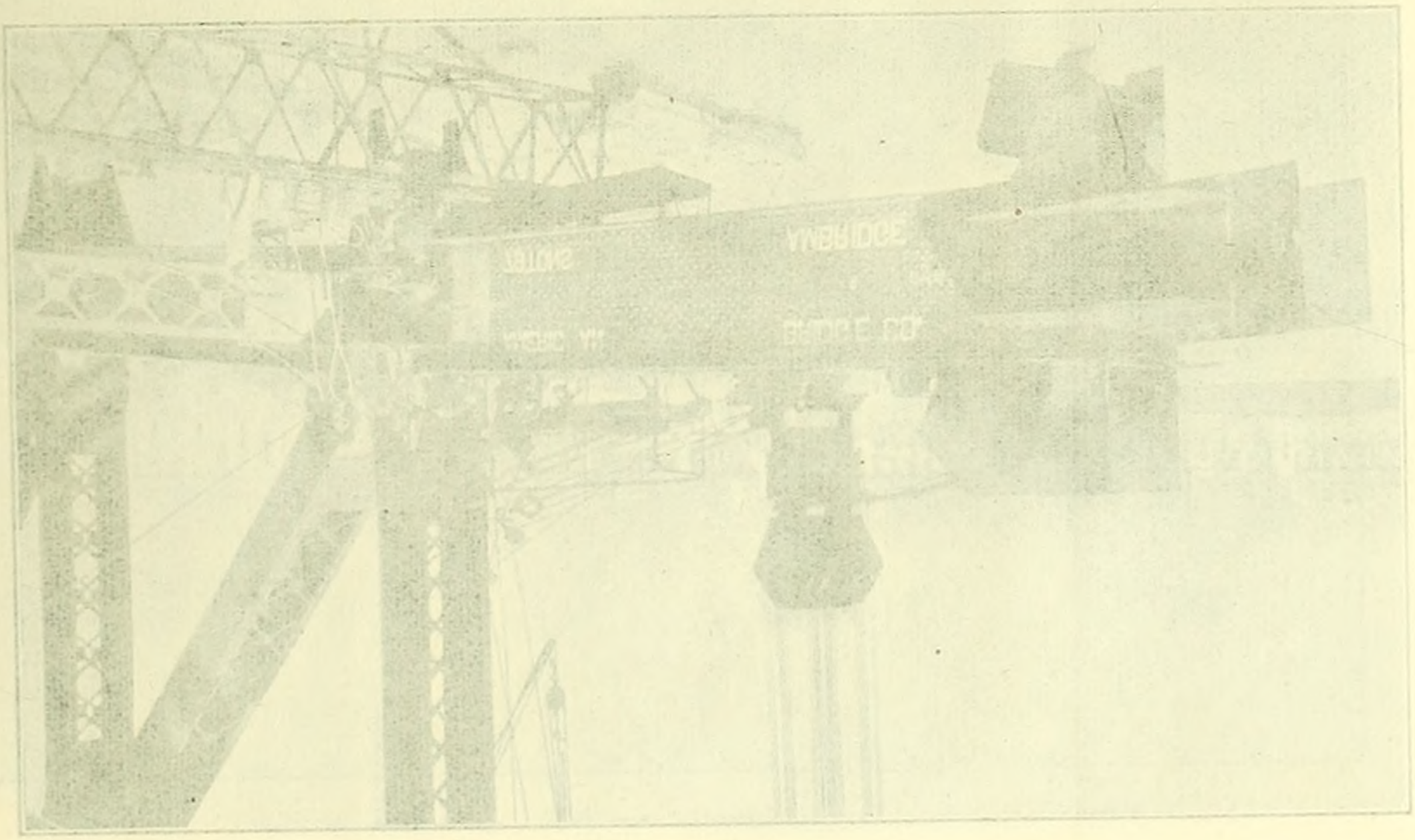


FIG. 33.—CONNECTING BOTTOM CHORD MEMBER, HELL GATE BRIDGE.

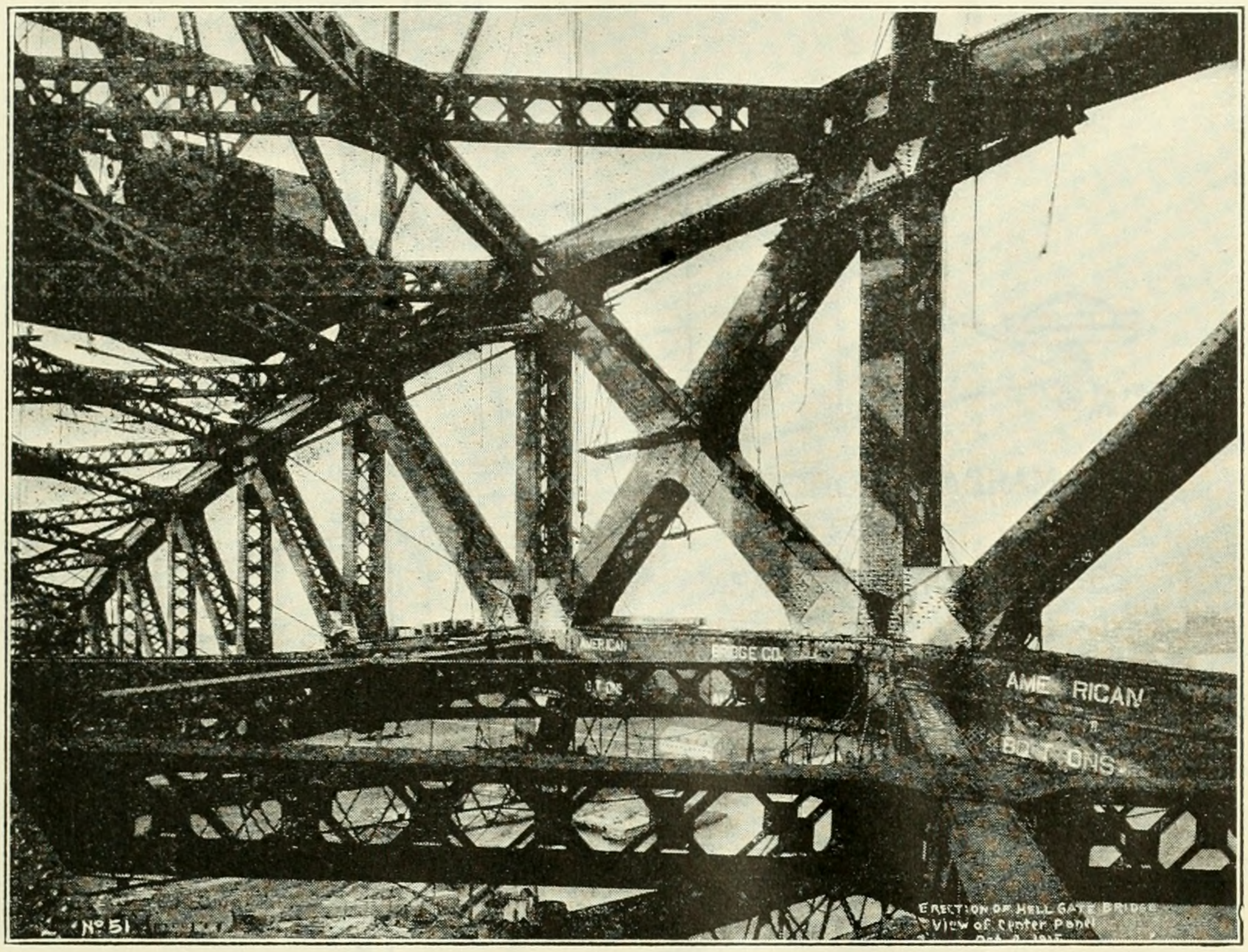


FIG. 35.—HELL GATE BRIDGE: CENTER PANEL COMPLETELY ERECTED, OCTOBER 4TH, 1915.

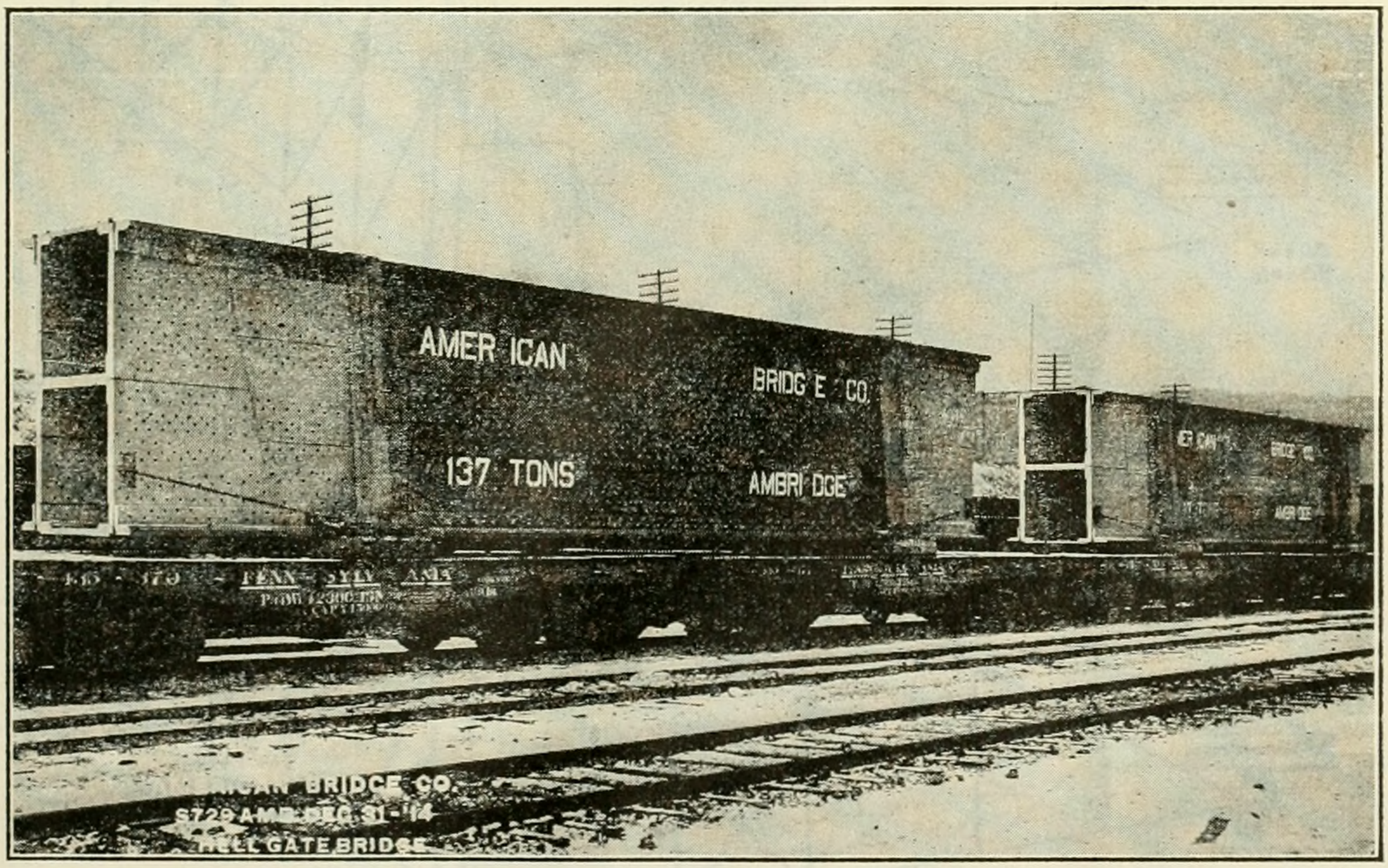


FIG. 36.—HELL GATE BRIDGE: TRANSPORTATION OF BOTTOM CHORD MEMBERS.

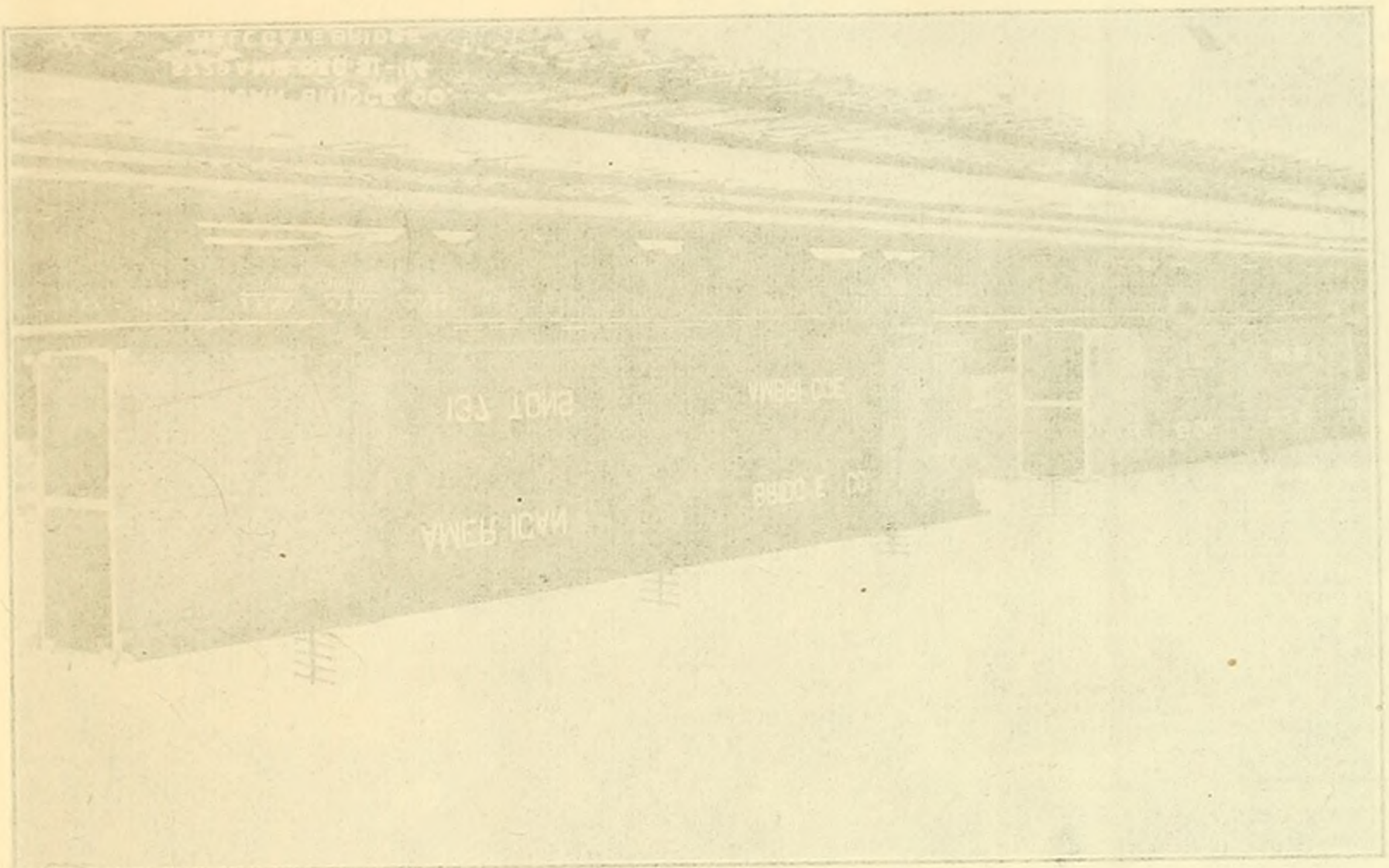


FIG. 31.—HELL GATE BRIDGE: TRANSPORTATION OF BOTTOM CHORD MEMBERS.

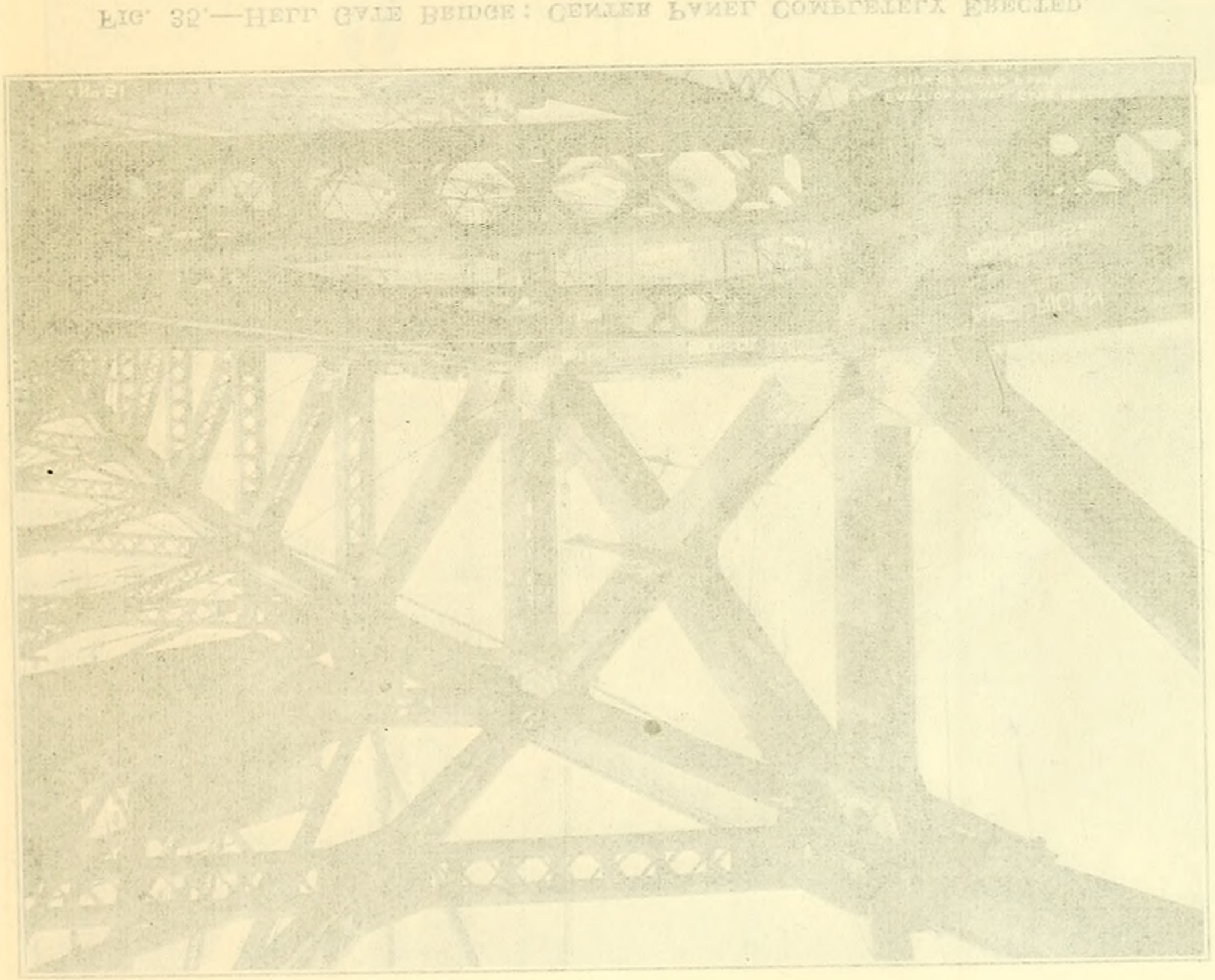
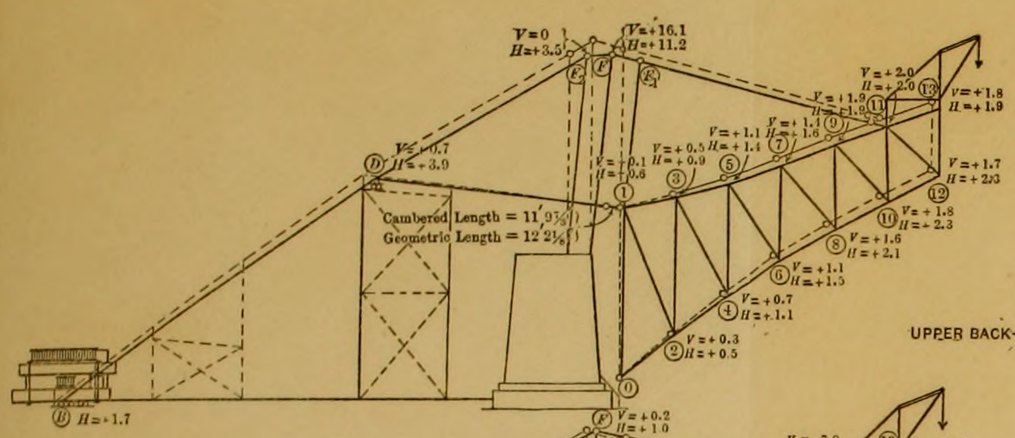
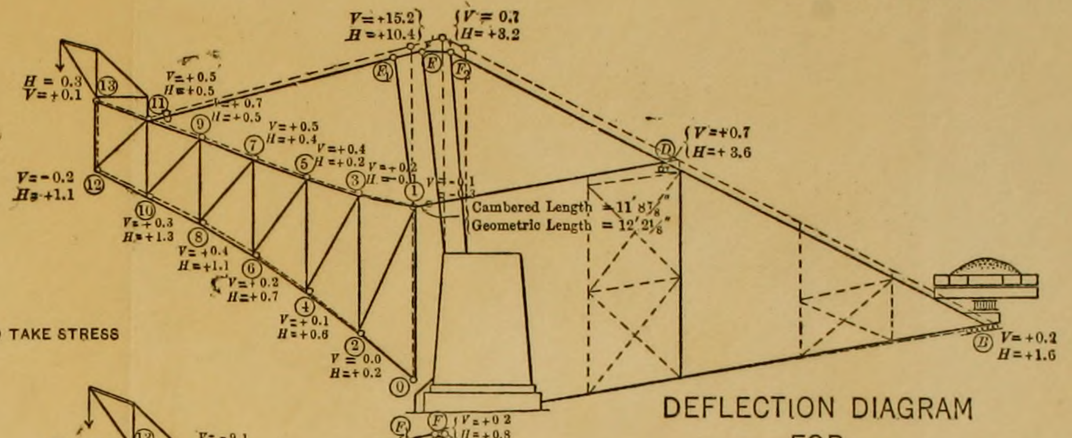


FIG. 32.—HELL GATE BRIDGE: CENTER PANEL COMPLETELY ERECTED.

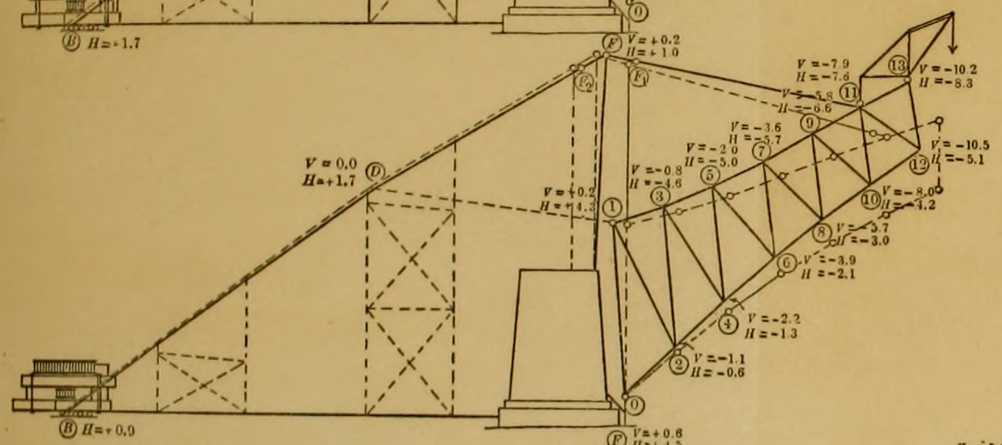


STAGE 6

UPPER BACK-STAY TIE CONNECTED AND JUST BEGINNING TO TAKE STRESS

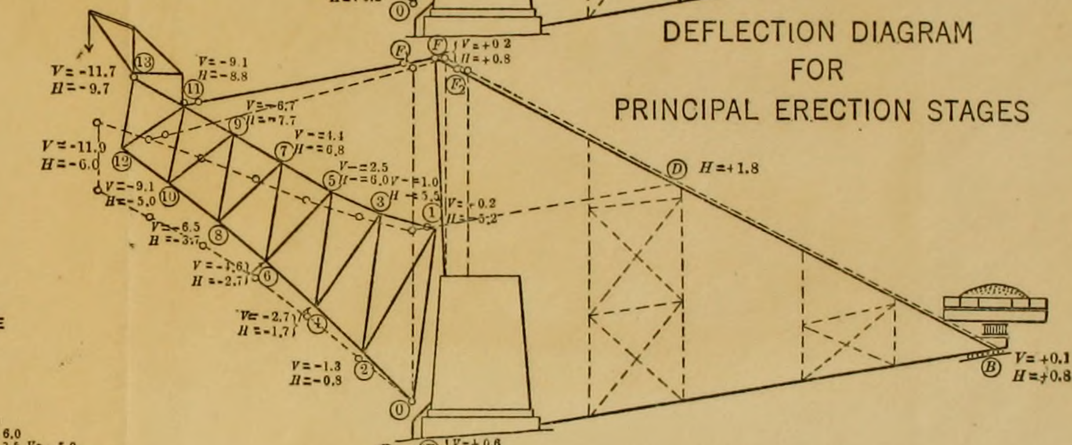


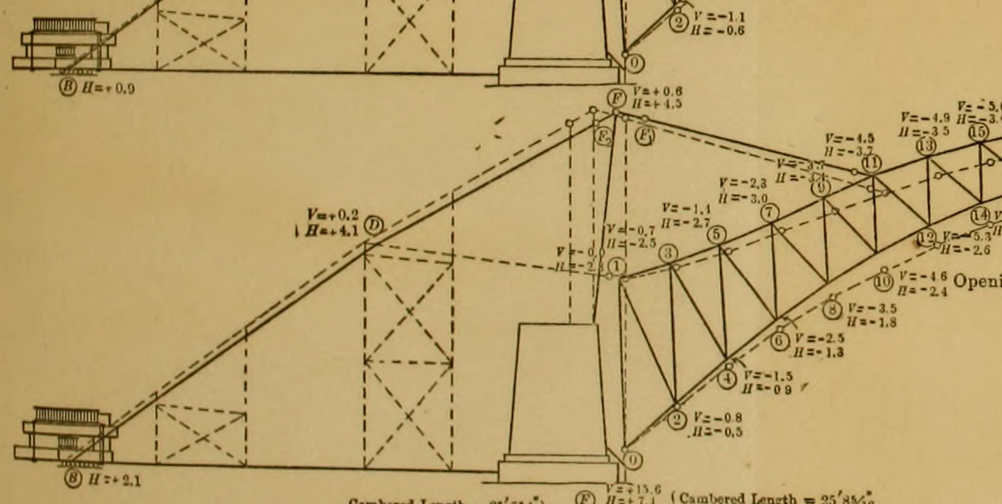
DEFLECTION DIAGRAM
FOR
PRINCIPAL ERECTION STAGES

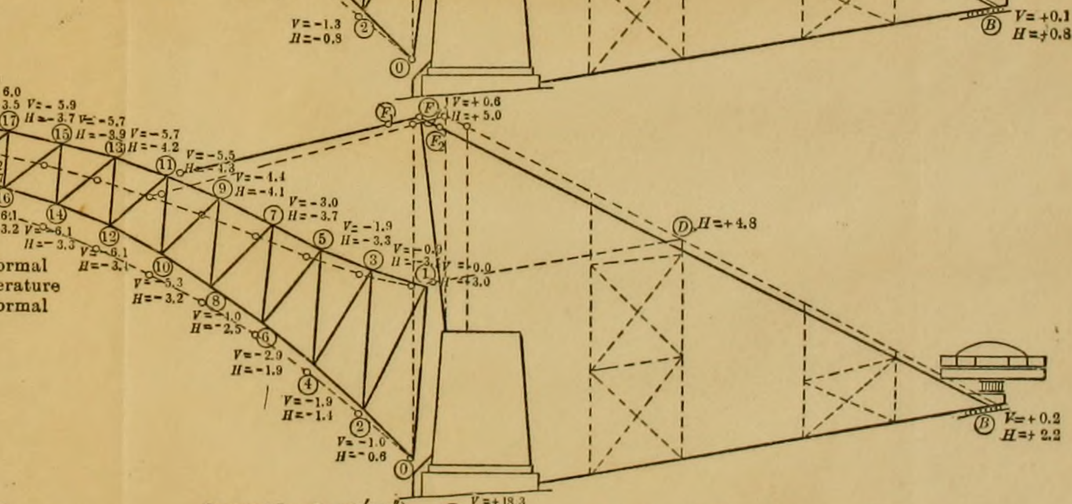


STAGE 7

AFTER DISCONNECTING LOWER BACK-STAY TIE
AND JACKING *F* TO FULL HEIGHT



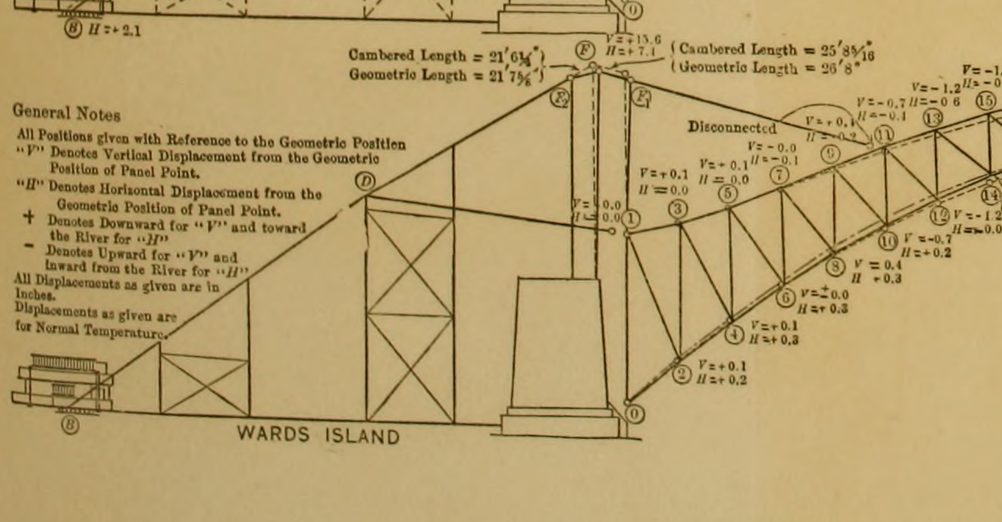


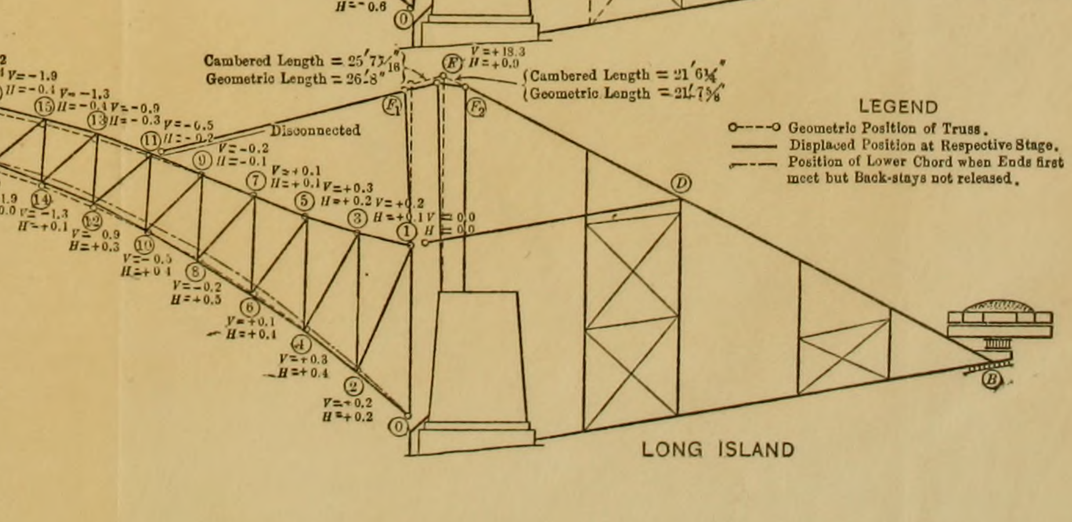


Opening at Panel Point 22 { 1.5 In. at 30° F. above Normal
4.8 In. at Normal Temperature
8.1 In. at 30° F. below Normal

STAGE 12 STAGE 12A

IMMEDIATELY BEFORE LOWERING TRUSSES





STAGE 13

ARCH FIRST SELF-SUPPORTING
A 3-HINGED ARCH CARRYING ERECTION LOADS

General Notes

All Positions given with Reference to the Geometric Position

"*V*" Denotes Vertical Displacement from the Geometric Position of Panel Point.

"*H*" Denotes Horizontal Displacement from the Geometric Position of Panel Point.

\+ Denotes Downward for "*V*" and toward the River for "*H*"

\- Denotes Upward for "*V*" and Inward from the River for "*H*"

All Displacements as given are in Inches.

Displacements as given are for Normal Temperature.

LEGEND

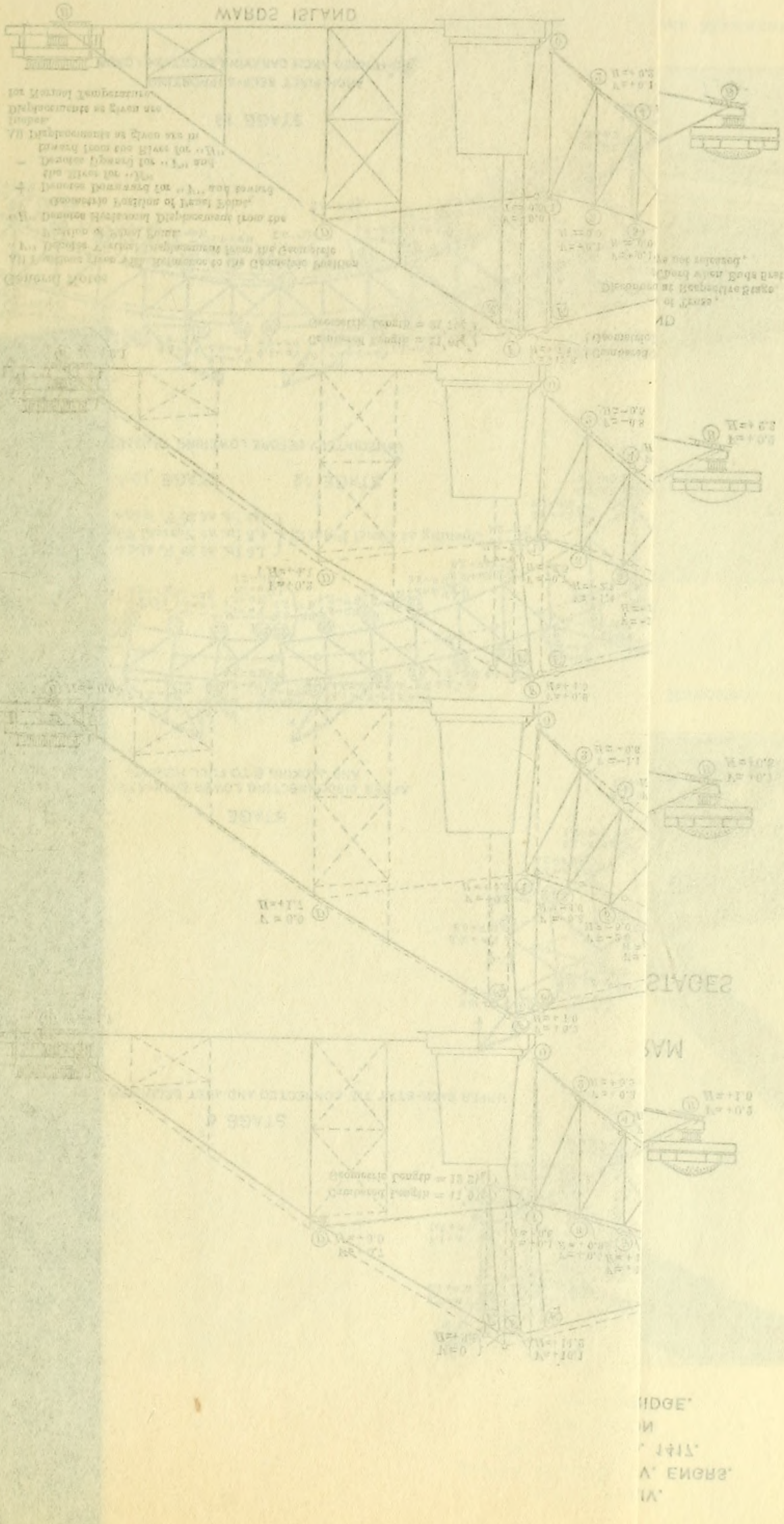
Geometric Position of Truss.

Displaced Position at Respective Stage.

Position of Lower Chord when Ends first meet but Back-stays not released.

WARDS ISLAND

LONG ISLAND



to the normal elevation required for Stage 12. To keep the amount of jacking within the range required for the closing operation, it was found necessary to make the eye-bar link, *l-g*, of the lower back-stay shorter than its geometric length by $5\frac{1}{4}$ in. on the Long Island side and by $4\frac{1}{4}$ in. on the Wards Island side.

During the first part of the movement from Stage 6 to Stage 7, during which the upper back-stay gradually took its full stress, the trusses changed their elastic form; during the second part, after the lower back-stays had been disconnected, the trusses merely revolved around their bearings. The intermediate stage is not shown in the deflection diagrams. The total vertical movement of Panel Point 11 during this operation was 9.6 in. on the Long Island side and 9.9 in. on the Wards Island side. It is interesting to note, also, the very considerable horizontal deflection of Points *F* at the top of the back-stay column during this operation, amounting to 10.4 in. on the Long Island side and 11.2 in. on the Wards Island side. Fig. 37 shows the theoretical movements of the hydraulic jacks and the positions of the pins, *F*, at the various critical stages.

Jacking Operation for Change from Lower to Upper Back-Stay.—When Erection Stage 6 (Plate XXXIV) had been reached, the upper back-stay was connected to the truss at Panel Point 11, with a slight play between the connecting pin and its bearing on the gusset-plates of the truss. The shoes at *F* were then jacked up until the pin at 11 got a firm bearing and the upper back-stay began to take stress (Fig. 37*a*). Jacking was then continued until the upper back-stay had taken full stress and the lower back-stay could be disconnected (Fig. 37*b*). After this the shoes were further raised to their normal elevation (Fig. 37*c*), corresponding to the required position of the trusses for Stage 7. The calculated total raising of the shoes was $22\frac{1}{2}$ in. The shims between the shoe and their original bearings on the post were inserted as fast as the jacking proceeded, and, after completion of the jacking, the jacks were released.

Actually, the procedure on the Long Island side differed somewhat in that the jacking operation was discontinued after the upper tie had taken about 75% of its stress. Erection was then continued until the seventh panel had been placed, after which the jacking operation was completed. The reason for this procedure was that, in changing from Stage 6 to Stage 7, the foundation pressure under the counter-

weight would have increased from 2.0 to 3.9 tons per sq. ft. This sudden increase might have caused excessive settlement, as the ground was comparatively soft. By adding the seventh panel and moving the traveler to Panel Point 15 before completing the jacking, it was possible to keep the foundation pressure within 3 tons.

A complete record of this jacking operation for both sides is given in Tables 4 and 5. It shows a remarkable coincidence between the jacking heights as calculated and as observed. The "effective" jacking height, that is, the amount of jacking from the moment the upper back-stay started to take stress until the stress in the lower back-stay became zero, was actually 11 in. (north truss) and $11\frac{3}{8}$ in. (south truss), on the Long Island side, and 13 in. (north truss) and 13 in. (south truss), on the Wards Island side, as compared with $10\frac{7}{8}$ in.

TABLE 4.—RECORD OF JACKING FOR CHANGING

Date.	Time.	Remarks.	Height of Shims. Observed: N. Truss. Inches.	Observed: S. Truss. Inches.	Calc. Inches.	Percentage of effective jacking completed.
Aug. 28.	.....	STAGE 6.—Lower stay fully stressed.....	3 5/8	3 5/8	.....	....
Aug. 31.	.....	Slight jacking to take out slack in upper stay.	4 1/4†	3 7/8†	6 3/8	0
		Continued jacking and stopped at.....	6	5 7/8	.....	14
Sept. 2.	8.45	Started jacking at.....	5 7/8	5 9/16	.....	13
	9.20	Continued jacking.....	14 1/2	14 1/8	.....	77
	9.33	Continued jacking.....	16 3/8	15 3/4	.....	90
	9.38	Continued jacking.....	17 1/2	16 5/8	.....	97
	9.41	Continued jacking and stopped at.....	17 5/8	17 1/8	19 1/2	100
	9.46	Jacks lowered to bearing on shims.....	16 3/8	16 1/4	.....	92
Sept. 8.	.....	Raised jacks—7 panels erected	17	16 1/2	.....	97
		Continued jacking to remove pins in lower stay	17 1/4	17 1/8	19 1/2	100
		Total effective jacking from 0 to full stress in upper stay.....	13	13 1/4	13 1/8	.....
		Continued jacking after removing pins in lower stay.....	22 7/16	22 3/8	22 1/2	.....
Sept. 9.	.....	Raised jack to level up pins over jacks and at Panel Point No. 11.....	22 5/8	21 7/8	22 1/2	.....

No allowance made for friction. † Interpolated, not observed. ‡ Stress falling rapidly.

(Long Island side) and 13⅛ in. (Wards Island side), respectively, as calculated. The tables also show a close agreement between the tension stresses in the eye-bar links of the upper and lower back-stays as calculated and as actually observed in the field by extensometer measurements. This proves the practicability of such measurements and their value as a means of checking stresses for similar operations. The discrepancy between the calculated jack pressures and the values observed on the pressure gauge is due to the friction between the plunger and the cylinder.

Jacking Operation for Closing Arch.—The closing of the trusses at the center and their transformation from cantilevers into three-hinged arches proceeded as follows: As the Erection Stages 12 (Wards Island side) and 12*A* (Long Island side) (Plate XXXIV) had been

BACK-STAYS—HELL GATE ARCH—WARDS ISLAND END.

Jack Pressure.		Tension in Lower Stay (Eye-bar Links).			Tension in Upper Stay (Eye-bar Links).			
		Observed:			Observed:			
Meas.	Calc.*	N. Truss.	S. Truss.	Calc.	N. Truss.	S. Truss.	Calc.	Remarks.
Pounds per square inch.	Pounds per square inch.	Pounds per square inch.	Pounds per square inch.	Pounds per square inch.	Pounds per square inch.	Pounds per square inch.	Pounds per square inch.	
0	0	19 500	19 950	18 160	0	0	0	Before any jacking, 6 panels erected.
.....	.....	19 500	19 950	18 160	0	0	0	Upper stay begins to act.
500	220	17 100	17 000	16 200	1 050	1 950	1 050	In this condition until Sept. 2d, 1915.
500	200	.....	.....	.....	.....	.....	.....	Traveler moved off jacks before starting.
1 300	1 200	11 400‡	.....	4 300	.....	.....	.....	Extensometer readings in lower stay.
1 500	1 400	.....	2 700	1 900	.....	.....	.....	Extensometer readings in lower stay.
1 700	1 500	900	.....	600	.....	.....	.....	Extensometer readings in lower stay.
1 725	1 550	.....	150	0	.....	.....	7 500	Lower stay eye-bars tested and found slack—Stage 7.
1 600	1 430	2 400	2 400	1 500	7 650	8 060	8 500	In this condition till Sept. 8th.
1 900	1 660	.....	.....	600	.....	.....	8 960	
2 000	1 710	150	.....	0	10 200	10 670	9 250	Pins removed to disconnect lower stay.
.....	.....	.....	.....	.....	.....	.....	.....	
2 050	1 710	.....	.....	.....	.....	.....	.....	
.....	.....	.....	.....	.....	.....	.....	.....	Stage 8.—Jacking completed.

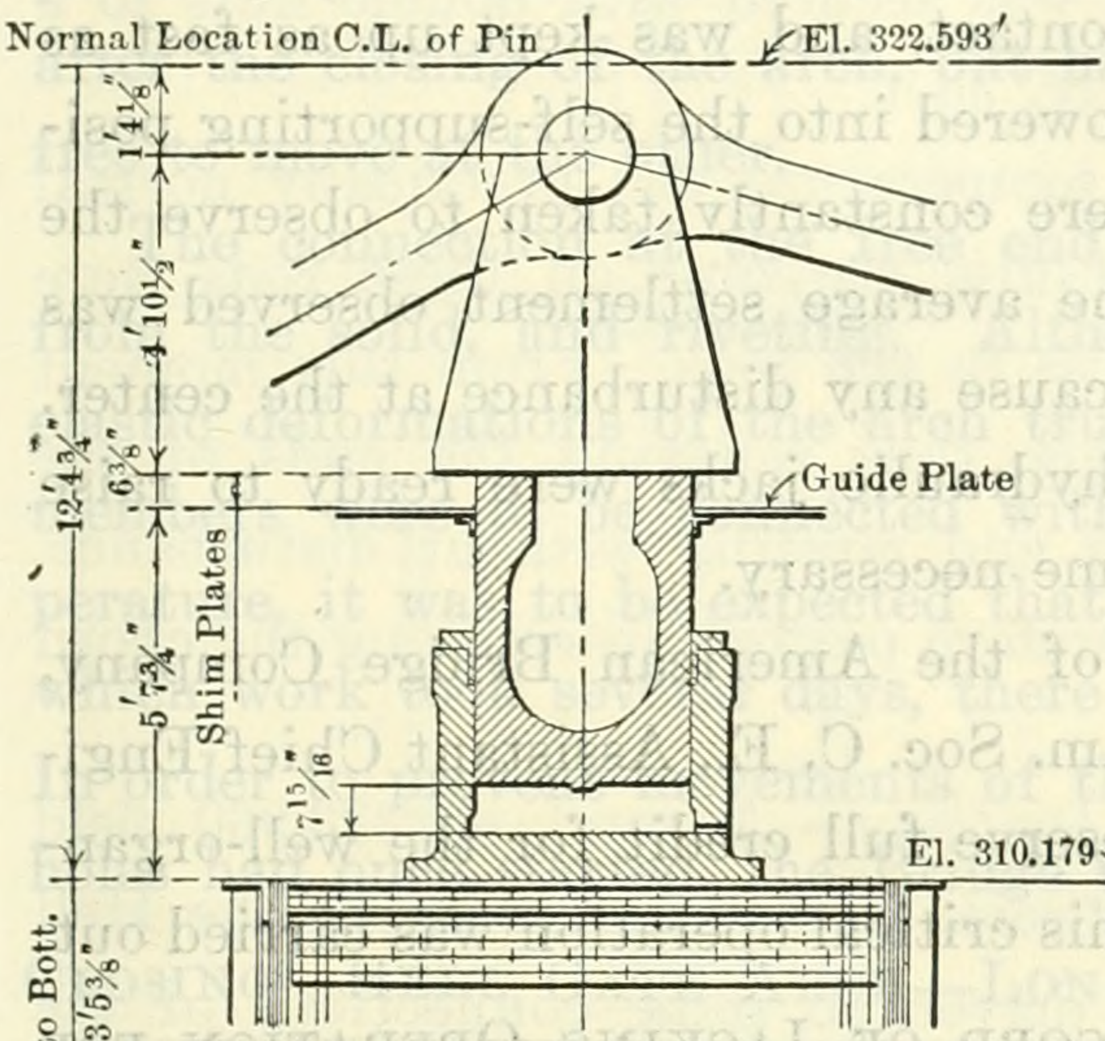
reached, the jack plungers were brought to bear against the shoes at Points *F*, which were in the position shown in Fig. 37*c*, and lifted them slightly so that the removal of the shim plates from under the shoes could be started. The jacks were then lowered until the bottom chords came into contact at Point 22, Wards Island side. After lowering a little farther, so as to produce a slight initial stress in the center chord, jacking was stopped. The connection was then made at 22, Wards Island side, with bolts and drift-pins. The jacks were then lowered farther, until the upper back-stay tie was relieved of stress (Fig. 37*d*).

Table 5 gives the record of this operation, and shows, again, a close agreement between the calculated and actual "effective" jacking heights, that is, the amount of jacking from the moment the ends of the two arms came to bear to the moment when the stress in the back-stays became zero. This height was actually 15 in. at both trusses on the Long Island side and $12\frac{1}{8}$ in. (north truss) and $12\frac{5}{16}$ in. (south truss) on the Wards Island side, as compared with $14\frac{3}{4}$ in. (Long Island side) and $12\frac{3}{16}$ in. (Wards Island side), respectively, as calculated. The opening at the center, before lowering started, was actually $4\frac{9}{16}$ in. at a temperature of 56° Fahr., as compared with $4\frac{3}{4}$ in. as calculated for the normal temperature of 60° Fahr. Panel Points 22 were found to be on an average 1 in. lower than the theoretical position, which is a very small deviation for such a great span. A difference in temperature of 10° would have been sufficient to produce that difference in elevation.

Weather conditions were very favorable for the closing operation, the sky having been covered and the temperature nearly constant at 56° Fahr. Great care had to be exercised to bring the ends of the two arms into perfect contact, and, after that, it was important to insure the simultaneous, proportionate lowering of all four jacks, as otherwise serious shearing stresses might have been caused in the center connection. This was secured by a telephone system through which constant communication was kept between the men in charge at the tops of the erection towers, where the jacks were operated, at the center of the arch, and at the main field office.

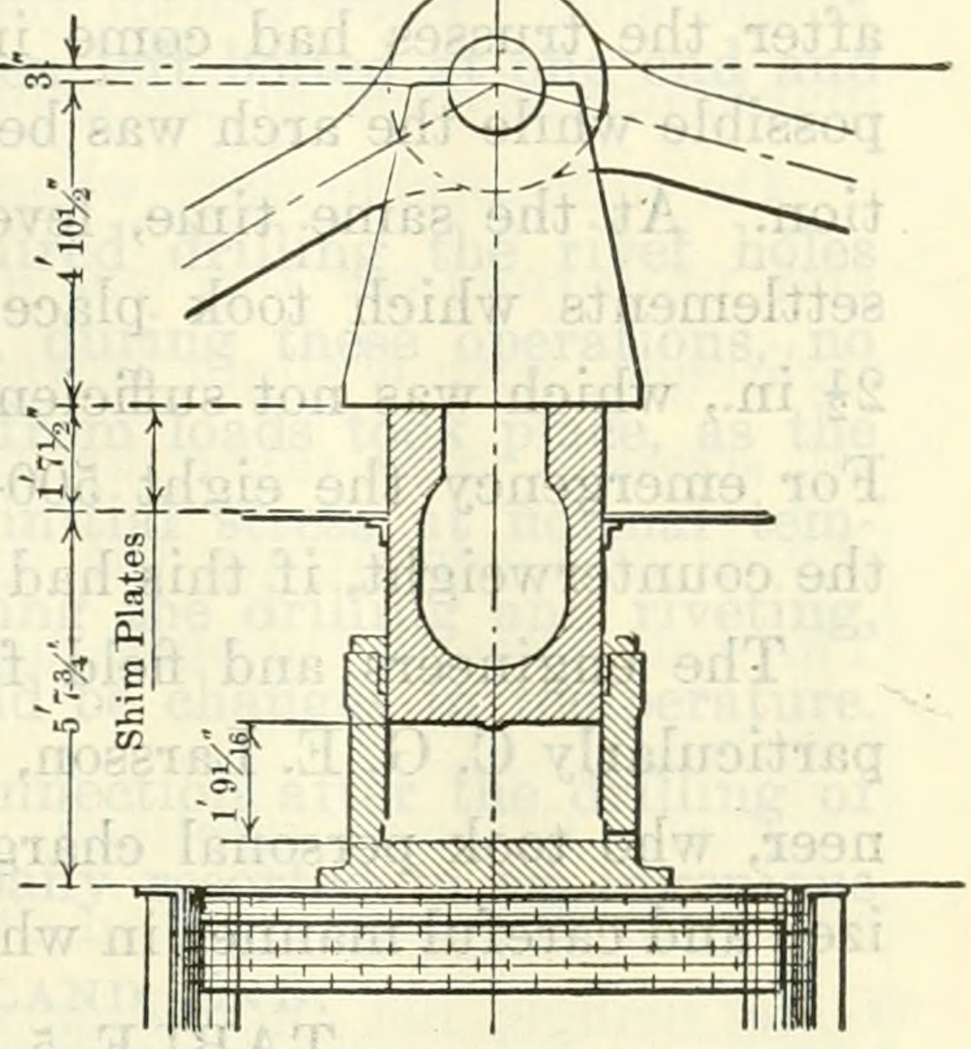
During the release of the back-stays the foundation pressure under the counterweights would, without reduction of the latter, have been gradually increased from 2.0 to 7.8 tons per sq. ft. This might have

DIAGRAM OF MOVEMENTS OF 3000-TON HYDRAULIC JACKS



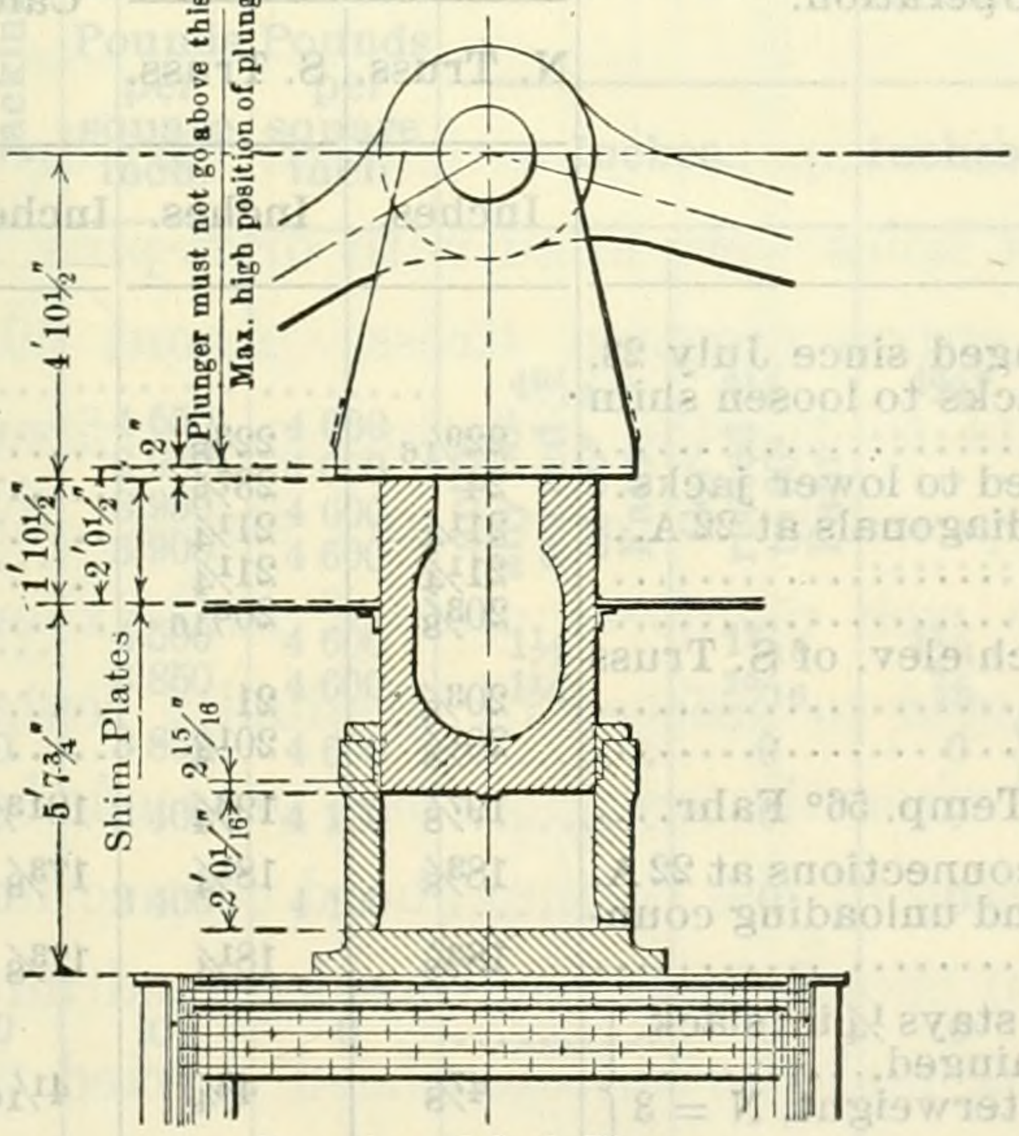
Location of Jack Plunger to connect Upper Back-stay at Panel Point No. 11

(*a*)



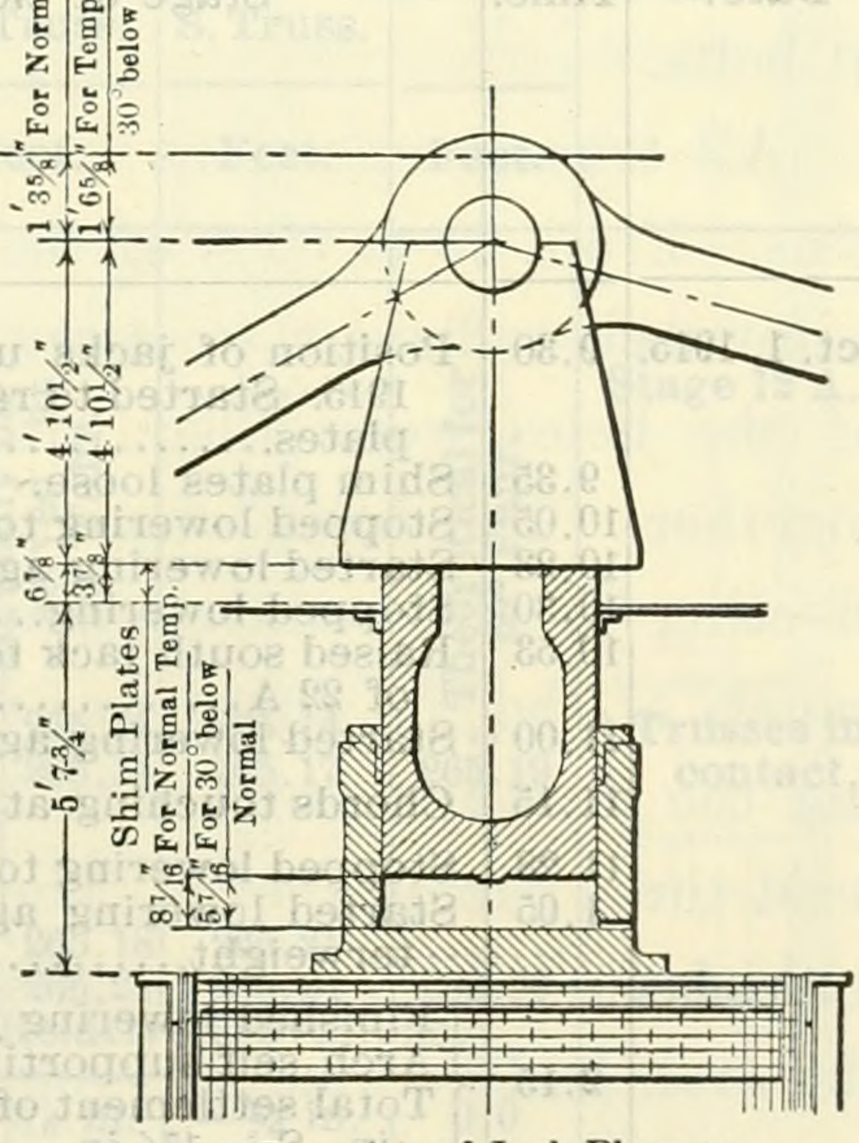
Location of Jack Plunger to disconnect Lower Back-stay at Panel Point No. 1

(*b*)



Location of Jack Plunger to continue erection after connection of Upper Back-stay at Panel Point No. 11. and disconnection of Lower Back-stay at Panel Point No. 1.

(*c*)



Location of Jack Plunger to disconnect Upper Back-stay at Panel Point No. 11.

(*d*)

FIG. 37.

caused excessive settlement of the comparatively soft ground on the Long Island side and interfered seriously with the closing operation. To relieve the foundation pressure, the removal of part of the earth fill from the counterweight box was started, therefore, immediately after the trusses had come into contact, and was kept up as fast as possible while the arch was being lowered into the self-supporting position. At the same time, levels were constantly taken to observe the settlements which took place. The average settlement observed was $2\frac{1}{2}$ in., which was not sufficient to cause any disturbance at the center. For emergency the eight 500-ton hydraulic jacks were ready to raise the counterweight, if this had become necessary.

The engineers and field force of the American Bridge Company, particularly C. G. E. Larsson, M. Am. Soc. C. E., Assistant Chief Engineer, who took personal charge, deserve full credit for the well-organized and careful manner in which this critical operation was carried out.

TABLE 5.—Record of Jacking Operation for

Date.	Time.	Stage of closing operation.	Height of Shims. Observed. N. Truss.	Observed. S. Truss.	Calc.
			Inches.	Inches.	Inches.
Oct. 1, 1915.	9.30	Position of jacks unchanged since July 23, 1915. Started to raise jacks to loosen shim plates	$22\frac{9}{16}$	$22\frac{3}{8}$	.....
	9.35	Shim plates loose. Started to lower jacks	24	$23\frac{7}{8}$	.....
	10.05	Stopped lowering to bolt diagonals at 22 A	$21\frac{1}{4}$	$21\frac{1}{4}$	.....
	10.23	Started lowering again	$21\frac{1}{4}$	$21\frac{1}{4}$	.....
	10.30	Stopped lowering	$20\frac{3}{8}$	$20\frac{3}{16}$	.....
	10.53	Raised south jack to match elev. of S. Truss of 22 A	$20\frac{3}{8}$	21	.....
	11.00	Started lowering again	$20\frac{1}{8}$	$20\frac{1}{4}$	.....
	11.15	Chords touching at 22 A, Temp. 56° Fahr.	$19\frac{7}{8}$	$19\frac{3}{4}$	$18\frac{13}{16}$
	11.28	Stopped lowering to bolt connections at 22 A.	$18\frac{3}{8}$	$18\frac{1}{4}$	$17\frac{3}{8}$
	1.05	Started lowering again and unloading counterweight	$18\frac{3}{8}$	$18\frac{1}{4}$	$17\frac{3}{8}$
	2.15	Finished lowering jacks, stays $\frac{1}{4}$ in. slack. Arch self-supporting, 3-hinged. Total settlement of counterweight, N = 3 in., S = $1\frac{7}{8}$ in	$4\frac{7}{8}$	$4\frac{3}{4}$	$4\frac{1}{16}$
Summary.....		Total lowering of jacks to make chords touch	$2\frac{11}{16}$	$2\frac{5}{8}$	$2\frac{5}{8}$‡
		Additional lowering to release stays	15	15	$14\frac{3}{4}$

* Discrepancy between actual and calculated due to trusses being lower than calculated before

Transforming Trusses from Three-Hinged Into Two-Hinged Arches.—The transformation of the trusses from three-hinged into two-hinged arches required the connection of the top chord and one of the diagonals of the center panel of each truss at normal temperature (60° Fahr.). These members had been erected immediately after the closing of the arch, but had been left bolted at one end and free to move at the other.

The connection at the free end required drilling the rivet holes from the solid, and riveting. Although, during these operations, no elastic deformations of the arch trusses from loads took place, as the members were to be connected without initial stress at normal temperature, it was to be expected that, during the drilling and riveting, which work took several days, there would be changes in temperature. In order to prevent movements of the connection after the drilling of holes had once started, the Bridge Company resorted to an ingenious

CLOSING—HELL GATE ARCH—LONG ISLAND END.

Percentage of effective jacking completed.	Jack Pressure. Observed.	Jack Pressure. Calc.†	Opening Bet. Bottom Chords at Point 22. Observed.		Opening Bet. Bottom Chords at Point 22. Theoretical.	Elev. of Panel Point 22 L. I. Observed.		Elev. of Panel Point 22 L. I. Calc.	Condition of arch.
	Pounds per square inch.	Pounds per square inch.	Actually observed at 56° Fahr.	Corrected to 60° Fahr.		N. Truss.	S. Truss.		
			Inches.		Inches.	Feet.	Feet.	Feet.	
.....	.....	.....	4 9/16	4 1/4	4 3/4*	.....	.....	Theoretical for 60° Fahr.	Stage 12 A.
.....	4 550	4 600			.....				
.....	3 950	4 600			.....				
.....	3 950	4 600			.....				
.....	3 900	4 600			.....				
.....	4 500	4 600	1 1/2	1 3/16	1 1/8				
.....	3 850	4 600	1 1/4	1 5/16	7/8				
0	3 850	4 600	.....	0	0	Corrected to 60° Fahr. 265.11	265.14	.....	Trusses in contact.
10	3 400	4 100	.....	0	0	Actually observed at 56° Fahr. 265.14	265.17	265.19	
10	3 400	4 100	.....	0	0	265.18	265.21	.....	
100	0	0	.....	0	0	265.21	265.24	265.30	3-hinged.
						Relative Elevations.			
						— 1 1/16 in.	— 3/4 in.	0.0	
.....	.....	.....	.....	.....	.....	.....	.....	.....	
.....	.....	.....	.....	.....	.....	.....	.....	.....	

position. † No correction for friction of jacks. ‡ Calculated for opening observed jacking.

device to hold the ends in place. A rod, 8 in. in diameter and 16 ft. long, of sufficient strength to resist any possible tension or compression from changes in temperature, was introduced into each top chord, extending across the open joint of the chord at Panel Point 23, Wards Island side. Each end of the rod was secured with double nuts to a diaphragm riveted to the chord (Fig. 38).

On a favorable day, when the temperature was practically normal and uniform over the whole structure, the nuts on these rods were tightened, with a slight initial stress in the rods. From this moment all stresses were taken by the rods, thus relieving the ends of the chords and permitting the drilling and riveting without disturbance. The rods were left in place, and therefore partook in resisting the chord stresses.

Field Riveting, Bolting, and Drifting.—A total of 333 960 field rivets, or about 17 per ton of steelwork, had to be driven in the Hell Gate Bridge, not including those driven in some connections of the back-stays. About two-thirds of this number, or 202 404, are $1\frac{1}{4}$ in. in diameter and have grips up to $9\frac{7}{8}$ in. Their shape has been described under "Workmanship and Fabrication." The connections of the back-stay trusses were in general made with 80% of drift-pins and 20% of bolts.

All truss connections were made temporarily with bolts and drift-pins. From 25 to 60% of the rivet holes were filled with drift-pins, or a number sufficient to carry the entire erection stress. About 50% of the holes were filled with bolts to tie the different parts firmly together. The bolts were not supposed to transmit any stress. The riveting of the connection of the web members and of the six end panels of the top chord was permitted to proceed by gradually replacing the pins and bolts with rivets after the erection had proceeded at least three panels ahead. The riveting of the bottom chord connections and of the middle panels of the top chord, however, was deferred until the arch had become self-supporting and the bottom chord carried the greater part of the dead load. The object was to allow the joints of these compression chords to come into full and forcible contact, and transmit a greater unit stress than the splicing material after the riveting had been completed. As the bottom chords had been planed at one end to a three-plane face, as explained previously, the two outer

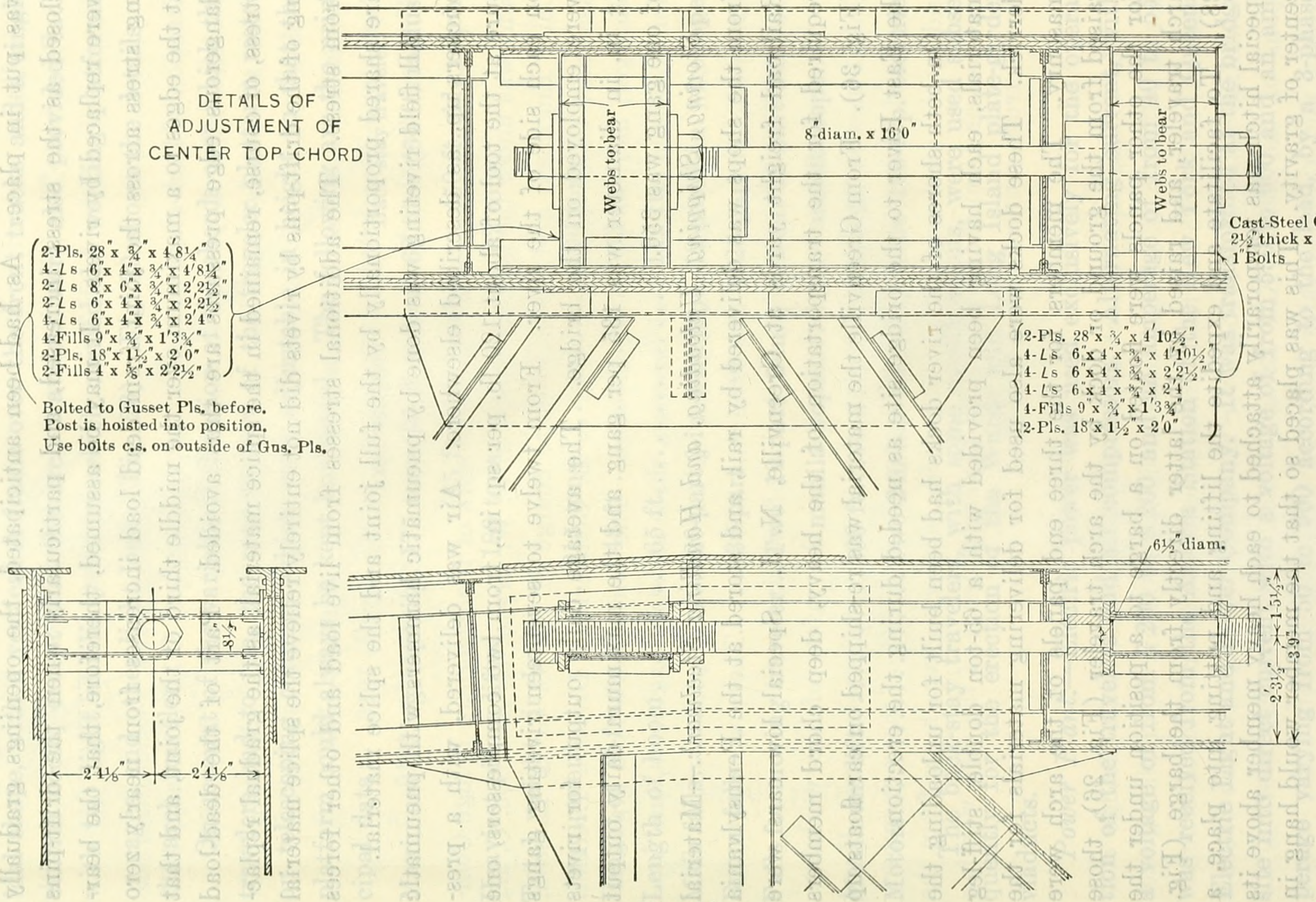

DETAILS OF ADJUSTMENT OF CENTER TOP CHORD
2-Pls. 28"x ¾"x 4'8¼"
4-Ls 6"x 4"x ¾"x 4'8¼"
2-Ls 8"x 6"x ¾"x 2'2½"
2-Ls 6"x 4"x ¾"x 2'2½"
4-Ls 6"x 4"x ¾"x 2'4"
4-Fills 9"x ¾"x 1'3¾"
2-Pls. 18"x 1½"x 2'0"
2-Fills 4"x ⅝"x 2'2½"
Bolted to Gusset Pls. before.
Post is hoisted into position.
Use bolts c.s. on outside of Gus. Pls.
Webs to bear
8"diam. x 16'0"
Webs to bear
Cast-Steel Collar
2½"thick x 10"long
1"Bolts
2-Pls. 28"x ¾"x 4'10½".
4-Ls 6"x 4"x ¾"x 4'10½"
4-Ls 6"x 4"x ¾"x 2'2½"
4-Ls 6"x 4"x ¾"x 2'4"
4-Fills 9"x ¾"x 1'3¾"
2-Pls. 18"x 1½"x 2'0"
8½"
2'4⅛"
2'4⅛"
6½"diam.
2'3½"
1'5½"
3'9"

FIG. 38.

thirds of each joint formed wedge-shaped openings when the member was put in place. As had been anticipated, the openings gradually closed as the stress increased, and particularly when the drift-pins were replaced by rivets. It may be assumed, therefore, that the bearing stress across the joint from dead load increases from nearly zero at the edges to a maximum over the middle third of the joint, and that dangerous edge pressures are thus avoided. Part of the dead-load stress, of course, remained in the splice material, as the gradual replacing of the drift-pins by rivets did not entirely relieve the splice material from stress. The additional stresses from live load and other forces are shared proportionally by the full joint and the splice material.

All field riveting was done by pneumatic hammers with pneumatic buckers-up, as described elsewhere. Air was delivered, with a pressure at the tool of about 120 lb. per sq. in., from two compressors, one on each side of the river. From twelve to seventeen riveting gangs were employed on the bridge. The average daily output for rivets $1\frac{1}{4}$ in. in diameter was 135 per gang and the maximum daily output of one gang was 356.

Storing, Shipping, Unloading, and Handling Steelwork.—Material from the shops was delivered by rail, and stored at the Pennsylvania Railroad freight yards at Greenville, N. J. Special low cars were required for the transportation of the heavy, deep chord members (Fig. 36). From Greenville the material was re-shipped on car-floats up the East River to the bridge site as needed during the erection.

On both shores of the river docks had been built for unloading the materials, each having been provided with a 65-ton double, stiff-leg derrick. These docks were also used for delivering materials for the masonry. The members for the three end panels of the arch were raised from the ground or dock by the arch traveler (Fig. 26), those for the other panels were floated on a barge to a position under the arch traveler, and raised by the latter directly from the barge (Fig. 28). To facilitate and expedite the lifting and putting into place, a special hitch was temporarily attached to each heavy member above its center of gravity. This was placed so that the member would hang in the same relative position as it was to occupy in the structure.

Travelers.—The four powerful creeper travelers, one pair on the back-stays and one pair on the arch proper, had been specially designed

and built for the erection of the arch bridge. They were of the pyramidal shape, with a vertical A-frame over the front trucks and a single central boom, except that the arch travelers were also provided each with two light auxiliary booms which carried the working platforms, or cages, and other light loads (Fig. 26). Fig. 39 shows the details of the arch traveler.

On account of the variable inclination of the chords over which the travelers had to run, the rear corners of the traveler platform rested on telescoping columns which were adjustable in height so that the platform could always be kept horizontal. The following are the principal data:

	Arch traveler.	Back-stay traveler.
Maximum lifting capacity.	175 tons (54 ft. radius) (45 ft. side reach).	62 tons (64 ft. radius) (50 ft. side reach)
Heaviest pieces lifted	180 tons	62 tons.
Total weight, inclusive of equipment	315 tons	166 tons.
Height of A-frame	48 ft.	53 ft.
Length of boom	65 ft.	112 ft.
Main falls	26-part $\frac{7}{8}$-in. wire rope.	12-part $\frac{3}{4}$-in. wire rope.
Boom falls	36-part " " "	26-part " " "
Falls for moving traveler	Two 12-part 2-in. manila rope	Four 12-part 2-in. manila rope.
Maximum lift	300 ft.	300 ft.
Motors	Two 240-h.p. electric	One 100-h.p. electric.

The back-stay travelers, with some modifications, were used subsequently for the erection of the Wards and Long Island plate-girder viaducts.

Power Plant.—Electric power was used exclusively for the operation of the travelers and air compressors. Alternating current, with a voltage of from 7 200 to 8 200 and an amperage of from 40 to 55, was received from the Astoria Station of the New York and Queens Electric Light and Power Company. It was transformed at the bridge site into direct current with a voltage of from 550 to 600 and an amperage varying from 50 to 900 according to the load, by a 3-phase 60-cycle Allis Chalmers motor generator set, with a capacity of about 500 h.p. Each of the two Ingersoll air compressors was driven by a 75-h.p. electric motor, and had a capacity of 225 cu. ft. of free air

compressed per minute to a pressure of at least 135 lb. per sq. in. at the cylinder.

Progress.—The following are the principal dates in connection with the erection of the Arch Bridge (see also Plate XXXIII):

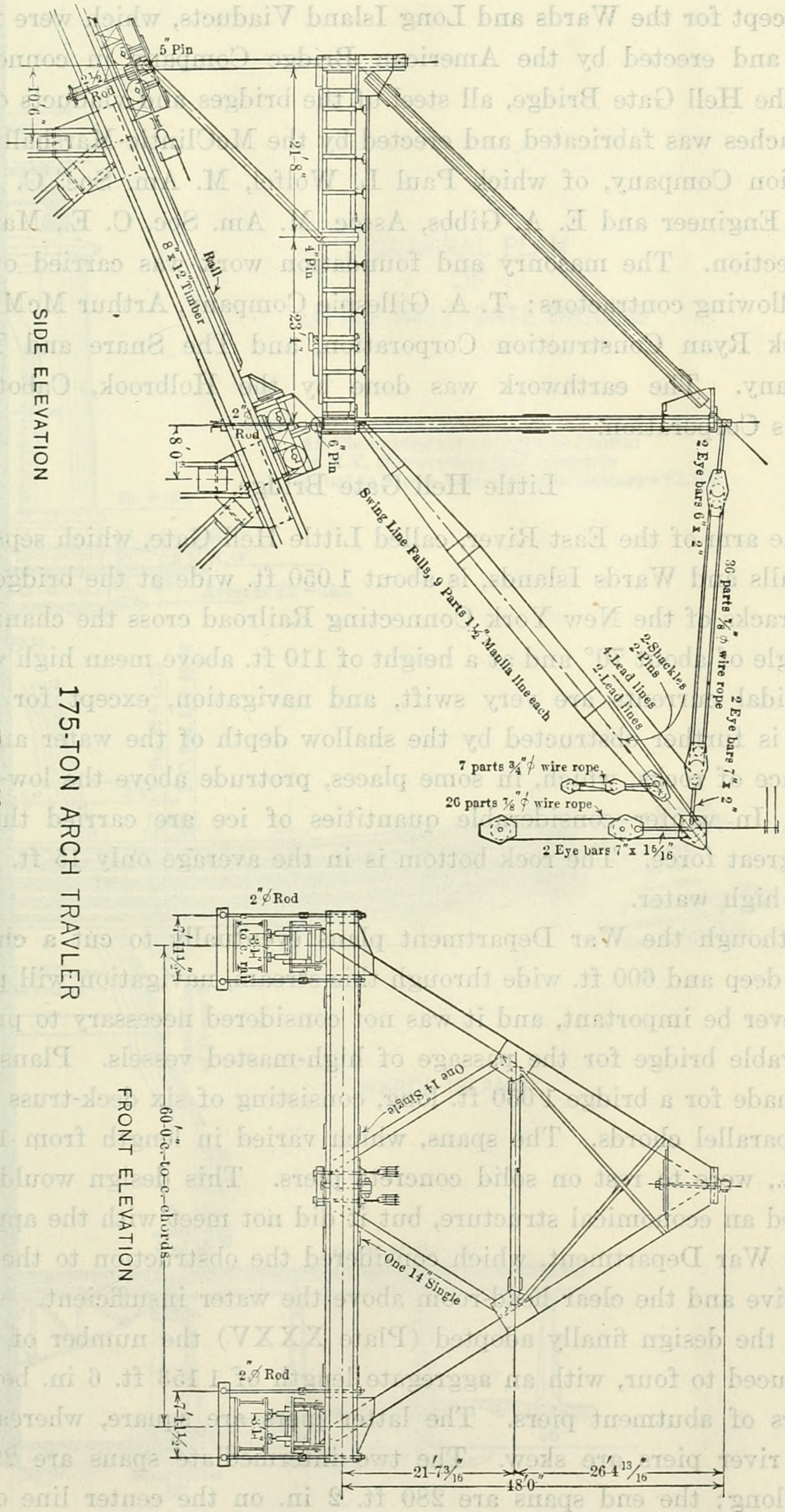
	Long Island side.	Wards Island side.
Erection of back-stays started	September, 1914	January, 1915
Erection of arch proper started	January, 1915	May, 1915
First six panels erected	June, 1915	August, 1915
Arch closed	October 1st, 1915	
Suspenders and floor system erected	January, 1916	
Back-stays dismantled	April, 1916	
Riveting completed	September, 1916	

Contractors' Organization.—The work of the American Bridge Company was under the general charge of C. W. Bryan, M. Am. Soc. C. E., Chief Engineer, and the immediate charge of Mr. C. G. E. Larsson, Assistant Chief Engineer. Mr. J. B. Gemberling, Division Erection Manager, had charge of the erection. The contractor's field organization consisted of a general foreman, three assistant engineers, one foreman, and an average daily force of 140 specially picked men. The maximum force at any one time was 270 men. There were two separate erection gangs, one on each side of the river.

Unusual precautions were taken for the safety of the workmen. Only 5 men lost their lives, mostly due to their own carelessness. Hand ropes and temporary wooden railings were provided on many members, and special steel cages with comfortable platforms were hung from the travelers or from the steelwork for the convenience of the men working on the truss connections (Fig. 33).

12.—Approaches.

The bridge and viaduct approaches to the Hell Gate Bridge consist of 2 735 lin. ft. of steel truss bridges, 10 818 lin. ft. of plate-girder viaducts, and 3 228 lin. ft. of embankment between reinforced concrete retaining walls, with arches over the streets. Except for the bascule spans of the Bronx Kill Bridge, which have open tie flooring, all bridges and viaducts carry ballasted tracks on concrete slabs, weighing 3 500 lb. per lin. ft. of single track. The live load, quality of steel, and permissible unit stresses are given under "Material" and "Loads and Unit Stresses."



175-TON ARCH TRAVLER

FIG. 39.

Except for the Wards and Long Island Viaducts, which were fabricated and erected by the American Bridge Company in connection with the Hell Gate Bridge, all steel of the bridges and viaducts of the approaches was fabricated and erected by the McClintic-Marshall Construction Company, of which Paul L. Wolfel, M. Am. Soc. C. E., is Chief Engineer and E. A. Gibbs, Assoc. M. Am. Soc. C. E., Manager of Erection. The masonry and foundation work was carried out by the following contractors: T. A. Gillespie Company, Arthur McMullen, Patrick Ryan Construction Corporation, and The Snare and Triest Company. The earthwork was done by the Holbrook, Cabot and Rollins Corporation.

Little Hell Gate Bridge.

The arm of the East River, called Little Hell Gate, which separates Randalls and Wards Islands, is about 1 050 ft. wide at the bridge site. The tracks of the New York Connecting Railroad cross the channel at an angle of about 70° and at a height of 110 ft. above mean high water. The tidal currents are very swift, and navigation, except for small craft, is further obstructed by the shallow depth of the water and the presence of rocks, which, in some places, protrude above the low-water level. In winter considerable quantities of ice are carried through with great force. The rock bottom is in the average only 15 ft. below mean high water.

Although the War Department plans eventually to cut a channel 24 ft. deep and 600 ft. wide through this stream, navigation will probably never be important, and it was not considered necessary to provide a movable bridge for the passage of high-masted vessels. Plans were first made for a bridge 1 050 ft. long, consisting of six deck-truss spans with parallel chords. The spans, which varied in length from 145 to 205 ft., were to rest on solid concrete piers. This design would have secured an economical structure, but it did not meet with the approval of the War Department, which considered the obstruction to the river excessive and the clear head-room above the water insufficient.

In the design finally adopted (Plate XXXV) the number of spans is reduced to four, with an aggregate length of 1 153 ft. 6 in. between centers of abutment piers. The latter piers are square, whereas the three river piers are skew. The two intermediate spans are 296 ft. 6 in. long; the end spans are 280 ft. 2 in. on the center line of the

PLATE XXXV.
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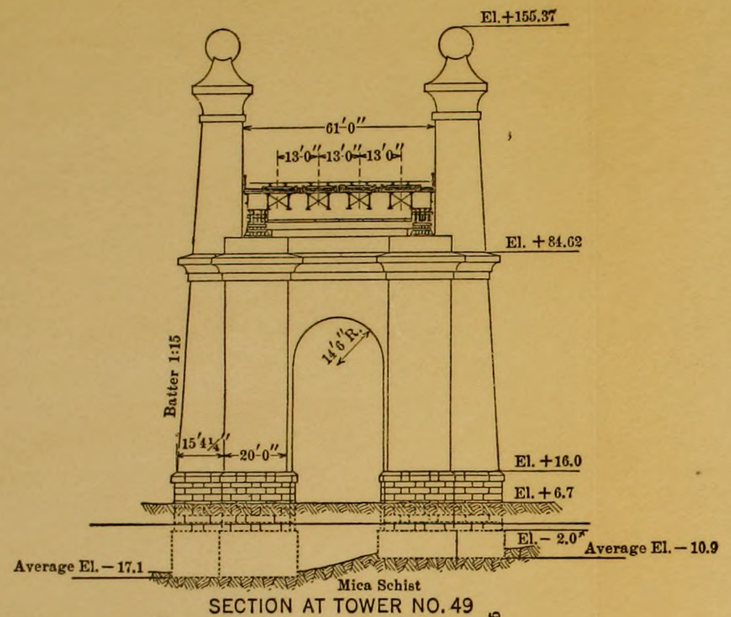
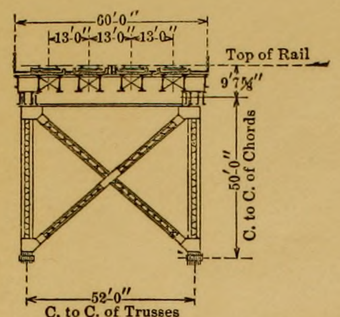
AMMANN ON
HELL GATE BRIDGE.

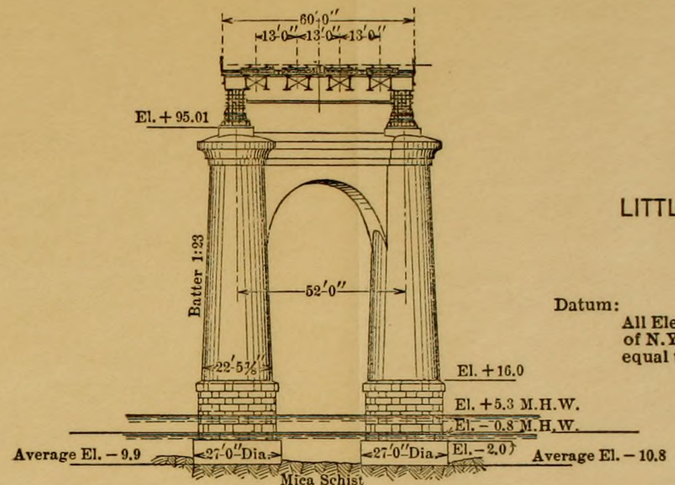
GENERAL PLAN OF LITTLE HELL GATE BRIDGE



SECTION AT TOWER NO. 49

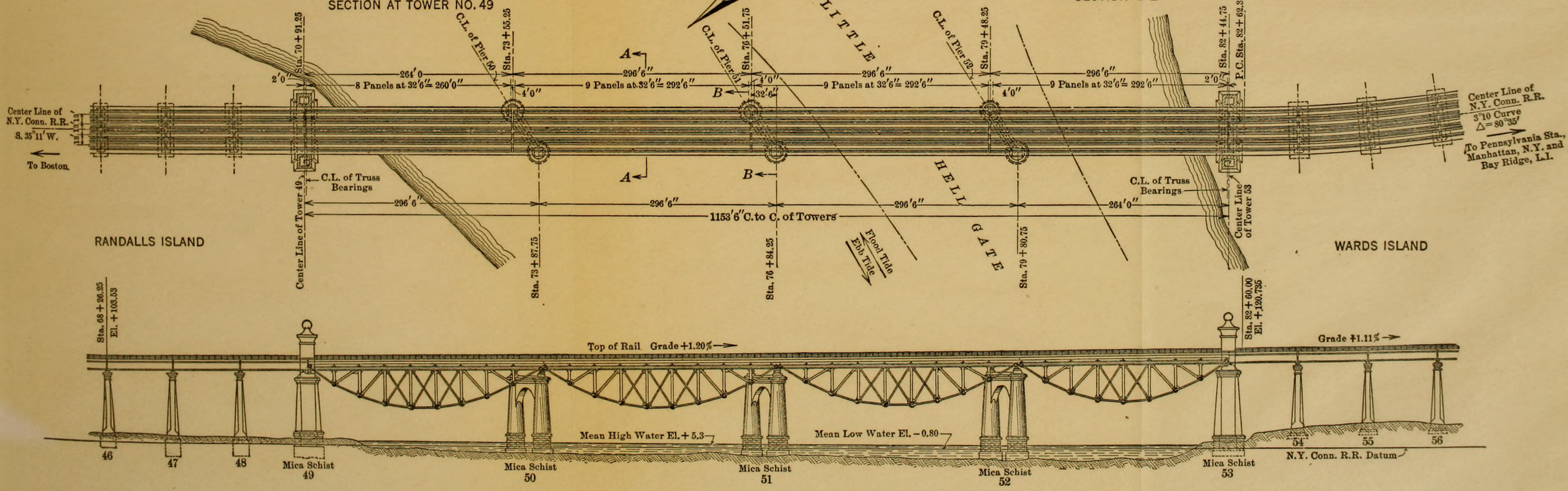


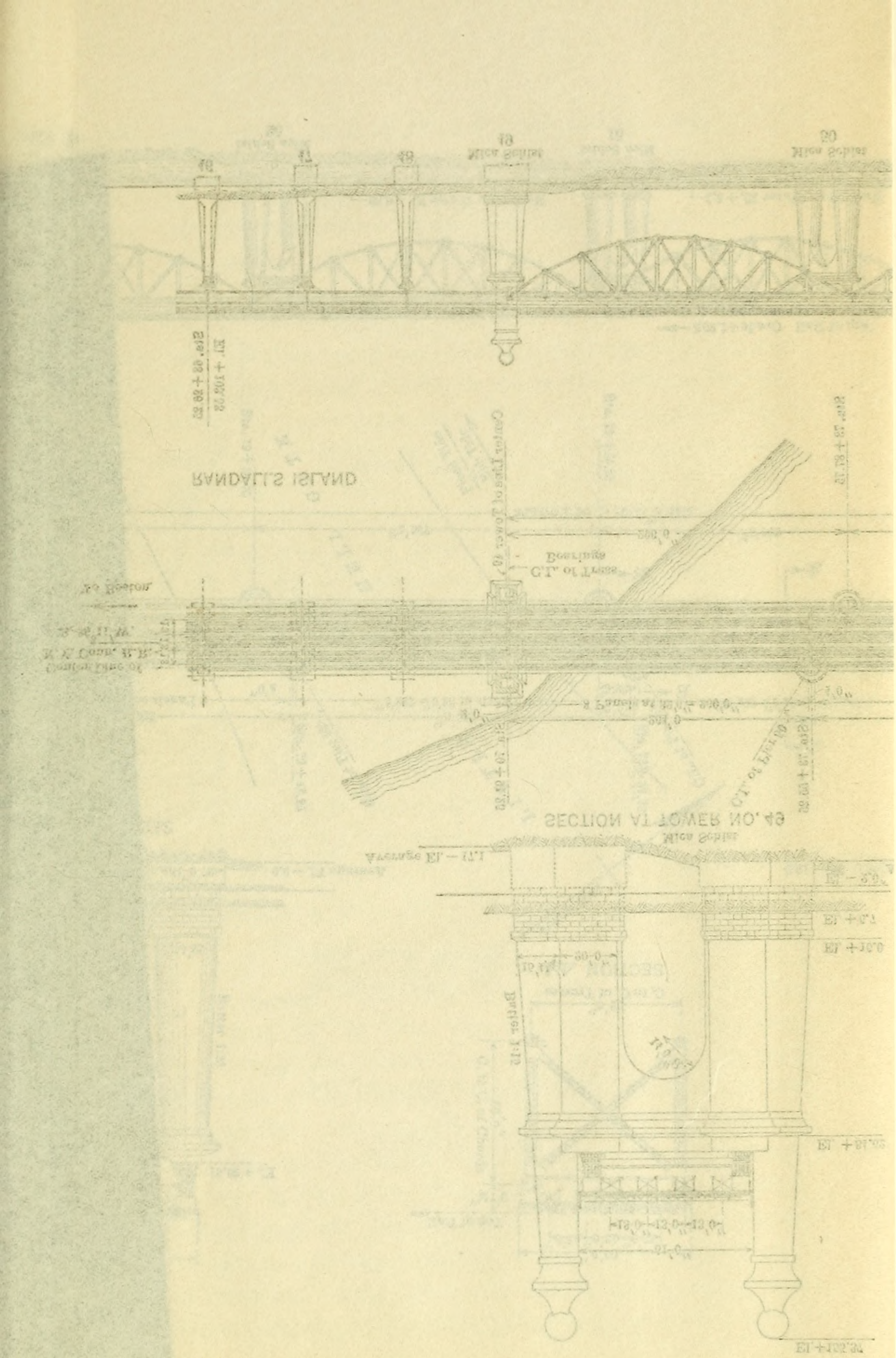
SECTION *A-A*



SECTION *B-B*

Datum:
All Elevations given refer to Datum of N.Y. Connecting R.R. which is equal to Mean Low Water at Battery = El. 0.0





bridge, or 296 ft. 6 in. on the center line of the longer truss, so that only two different sizes of trusses were required. The deck trusses, two for each span, were made of the bowstring type so as to give additional clear height near the piers for higher vessels which might occasionally pass underneath. The river piers were reduced in size, each to two cylindrical shafts 25 ft. in diameter at the water line, so as to offer the least resistance to the flow of water and ice. A through-truss bridge would have given more clearance, but would have been considerably more expensive on account of the greater width, and would not have harmonized in appearance with the approach viaducts as well as the adopted type. The architecture of the piers and abutments is similar to that of the viaduct piers. The slender, slightly battered river piers, in combination with the graceful outline of the trusses, give the bridge a very pleasing appearance (Fig. 40). The ends of this important structure are befittingly marked by towers extending above the track floor at each abutment.

All foundations are on solid rock, and were placed by open coffer-dams. The piers and abutments are of concrete, except that between Elevation 10.7 ft. above mean high water and 1.2 ft. below mean low water the concrete is protected against disintegration by granite facing.

The two trusses are 52 ft. apart on centers, and have a greatest depth of 50 ft. at the center. The bottom chord has a parabolic shape, and is made up of from ten to thirteen lines of structural-steel eye-bars, 16 in. wide, up to $2\frac{1}{4}$ in. thick, and more than 38 ft. long, connected by 16-in. pins. All other members are riveted. Eye-bars were selected in this case, as they effected a considerable saving over riveted chords. The type of truss is economical, as the chords are of nearly uniform section and the web members are very light. The secondary stresses in the end panels are comparatively large, but those from dead load have been practically eliminated by making the joints and angles between members in accordance with the geometric form of truss and swinging the spans clear of falsework before the connections were riveted. The trusses are connected by a rigid top lateral system and also by stiff sway-frames at each vertical post. No lateral system is provided along the bottom chord, as it was considered preferable to have the wind force, which acts along this chord, transmitted to the lateral system between the stiffer top chords.

The two end spans have fixed bearings over the abutments; all bearings over the piers are movable, an expansion joint being provided over the center pier. With this arrangement the piers could be made considerably narrower, as they do not have to resist the longitudinal force from braking. To avoid the transmission of any longitudinal force to the piers through friction, the bearings are provided with cast-steel rockers of the unusual height of 24 in. The relative movement of the two bearings at the center pier is as much as ± 6 in.

The four spans were erected successively on unusually heavy timber falsework (Fig. 41). A total of 360 000 ft. b.m. of timber was required for each span. Each panel point was supported by a double bent strongly braced transversely and longitudinally. The piles had to be braced above and below water level on account of their small penetration and the strong current of the river. The steel was handled with a steel gantry traveler, which weighed 275 tons, inclusive of equipment. Erection was started in February, and completed in November, 1915.

The bridge contains 25 700 cu. yd. of masonry and 11 250 tons of steelwork. The cost of construction, inclusive of tracks, was approximately $990 000.

Bronx Kill Bridge.

The tracks of the New York Connecting Railroad cross Bronx Kill, which separates Randalls Island from the main land, at an angle of about 72° with the channel and at a height of 75 ft. above mean high water. Bronx Kill is here about 600 ft. wide between shore lines, and 320 ft. between bulkhead lines as established by the War Department. Under present conditions, it is navigable only for small boats. The river bottom consists of various strata of river mud, sand, gravel, and clay overlying the gneiss bed-rock, the surface of which falls from both shores to a maximum depth of about 110 ft. below mean high water under the north abutment. According to plans of the War Department, Bronx Kill is to be improved by dredging a channel 24 ft. deep and 480 ft. wide, principally for the purpose of decreasing the excessive tidal currents in the East River, and incidentally to provide a direct passageway for deep-draft vessels between the Harlem River and the upper East River. For this reason the War Department required a movable bridge having two clear openings, of at least 120 ft. each, at right angles to the channel.

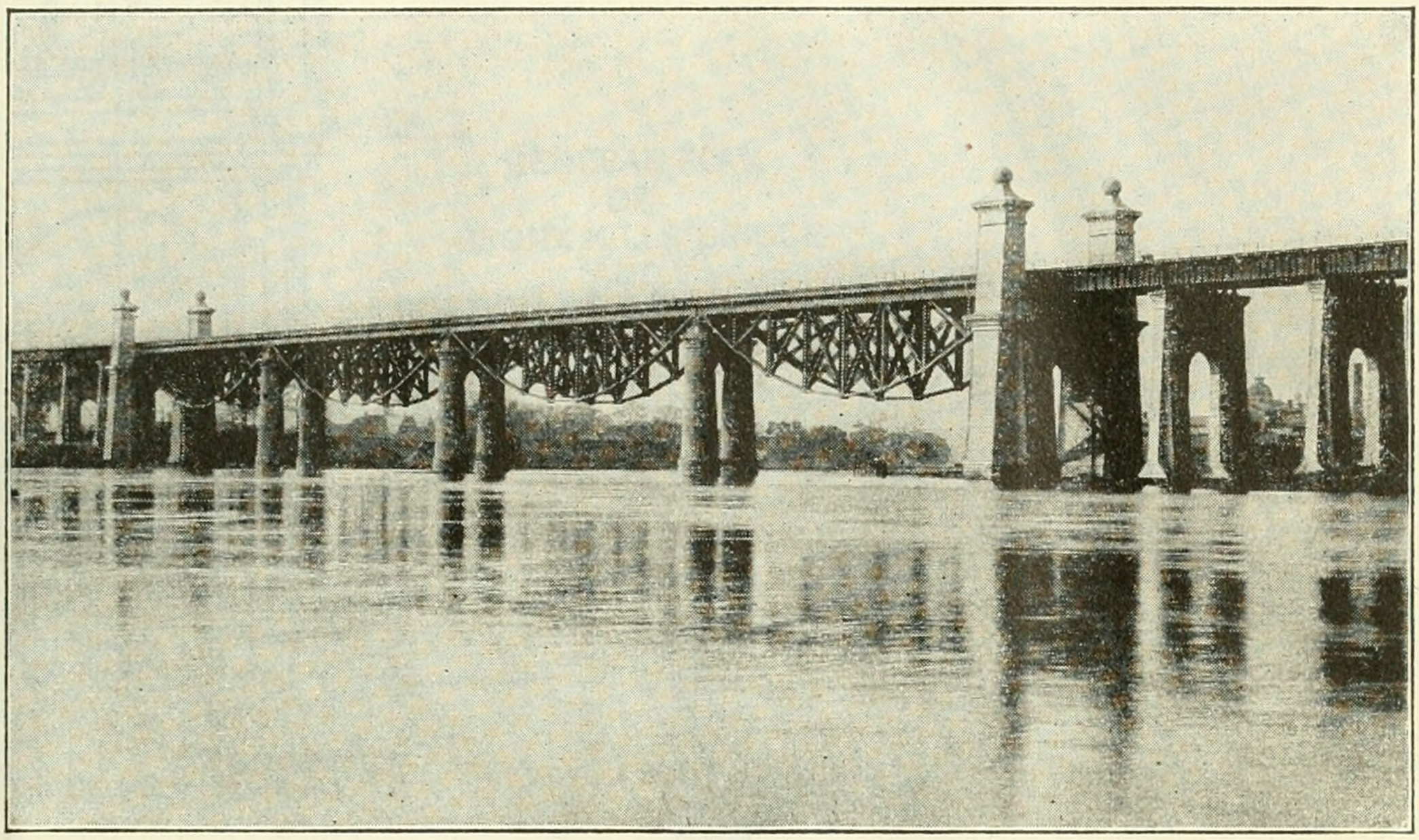


FIG. 40.—LITTLE HELL GATE BRIDGE.

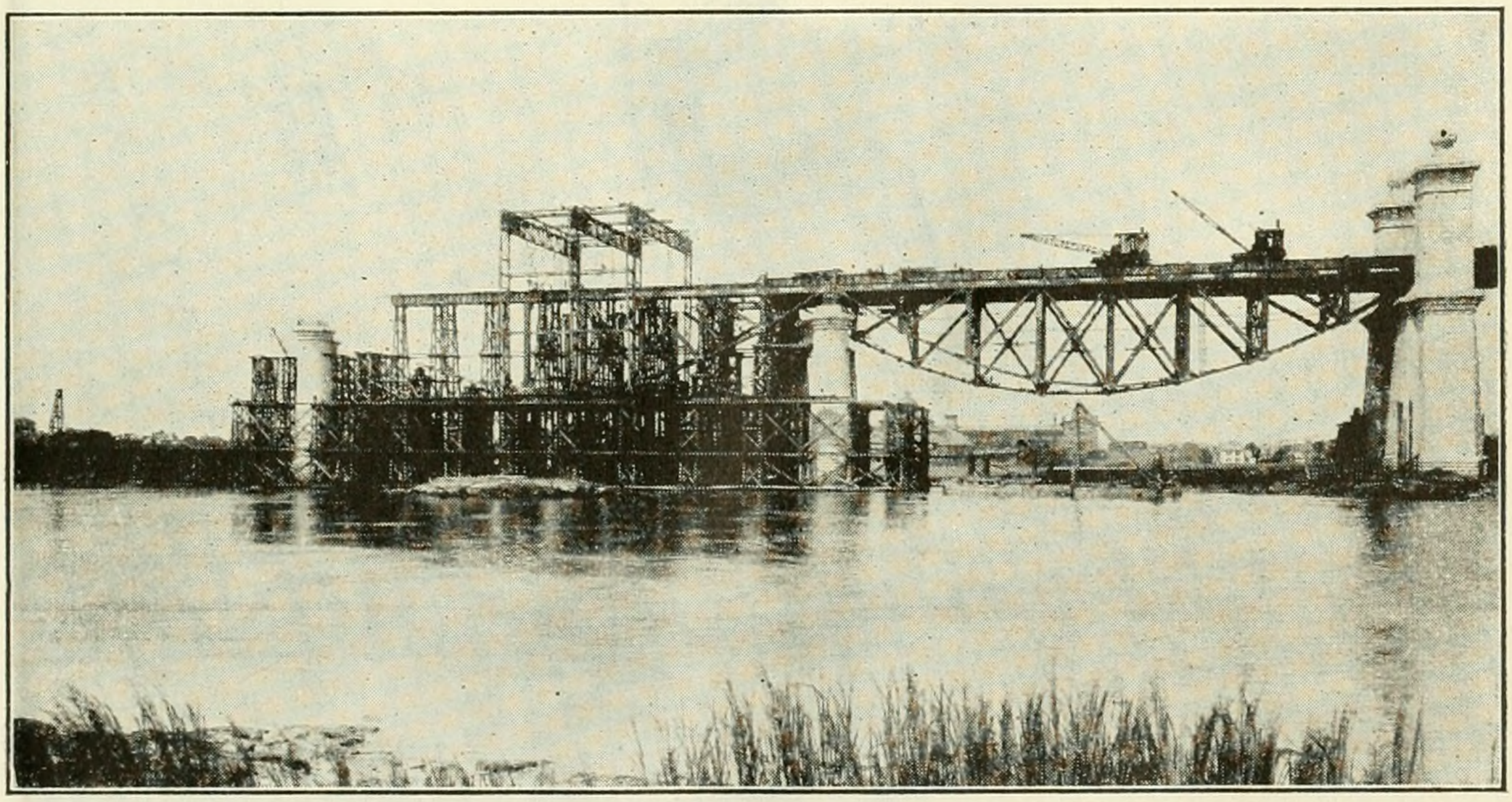


FIG. 41.—ERECTION OF LITTLE HELL GATE BRIDGE.

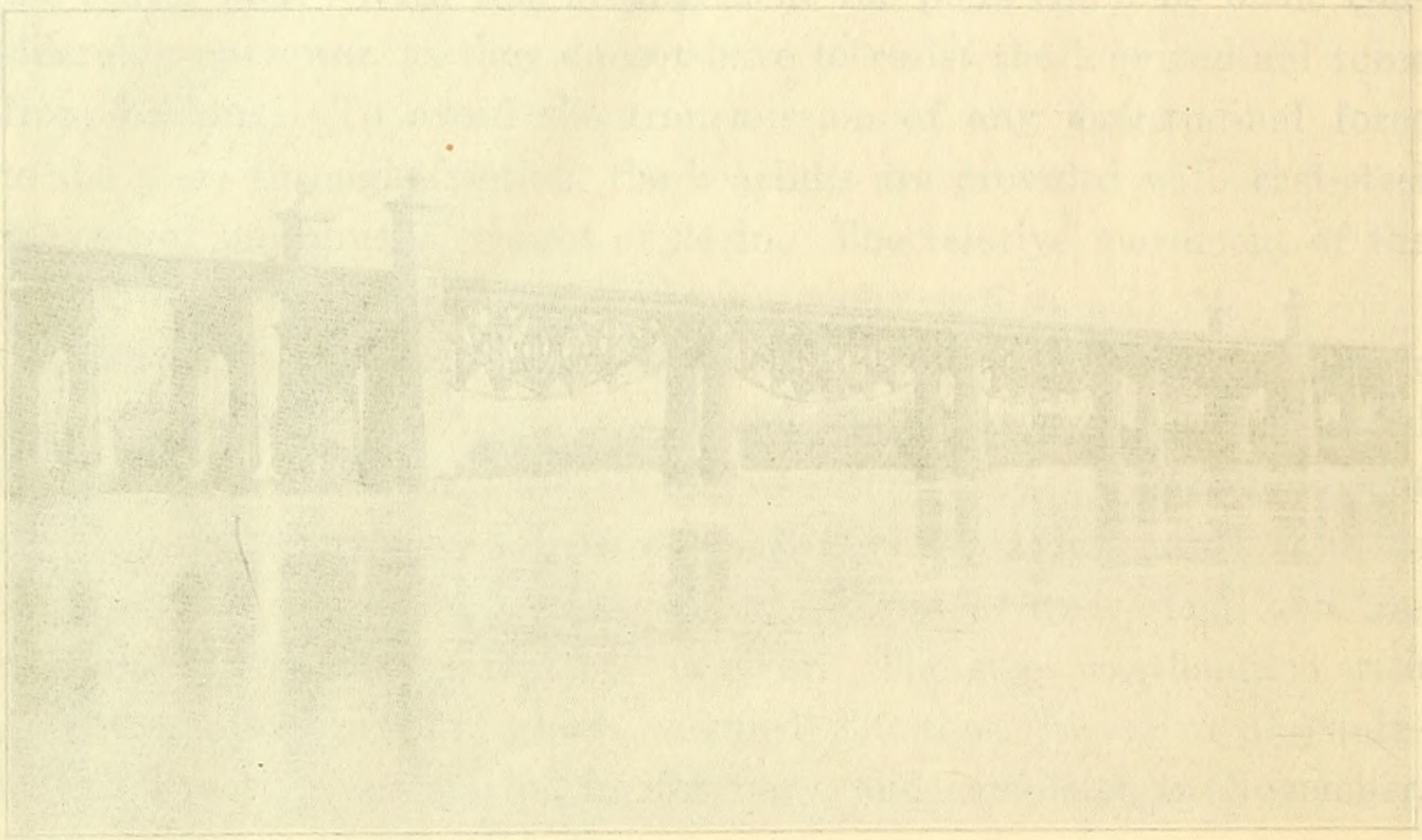


FIG. 40.—LITTLE HELL GATE BRIDGE.

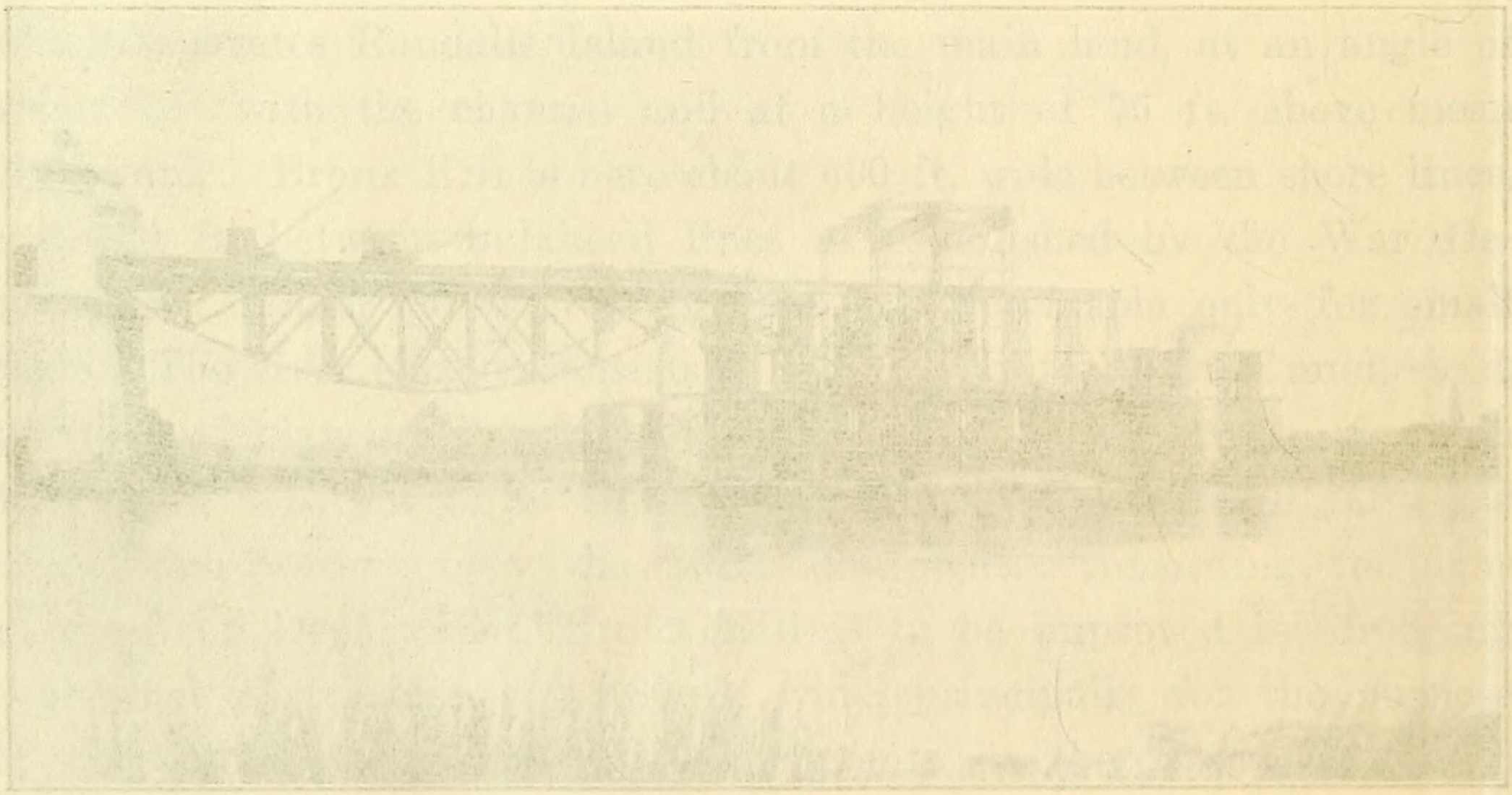
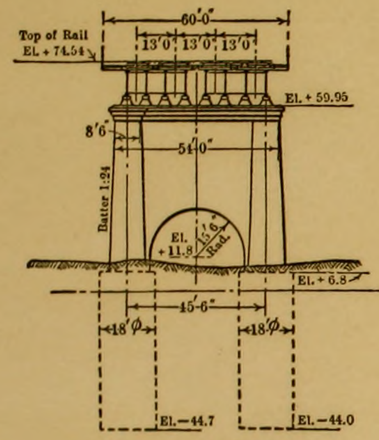
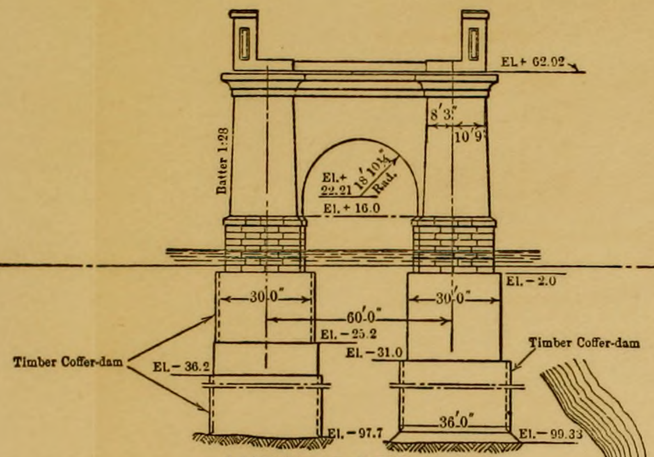


FIG. 41.—ERECTION OF LITTLE HELL GATE BRIDGE.

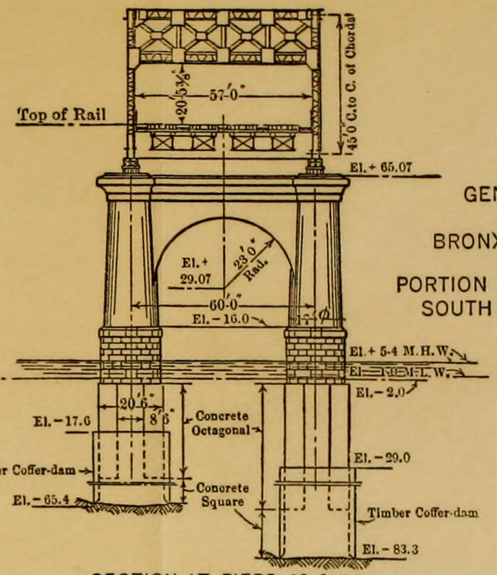
GENERAL PLAN OF BRONX KILLS BRIDGE AND PORTION OF BRONX VIADUCT SOUTH OF 132D STREET



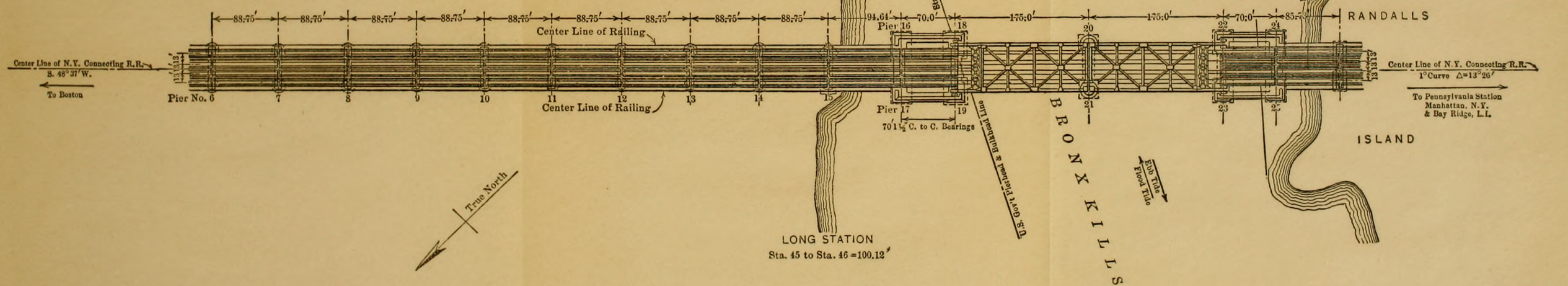
SECTION AT PIER 13

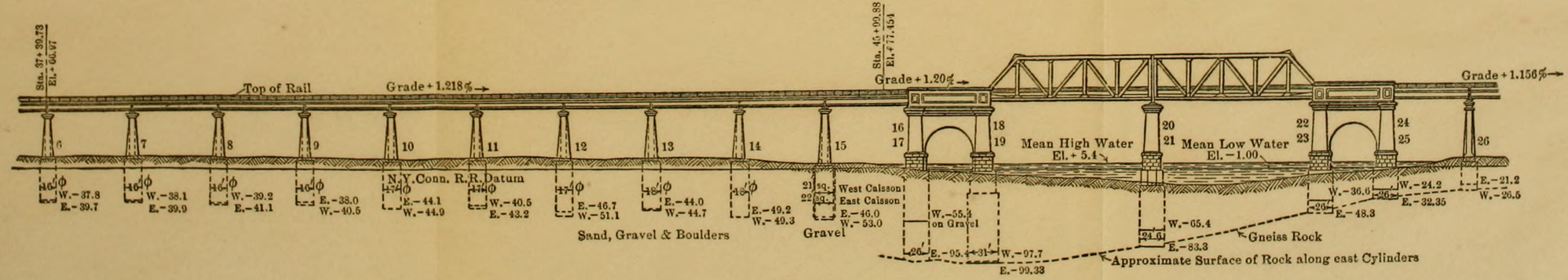


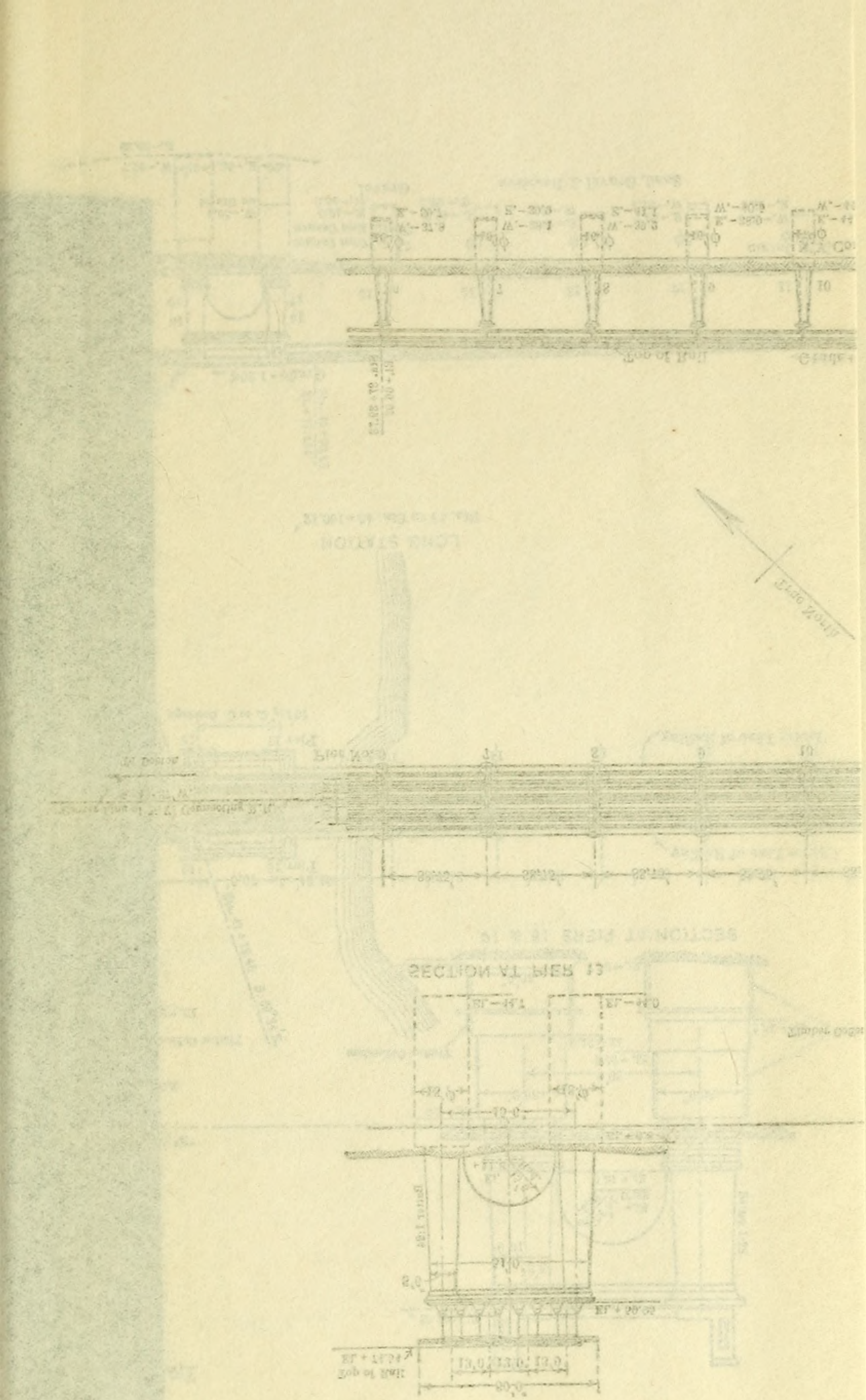
SECTION AT PIERS 18 & 19



SECTION AT PIERS 20 & 21







The available height of 75 ft. from top of rail to mean high water would have been sufficient for a deck bridge, which would have been more economical than a through structure. The latter type was adopted, however, in order to provide sufficient clear height for all ordinary navigation to pass underneath, and thus restrict the frequency of opening the bridge and the consequent delay to railroad traffic. This consideration was important in view of the prospective heavy traffic, and on account of the heavy grade, on which it is difficult to start trains. Comparative designs were made of various types of movable bridges, including the horizontal draw, the Scherzer rolling lift, and the Strauss trunnion bascule.

The horizontal draw-bridge would have necessitated a very wide pier which would have obstructed the channel excessively and would have required very costly foundations. The Strauss bascule was finally adopted because it was found to be cheapest in first cost, that is, without operating machinery, counterweights, etc., which may not have to be put in for many years. The only parts of the moving mechanism which had to be provided are the trunnions and their bearings, and certain connections for the future attachment of the counterweight trusses.

The bridge as built (Plate XXXVI) has two leaves, each 175 ft. long between centers of trunnion bearings and middle pier. Each leaf acts as a single span when closed. All four tracks are carried between the two trusses, which are 60 ft. apart on centers and 45 ft. deep. The trusses are fully riveted.

Each abutment consists of four rectangular piers, connected in pairs longitudinally and transversely by arched concrete walls, and thus forming a rectangular enclosure which will contain the counterweights and hide them from view when the bridge is closed. The two transverse walls support a 70-ft. plate-girder span which now carries the tracks and, ultimately, will carry the operating machinery. The intermediate pier consists of two circular shafts joined by a light concrete arch. The abutments and pier are architecturally treated to conform in appearance to the adjoining viaduct piers (Fig. 42).

All piers were sunk by the pneumatic process, with timber caissons, to depths of from 30 to 105 ft. below mean high water. At the water level the concrete is protected with granite facing in a manner similar to that adopted for the piers of the Little Hell Gate Bridge. The

steel trunnions, around which the trusses will turn, are of exceptional size, being 30 in. in diameter and 10½ ft. long. The tracks rest on open tie flooring securely fastened to the steel floor system.

The bridge was erected on heavy timber falsework, one span at a time, 256 000 ft. b.m. of lumber having been used in each span. The members were handled with the 50-ton derrick car and a locomotive crane, and these were used for the erection of the adjoining viaducts. Erection was started in August, 1914, and completed in March, 1915. The principal quantities in this bridge are:

Masonry	28 300 cu. yd.
Steelwork:	
Two 70-ft. plate-girder spans.....	470 tons
Two 175-ft. bascule spans (exclusive of machinery and counterweights)....	3 105 tons
Total	3 575 tons

Counterweights, operating machinery, and their supports are estimated to require about 750 tons of steel and 1 300 cu. yd. of concrete filling. The cost, exclusive of counterweights and operating machinery, but inclusive of tracks, was approximately $800 000.

Bronx Viaduct North of 132d Street.

General Conditions.—The design and construction of the portion of The New York Connecting Railroad designated as "Bronx Viaduct north of 132d Street", extending from 132d Street, where the New Haven and the Connecting Roads converge, to the crossing over the New York and Harlem Railroad, about 380 ft. north of 141st Street (Plate XXXVII), presented considerable difficulties on account of the unfavorable character of the soil and the proximity of the four New Haven tracks, with a siding along the eastern side of the New York Connecting Railroad from 132d Street to the De La Vergne Machine Works at 137th Street. The two easterly New Haven tracks were originally located alongside the two westerly tracks and, therefore, had to be shifted eastward, to make place for the construction of the two westerly New York Connecting tracks. This had to be done without interruption of traffic on the New Haven tracks. Borings along this entire section indicated a top layer of gravel, coarse sand,

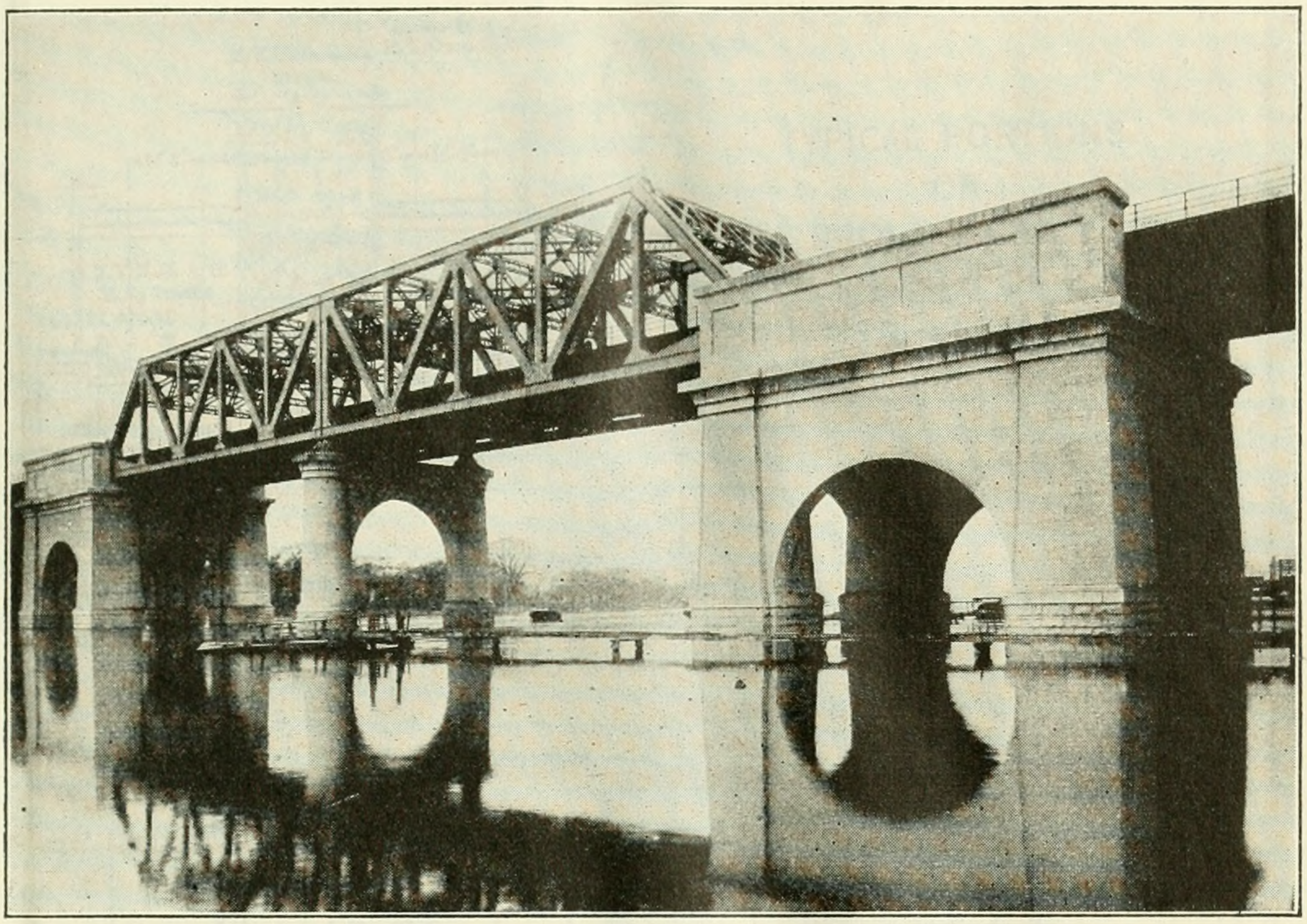


FIG. 42.—BRONX KILL BRIDGE.

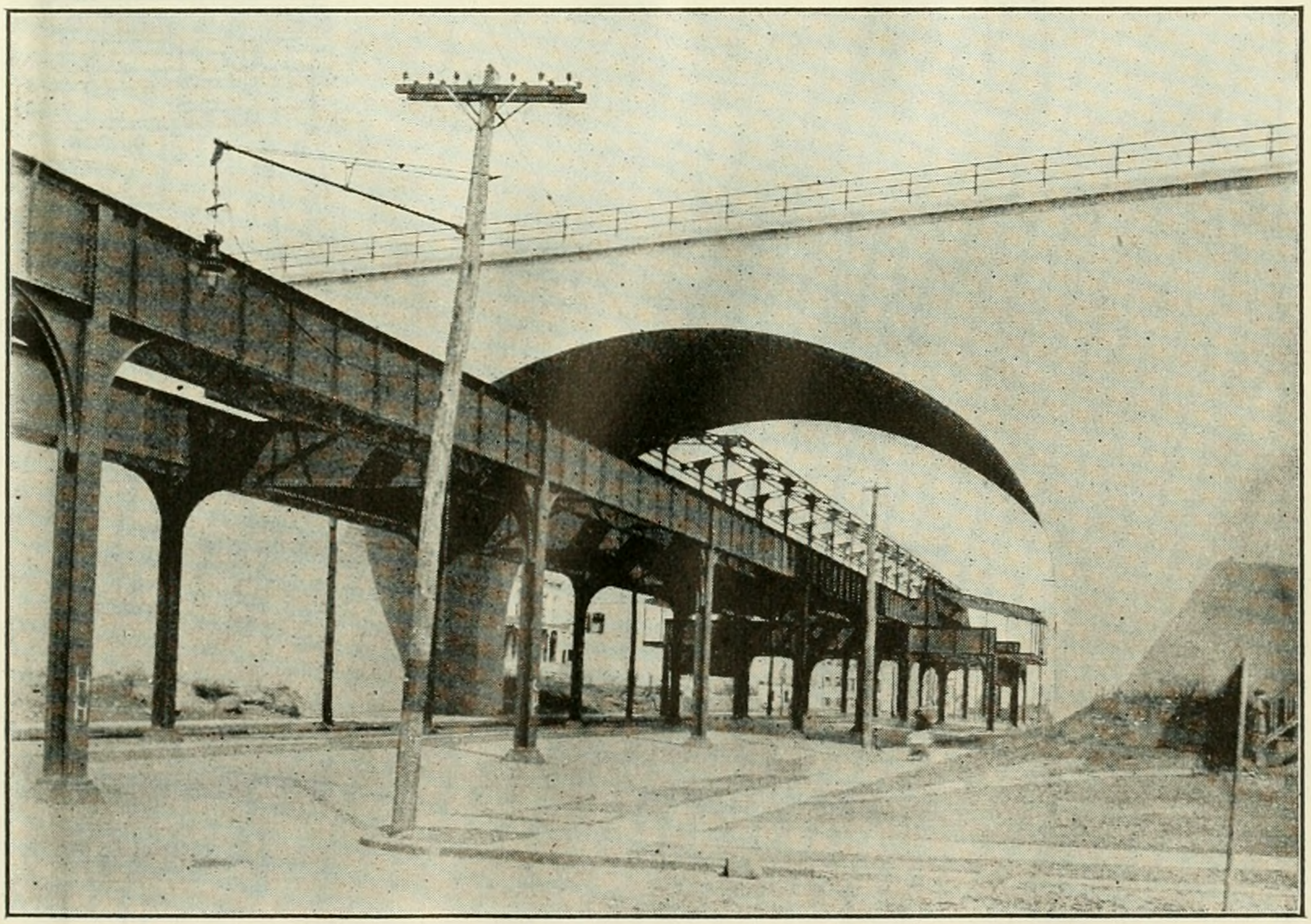


FIG. 43.—CONCRETE ARCH OVER DEBEVOISE (SECOND) AVENUE, LONG ISLAND CITY.

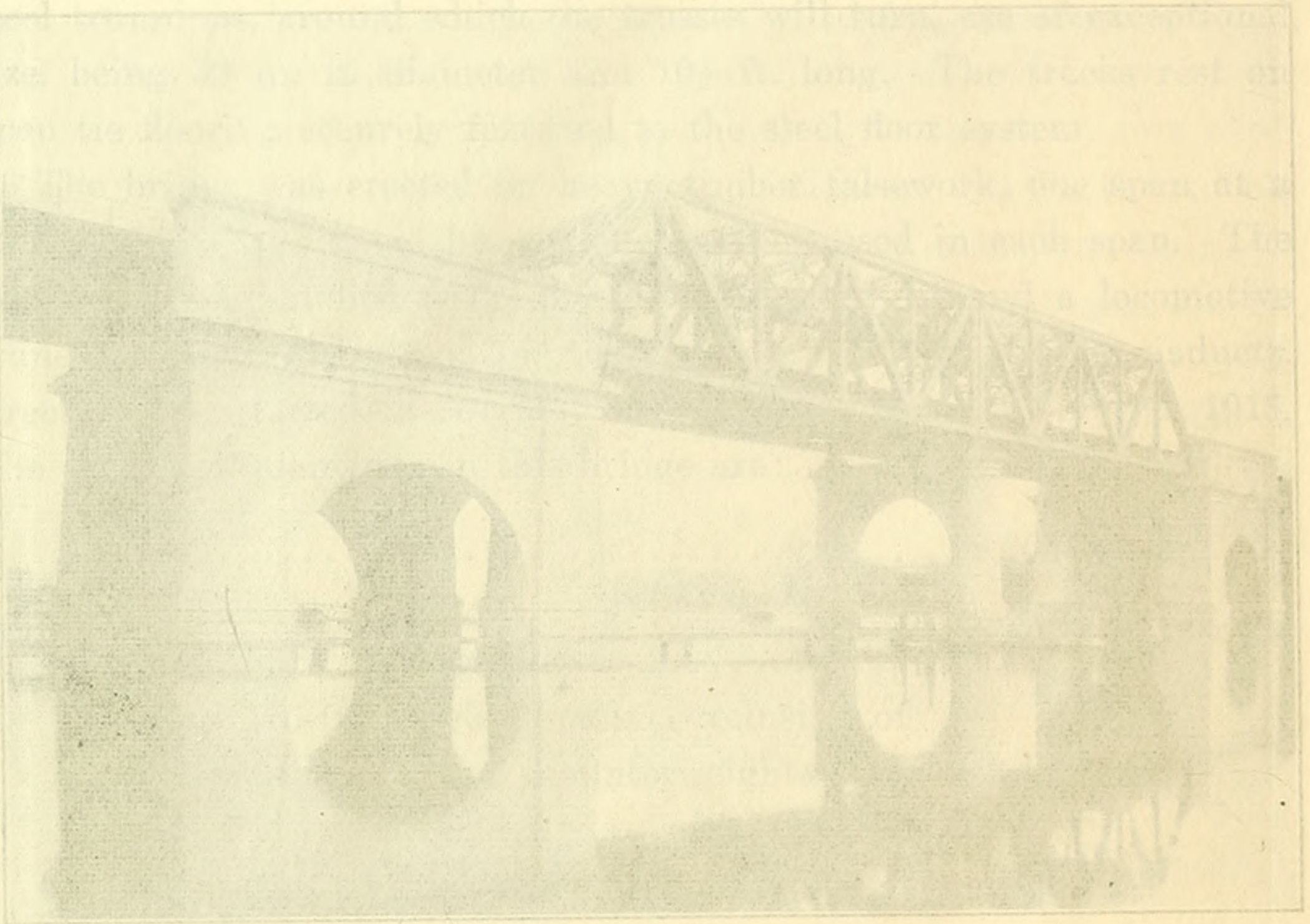


PLATE XXXVII.
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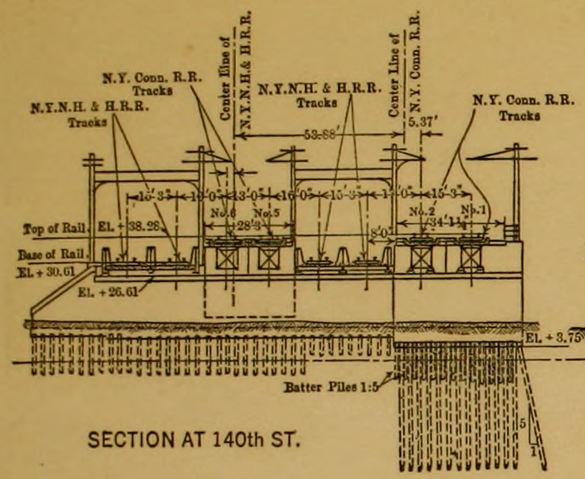
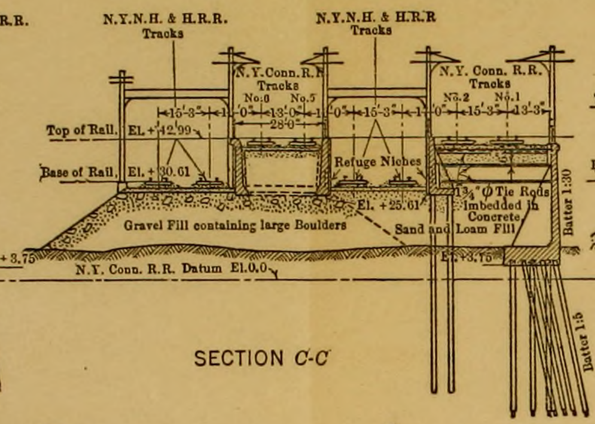
VOL. LXXXII, No. 1417.
AMMANN ON
HELL GATE BRIDGE.

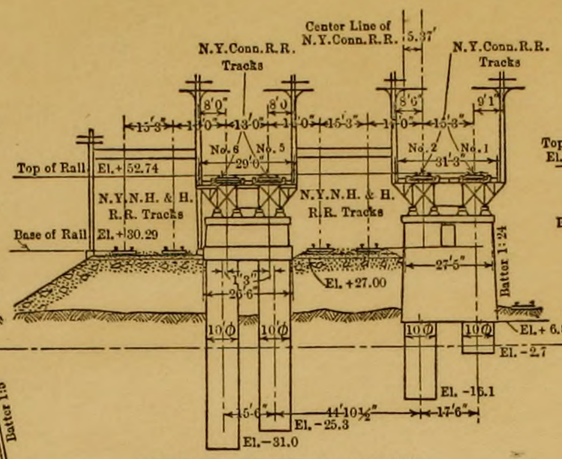
TYPICAL PORTIONS OF BRONX VIADUCT NORTH OF 132d STREET



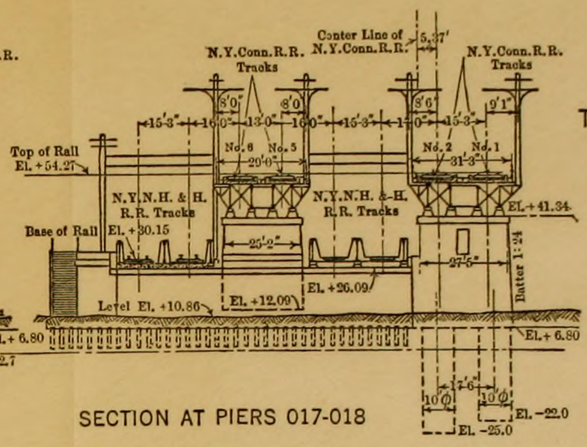
SECTION AT 140th ST.



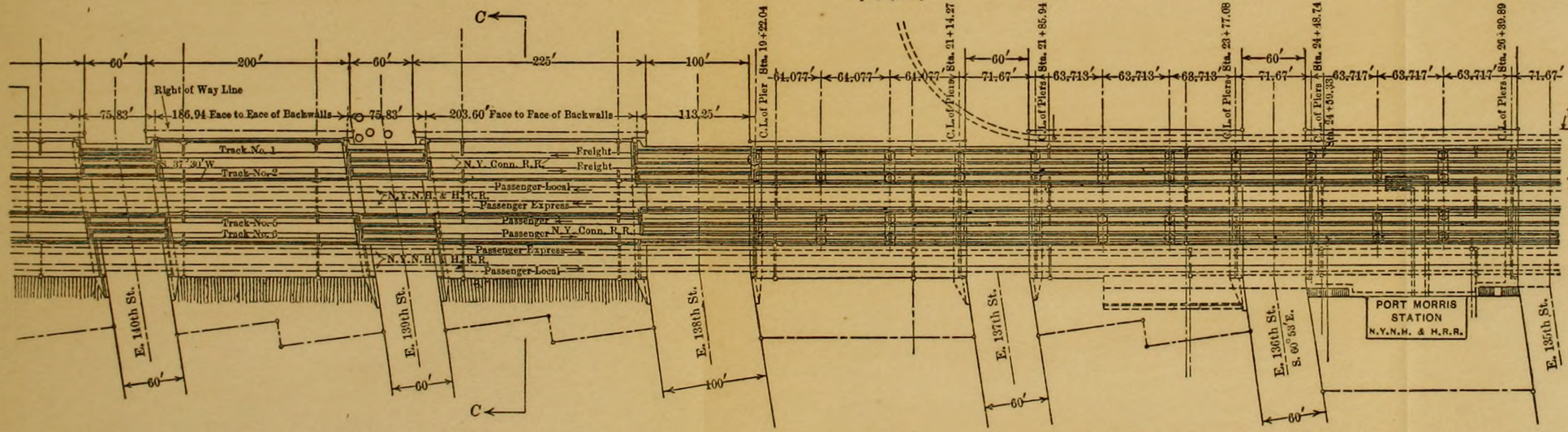
SECTION C-C

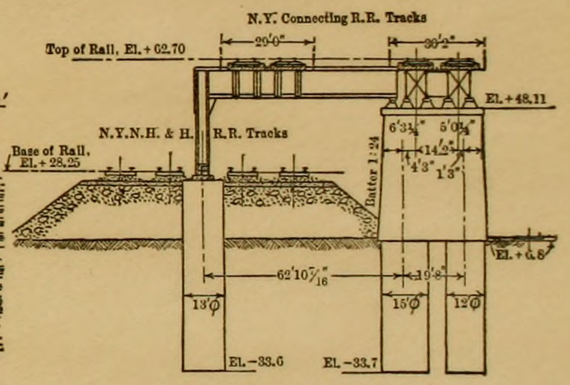


SECTION AT PIERS 021-022

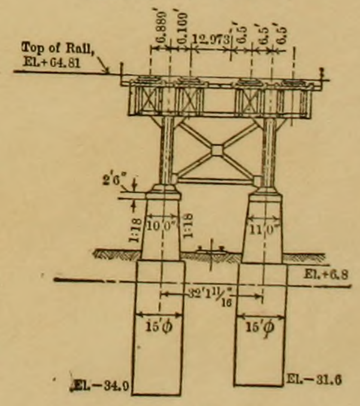


SECTION AT PIERS 017-018

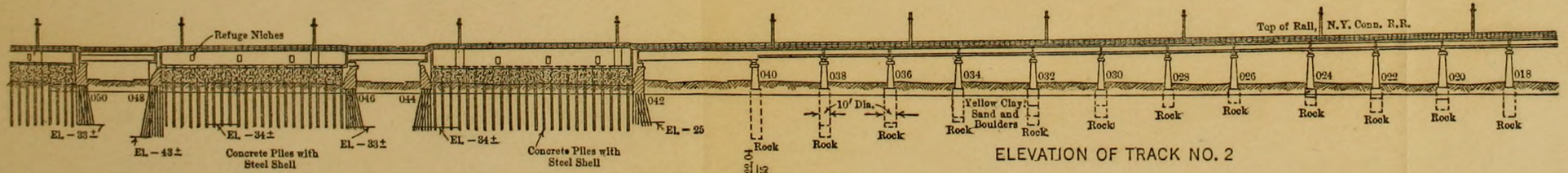




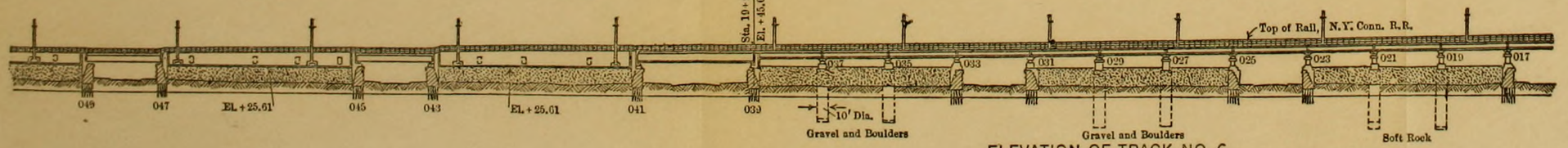
SECTION AT BENT 2



SECTION AT BENT 4



ELEVATION OF TRACK NO. 2



ELEVATION OF TRACK NO. 6

and ashes, from 6 to 12 ft. deep, below this a soft stratum of silt and mud from 12 to 25 ft. deep, resting on a harder stratum of sand and gravel. Only under the easterly tracks near 136th Street does the rock rise to a few feet below the surface.

In 1906, or about 6 years previous to the construction of the Connecting Railroad, the New Haven Railroad had raised its four tracks on an embankment, 65 ft. wide at the top and about 20 ft. above the original ground surface. This embankment consists of cinders, earth, gravel, and a great quantity of large boulders; the latter have gradually settled several feet into the original soft ground and have caused it to heave on the sides. Subsequently, this embankment had to be widened to 95 ft. at the top, when the two easterly tracks were shifted to their present location. The vicinity of this portion of the line is built up mainly with industrial buildings, for which reason no particular consideration had to be given to the architectural treatment of the structure.

Embankment Portion.—On the portion of the Bronx Viaduct between the crossing over the New York and Harlem Railroad and 138th Street, it was found most economical to place the four New York Connecting tracks on fill between reinforced concrete retaining walls, the streets being crossed on deck plate-girder spans. The two retaining walls which enclose the fill under the two westerly tracks—Nos. 5 and 6—rest directly on the New Haven embankment. They are connected by transverse walls, about 20 ft. apart, and thus form, in each block between streets, a monolithic cellular box without bottom. Its maximum height is 19 ft. This box, with the fill between, forms a compact unit which exerts a practically uniform pressure of not more than $1\frac{1}{2}$ tons per sq. ft. on the embankment beneath. Settlements of the retaining walls were expected, but they can have no serious consequences. Since their completion in 1914 a maximum settlement of 8 in. has been observed, without any sign of cracks in the walls. The abutments of the street crossings are independent of the retaining walls, and rest on timber piles. This plan was simple, and proved to be more economical than any other that could have been used. Piles under the retaining walls would have been impracticable on account of the boulder formation of the New Haven embankment, and the expense of any other foundation reaching down to hard soil would have been excessive.

The conditions under the two easterly New York Connecting tracks —Nos. 1 and 2—were different. There, no embankment existed on top of the original surface. The easterly retaining wall under Track No. 1 was carried down to about 7 ft. below the ground surface or 2 ft. below the probable future ground-water level, and was placed on timber piles, from 35 to 45 ft. long, driven to refusal into the hard strata of sand and gravel. The maximum pressure per pile is approximately 20 tons. The westerly retaining wall under Track No. 2 was to have been placed on the new fill between the original New Haven embankment and the retaining wall under Track No. 1. After this fill, which consists largely of sand and loam without boulders, had been placed, it settled considerably, and it was deemed inadvisable to place the wall thereon. Therefore, hollow steel piles, 12 in. in diameter and about 60 ft. long, with cast-steel shoes, were driven through the fill into the hard soil. After being cut off at the proper elevation, the piles were filled with concrete. The wall was then placed on top of these piles. The maximum pressure per pile is 45 tons. To prevent any lateral motion of this wall, due to greater earth pressure on the east side, it is connected to the wall under Track No. 1 with steel tie-rods encased in concrete.

Both retaining walls under Tracks Nos. 1 and 2 are of L-shape and are stiffened by vertical buttresses, 8 ft. apart. They are heavily reinforced with steel rods. This portion of the Bronx Viaduct, about 1 173 ft. long, contains approximately 16 000 cu. yd. of earth fill, 135 000 lin. ft. of timber piles, 8 800 lin. ft. of steel piles, 12 000 cu. yd. of concrete masonry, and 855 tons of structural steelwork. The cost of construction, inclusive of tracks but exclusive of electrification, was about $230 per lin. ft. of 4-track road.

Viaduct Portion.—The embankment type of construction, as previously described, was not practicable south of 138th Street, because, on account of the greater height, the foundation pressures would have become excessive. The plate-girder type of viaduct, with concrete piers, was adopted as being most suitable (Plate XXXVII). The spans are generally about 64 ft. long, except over the streets, where they had to be from 72 to 112 ft. The piers are of plain rectangular shape, with simple square copings.

To be safe, the foundations for the piers had to go down through the silt to the hard strata of sand and gravel or rock. Piles would

have been the least expensive foundation, but, in view of the considerable depth of soft, yielding material, they would not have afforded sufficient stability against lateral forces or lateral motion which might be caused by any disturbance of the soft ground in the future construction of adjacent foundations. Furthermore, piles could not have been driven through the boulder formation of the New Haven embankment under the two westerly tracks. Therefore, the more expensive, but securer type of foundation, consisting of two cylindrical concrete caissons under each pier, was adopted. The cylinders are from 10 to 15 ft. in diameter and were sunk partly by open dredging, and partly by the pneumatic process, to depths of from 12 to 55 ft. below the surface. The greatest pressure on the foundations is approximately 8 tons per sq. ft.

A deviation from the foregoing type of viaduct was necessitated at 133d and 132d Streets, where the New Haven and New York Connecting Railroads diverge. On account of insufficient space for piers, it was necessary to frame the longitudinal girders of the two westerly tracks into cross-girders supported by steel columns. The plate-girder viaduct portion, 1 736 ft. long, contains 14 800 cu. yd. of concrete masonry and 7 080 tons of steel work. The cost of construction was about $365 per lin. ft. of 4-track road.

Bronx Viaduct South of 132d Street, Randalls, Wards, and Long Island Viaducts.

The general conditions which affected the design of the viaducts in the Bronx section south of 132d Street, on Randalls and Wards Islands, and on Long Island north of Lawrence Street, are similar, with the exception of the character of the soil. The fact that these four sections are prominently exposed to view called for a uniform and pleasing appearance, in harmony with the monumental character of the arch bridge which they flank on both sides. The type of viaduct adopted consists of deck plate-girder spans resting on arched concrete piers. The span length varies from 72 to 94 ft., except for a single span over Van Alst Avenue in Long Island, which is 130 ft. The lengths chosen are the most economical, except on the Long Island viaduct, where they were largely determined by the location of the many streets which had to be crossed.

On the Bronx Viaduct the piers rest on cylinder caissons, from 15 to 18 ft. in diameter, one under each leg of the pier. These cylinders were sunk partly by open dredging, and partly under air pressure, to depths varying from 45 to 60 ft. below the surface, through soft silt and mud to a hard stratum of sand and gravel or solid rock (Plate XXXVII). On all other viaduct sections the piers have ordinary footings built in open excavation. On the Long Island Viaduct, where the soil has various formations of sand, gravel, boulders, loam, and clay, partly saturated with water which is likely to be drained off, all the footings had to be spread considerably so as to decrease the bearing pressure to the safe value of about $2\frac{1}{2}$ tons per sq. ft., and had to go to depths as great as 34 ft. below the surface.

On Wards and Randalls Islands the piers have comparatively shallow foundations, either on solid rock, or on hard strata of sand and gravel, or hardpan, which permitted pressures of 4 tons per sq. ft. and greater.

The proximity of the railroad to residential districts and to public parks made it desirable to restrict the noise from trains. Embankments between retaining walls were out of the question on account of the great height. Had soil conditions been favorable throughout, a viaduct consisting of a series of solid concrete arches would have been most suitable, as regards appearance and restriction of noise. This type is expensive in first cost, but is more durable, and less costly to maintain, than a plate-girder viaduct, the steel superstructure of which may have to be replaced or strengthened at some future date, and will require frequent painting. Concrete arches, however, require solid, unyielding foundations, in order to prevent dangerous and unsightly cracks, and such foundations could not be had on every section at reasonable cost.

Comparison of Three Designs of Plate-Girder Viaducts.

Fig. 44 shows typical portions of two preliminary designs for the plate-girder viaducts, and the design adopted. Fig. 44(*a*) represents the typical American trestle viaduct with alternate tower spans, 40 ft. long, and intermediate spans of 80 ft. This type, the advantages of which were cheapness and rapidity of erection, is now gradually being displaced by types of greater rigidity and durability, and better appearance. It would have been inappropriate and inadequate for the approaches to the Hell Gate Bridge. Fig. 44(*b*) represents the

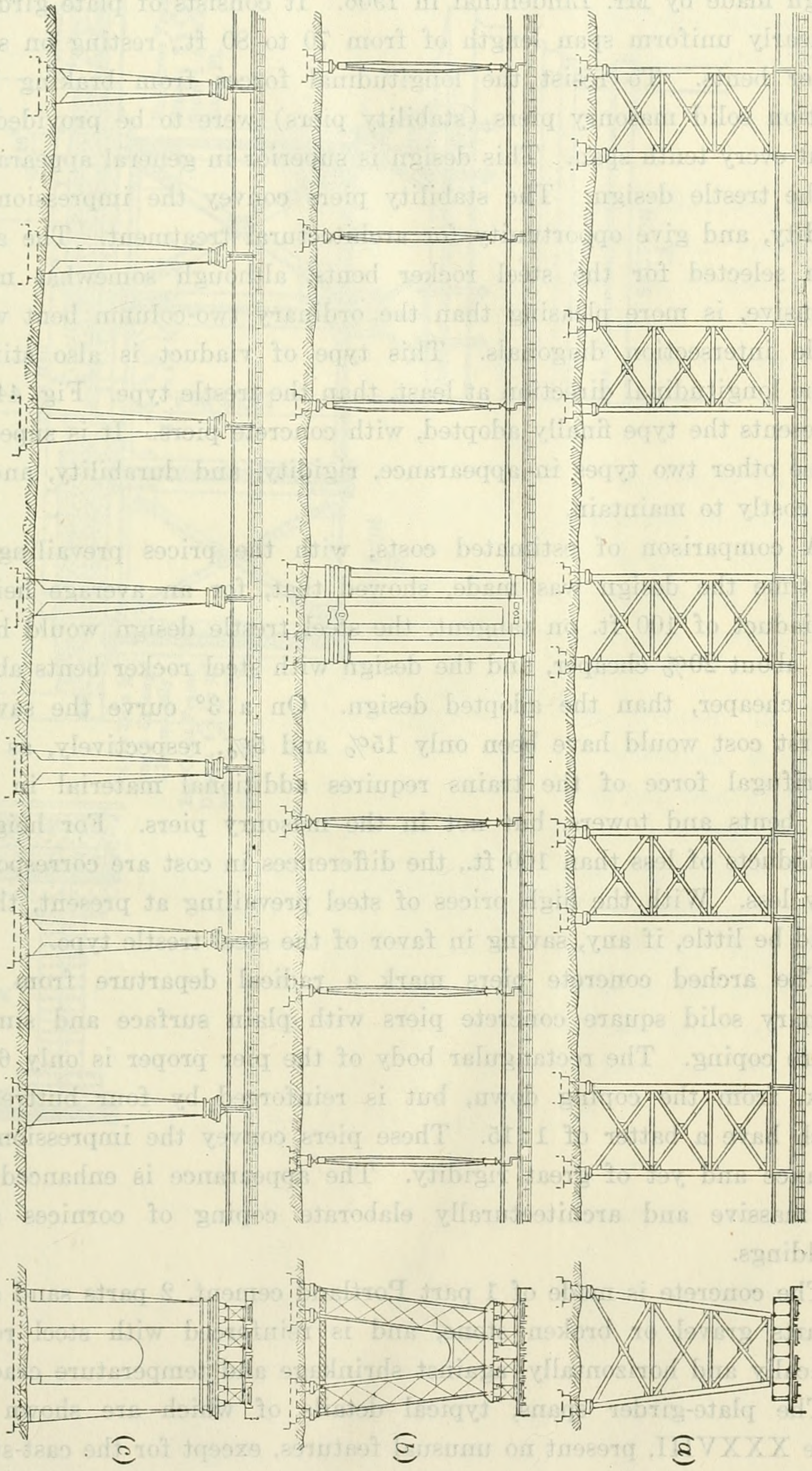


FIG. 44. COMPARISON OF THREE DESIGNS OF PLATE GIRDER VIADUCTS

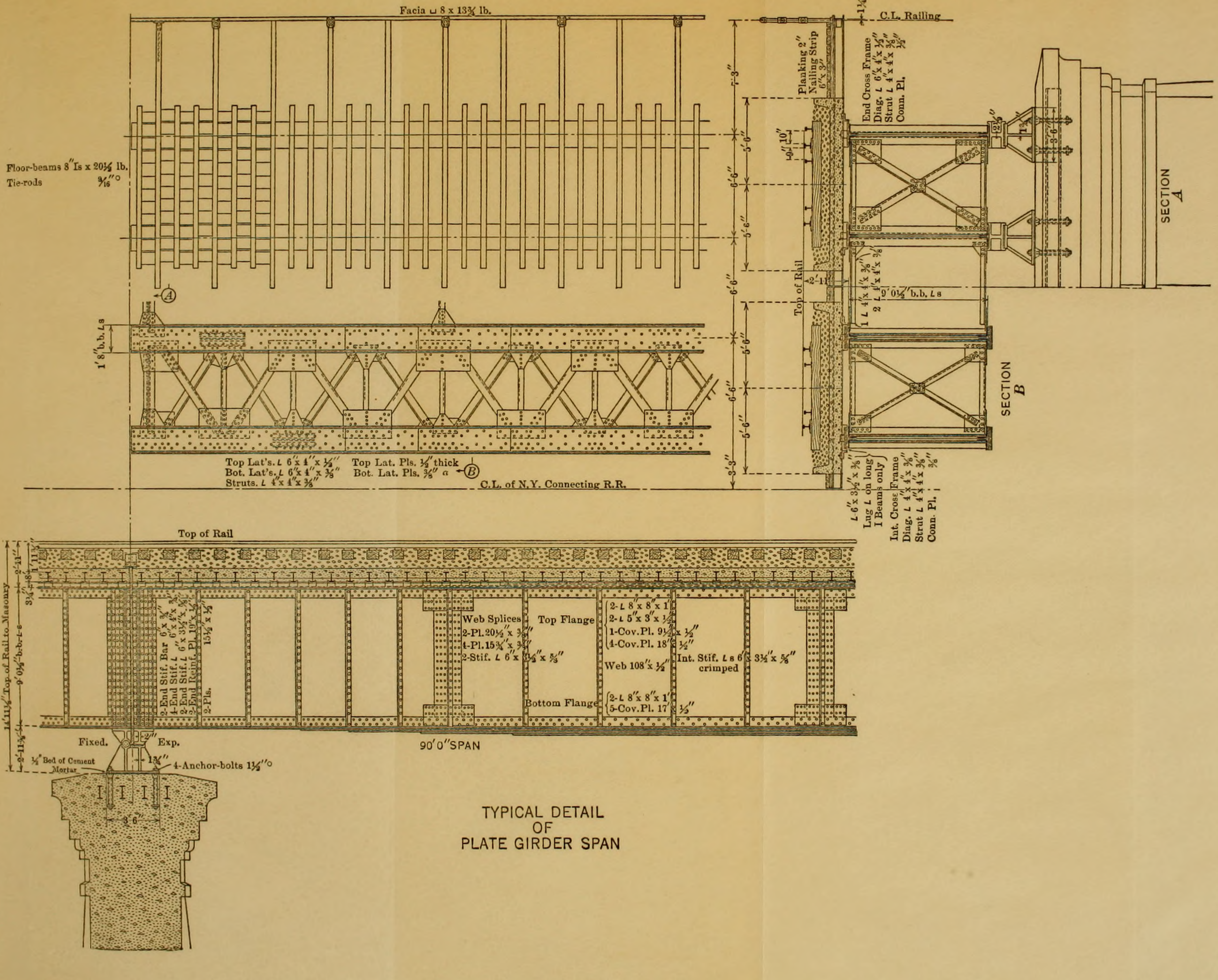
design made by Mr. Lindenthal in 1906. It consists of plate girders, of nearly uniform span length of from 70 to 80 ft., resting on steel rocker bents. To resist the longitudinal forces from braking and traction solid masonry piers (stability piers) were to be provided at about every tenth span. This design is superior in general appearance to the trestle design. The stability piers convey the impression of rigidity, and give opportunity for architectural treatment. The arch form selected for the steel rocker bents, although somewhat more expensive, is more pleasing than the ordinary two-column bent with single intersection diagonals. This type of viaduct is also stiffer, in the longitudinal direction at least, than the trestle type. Fig. 44(*c*) represents the type finally adopted, with concrete piers. It is superior to the other two types in appearance, rigidity, and durability, and is less costly to maintain.

A comparison of estimated costs, with the prices prevailing at the time the design was made, showed that, for an average height of viaduct of 100 ft. on tangent, the steel trestle design would have been about 20% cheaper, and the design with steel rocker bents about 10% cheaper, than the adopted design. On a 3° curve the saving in first cost would have been only 15% and 5%, respectively, as the centrifugal force of the trains requires additional material in the steel bents and towers, but not in the masonry piers. For heights of viaducts of less than 100 ft., the differences in cost are correspondingly less. With the high prices of steel prevailing at present, there would be little, if any, saving in favor of the steel trestle type.

The arched concrete piers mark a radical departure from the ordinary solid square concrete piers with plain surface and simple square coping. The rectangular body of the pier proper is only 6 ft. thick, from the coping down, but is reinforced by four buttresses which have a batter of 1:15. These piers convey the impression of elegance and yet of great rigidity. The appearance is enhanced by the massive and architecturally elaborate coping of cornices and mouldings.

The concrete is made of 1 part Portland cement, 2 parts sand and 4 parts gravel or broken stone, and is reinforced with steel rods, vertically and horizontally, against shrinkage and temperature cracks.

The plate-girder spans, typical details of which are shown in Plate XXXVIII, present no unusual features, except for the cast-steel



TYPICAL DETAIL
OF
PLATE GIRDER SPAN

bearings, which were designed to keep the reaction from one-sided loading as close as possible to the center of the pier. Special attention was also given to efficient web splices.

Erection of Plate-Girder Viaducts.

The erection of the viaducts presented no unusual difficulties, and the great number of nearly uniform spans, with the large tonnage involved, afforded opportunity for economical and rapid erection. The Bronx and Randalls Island Viaducts, for which the McClintic-Marshall Construction Company had the contract, were erected by a 50-ton steel derrick car, in some operations assisted by a 50-ton locomotive crane (Fig. 45).

All material for these viaducts was delivered over temporary tracks laid on the finished portion of the viaduct. Where possible, the girders were shipped and erected riveted up in pairs at the shop. A remarkable record was made on March 8th, 1915, when, after careful preparation, twenty-two single-track spans, with an aggregate weight of 1 504 tons, were put in place in a single 8-hour day.

A somewhat different method was used in the erection of the Wards and Long Island Viaducts, for which the American Bridge Company had the contract. After the Hell Gate Arch had been closed, and as the temporary back-stays were being dismantled, the plate girders, about 50% of which had formed part of the back-stays and counterweights, were distributed on the ground along the viaducts by using a locomotive crane running on a temporary track. The two 65-ton steel travelers, which had previously been used for the erection and dismantling of the back-stays of the arch bridge, were set up at the ends of the viaducts and proceeded toward the Hell Gate Bridge, raising the girders singly from the ground (Fig. 46).

Quantities, Weights, and Cost of Viaducts.

Table 6 gives the principal dimensions and quantities and the cost per linear foot for the different viaduct sections.

The weight of the steelwork (exclusive of I-beams in flooring), in pounds per linear foot, of single-track plate-girder spans, can be expressed approximately by the formula:

$$W = 350 - 17\ l$$

for span lengths of $l = 72$ to 94 ft.

TABLE 6.—Dimensions, Quantities, Cost, Etc., of Viaducts.

Section.	Bronx Viaduct South of 132d St.	Randalls Island Viaduct.	Wards Island Viaduct.	Long Island Viaduct.
Length of section, in feet	1 071	1 965	2 654	2 868
Approximate average height of rail above ground surface, in feet	60	80	110	90
Type of foundation	Caissons 45 to 60 feet deep.	Mostly shallow and narrow footings.	Mostly shallow and narrow footings.	Mostly deep and wide footings.
Average span length, in feet	89.0	82.0	88.5	85.5
Total quantity of concrete in piers, in cubic yards	17 500	29 800	60 300	77 900
Total weight of steelwork, in tons	4 375	7 610	11 190	11 980
Approximate cost per linear foot of viaduct, inclusive of tracks	$445	$370	$445	$500

Deck Truss Bridges of Long Island Eastern Viaduct.

Potter Avenue, and the streets between Steinway and Flushing Avenues, in Long Island City, are crossed by deck-truss bridges aggregating 1 231 ft. in length. On account of excessive span length and limited height, concrete arches, such as were built over the other streets on the Long Island Eastern Viaduct, would have been impracticable. Potter Avenue, which is 80 ft. wide, is crossed by the railroad at an angle of only 19 degrees. The distance between street lines in the direction of the tracks is about 250 ft., and the rails are only 50 ft. above street level. A skew steel bridge, about 270 ft. long, with abutments parallel to the street lines, would have been a very unsatisfactory design, and probably as expensive as the one adopted, although lighter in steelwork.

The bridge, as built, consists of three square deck-truss spans, each 135 ft. long (Plate XXXIX and Fig. 47). The two ends are supported on concrete abutments, which form a monolithic structure with the embankment side-walls. Two heavy steel rocker bents, each consisting of two columns and a cross-girder, form the intermediate supports. Two of the columns are placed in the center of the street, on permission secured from the city. Each span has four trusses, one for each track. They are $18\frac{1}{2}$ ft. deep, and 13 ft. 9 in. apart on centers. The trusses have fixed bearings on one abutment and expansion bearings on the other, the intermediate steel bents acting as rockers. The longitudinal forces from all spans, therefore, are transmitted to one end, and all temperature expansion takes place toward

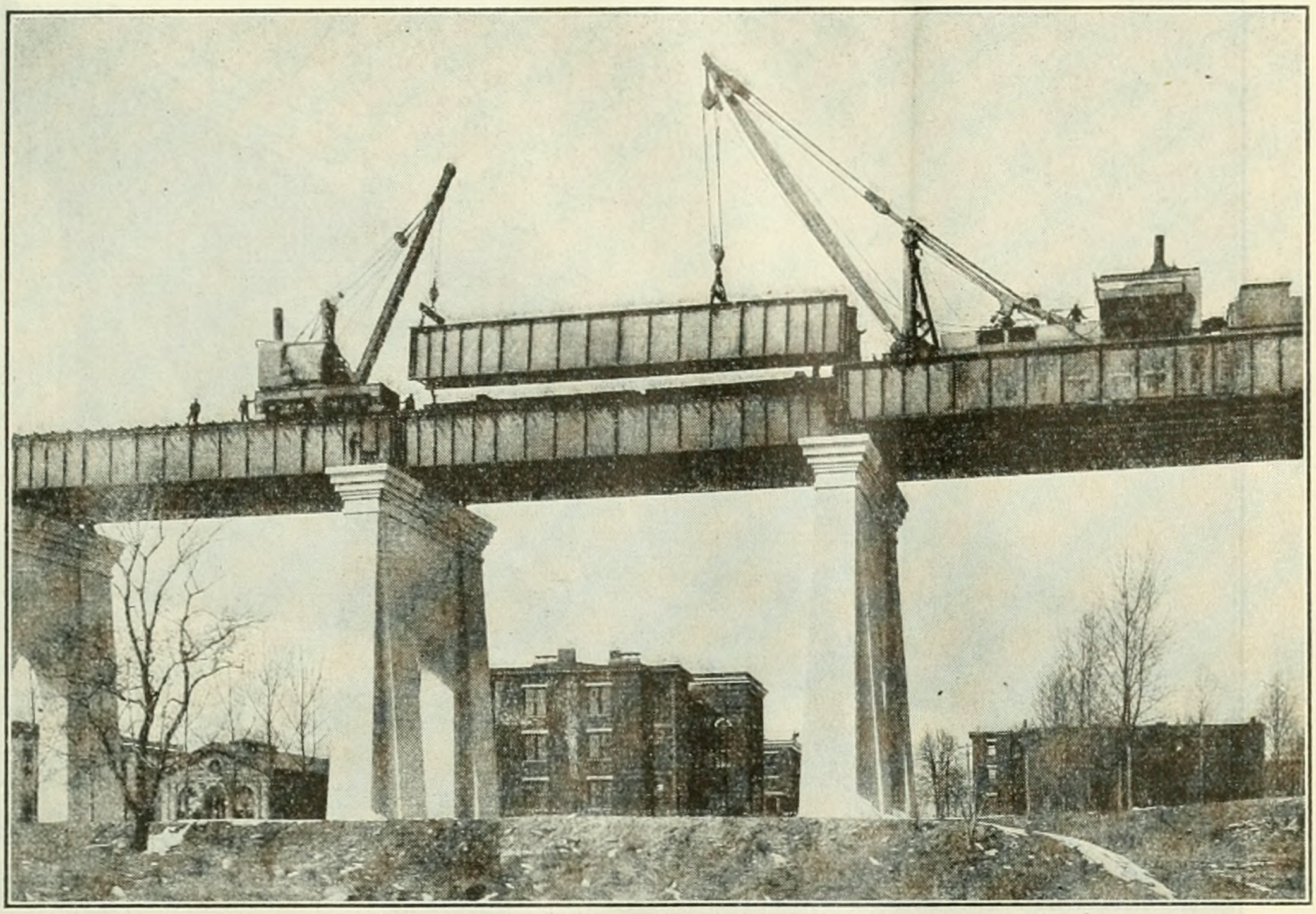


FIG. 45.—ERECTION OF RANDALLS ISLAND VIADUCT.

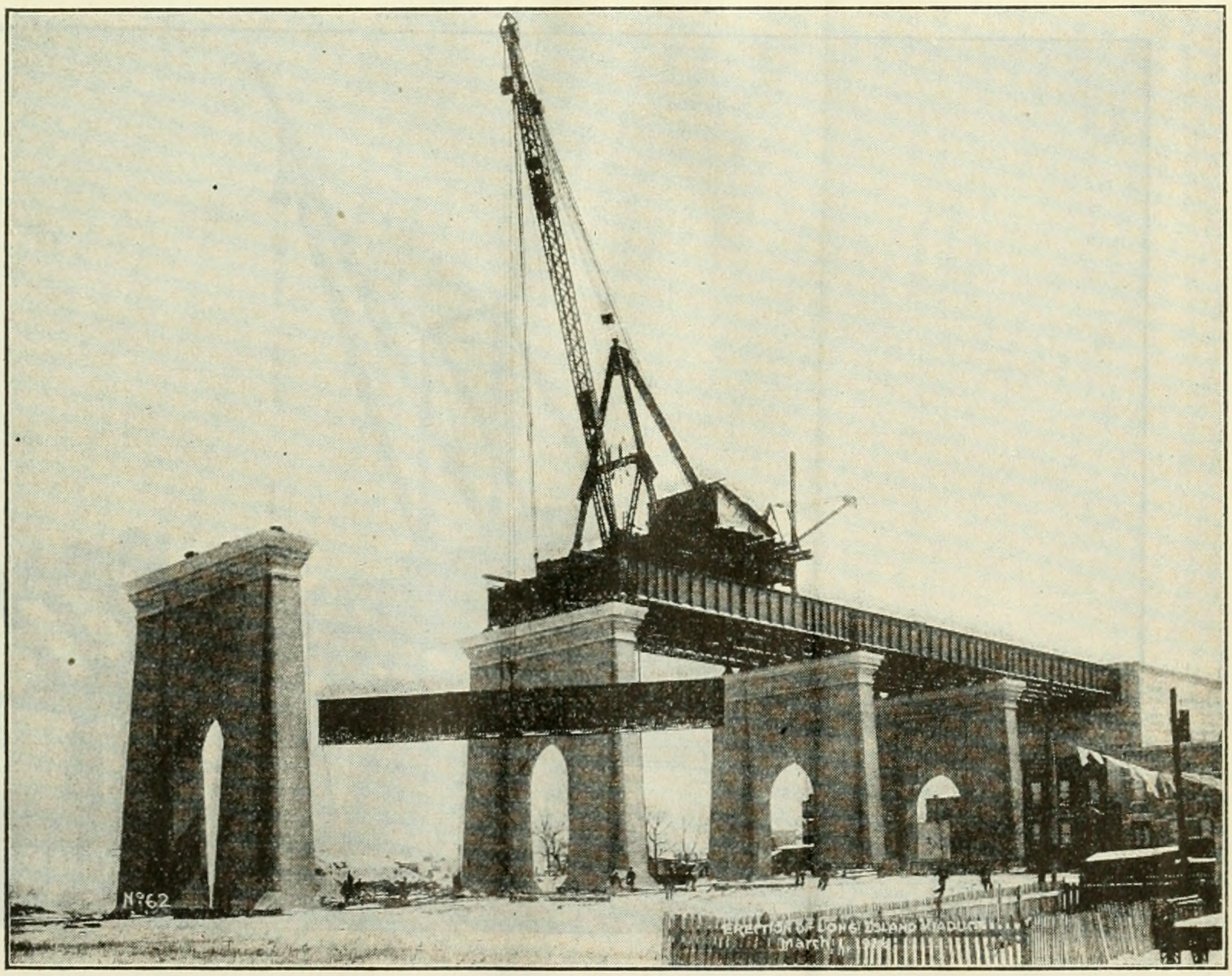


FIG. 46.—ERECTION OF LONG ISLAND VIADUCT.

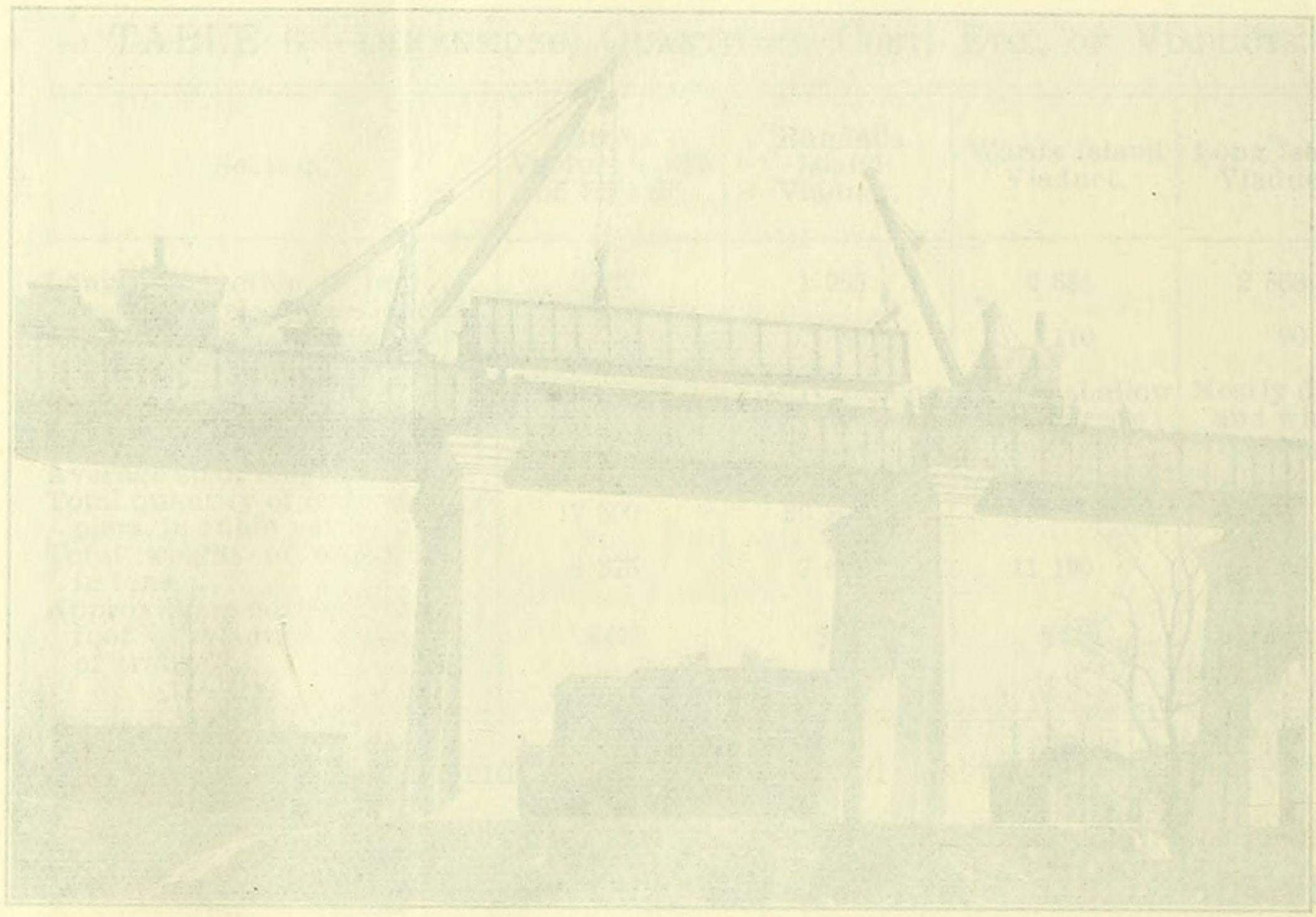


FIG. 45.—ERECTION OF RANDALLS ISLAND VIADUCT.

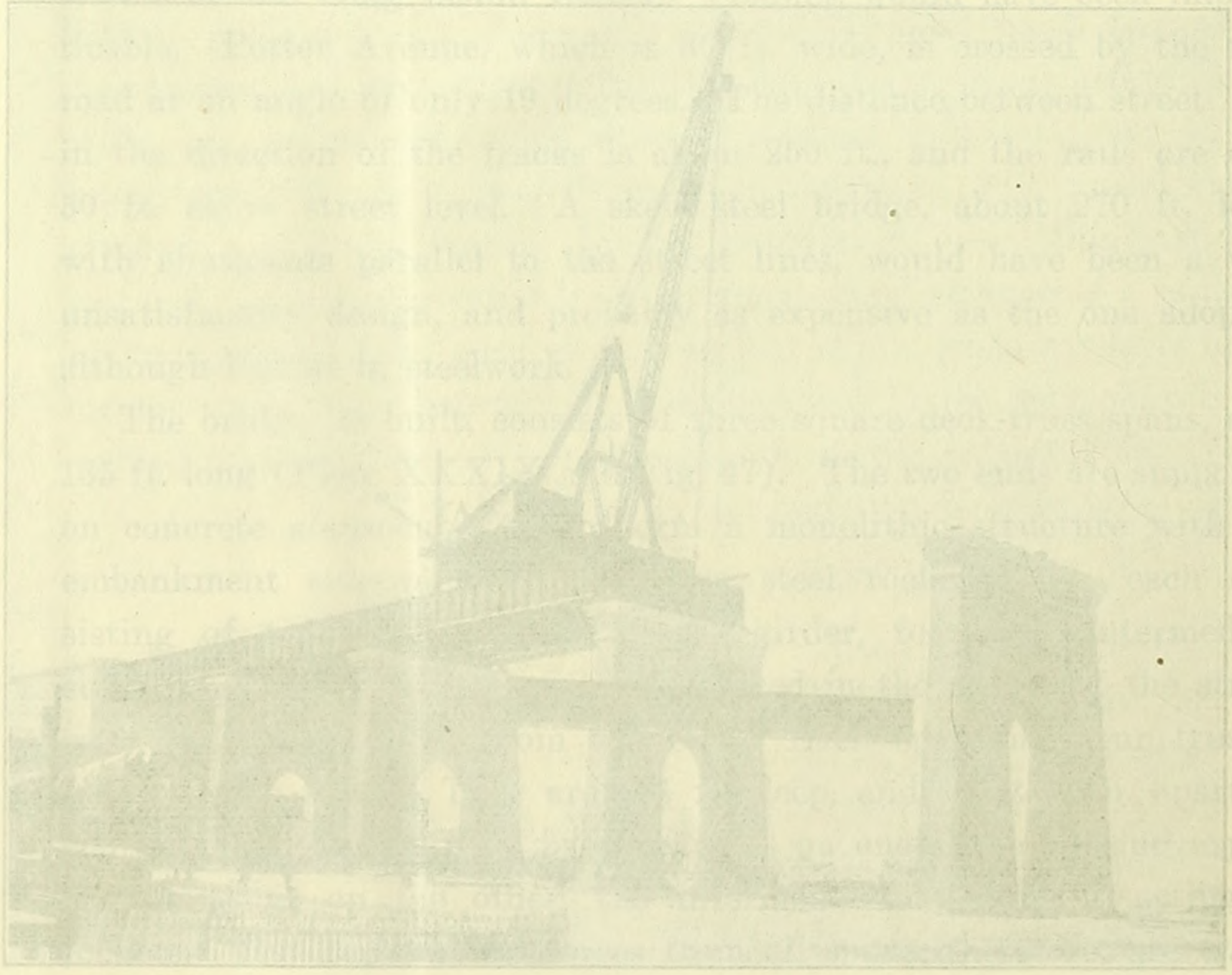


FIG. 46.—ERECTION OF LONG ISLAND VIADUCT.

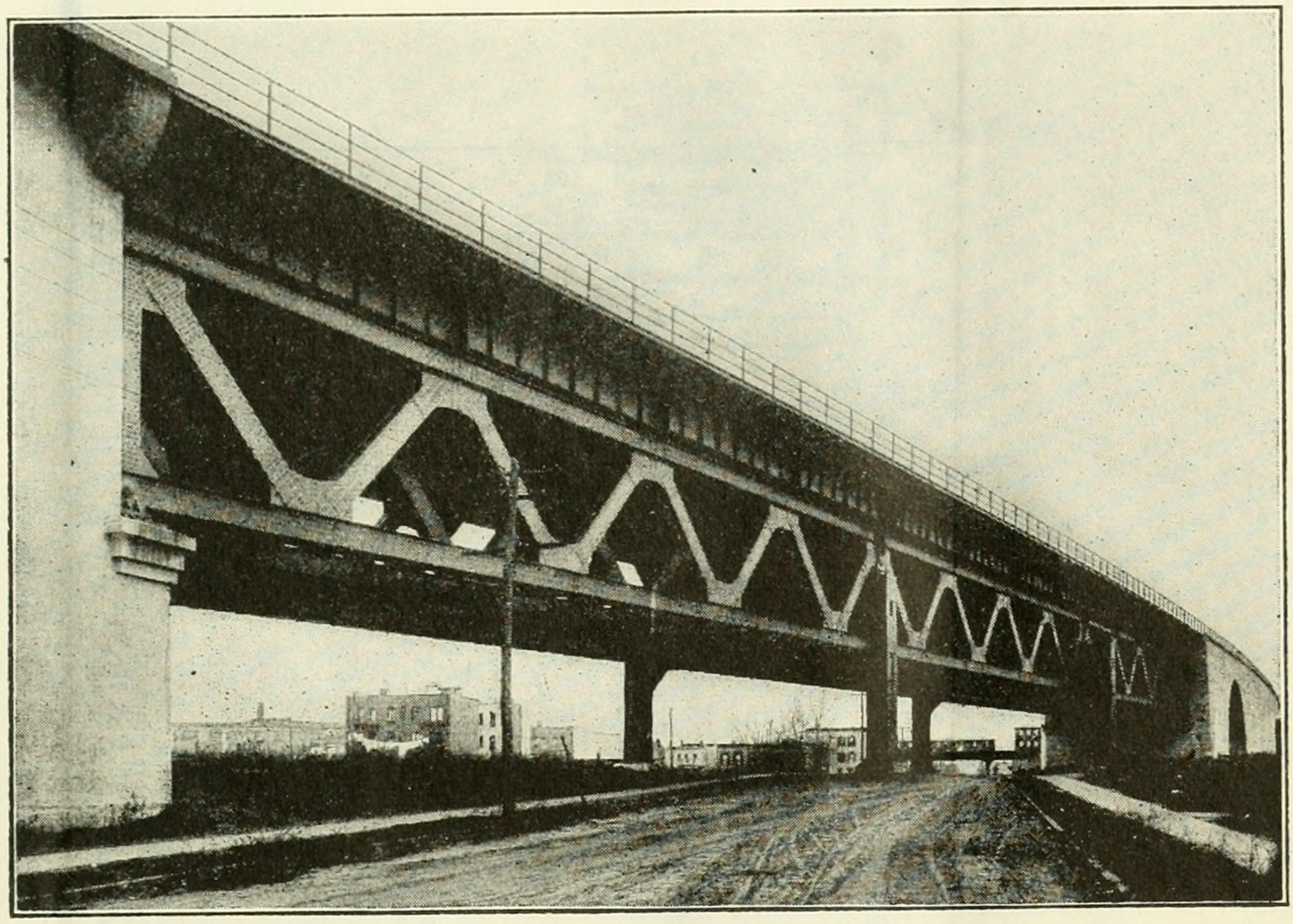


FIG. 47.—POTTER AVENUE CROSSING.

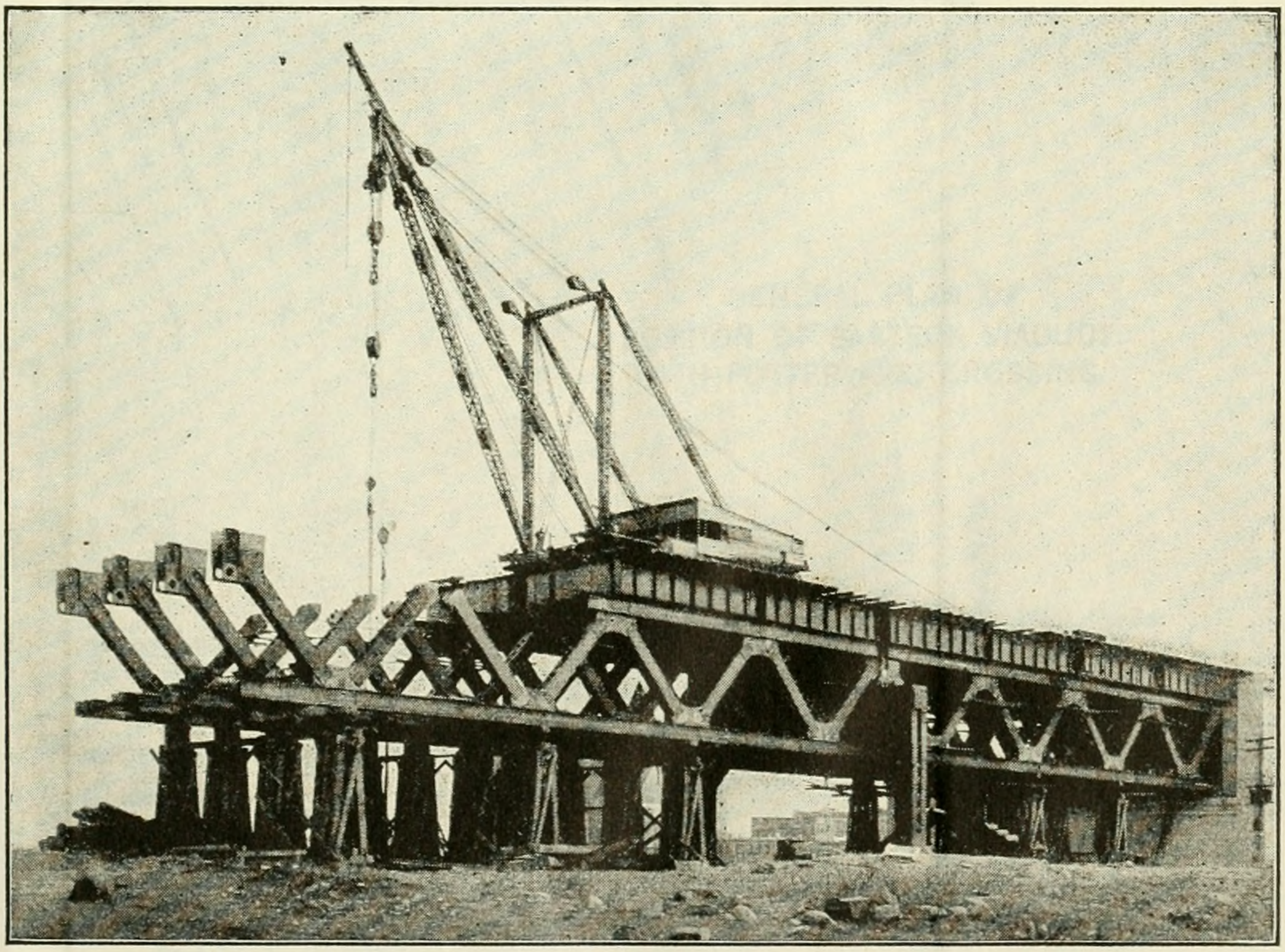


FIG. 48.—ERECTION OF POTTER AVENUE CROSSING.

FIG. 48.—ERECTION OF POTTER TRUSS CROSSING.

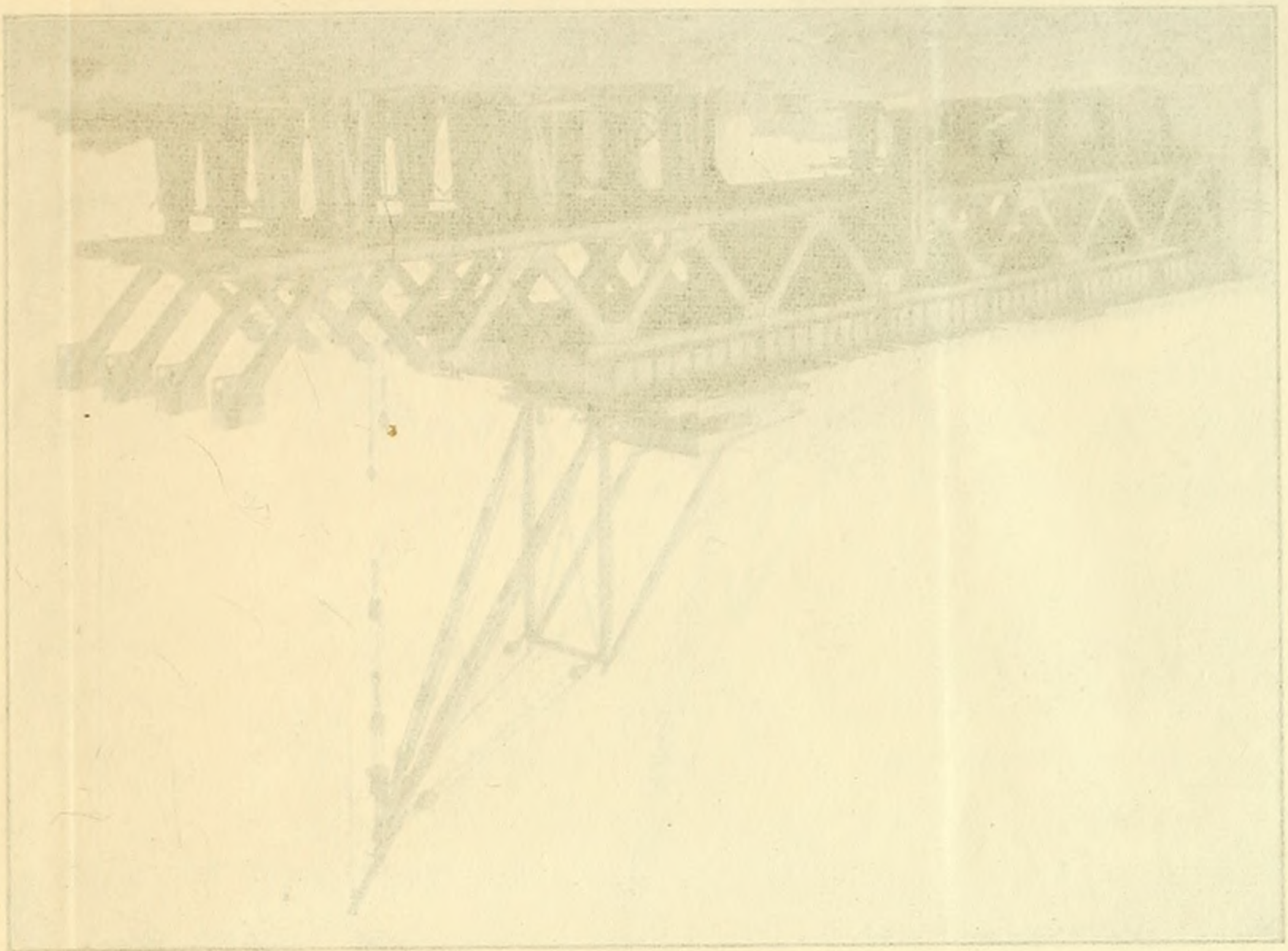
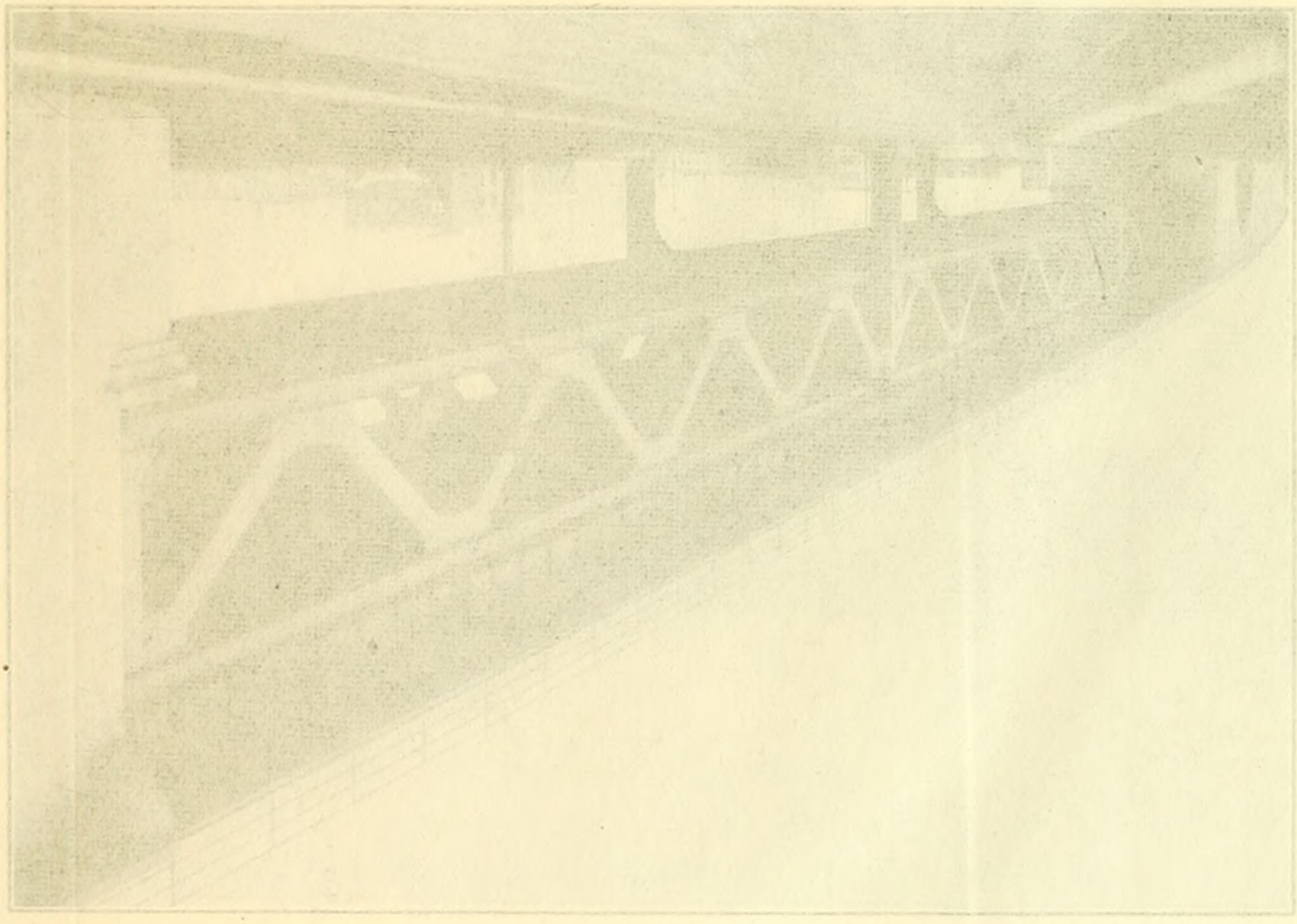
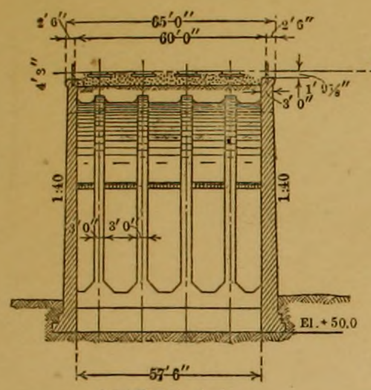
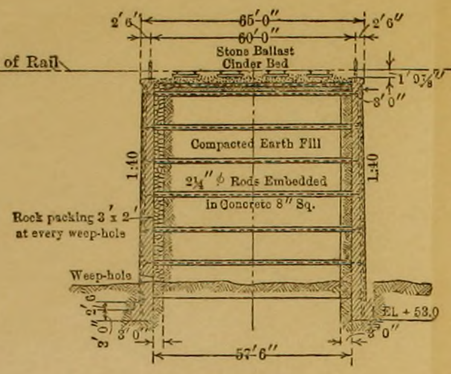


FIG. 49.—POTTER TRUSS CROSSING.





SECTION *A A*



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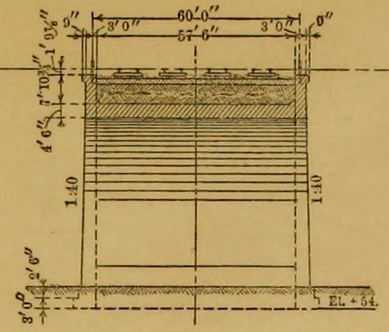

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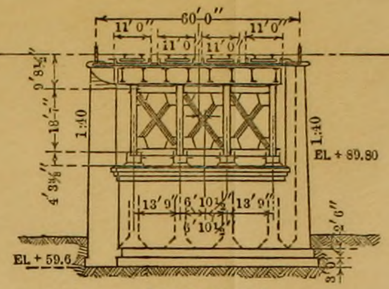
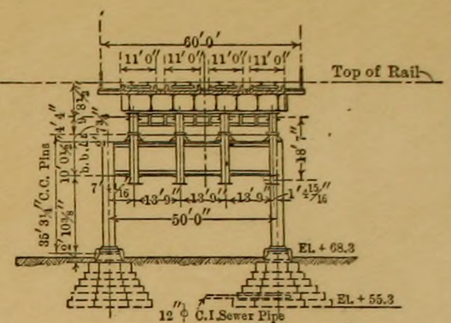


SECTION *D D*



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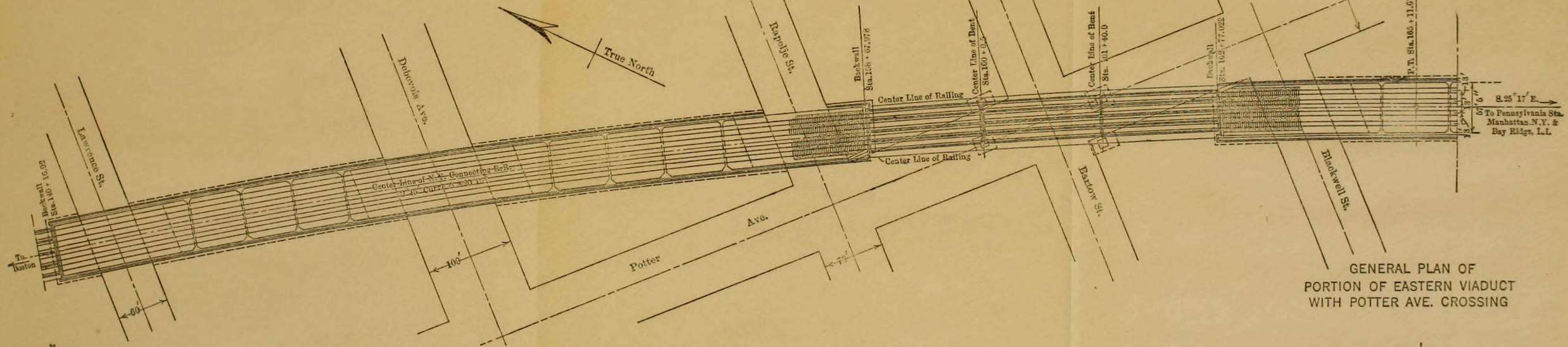
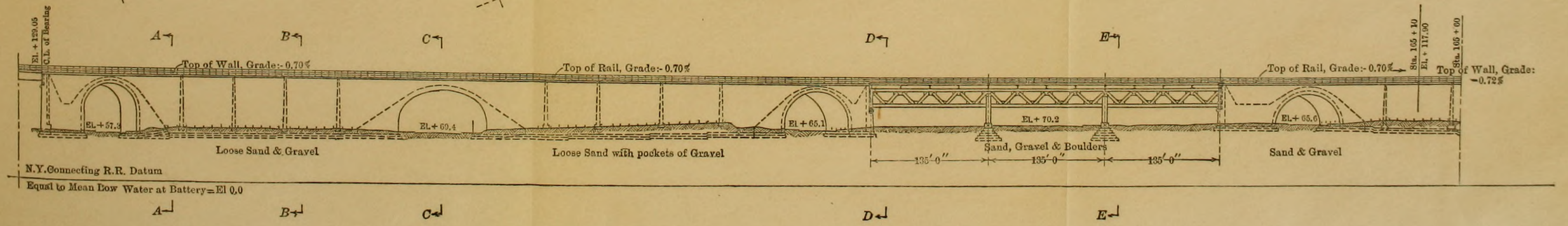


GENERAL PLAN OF
PORTION OF EASTERN VIADUCT
WITH POTTER AVE. CROSSING



the other end. Over the steel bents the trusses are supported at their top chord points by cast-steel pin bearings, which in turn are supported by the cross-girders of the bent. Each of two adjoining trusses turns independently on its bearing, and the bottom chord member opposite the bearing is free to slide at its end, so that the trusses act as simple spans.

The columns are provided with pin bearings at the bottom, so as to be free from bending stresses from longitudinal forces or temperature expansion of the trusses. Transversely, however, the columns, with the cross-girders, form rigid portals which transmit the reactions from the lateral forces to the foundations. The cross-girders are double web-girders, 10 ft. deep, and weigh 130 tons each. They were shipped in three sections. The floor-beams rest on top of the trusses, and the stringers are framed into them. To avoid stresses in the floor-beams, due to unequal deflections of the trusses, all floor-beams except those over supports are interrupted between the two interior trusses.

The design of the truss bridges, between Steinway and Flushing Avenues, was governed by similar conditions, and is similar in every respect to that of the Potter Avenue Crossing, except that all intermediate supports are solid concrete piers instead of steel bents, and are on railroad property. The spans vary in length from 120 ft. to 165 ft. 11 in. The depth of the trusses is only 16 ft. 4 in., which was limited by the required minimum height of 16 ft. above the street level. The trusses, therefore, are unusually heavy.

All these truss bridges were erected on heavy timber bents by an 80-ton steel traveler (Fig. 48). Erection was started in October, 1914, and completed in April, 1915. Potter Avenue Crossing contains approximately 4 000 cu. yd. of concrete masonry and 3 722 tons of steel work, and the cost of construction, inclusive of tracks, was about $275 000, or $675 per lin. ft. The Steinway-Flushing Avenue Crossing contains approximately 7 600 cu. yd. of masonry and 6 526 tons of steelwork, and its cost of construction, inclusive of tracks, was about $500 000, or $565 per lin. ft.

Embankment Portion of Long Island Eastern Viaduct.

Except for the steel truss bridges over certain streets, as described before, the Eastern Viaduct, which extends from Lawrence Street to Stemler Street in Long Island City, a total length of 3 500 ft.,

consists of a novel type of embankment, from 30 to 65 ft. in height above ground. Seven streets are crossed by reinforced concrete arches which form a monolithic structure with the retaining walls of the embankment. (Plate XXXIX and Fig. 43.)

The embankment consists of two longitudinal reinforced concrete retaining walls, connected and held in relative position by horizontal steel tie-rods, which are embedded individually in a shell of concrete for protection against corrosion. These rods resist the pressure from the earth fill. For additional stability, the two walls are connected by thin cross-walls, about 50 ft. apart. The arches consist of a comparatively thin barrel reinforced by vertical ribs. The fill is mixed clay, sand, and gravel, carefully placed in 12-in. crowned layers, and thoroughly tamped, so as to form a uniform compact mass which exerts a comparatively small pressure on the retaining walls. It is thoroughly drained by chimneys of rock packing which extend along the walls from the top of the fill to the weep-holes at the bottom.

The walls and arches have perfectly plain surfaces and a simple coping. No attempt has been made at architectural treatment, because the territory in the vicinity is being built up mostly by industrial buildings which hide that portion of the railroad from prominent view. This embankment construction is considerably cheaper than the ordinary type, which consists of a fill between two independent gravity walls. For a height of 50 ft. the latter type would have cost from 30 to 40% more.

A plate-girder viaduct with concrete piers, such as was used north of Lawrence Street and over the island, would also have been more expensive than the adopted type of embankment. For heights exceeding about 65 ft., however, the plate-girder viaduct became cheaper.

The embankment contains approximately 70 000 cu. yd. of concrete masonry, 160 000 cu. yd. of earth fill, and 1 000 tons of steel reinforcement.

The cost of construction per linear foot of embankment, inclusive of tracks, was approximately $470 for a height of 65 ft. and $280 for a height of 35 ft., or $(6.5\ h - 50)$ dollars per square foot of elevation, h being the height of rail above the ground line.

13.—Track Floor Construction.

The franchise required a ballasted roadbed on the Bronx Section, north of Bronx Kill, and on the Long Island Section, south of Hell

Gate. A solid ballasted floor, owing to its advantages over the open tie floor, namely, more uniform roadbed, less noise and impact, smaller cost of maintenance, and greater safety in case of derailment or fire, was adopted throughout, however, except for the bascule spans of the Bronx Kill Bridge, although it involved a considerable additional initial expenditure over the open tie floor.

Various types of solid floor construction were considered. Previous to construction a wooden floor, consisting of framed, treated ties, laid closely together, appeared to be most suitable, owing to its lightness and, as it seemed then, its comparatively small cost. By the time construction started, however, the prices of framed and treated timber had increased considerably, and a more durable and fire-proof, although somewhat heavier and slightly more expensive, concrete floor construction was adopted.

A reinforced concrete slab, in which the compressive strength of the concrete and the bond between the steel rods and the concrete are relied on to resist the stresses, was not considered advisable, in view of the limited experience as regards the durability of such a construction under the heavy impact to which the floor would be subjected.

The type finally adopted (Fig. 49) consists of 8-in. I-beams, about 15 in. from center to center, placed across the two stringers or girders of each track. These beams are tied together near the bottom with $\frac{5}{8}$-in. tie-rods, about 10 in. apart, and thus form, by themselves, a comparatively rigid steel skeleton. The axle load of 70 000 lb., plus 200% impact, was assumed as distributed over three beams, or an equivalent length of track of twice the height from top of rail to top of I-beams.

The concrete, which is placed between and from $2\frac{3}{4}$ to $3\frac{1}{2}$ in. above the beams, but flush with the bottom surface of the beams, forms merely an encasement. To prevent transverse temperature cracks at the top surfaces, the concrete is further reinforced by longitudinal rods resting on, and tied to, the steel beams. The ballast is held on each side of the track by parapet walls, which are also well reinforced against temperature or shrinkage cracks. The I-beams served as ties for the construction tracks, and thereby saved the cost of temporary timber ties. Plain steel sheets, $\frac{1}{8}$ in. thick, wedged against the bottom of the beams, constituted simple forms for the bottom surface of the concrete. The space between the concrete troughs is utilized for

footwalks and for the six-duct concrete conduit construction. Expansion joints in the concrete troughs are provided at all expansion joints in the steel superstructure. These joints are covered by T-shaped steel dams, bedded on a thin layer of asphalt.

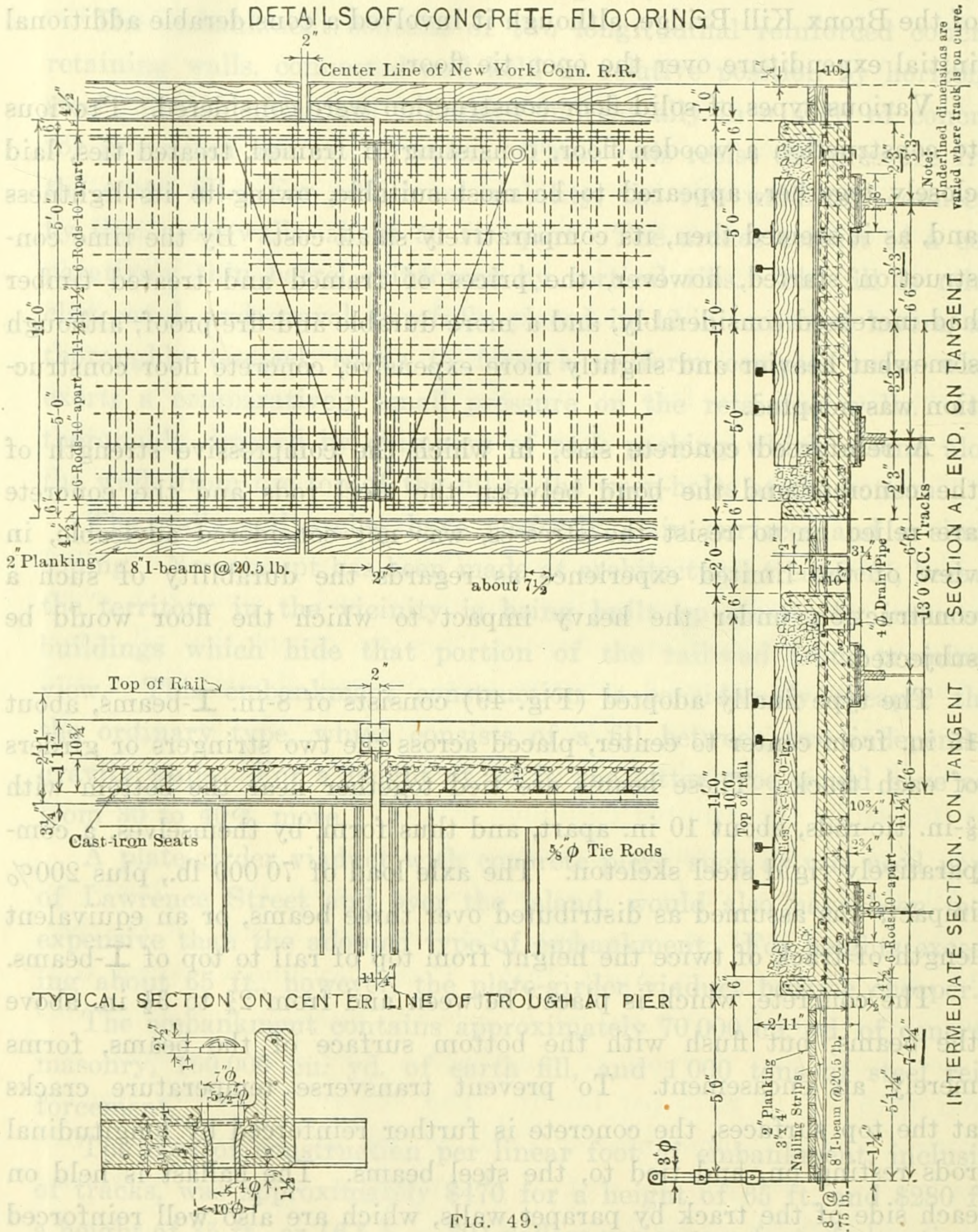


FIG. 49.

Care was taken to secure a dense concrete by using a mixture of 1 part Portland cement, 2 parts of well-graded sand, and 4 parts of broken limestone, the latter composed of 75% of ¾-in. stone and 25% of screenings. This concrete, tested on 4-in. cubes, showed an average compressive strength of about 3 500 lb. per sq. in. at the age of 28

days. The top of the concrete slab was carefully troweled to a smooth finish. No water-proofing material was placed thereon.

For efficient drainage, the top surface was given a transverse slope of $1\frac{1}{2}$ in. in 10 ft., and 4-in. cast-iron drain pipes, with strainers, were placed about 15 ft. apart. On the street crossings, these pipes discharge into steel gutters which lead to 6-in. down-spouts at the piers or abutments.

The sidewalks and the walks between the tracks are of 2-in. wooden planks resting on extensions of the I-beams. The flooring contains, per linear foot of 4-track structure, 1.5 cu. yd. of concrete, 1 200 lb. of steel, and 40 ft. b. m. of timber, and cost about $53 per lin. ft. The I-beams were furnished and erected by the contractors for the steelwork. The concrete and the timber flooring were placed on contract by Fraser, Brace and Company and The Snare and Triest Company.

14.—Engineering Organization.

The New York Connecting Railroad has been built under the direction of Mr. Samuel Rea as President and Mr. A. T. County, Assistant to the President. Mr. Gustav Lindenthal, Consulting Engineer and Architect, prepared the plans for the East River Bridge Division, and, as Chief Engineer, directed their execution. During construction the Chief Engineer was assisted by an engineering staff of ninety-five members.

O. H. Ammann, M. Am. Soc. C. E., Assistant Chief Engineer, had general charge of the office, field, and inspection work, H. W. Hudson, M. Am. Soc. C. E., Construction Engineer, was in direct charge of the field operations, in which he was assisted, in the earlier stages of the work, by three Resident Engineers, George W. Philips, Assoc. M. Am. Soc. C. E., R. T. Robinson, Assoc. M. Am. Soc. C. E., and S. D. Heed, Assoc. M. Am. Soc. C. E., and later by Mr. S. D. Heed as Assistant Construction Engineer. D. B. Steinman, Assoc. M. Am. Soc. C. E., Special Assistant Engineer, attended to computations and strain measurements, and Mr. W. A. Cuenot, Assistant Engineer, to the drafting and checking of the plans and shop drawings.

A very thorough inspection was exercised over all materials. All cement which went into the work was tested at the Company's laboratory, in charge of Mr. G. B. MacWhinney, Assistant Engineer. The steel was tested and inspected at the mills and at the various shops by a corps of fifteen inspectors, directed successively by Mr. J. C.

Naegeley, as Engineer of Inspection, and Messrs. William E. Crane and R. E. McGough, as Chief Inspectors.

Conclusion.

Some of the broader engineering questions which suggest themselves in the design and execution of the structure forming the subject of this paper may be summarized as follows:

A great engineering work cannot be spontaneously created in its final, perfect form, but has to grow and develop gradually, in its entirety as well as in its constituent parts. Although the layman can only judge such a work in the light of an accomplished fact, the engineer must ever be conscious that it is only through extensive and laborious preliminary studies, and untiring efforts to improve, that he can hope to achieve a perfect work.

In the execution of a great and complex engineering or scientific undertaking, collaboration of experts in various fields is essential, but a great structure of monumental character must be the product of an individual creative and directive mind.

A great structure cannot be the result of a set of rules and specifications, nor of elaborate mathematical computations. Such a work requires wide experience and sound judgment, and therefore, should be entrusted only to engineers of high professional attainments and reputation.

Throughout this paper the importance of rigidity in bridge construction has been pointed out. Rigidity insures greater durability and safety. There are remarkable examples of structures which have stood up under excessive strains under which they would have failed had it not been for the rigidity of their members or connections. Large bridges must be built for generations to come. Engineers to-day cannot afford to build important structures cheaply, to serve their purpose for the time being, and incur the risk of having to replace them after a short period of usefulness.

Emphasis has been laid on the appearance of the structures described. Engineering structures are still regarded by many engineers as mere works of utility, which deserve no consideration in architectural or artistic treatment. So long as this opinion prevails, the Engineering Profession will not lift itself to a higher plane, and it is even running the risk of being relegated to second place—or after the architect—in the creation of such monumental structures as properly belong in its domain.

APPENDIX A

Calculation of Dead-Load, Live-Load, and Temperature Stresses in Arch Trusses of Hell Gate Bridge.

1.—Influence Line for Horizontal Reaction of 2-Hinged Arch.

In calculating the influence line for horizontal reaction, the following analytical method has been applied:

A.—Calculation of Values of Δs, $\Delta \alpha$, and Δl.— If the arch is considered a simple span fixed at A and free to move at B (Fig. 50) and if, for any one member of the truss,

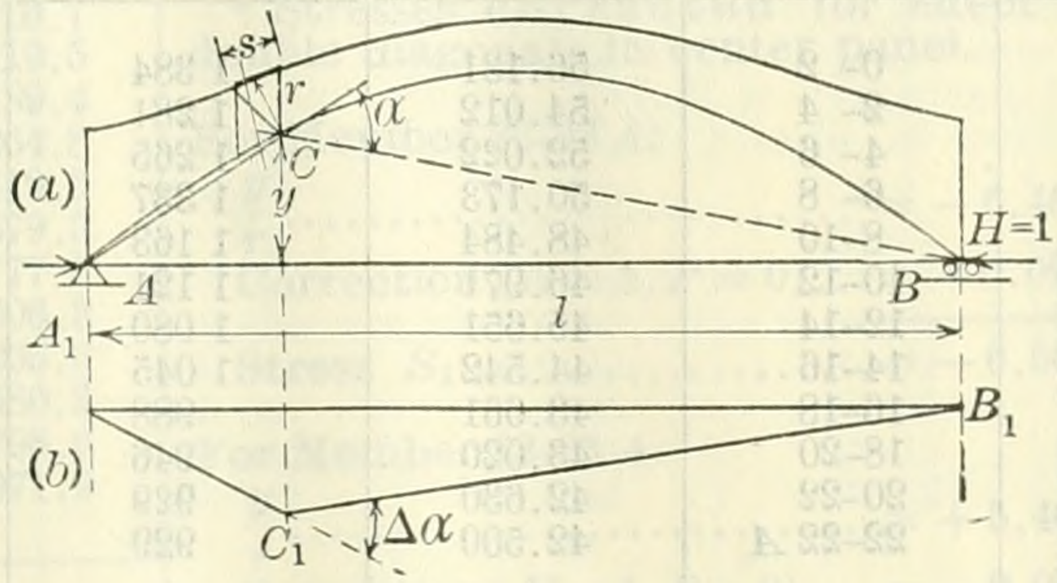


FIG. 50.

A = area of its gross section, in square inches;
s = its length, in feet;
r = its lever arm (perpendicular distance from its center of moments, C), in feet;
y = ordinate of its center of moments, above line connecting hinges, in feet;

$S_1 = \frac{y}{r}$ = stress in the member due to the sole application of a horizontal force of unity at B, and E = modulus of elasticity (30 000 000 lb. per sq. in.); then the axial deformation of the member is, in feet,

$$\Delta s = \frac{S_1 s}{A E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If it is assumed that only this one member is elastic, the angle, α, between the lines, $A\ C$ and $B\ C$, will change by an amount

$$\Delta \alpha = \frac{\Delta s}{r} \text{ (arc measure)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

that is, the elastic line is a triangle, $A_1\ C_1\ B_1$ (Fig. 50 (b)) the sides of which, $A_1\ C_1$ and $B_1\ C_1$, form the angle $\Delta \alpha$.

The point, B, moves horizontally, that is, the span length, l, changes by an amount, in feet, equal to

$$\Delta l = \frac{y}{r} \Delta s = y \Delta \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The sum, $\Sigma \Delta l$, of the values Δl, for all truss members gives the total horizontal movement of the point, B, due to the sole application of the horizontal load of unity at B.

Tables 7 and 8 show the calculation of the values, Δs, $\Delta \alpha$, and Δl, for each truss member, and also the sum, $\Sigma \Delta l$. For convenience

TABLE 7.—DETERMINATION OF VALUES

	Member.	(1) Length, s, in feet.	(2) Gross Area, A, in square inches.	(3) Lever Arm, r, in feet.	(4) Ordinate of Center of Moments, y, in feet.	(5) Stress $S_1 = \frac{y}{r}$
Bottom Chord.	0- 2	56.131	1 384	— 106.00	+ 140.00	— 1.3208
	2- 4	54.012	1 281	— 88.86	+ 149.60	— 1.6835
	4- 6	52.022	1 265	— 75.92	+ 162.93	— 2.1461
	6- 8	50.173	1 337	— 67.76	+ 180.00	— 2.6562
	8-10	48.484	1 163	— 62.38	+ 197.83	— 3.1714
	10-12	46.971	1 121	— 57.39	+ 213.43	— 3.7180
	12-14	45.651	1 080	— 52.88	+ 226.80	— 4.2890
	14-16	44.542	1 045	— 48.92	+ 237.94	— 4.8639
	16-18	43.661	988	— 45.60	+ 246.85	— 5.4134
	18-20	43.020	946	— 43.01	+ 253.54	— 5.8949
	20-22	42.630	929	— 41.21	+ 258.00	— 6.2606
	22-22 A	42.500	929	— 40.22	+ 260.22	— 6.5669 *
Top Chord.	1- 3	43.571	315	+ 110.16	+ 36.67	+ 0.3328
	3- 5	44.541	315	+ 88.67	+ 70.00	+ 0.7814
	5- 7	45.800	315	+ 74.24	+ 100.00	+ 1.3470
	7- 9	46.089	329	+ 65.62	+ 126.67	+ 1.9303
	9-11	45.273	385	+ 59.54	+ 150.00	+ 2.5193
	11-13	44.553	385	+ 54.18	+ 170.00	+ 3.1377
	13-15	43.936	385	+ 49.60	+ 186.67	+ 3.7634
	15-17	43.424	385	+ 45.85	+ 200.00	+ 4.3651
	17-19	43.023	385	+ 43.01	+ 210.00	+ 4.8826
	19-21	42 733	385	+ 41.11	+ 216.67	+ 5.2704
	21-23	42.558	329	+ 40.16	+ 220.00	+ 5.4781
	23-23 A	42.500	329	+ 40.22	+ 220.00	+ 5.3729 *

in calculation, the foregoing values have been determined in units of 1 000 E. (See Columns 6, 7, 8, and 9 of Tables 7 and 8.)

B.—Determination of Elastic Curve of Arch Truss.—As can easily be proved, the elastic line, $A_1\ C_1\ B_1$ (Fig. 50 (b)), assuming again only the one member elastic, is identical with the moment diagram of a simple span, $A\ B$, due to the sole application of a vertical load, $\Delta\ \alpha$, at the center of moments, C, of the member in question.

From this rule, if applied to every truss member, it follows that the elastic line of the arch truss due to the sole application of a horizontal load of unity at B is identical with the moment diagram due to the application of the values, $\Delta\ \alpha$, as vertical loads, called "Elastic Loads", at the respective centers of moments of the truss members.

The center of moments of a chord member, and therefore the corresponding elastic load, $\Delta\ \alpha$, is always at a panel point.

The center of moments of a web member, in general, is not at a panel point, nor is it always within the span length, and, therefore, it

Δs, $\Delta \alpha$, and Δl, FOR CHORD MEMBERS.

(6) $1\,000\,E\,\Delta s = 1\,000\,\frac{S_1 s}{A}$	(7) $1\,000\,E\,\Delta \alpha = 1\,000\,\frac{E\,\Delta s}{r}$	(8) $1\,000\,E\,\Delta l = \frac{y}{r}\,1\,000\,E\,\Delta s = y\,1\,000\,E\,\Delta \alpha$	(9) Notes.
— 53.57	+ 0.505	+ 70.7	* Stresses corrected for effect of double diagonals in center panel.
— 70.98	+ 0.799	+ 119.5	
— 88.26	+ 1.163	+ 189.4	
— 99.68	+ 1.471	+ 264.8	For Member 22-22 A:
—132.21	+ 2.119	+ 419.3	$\frac{y}{r}$ ..... $= -6.4699$
—156.19	+ 2.715	+ 579.5	Correction ($H=1, P=0$) .. $= -0.0970$
—181.29	+ 3.428	+ 777.6	Stress S_1 ..... $= -6.5669$
—206.90	+ 4.229	+ 1 006.3	For Member 23-23 A:
—239.22	+ 5.246	+ 1 295.0	$\frac{y}{r}$ ..... $= +5.4699$
—268.08	+ 6.283	+ 1 580.3	Correction ($H=1, P=0$)... $= -0.0970$
—287.29	+ 6.971	+ 1 798.6	Stress S_1..... $= +5.3729$
—300.42	+ 2 × 3.735	+ 2 × 971.9	
	$1\,000\,E\,\frac{\Sigma\,\Delta l}{2} =$	+ 9 071.9	
$1\,000\,E\,\Sigma\,\Delta l$ for bottom chords =		+ 18 143.8	
+ 46.04	+ 0.418	+ 15.3	
+111.63	+ 1.259	+ 88.1	
+195.85	+ 2.638	+ 263.8	
+270.41	+ 4.121	+ 522.0	
+296.25	+ 4.976	+ 746.4	
+363.10	+ 6.702	+ 1 139.3	
+429.48	+ 8.659	+ 1 616.3	
+491.99	+ 10.731	+ 2 146.1	
+545.62	+ 12.686	+ 2 664.0	
+584.99	+ 14.230	+ 3 083.2	
+708.62	+ 17.645	+ 3 881.9	
+693.95	+ 2 × 8.627	+ 2 ×1 897.5	
	$1\,000\,E\,\frac{\Sigma\,\Delta l}{2} =$	+ 18 063.9	
$1\,000\,E\,\Sigma\,\Delta l$ for top chords =		+ 36 127.8	

is more convenient to substitute for the load, $\Delta \alpha$, applied at the center of moments of a web member, two vertical panel loads as follows:

Take, for instance, the diagonal, $D\,E$ (Fig. 51), the center of moments of which is at C. The elastic line, considering only $D\,E$ elastic, is a broken line, $A_1\,D_1\,E_1\,B_1$, the segments of which, $A_1\,D_1$ and $B_1\,E_1$, intersect at C_1 vertically below C, and enclose the angle, $\Delta \alpha$. It can easily be proved that this line is identical with the moment diagram due to the vertical loads, $\Delta \alpha' = \Delta \alpha \left[\frac{x}{\lambda} - (n+1)\right]$ and $\Delta \alpha'' = \Delta \alpha \left(\frac{x}{\lambda} - n\right)$, applied at the panel points, D and E,

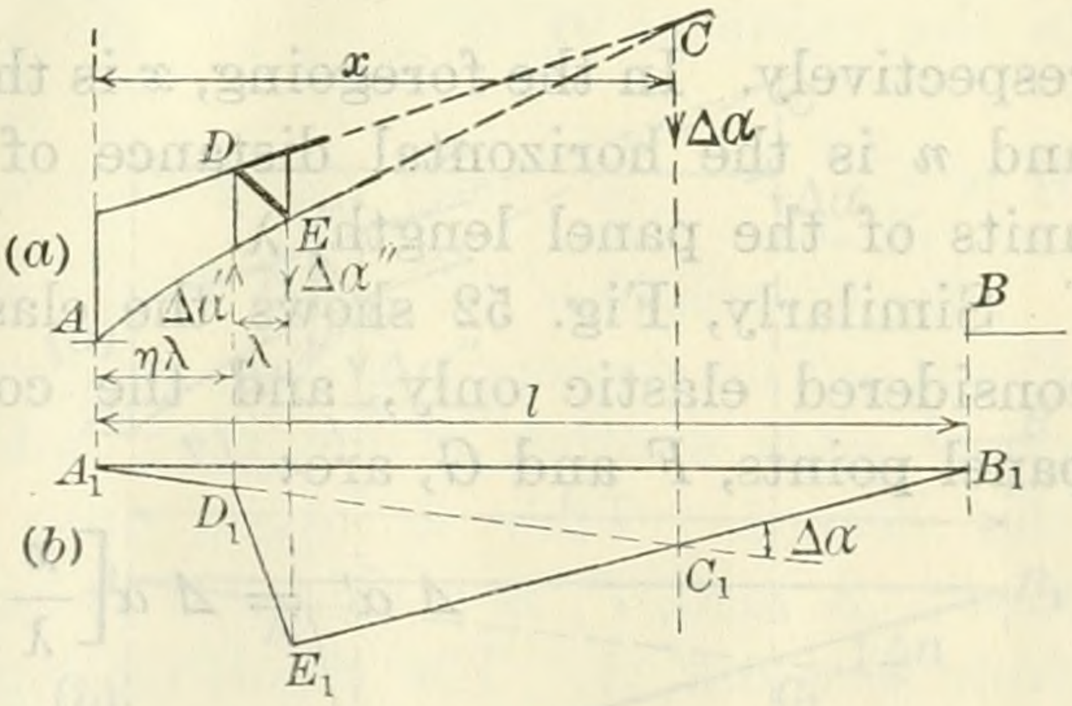


FIG. 51.

TABLE 8.—DETERMINATION OF VALUES,

	Member.	(1) Length, s, in feet.	(2) Gross area, A, in square inches.		(3) Lever arm, r, in feet.	(4) Ordinate of center of moments, y, in feet.	(5) Stress, $S_1 = \frac{y}{r}$
DIAGONALS.	1- 2	111.732	235.5		— 222.2	+ 189.6	— 0.8535
	3- 4	90.235	228.4		— 247.2	+ 224.9	— 0.9097
	5- 6	75.937	201.8		— 321.8	+ 285.6	— 0.8875
	7- 8	68.197	129.8		— 401.5	+ 341.4	— 0.8504
	9-10	63.984	129.8		— 387.7	+ 341.4	— 0.8805
	11-12	60.765	129.8		— 380.1	+ 341.3	— 0.8981
	13-14	58.454	129.8		— 383.1	+ 341.3	— 0.8907
	15-16	56.971	168.8		— 405.2	+ 341.2	— 0.8422
	17-18	56.251	168.8		— 465.6	+ 341.5	— 0.7335
	19-20	56.266	196.0		— 615.9	+ 341.5	— 0.5545
	21-22	57.011	196.0		— 1 113.6	+ 340.5	— 0.3050
	23-22	58.514	129.8				+ 0.1335*
VERTICALS.	0- 1	140.000	312.0	Average	+ 42.5	+ 36.7	+ 0.8628
	2- 3	112.933	255.0		+ 202.2	+ 195.3	+ 0.9656
	4- 5	92.930	235.0		+ 237.3	+ 237.3	+ 1.0013
	6- 7	80.000	151.0		+ 354.3	+ 322.3	+ 0.9097
	8- 9	71.163	137.0		+ 549.6	+ 428.4	+ 0.7795
	10-11	63.430	126.1		+ 612.7	+ 438.3	+ 0.7154
	12-13	56.800	126.1		+ 732.2	+ 457.1	+ 0.6244
	14-15	51.273	126.1		+ 993.7	+ 498.4	+ 0.5016
	16-17	46.850	126.1		+ 1 826.7	+ 629.8	+ 0.3448
	18-19	43.540	126.1		— 80 442.0	— 12 410.	+ 0.1543
	20-21	41.333	126.1		— 1 556.5	+ 94.6	— 0.0608
	22-23	40.220	126.1		— 769.7	+ 220.0	— 0.3776†

respectively. In the foregoing, x is the horizontal distance of C from A, and n is the horizontal distance of the panel point, D, from A, in units of the panel length, λ.

Similarly, Fig. 52 shows the elastic line, if the vertical, $F\ F'$, is considered elastic only, and the corresponding elastic loads at the panel points, F and G, are:

$$\Delta\ \alpha' = \Delta\ \alpha\left[\frac{x}{\lambda} - (n+1)\right]$$

$$\Delta\ \alpha'' = \Delta\ \alpha\left(\frac{x}{\lambda} - n\right).$$

Tables 9 and 10 (Plate XL) show the calculation of the values, $\Delta\ \alpha'$ and $\Delta\ \alpha''$, again in units of 1 000 E. The resultant elastic panel loads from all truss members are given in Table 12. The moments due to these resultant elastic panel loads have been determined analytically

Δs, $\Delta \alpha$, AND Δl, FOR WEB MEMBERS.

(6) $1\,000\,E\,\Delta s = 1\,000\,\frac{S_1 s}{A}$	(7) $1\,000\,E\,\Delta \alpha = 1\,000\,\frac{E\,\Delta s}{r}$	(8) $1\,000\,E\,\Delta l = \frac{y}{r} 1\,000\,E\,\Delta s = y\,1\,000\,E\,\Delta \alpha$	(9) Notes.
— 404.9	+ 1.8224	+ 345.6	* The stress, S_1, in the center diagonals has no influence on the deflection line.
— 359.4	+ 1.4551	+ 327.0	† The stress in 22–23 is corrected for effect of double diagonals in center panel.
— 334.0	+ 1.0379	+ 296.4	
— 446.8	+ 1.1126	+ 379.9	
— 434.0	+ 1.1194	+ 382.2	
— 420.4	+ 1.1062	+ 377.6	
— 401.1	+ 1.0470	+ 357.3	$\frac{y}{r}$ ..... = −0.2858
— 284.3	+ 0.7016	+ 239.4	
— 244.4	+ 0.5250	+ 179.3	Correction ($H = 1$, $P = 0$)... = −0.0918
— 159.2	+ 0.2585	+ 88.3	
— 88.7	+ 0.0797	+ 27.1	Stress S_1..... = −0.3776
	$1\,000\,E\,\frac{\Sigma\,\Delta l}{2}$ =	+ 3 000.1	
$1\,000\,E\,\Sigma\,\Delta\,l$ for diagonals =		+ 6 000.2	
+ 387.1	+ 9.109	+ 334.0	
+ 427.6	+ 2.114	+ 412.9	
+ 396.0	+ 1.671	+ 396.5	
+ 482.0	+ 1.360	+ 438.5	
+ 404.9	+ 0.800	+ 315.6	Summary of values..... $1\,000\,E\,\Sigma\,\Delta\,l$
+ 359.9	+ 0.737	+ 257.5	
+ 281.2	+ 0.384	+ 175.6	Bottom chord ..... = 18 143.8
+ 203.9	+ 0.205	+ 102.3	Top chord..... = 36 127.8
+ 128.1	+ 0.0701	+ 44.2	Diagonals..... = 6 000.2
+ 53.3	− 0.00066	+ 8.2	Verticals..... = 5 041.8
− 19.9	+ 0.0128	+ 1.2	
− 120.4	+ 0.156	+ 34.4	Total.....$1\,000\,E\,\Sigma\,\Delta\,l$ = 65 313.6
	$1\,000\,E\,\frac{\Sigma\,\Delta\,l}{2}$ =	+ 2 520.9	
$1\,000\,E\,\Sigma\,\Delta\,l$ for verticals =		+ 5 041.8	

in Table 12 by the usual method of shears and moment increments. The ordinate, δ, of the elastic line at any panel point, is equal to the corresponding moment from the elastic panel loads.

C.—Determination of Influence Line for Horizontal Reaction.—The influence ordinates for the horizontal reaction were determined in Table 12 by dividing the corresponding ordinates, δ, of the elastic line by the constant, $\Sigma\,\Delta\,l$ ($\Sigma\,\Delta\,l$ = horizontal deflection of the point, B, due to a horizontal load of unity at B, as obtained in Table 8, Column 9).

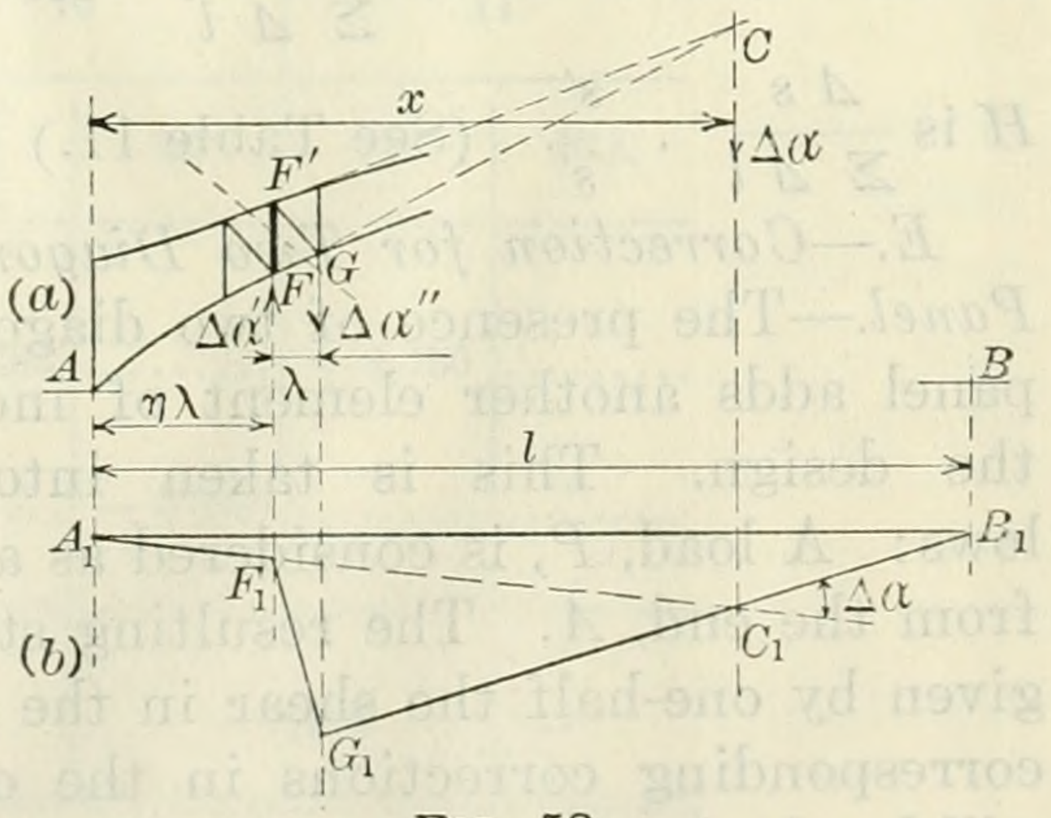


FIG. 52.

This is derived as follows: Assume the arch to be a simple span, $A\ B$, fixed at A and free to move at B (Fig. 53). According to Maxwell's Principle of Reciprocity, the vertical deflection, δ, at any point, C, due to a horizontal force of unity at B (Fig. 53 (a) and (b)), is equal to the horizontal deflection, ϵ, of the point, B, due to a vertical force of unity at C (Fig. 53 (c)), that is $\delta = \epsilon$.

In order to transform the simple span into a two-hinged arch, the horizontal reaction, H, has to overcome the horizontal deflection, ϵ, and we have, therefore, the relation, $\frac{H}{H \text{ unity}} = \frac{\varepsilon}{\Sigma\ \Delta\ l}$; or, as $\varepsilon = \delta$,

$$H = \frac{\delta}{\Sigma\ \Delta\ l}.$$

D.—Corrections for the Verticals, 0-1, 2-3, 4-5.—In the foregoing it has been assumed that the vertical load of unity is applied at the bottom chord panel points. As the live load is actually applied at the floor level, the following correction has to be made in the influence ordinates for H below the panel points, 0, 2, and 4.

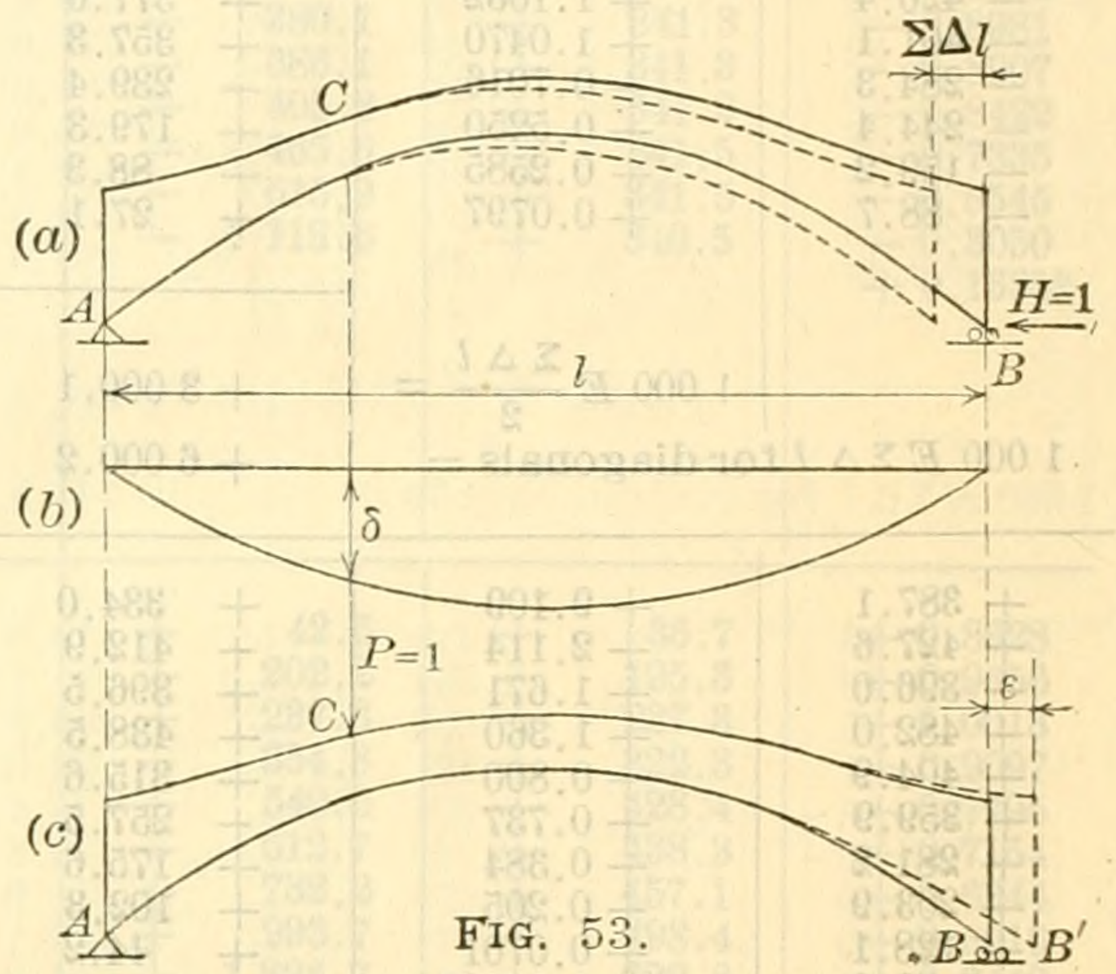


FIG. 53.

Let s be the total length of the vertical and s' its length below the floor (Fig. 54). As the load is to be applied at C' instead of C, the numerator, δ, in the foregoing formula, $H = \frac{\delta}{\Sigma\ \Delta\ l}$, must be corrected to represent the deflection of C' instead of C. If δ is the deflection at C, $\delta + \Delta\ s'$ is the deflection at C'. The resulting change in the value of H is $\frac{\Delta\ s'}{\Sigma\ \Delta\ l}$. As $\Delta\ s' = \Delta\ s\ .\ \frac{s'}{s}$, the correction for H is $\frac{\Delta\ s}{\Sigma\ \Delta\ l}\ .\ \frac{s'}{s}$. (See Table 11.)

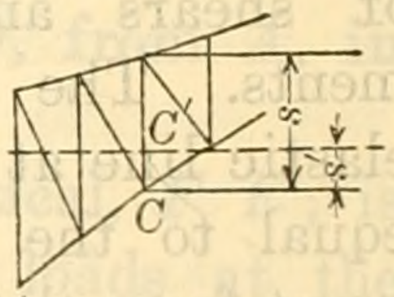


FIG. 54.

E.—Correction for Two Diagonals in the Center Panel.—The presence of two diagonals in the center panel adds another element of indeterminateness to the design. This is taken into account as follows: A load, P, is considered as acting at any point distant m panels from the end, A. The resulting stress in each center diagonal is that given by one-half the shear in the panel plus a correction, X, and the corresponding corrections in the other members of the center panel will be the horizontal and vertical components of X. The value which X must have in order to make both diagonals fit into their panel frame is then given by writing out and solving the equation, $\Delta d_1 + \Delta d_2 = (\Delta u + \Delta l)$ as $B + (\Delta v_1 + \Delta v_2)$ sin. B, where Δu, Δl, Δv, and Δd

TABLE 9.—DETERMINATION OF ELASTIC PANEL LOADS, $\Delta\alpha' + \Delta\alpha''$, FOR DIAGONALS.

Panel length, $\lambda = 42.5$ ft.

Member.	(1) 1-2		(2) 3-4		(3) 5-6		(4) 7-8		(5) 9-10		(6) 11-12		(7) 13-14		(8) 15-16		(9) 17-18		(10) 19-20		(11) 21-22		(12) 23-22A	
$1\,000\,E\,\Delta\alpha$ from Table 8	+ 1.8224		+ 1.4551		+ 1.0379		+ 1.1126		+ 1.1194		+ 1.062		+ 1.0470		+ 0.7016		+ 0.5250		+ 0.2585		+ 0.0797			
x	+ 219.83		+ 282.45		+ 390.45		+ 512.24		+ 561.12		+ 619.10		+ 691.75		+ 790.20		+ 941.55		+ 1 220.89		+ 2 003.08		∞	
n	0		1		2		3		4		5		6		7		8		9		10		11	
Panel point.	0	2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16	18	18	20	20	22		
$1\,000\,E\,\Delta\alpha'$ $= 1\,000\,E\,\Delta\alpha\left[\frac{x}{\lambda} - (n+1)\right]$	− 7.603	.....	− 6.756	.....	− 6.480	.....	− 8.962	.....	− 9.182	.....	− 9.477	.....	− 9.712	.....	− 7.432	.....	− 6.906	.....	− 4.840	.....	− 2.879			
$1\,000\,E\,\Delta\alpha''$ $= 1\,000\,E\,\Delta\alpha\left[\frac{x}{\lambda} - n\right]$	.....	+ 9.425	.....	+ 8.210	.....	+ 7.460	.....	+ 10.075	.....	+ 10.302	.....	+ 10.583	.....	+ 10.759	.....	+ 8.183	.....	+ 7.431	.....	+ 5.099	.....	+ 2.959		
Total elastic panel load, $1\,000\,E\,[\Delta\alpha' + \Delta\alpha'']$	− 7.603	+ 2.670		+ 1.788		− 1.502		+ 0.893		+ 0.825		+ 0.871		+ 3.327		+ 1.227		+ 2.591		+ 2.220		+ 2.959		

TABLE 10.—DETERMINATION OF ELASTIC PANEL LOADS, $\Delta\alpha' + \Delta\alpha''$, FOR VERTICALS.

Panel length, $\lambda = 42.5$ ft.

Member.	(1) 0-1		(2) 2-3		(3) 4-5		(4) 6-7		(5) 8-9		(6) 10-11		(7) 12-13		(8) 14-15		(9) 16-17		(10) 18-19		(11) 20-21		(12) 22-23	
$1\,000\,E\,\Delta\alpha$ from Table 8	+ 9.1090		+ 2.1145		+ 1.6708		+ 1.3605		+ 0.7367		+ 0.5874		+ 0.3841		+ 0.2052		+ 0.0701		− 0.0007		+ 0.0128		+ 0.1565	
x	+ 42.50		+ 244.74		+ 321.98		+ 481.77		+ 719.61		+ 825.18		+ 987.16		+ 1 291.22		+ 2 116.72		− 80 066.0		− 1 131.54		− 302.21	
n	0		1		2		3		4		5		6		7		8		9		10		11	
Panel point.	0	2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16	18	18	20	20	22	22	22A
$1\,000\,E\,\Delta\alpha'$ $= 1\,000\,E\,\Delta\alpha\left[\frac{x}{\lambda} - (n+1)\right]$	0	.....	− 7.947	.....	− 7.646	.....	− 9.980	.....	− 8.790	.....	− 7.880	.....	− 6.233	.....	− 4.593	.....	− 2.944	.....	− 1.254	.....	+ 0.482	.....	+ 0.156	.....
$1\,000\,E\,\Delta\alpha''$ $= 1\,000\,E\,\Delta\alpha\left[\frac{x}{\lambda} - n\right]$	.....	+ 9.109	.....	+ 10.062	.....	+ 9.315	.....	+ 11.341	.....	+ 9.527	.....	+ 8.467	.....	+ 6.617	.....	+ 4.798	.....	+ 3.014	.....	+ 1.255	.....	− 0.469	.....	.....
Total elastic panel load, $1\,000\,E\,[\Delta\alpha' + \Delta\alpha'']$	0	+ 1.162		+ 2.416		− 0.665		+ 2.551		+ 1.647		+ 2.234		+ 2.024		+ 1.854		+ 1.760		+ 1.737		− 0.313		

	Panel point.	0	8	9	1	4
		0	1·0	8·8	2·9	(3)
	Member.	(1)	(2)	(3)	(4)	
$= \frac{1}{1000} \Sigma \Delta_0 \left[\frac{\chi}{\alpha} - \mu \right] \dots$			+ 292·8	− 287·8	+ 158·8	− 68·0
$= \frac{1}{1000} \Sigma \Delta_0 \left[\frac{\chi}{\alpha} - (1+\mu) \right] \dots$						
	χ		0	1	2	3
	α		+ 68·28	+ 67·48	+ 15·28	+ 1·28
	Total elastic panel load.		− 807·7	+ 618·4	+ 381·1	

Panel length, λ = 45·2 ft.

	Panel point.	0	8	9	1	4
		0	1·0	8·8	2·9	(3)
	Member.	(1)	(2)	(3)	(4)	
$= \frac{1}{1000} \Sigma \Delta_0 \left[\frac{\chi}{\alpha} - \mu \right] \dots$			+ 292·8	− 287·8	+ 158·8	− 68·0
$= \frac{1}{1000} \Sigma \Delta_0 \left[\frac{\chi}{\alpha} - (1+\mu) \right] \dots$						
	χ		0	1	2	3
	α		+ 68·28	+ 67·48	+ 15·28	+ 1·28
	Total elastic panel load.		− 807·7	+ 618·4	+ 381·1	

Panel length, λ = 45·2 ft.

TABLE 11.—Correction of Influence Ordinates for Horizontal Reaction Due to Loads Applied to Verticals at Floor Level.

$1\,000\ E \Sigma \Delta l = 65\,313.6$.

Member.	(1) Total length, s, in feet.	(2) Length below floor, s' in feet.	(3) $\frac{1\,000\ E \Delta s}{1\,000\ S s} = \frac{}{A}$. From Table 8.	(4) Ratio $\frac{A}{A'}$.	(5) Correction. $\frac{1\,000\ E \Delta s}{1\,000\ E \Sigma \Delta l} \times \frac{s'}{s} \times \frac{A}{A'}$.
0–1	140.00	95	387.13	$\frac{312}{315} = 0.99$	0.00396
2–3	112.98	60	427.64	$\frac{255}{281} = 0.91$	0.00315
4–5	92.95	26	395.95	$\frac{235}{267} = 0.88$	0.00149

denote the elastic elongation of the top, bottom, vertical, and diagonal members, respectively, of the center panel, and is the inclination of the diagonals to the horizontal. We thus obtain

$$X = 0.1335\ H - 0.01538\ m\ P.$$

Consequently, for the sole application of $H = 1$, $X = +\ 0.1335$ is the stress in each center diagonal, and the resulting corrections in the other members of the center panel are included in Tables 7 and 8. The corrected value of the influence line for H is thus found, although the effect on H of the foregoing correction proves to be quite negligible.

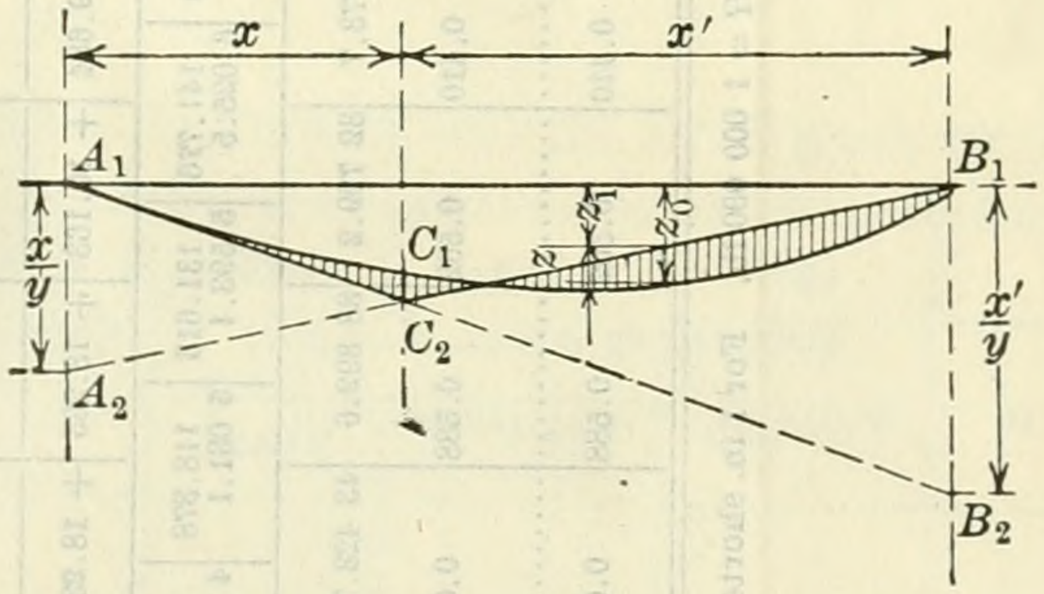


FIG. 55.

Substituting the resulting values of H with the corresponding values of m in the foregoing equation, we obtain the influence values of X for a unit load at the successive panel points of the span. These values are tabulated as the influence ordinates for the center diagonal in Table 15 (Plate XLII), and the corresponding corrections are tabulated for the other members of the center panel in Tables 13 and 14 (Plate XLI), and Table 16 (Plate XLII). All the remaining members of the truss are unaffected by the double center diagonals.

2.—Influence Lines for Arch Truss Members.

The influence ordinates for the stresses in the members of the arch truss were determined analytically as follows:

Assuming the truss to be a simple span, the influence line for the stress in any member has two straight segments, $A_1\ B_2$ and $B_1\ A_2$ (Fig. 55), which intersect at C_2 vertically below the center of moments, C, of the member, and have the ordinates, A_1, $A_2 = \frac{x}{r}$, and B_1, $B_2 = \frac{x'}{r}$.

TABLE 12.—Determination of Influence Ordinates for Horizontal Reaction.

Panel Length, $\lambda = 42.5$ ft. $1\,000\,E\,\Sigma\,\Delta\,l = 65\,313.6$.

	Panel point.	(1) 0	(2) 2	(3) 4	(4) 6	(5) 8	(6) 10	(7) 12	(8) 14	(9) 16	(10) 18	(11) 20	(12) 22	(13) Sum of elastic panel loads for ½ truss.
Elastic panel loads. $1\,000\,E\,\Delta\,a$.	Bottom chord. From Table 7	.....	+ 0.799	+ 1.163	+ 1.471	+ 2.19	+ 2.715	+ 3.428	+ 4.229	+ 5.246	+ 6.233	+ 6.971	+ 3.735	
	Top chord. From Table 7.	.....	+ 0.418	+ 1.259	+ 2.638	+ 4.121	+ 4.976	+ 6.702	+ 8.659	+ 10.731	+ 12.686	+ 14.230	+ 17.645 + 8.627	
$1\,000\,E\,(\Delta\,a' + \Delta\,a'')$	Diagonals. From Table 9.	.....	+ 2.670	+ 1.788	− 1.502	+ 0.893	+ 0.825	+ 0.871	+ 3.327	+ 1.227	+ 2.591	+ 2.220	+ 2.959	
	Verticals. From Table 9.	.....	+ 1.162	+ 2.416	− 0.665	+ 2.551	+ 1.647	+ 2.234	+ 2.024	+ 1.854	+ 1.760	+ 1.737	− 0.313	
	Total elastic panel load...	.....	+ 5.049	+ 6.626	+ 0.942	+ 9.684	+ 10.163	+ 13.235	+ 18.239	+ 19.058	+ 23.270	+ 25.158	+ 32.653	+ 165.077
	Shear = V. Moment increment = $V\lambda$	165.077 7 015.9	160.028 6 801.3	153.402 6 519.5	151.460 6 437.0	141.776 6 025.5	131.613 5 593.4	118.378 5 031.1	100.139 4 255.9	81.081 3 445.9	57.811 2 457.0	32.653 1 387.8		
	Moment = ordinate of elastic line = $1\,000\,E\,\delta$	0	7 015.9	13 817.2	20 336.7	26 773.7	32 799.2	38 892.6	43 423.7	47 679.6	51 125.5	53 582.5	54 970.3	
	Influence ordinates for horizontal reaction $= \frac{1\,000\,E\,\delta}{1\,000\,E\,\Sigma\,\Delta\,l}$	0	0.1074	0.2115	0.311	0.410	0.502	0.588	0.665	0.730	0.783	0.820	0.841	
	Correction for loads at floor level. From Table 11.	0.0040	0.0032	0.0015	.....	.....	.....	.....	.....	.....	.....	.....	.....	Sum of ordinates for ½ truss.
	Total influence ordinates for horizontal reaction.	0.004	0.111	0.213	0.311	0.410	0.502	0.588	0.665	0.730	0.783	0.820	0.841	5.978

Note.—$1\,000\,E\,\Sigma\,\Delta\,l = 65\,313.6$, therefore, $\frac{65\,313.6 \times 12}{30\,000} = 26.125$ in., shortening for $H = 1\,000\,000$ lb. For 1 in. shortening, $H = \frac{1\,000\,000}{26.125} = 38\,278$ lb.

TABLE 13.—Determination of Influence Ordinates.—For Bottom Chord. Live Load per Panel = 510 000 lb. = 1 000 P. All Stresses Given in Units of 1 000 lb.

+ Denotes tension.
— Denotes compression.

Member.	(1) X, in feet.	(2) X', in feet.	(3) $\frac{x}{y}$	(4) $\frac{x'}{y}$	(5) $\frac{y}{r}$	(6)	Panel Points.—All Values in These Columns are Influence Ordinates, and are Expressed in Thousandths of a Unit. 0	2	4	6	8	10	12	14	16	18	20	22	22	20	18	16	14	12	10	8	6	4	2	0	(31) Influence Area, $Z = \Sigma z$. Positive. +	(32) Negative. —	(33) Total.	(34) Live-Load Stress = $P \frac{y}{r} \Sigma z$. +	(35) —
Influence ordinate. Hor. reaction......	....	....	....	....	....	Z_0	+ 4	+ 111	+ 213	+ 311	+ 410	+ 502	+ 588	+ 665	+ 730	+ 783	+ 820	+ 841	+ 841	+ 820	+ 783	+ 730	+ 665	+ 588	+ 502	+ 410	+ 311	+ 213	+ 111	+ 4	....	....	....	....	....
						Z_1	0	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....					
0-2...........	0	977.5	0.000	− 6.982	− 1.321	$Z = Z_0 + Z_1$	+ 4	+ 111	+ 213	+ 311	+ 410	+ 502	+ 588	+ 665	+ 730	+ 783	+ 820	+ 841	+ 841	+ 820	+ 783	+ 730	+ 665	+ 588	+ 502	+ 410	+ 311	+ 213	+ 111	+ 4	11.962	0.	+ 11.962	....	8 057
						Z_1	0	− 272	− 259	− 247	− 235	− 222	− 210	− 198	− 185	− 173	− 161	− 148	− 136	− 124	− 111	− 99	− 86	− 74	− 62	− 49	− 37	− 25	− 12	0					
2-4...........	42.5	935.0	− 0.284	− 6.250	− 1.684	$Z = Z_0 + Z_1$	+ 4	− 161	− 46	+ 64	+ 175	+ 280	+ 378	+ 467	+ 545	+ 610	+ 659	+ 693	+ 705	+ 696	+ 672	+ 631	+ 579	+ 514	+ 440	+ 361	+ 274	+ 188	+ 99	+ 4	9.021	0.192	+ 8.829	165	7 746
						Z_1	0	− 238	− 476	− 454	− 431	− 408	− 386	− 363	− 340	− 318	− 295	− 272	− 250	− 227	− 204	− 182	− 159	− 136	− 113	− 91	− 68	− 45	− 23	0					
4-6...........	85.0	892.5	− 0.522	− 5.478	− 2.146	$Z = Z_0 + Z_1$	+ 4	− 127	− 263	− 148	− 21	+ 94	+ 202	+ 302	+ 390	+ 465	+ 525	+ 569	+ 591	+ 593	+ 579	+ 548	+ 506	+ 452	+ 389	+ 319	+ 243	+ 168	+ 88	+ 4	7.019	0.543	+ 6.476	594	7 682
						Z_1	0	− 205	− 411	− 616	− 585	− 554	− 524	− 493	− 462	− 431	− 400	− 370	− 339	− 308	− 277	− 246	− 216	− 185	− 154	− 123	− 92	− 62	− 31	0					
6-8...........	127.5	850.0	− 0.708	− 4.722	− 2.656	$Z = Z_0 + Z_1$	+ 4	− 94	− 198	− 305	− 175	− 52	+ 64	+ 172	+ 268	+ 352	+ 420	+ 471	+ 502	+ 512	+ 506	+ 484	+ 449	+ 403	+ 348	+ 287	+ 219	+ 151	+ 80	+ 4	5.678	0.808	+ 4.870	1 095	7 692
						Z_1	0	− 177	− 355	− 532	− 710	− 672	− 635	− 598	− 560	− 523	− 486	− 448	− 411	− 374	− 336	− 299	− 262	− 224	− 187	− 149	− 112	− 75	− 37	0					
8-10...........	170.0	807.5	− 0.859	− 4.082	− 3.171	$Z = Z_0 + Z_1$	+ 4	− 66	− 142	− 221	− 300	− 170	− 47	+ 67	+ 170	+ 260	+ 334	+ 393	+ 430	+ 446	+ 447	+ 431	+ 403	+ 364	+ 315	+ 261	+ 199	+ 138	+ 74	+ 4	4.722	0.930	+ 3.792	1 504	7 638
						Z_1	0	− 156	− 312	− 467	− 623	− 779	− 736	− 693	− 649	− 606	− 563	− 519	− 476	− 433	− 390	− 346	− 303	− 260	− 216	− 173	− 129	− 86	− 43	0					
10-12...........	212.5	765.0	− 0.996	− 3.584	− 3.719	$Z_1 = Z_0 + Z_1$	+ 4	− 45	− 99	− 156	− 213	− 277	− 148	− 28	+ 81	+ 177	+ 257	+ 322	+ 365	+ 387	+ 393	+ 384	+ 362	+ 328	+ 286	+ 237	+ 182	+ 127	+ 68	+ 4	3.950	0.954	+ 2.996	1 809	7 492
						Z_1	0	− 139	− 277	− 416	− 554	− 692	− 831	− 782	− 733	− 684	− 635	− 587	− 538	− 489	− 440	− 391	− 342	− 293	− 244	− 196	− 147	− 98	− 49	0					
12-14...........	255.0	722.5	− 1.124	− 3.186	− 4.239	$Z = Z_0 + Z_1$	+ 4	− 28	− 64	− 105	− 144	− 190	− 243	− 117	− 3	+ 99	+ 185	+ 254	+ 303	+ 331	+ 343	+ 339	+ 323	+ 295	+ 258	+ 214	+ 164	+ 115	+ 62	+ 4	3.288	0.891	+ 2.397	1 949	7 192
						Z_1	0	− 124	− 249	− 373	− 497	− 621	− 746	− 870	− 815	− 761	− 707	− 652	− 598	− 544	− 489	− 435	− 381	− 326	− 272	− 217	− 163	− 109	− 54	0					
14-16...........	297.5	680.0	− 1.250	− 2.858	− 4.864	$Z = Z_0 + Z_1$	+ 4	− 13	− 36	− 62	− 87	− 119	− 158	− 205	− 85	+ 22	+ 113	+ 189	+ 243	+ 276	+ 294	+ 295	+ 284	+ 262	+ 230	+ 193	+ 148	+ 104	+ 57	+ 4	2.706	0.755	+ 1.951	1 873	6 712
						Z_1	0	− 112	− 225	− 337	− 449	− 561	− 674	− 786	− 898	− 838	− 779	− 719	− 659	− 599	− 539	− 479	− 419	− 359	− 300	− 240	− 180	− 120	− 60	0					
16-18...........	340.0	637.5	− 1.377	− 2.582	− 5.413	$Z = Z_0 + Z_1$	+ 4	− 1	− 12	− 26	− 39	− 59	− 86	− 121	− 168	− 55	+ 41	+ 122	+ 182	+ 221	+ 244	+ 251	+ 246	+ 229	+ 203	+ 170	+ 131	+ 93	+ 51	+ 4	2.178	0.555	+ 1.623	1 532	6 013
						Z_1	0	− 102	− 204	− 306	− 408	− 510	− 612	− 714	− 816	− 918	− 853	− 787	− 722	− 656	− 590	− 525	− 459	− 394	− 328	− 262	− 197	− 131	− 66	0					
18-20...........	382.5	595.0	− 1.509	− 2.347	− 5.895	$Z = Z_0 + Z_1$	+ 4	+ 9	+ 9	+ 5	+ 2	− 8	− 24	− 49	− 86	− 135	− 33	+ 54	+ 119	+ 164	+ 193	+ 205	+ 206	+ 194	+ 174	+ 148	+ 114	+ 82	+ 45	+ 4	1.718	0.324	+ 1.394	974	5 165
						Z_1	0	− 93	− 186	− 279	− 372	− 466	− 559	− 652	− 745	− 838	− 931	− 859	− 788	− 716	− 645	− 573	− 501	− 430	− 358	− 287	− 215	− 143	− 72	0					
20-22...........	425.0	552.5	− 1.647	− 2.142	− 6.261	$Z = Z_0 + Z_1$	+ 4	+ 18	+ 27	+ 32	+ 38	+ 36	+ 29	+ 13	− 15	− 55	− 111	− 18	+ 53	+ 104	+ 138	+ 157	+ 164	+ 158	+ 144	+ 123	+ 96	+ 70	+ 39	+ 4	1.438	0.188	+ 1.250	600	4 591
						Z_1	0	− 82	− 163	− 245	− 327	− 408	− 490	− 572	− 653	− 735	− 817	− 898	− 898	− 817	− 735	− 653	− 572	− 490	− 408	− 327	− 245	− 168	− 82	0					
22-22A.........	488.75	488.75	− 1.878	− 1.878	− 6.470	Correct Z_2	+ 0.06	− 0.07	− 0.27	− 0.52	− 0.75	− 1.10	− 1.55	− 2.12	− 2.86	− 3.81	− 4.97	− 6.37	− 6.37	− 4.97	− 3.81	− 0.86	− 2.12	− 1.55	− 1.10	− 0.75	− 0.52	− 0.27	− 0.07	+ 0.06	1.252	0.128	+ 1.124	422	4 131
						$Z = Z_0 + Z_1 + Z_2$	+ 5	+ 29	+ 50	+ 65	+ 82	+ 93	+ 96	+ 91	+ 74	+ 44	− 2	− 63	− 63	− 2	+ 44	+ 74	+ 91	+ 96	+ 93	+ 82	+ 65	+ 50	+ 29	+ 5					

Stress 22-22A $= -6.47\,\eta_H + 0.5283\,m - \eta_x \cdot \cos\beta = -6.47\,(\eta_H - 0.08165\,m + 0.1123\,\eta_x) = -6.47\,(Z_0 + Z_1 + Z_2)$.

TABLE 14.—Determination of Influence Ordinates.—For Top Chord. Live Load per Panel = 510 000 lb. = 1 000 P. All Stresses Given in Units of 1 000 lb.

+ Denotes tension.
— Denotes compression.

Member.	(1) X, in feet.	(2) X', in feet.	(3) $\frac{x}{y}$	(4) $\frac{x'}{y}$	(5) $\frac{y}{r}$	(6)	Panel Points.—All Values in These Columns Are Influence Ordinates, and Are Expressed in Thousandths of a Unit. 0	2	4	6	8	10	12	14	16	18	20	22	22	20	18	16	14	12	10	8	6	4	2	0	(31) Influence Area, $Z = \Sigma z$. Positive. +	(32) Negative. —	(33) Total.	(34) Live-Load Stress = $P \frac{y}{r} \Sigma z$. +	(35) —
Influence ordinate. Hor. reaction......	....	....	....	....	....	Z_0	+ 4	+ 111	+ 213	+ 311	+ 410	+ 502	+ 588	+ 665	+ 730	+ 783	+ 820	+ 841	+ 841	+ 820	+ 783	+ 730	+ 665	+ 588	+ 502	+ 410	+ 311	+ 213	+ 111	+ 4	....	....	....	....	....
						Z_1	0	− 1 109	− 1 059	− 1 008	− 958	− 907	− 857	− 807	− 756	− 706	− 655	− 605	− 555	− 504	− 454	− 403	− 353	− 302	− 252	− 202	− 151	− 101	− 50	0					
1-3...........	42.5	935.0	− 1.159	− 25.500	+ 0.333	$Z = Z_0 + Z_1$	+ 4	− 998	− 846	− 697	− 548	− 405	− 269	− 142	− 26	+ 77	+ 165	+ 236	+ 286	+ 316	+ 329	+ 327	+ 312	+ 286	+ 250	+ 208	+ 160	+ 112	+ 61	+ 4	3.119	3.916	− 0.797	530	665
						Z_1	0	− 551	− 1 109	− 1 056	− 1 008	− 950	− 898	− 845	− 792	− 739	− 686	− 634	− 581	− 528	− 475	− 422	− 370	− 317	− 264	− 211	− 158	− 106	− 53	0					
3-5...........	85.0	892.5	− 1.214	− 12.750	+ 0.789	$Z = Z_0 + Z_1$	+ 4	− 443	− 896	− 745	− 593	− 448	− 310	− 180	− 62	+ 44	+ 134	+ 207	+ 260	+ 293	+ 308	+ 308	+ 295	+ 271	+ 238	+ 199	+ 153	+ 107	+ 58	+ 4	2.865	3.662	− 0.797	1 153	1 474
						Z_1	0	− 370	− 739	− 1 109	− 1 053	− 998	− 942	− 887	− 832	− 776	− 721	− 665	− 610	− 554	− 499	− 444	− 388	− 333	− 277	− 222	− 166	− 111	− 55	0					
5-7...........	127.5	850.0	− 1.275	− 8.500	+ 1.347	$Z = Z_0 + Z_1$	+ 4	− 259	− 526	− 798	− 643	− 496	− 354	− 222	− 102	+ 7	+ 99	+ 176	+ 231	+ 266	+ 284	+ 286	+ 277	+ 255	+ 225	+ 188	+ 145	+ 102	+ 56	+ 4	2.598	3.395	− 0.797	1 785	2 332
						Z_1	0	− 277	− 554	− 832	− 1 109	− 1 050	− 992	− 934	− 875	− 816	− 759	− 700	− 642	− 584	− 525	− 467	− 408	− 350	− 292	− 234	− 175	− 117	− 58	0					
7-9...........	170.0	807.5	− 1.342	− 6.375	+ 1.930	$Z = Z_0 + Z_1$	+ 4	− 166	− 341	− 521	− 699	− 548	− 404	− 269	− 145	− 34	+ 61	+ 141	+ 199	+ 236	+ 258	+ 263	+ 257	+ 238	+ 210	+ 176	+ 136	+ 96	+ 53	+ 4	2.317	3.114	− 0.797	2 281	3 066
						Z_1	0	− 222	− 443	− 665	− 887	− 1 109	− 1 047	− 986	− 924	− 862	− 801	− 739	− 678	− 616	− 554	− 493	− 431	− 370	− 308	− 246	− 185	− 123	− 62	0					
9-11...........	212.5	765.0	− 1.417	− 5.100	+ 2.519	$Z = Z_0 + Z_1$	+ 4	− 111	− 230	− 354	− 477	− 607	− 459	− 321	− 194	− 79	+ 19	+ 102	+ 163	+ 204	+ 229	+ 237	+ 234	+ 218	+ 194	+ 164	+ 126	+ 90	+ 49	+ 4	2.025	2.822	− 0.797	2 602	3 626
						Z_1	0	− 185	− 370	− 554	− 739	− 924	− 1 109	− 1 043	− 978	− 913	− 848	− 783	− 717	− 652	− 587	− 522	− 456	− 391	− 326	− 261	− 196	− 130	− 65	0					
11-13...........	255.0	722.5	− 1.500	− 4.25	+ 3.138	$Z = Z_0 + Z_1$	+ 4	− 74	− 157	− 243	− 329	− 422	− 521	− 378	− 248	− 130	− 28	+ 58	+ 124	+ 168	+ 196	+ 208	+ 209	+ 197	+ 176	+ 149	+ 115	+ 83	+ 46	+ 4	1.724	2.519	− 0.797	2 759	4 031
						Z_1	0	− 158	− 317	− 476	− 634	− 792	− 950	− 1 109	− 1 039	− 970	− 901	− 831	− 762	− 693	− 624	− 554	− 485	− 416	− 347	− 277	− 208	− 139	− 69	0					
13-15...........	297.5	680.0	− 1.593	− 3.643	+ 3.763	$Z = Z_0 + Z_1$	+ 4	− 47	− 104	− 165	− 224	− 290	− 362	− 444	− 309	− 187	− 81	+ 10	+ 79	+ 127	+ 159	+ 176	+ 180	+ 172	+ 155	+ 133	+ 103	+ 74	+ 42	+ 4	1.410	2.207	− 0.797	2 706	4 236
						Z_1	0	− 139	− 277	− 416	− 554	− 698	− 832	− 970	− 1 109	− 1 035	− 961	− 887	− 813	− 739	− 665	− 591	− 517	− 443	− 370	− 296	− 222	− 148	− 74	0					
15-17...........	340.0	637.5	− 1.700	− 3.188	+ 4.365	$Z = Z_0 + Z_1$	+ 4	− 28	− 64	− 105	− 144	− 191	− 244	− 305	− 379	− 252	− 141	− 46	+ 28	+ 81	+ 118	+ 139	+ 148	+ 145	+ 132	+ 114	+ 89	+ 65	+ 37	+ 4	1.092	1.889	− 0.797	2 429	4 202
						Z_1	0	− 123	− 246	− 370	− 493	− 616	− 739	− 862	− 986	− 1 109	− 1 030	− 950	− 871	− 792	− 713	− 634	− 554	− 475	− 396	− 317	− 238	− 158	− 79	0					
17-19...........	382.5	595.0	− 1.821	− 2.833	+ 4.883	$Z = Z_0 + Z_1$	+ 4	− 12	− 33	− 59	− 83	− 114	− 151	− 197	− 256	− 326	− 210	− 109	− 30	+ 28	+ 70	+ 96	+ 111	+ 113	+ 106	+ 93	+ 73	+ 55	+ 32	+ 4	0.774	1.571	− 0.797	1 927	3 912
						Z_1	0	− 111	− 222	− 333	− 444	− 554	− 665	− 776	− 887	− 998	− 1 109	− 1 024	− 938	− 853	− 768	− 683	− 597	− 512	− 427	− 341	− 256	− 171	− 85	0					
19-21...........	425.0	552.5	− 1.962	− 2.550	+ 5.270	$Z = Z_0 + Z_1$	+ 4	− 0	− 9	− 22	− 34	− 52	− 77	− 111	− 157	− 215	− 289	− 183	− 97	− 33	+ 15	+ 47	+ 68	+ 76	+ 75	+ 69	+ 55	+ 42	+ 26	+ 4	0.475	1.272	− 0.797	1 277	3 419
						Z_1	0	− 101	− 202	− 302	− 403	− 504	− 605	− 706	− 806	− 907	− 1 008	− 1 109	− 1 016	− 924	− 832	− 739	− 647	− 554	− 462	− 370	− 277	− 185	− 92	0					
21-23...........	467.5	510.0	− 2.125	− 2.319	+ 5.478	$Z = Z_0 + Z_1$	+ 4	+ 10	+ 11	+ 9	+ 7	− 2	− 17	− 41	− 76	− 124	− 188	− 268	− 175	− 104	− 49	− 9	+ 18	+ 34	+ 40	+ 40	+ 34	+ 28	+ 19	+ 4	0.252	1.049	− 0.797	704	2 931
						Z_1	0	− 96.6	− 193.2	− 289.8	− 386.4	− 483	− 579.5	− 676.1	− 772.7	− 869.3	− 965.9	− 1 062.5	− 1 062.5	− 965.9	− 869.3	− 772.7	− 676.1	− 579.5	− 483	− 386.4	− 289.8	− 193.2	− 96.6	0					
23-23 A.........	488.75	488.75	− 2.222	− 2.222	+ 5.470	Correct Z_2	− 0.07	+ 0.1	+ 0.3	+ 0.6	+ 0.9	+ 1.3	+ 1.8	+ 2.5	+ 3.4	+ 4.5	+ 5.9	+ 7.5	+ 7.5	+ 5.9	+ 4.5	+ 3.4	+ 2.5	+ 1.8	+ 1.8	+ 0.9	+ 0.6	+ 0.3	+ 0.1	− 0.07	0.221	0.963	− 0.742	617	2 686
						$Z = Z_0 + Z_1 + Z_2$	+ 4	+ 14	+ 20	+ 22	+ 24	+ 20	+ 10	− 9	− 39	− 82	− 140	− 214	− 214	− 140	− 82	− 39	− 9	+ 10	+ 20	+ 24	+ 22	+ 20	+ 14	+ 4					

Stress 23-23A $= +5.47\,(\eta_H - 0.0966\,m - 0.1328\,\eta_x) = +5.47\,(Z_0 + Z_1 + Z_2)$.

Between ordinates.	(1) X, in feet.	(2) X', in feet.	(3) $\frac{x}{y}$	(4) $\frac{x'}{y}$	(5) $\frac{y}{r}$	(6)		0
Before reaction.....						$\Sigma_0 = \Sigma_0 + Z_0$		+ 4
0-2	0	977.5	0.000	— 6.382	— 1.321	$\Sigma_1 = \Sigma_0 + Z_1$		+ 0
2-4	42.5	935.0	— 0.284	— 6.230	— 1.864	$\Sigma_2 = \Sigma_1 + Z_2$		+ 0
4-6	85.0	892.5	— 0.529	— 5.479	— 2.146	$\Sigma_3 = \Sigma_2 + Z_3$		+ 0
6-8	127.5	850.0	— 0.708	— 4.725	— 2.356	$\Sigma_4 = \Sigma_3 + Z_4$		+ 0
8-10	170.0	807.5	— 0.859	— 4.089	— 3.171	$\Sigma_5 = \Sigma_4 + Z_5$		+ 0
10-12	212.5	765.0	— 0.996	— 3.581	— 3.719	$\Sigma_6 = \Sigma_5 + Z_6$		+ 0
12-14	255.0	722.5	— 1.124	— 3.186	— 4.289	$\Sigma_7 = \Sigma_6 + Z_7$		+ 0
14-16	297.5	680.0	— 1.250	— 2.858	— 4.864	$\Sigma_8 = \Sigma_7 + Z_8$		+ 0
16-18	340.0	637.5	— 1.377	— 2.563	— 5.413	$\Sigma_9 = \Sigma_8 + Z_9$		+ 0
18-20	382.5	595.0	— 1.509	— 2.287	— 5.885	$\Sigma_{10} = \Sigma_9 + Z_{10}$		+ 0
20-22	425.0	552.5	— 1.647	— 2.128	— 6.301	$\Sigma_{11} = \Sigma_{10} + Z_{11}$		+ 0
21-22A	468.75	485.75	— 1.818	— 1.978	— 6.470	Correct Z_{11}		+ 0.06
						$\Sigma = \Sigma_0 + Z_1 + Z_2$		+ 5

Between ordinates.	(1) X, in feet.	(2) X', in feet.	(3) $\frac{x}{y}$	(4) $\frac{x'}{y}$	(5) $\frac{y}{r}$	(6)		0
Before reaction.....						$\Sigma_0 = \Sigma_0 + Z_0$		+ 4
1-3	42.5	985.0	— 1.159	— 25.500	+ 0.389	$\Sigma_1 = \Sigma_0 + Z_1$		+ 4
3-5	85.0	892.5	— 1.314	— 12.750	+ 0.739	$\Sigma_2 = \Sigma_1 + Z_2$		+ 4
5-7	127.5	850.0	— 1.275	— 8.500	+ 1.347	$\Sigma_3 = \Sigma_2 + Z_3$		+ 4
7-9	170.0	807.5	— 1.142	— 6.375	+ 1.830	$\Sigma_4 = \Sigma_3 + Z_4$		+ 4
9-11	212.5	765.0	— 1.417	— 5.100	+ 2.319	$\Sigma_5 = \Sigma_4 + Z_5$		+ 4
11-13	255.0	722.5	— 1.300	— 4.25	+ 2.738	$\Sigma_6 = \Sigma_5 + Z_6$		+ 4
13-15	297.5	680.0	— 1.398	— 3.643	+ 3.168	$\Sigma_7 = \Sigma_6 + Z_7$		+ 4
15-17	340.0	637.5	— 1.700	— 3.188	+ 4.305	$\Sigma_8 = \Sigma_7 + Z_8$		+ 4
17-19	382.5	595.0	— 1.891	— 2.833	+ 4.903	$\Sigma_9 = \Sigma_8 + Z_9$		+ 4
19-21	425.0	552.5	— 1.896	— 2.550	+ 5.370	$\Sigma_{10} = \Sigma_9 + Z_{10}$		+ 4
21-21A	457.5	510.0	— 2.125	— 2.319	+ 5.478	$\Sigma_{11} = \Sigma_{10} + Z_{11}$		+ 4
21A-23 A	489.75	484.75	— 2.222	— 2.222	+ 5.470	Correct Z_{11}		+ 0.07
						$\Sigma = \Sigma_0 + Z_1 + Z_2$		+ 5

+ Denotes tension.
− Denotes compression.

TABLE 15.—Determination of Influence Ordinates.—For Diagonals. Live Load per Panel = 510 000 lb. = 1 000 P. All Stresses Given In Units of 1 000 lb.

Panel Points.—All Values in These Columns are Influence Ordinates, and are Expressed in Thousandths of a Unit.

Member.	(1) X, in feet.	(2) X', in feet.	(3) $\frac{x}{y}$	(4) $\frac{x'}{y}$	(5) $\frac{y}{r}$	(6)	0	2	4	6	8	10	12	14	16	18	20	22	22	20	18	16	14	12	10	8	6	4	2	0	(31) Influence, Area $Z = \Sigma z$. Positive. +	(32) Negative. −	(33) Total.	(34) Live-Load Stress $= P \frac{y}{r} \Sigma z$. +	(35) −
Influence ordinate. Hor. reaction	....	....	....	....	....	Z_0	+4	+111	+213	+311	+410	+502	+588	+665	+730	+783	+820	+841	+841	+820	+783	+730	+665	+588	+502	+410	+311	+213	+111	+4	....	....	....	....	....
1-2	219.83	757.67	−1.159	−3.995	−0.854	Z_1	0	−1 109	−1 058	−1 008	−958	−907	−856	−806	−756	−706	−655	−605	−554	−504	−455	−403	−353	−302	−252	−202	−151	−101	−50	0	3.119	3.916	−0.797	1 705	1 358
						$Z = Z_0 + Z_1$	+4	−998	−845	−697	−548	−405	−268	−141	−26	+77	+165	+236	+287	+316	+328	+327	+312	+286	+250	+208	+160	+112	+61	+4					
3-4	282.45	695.05	−1.256	−3.089	−0.910	Z_1	0	−134	−1 147	−1 092	−1 038	−983	−929	−874	−819	−765	−710	−655	−601	−546	−492	−437	−382	−328	−273	−218	−164	−109	−55	0	2.680	3.477	−0.797	1 613	1 243
						$Z = Z_0 + Z_1$	+4	−23	−934	−781	−628	−481	−341	−209	−89	+18	+110	+186	+240	+274	+291	+293	+283	+260	+229	+192	+147	+104	+56	+4					
5-6	390.45	587.05	−1.367	−2.056	−0.888	Z_1	0	−89	−179	−1 189	−1 129	−1 070	−1 011	−951	−892	−832	−773	−713	−654	−594	−535	−476	−416	−357	−297	−238	−178	−119	−59	0	2.260	3.057	−0.797	1 384	1 023
						$Z = Z_0 + Z_1$	+4	+22	+34	−878	−719	−568	−423	−286	−162	−49	+47	+128	+187	+226	+248	+254	+249	+231	+205	+172	+133	+94	+52	+4					
7-8	512.24	465.26	−1.500	−1.363	−0.850	Z_1	0	−59	−118	−178	−1 239	−1 174	−1 109	−1 044	−979	−913	−848	−783	−718	−652	−587	−522	−457	−392	−327	−261	−196	−130	−65	0	1.944	2.741	−0.797	1 189	843
						$Z = Z_0 + Z_1$	+4	+52	+95	+133	−829	−672	−521	−379	−249	−130	−28	+58	+123	+168	+196	+208	+208	+196	+175	+149	+115	+83	+46	+4					
9-10	561.12	416.38	−1.643	−1.220	−0.880	Z_1	0	−53	−106	−159	−212	−1 286	−1 215	−1 144	−1 072	−1 001	−929	−857	−786	−715	−643	−572	−500	−429	−357	−286	−215	−143	−72	0	1.692	2.489	−0.797	1 118	760
						$Z = Z_0 + Z_1$	+4	+58	+107	+152	+198	−784	−627	+479	−342	−218	−109	−16	+55	+105	+140	+158	+165	+159	+145	+124	+96	+70	+39	+4					
11-12	619.10	358.40	−1.814	−1.050	−0.898	Z_1	0	−46	−91	−137	−183	−228	−1 341	−1 262	−1 183	−1 104	−1 025	−946	−867	−789	−710	−631	−552	−473	−394	−315	−237	−158	−79	0	1.555	2.352	−0.797	1 077	712
						$Z = Z_0 + Z_1$	+4	+65	+122	+174	+227	+274	−753	−597	−453	−321	−205	−105	−26	+31	+73	+99	+113	+115	+108	+95	+74	+55	+32	+4					
13-14	691.75	285.75	−2.027	−0.837	−0.891	Z_1	0	−36	−73	−109	−146	−182	−218	−1 410	−1 322	−1 234	−1 146	−1 058	−969	−881	−793	−705	−617	−529	−441	−353	−265	−176	−88	0	1.606	2.403	−0.797	1 092	730
						$Z = Z_0 + Z_1$	+4	+75	+140	+202	+264	+320	+370	−745	−592	−451	−326	−217	−128	−61	−10	+25	+48	+59	+61	+57	+46	+37	+23	+4					
15-16	790.20	187.30	−2.316	−0.549	−0.842	Z_1	0	−24	−48	−72	−95	−119	−143	−167	−1 510	−1 410	−1 309	−1 208	−1 108	−1 007	−907	−806	−705	−604	−503	−403	−302	−201	−101	0	2.024	2.821	−0.797	1 212	870
						$Z = Z_0 + Z_1$	+4	+87	+165	+239	+315	+383	+445	+498	−780	−627	−489	−367	−267	−187	−124	−76	−40	−16	−1	+7	+9	+12	+10	+4					
17-18	941.55	35.95	−2.757	−0.105	−0.734	Z_1	0	−5	−9	−14	−18	−23	−27	−32	−37	−1 678	−1 558	−1 438	−1 318	−1 199	−1 079	−959	−839	−719	−599	−480	−360	−240	−120	0	3.174	3.971	−0.797	1 485	1 157
						$Z = Z_0 + Z_1$	+4	+106	+204	+297	+392	+479	+561	+633	+693	−895	−738	−597	−477	−379	−296	−229	−174	−131	−97	−70	−49	−27	−9	+4					
19-20	1 220.89	243.39	−3.575	+0.713	−0.554	Z_1	0	+31	+62	+93	+124	+155	+186	+217	+248	+279	−2 021	−1 865	−1 710	−1 554	−1 399	−1 244	−1 088	−983	−777	−622	−467	−311	−155	0	5.431	6.228	−0.797	1 762	1 536
						$Z = Z_0 + Z_1$	+4	+142	+275	+404	+534	+657	+774	+882	+978	+1 062	−1 201	−1 024	−869	−734	−616	−514	−423	−345	−275	−212	−156	−98	−44	+4					
21-22	2 003.46	1 025.53	−5.884	+3.012	−0.305	Z_1	0	+131	+262	+393	+524	+655	+786	+917	+1 048	+1 179	+1 310	−3 070	−2 814	−2 559	−2 303	−2 047	−1 791	−1 535	−1 279	−1 028	−768	−512	−256	0	11.799	12.596	−0.797	1 959	1 835
						$Z = Z_0 + Z_1$	+4	+244	+475	+704	+934	+115.7	+1 374	+1 582	+1 778	+1 962	+2 130	−2 229	−1 973	−1 739	−1 520	−1 317	−1 126	−947	−777	−613	−457	−299	−145	+4					
23-22	∞	....	....	....	....	Z_1	0	+31.63	+63.25	+94.88	+126.51	+158.14	+189.76	+221.39	+253.02	+284.64	+316.27	+347.90	−347.90	−316.27	−284.64	−253.02	−221.39	−189.76	−158.14	−126.51	−94.88	−63.25	−31.63	0	2.221	1.786	−0.435	911	1 133
						Z_2	−0.5	+0.6	+2.4	+4.6	+6.7	+9.8	+13.8	+18.9	+25.5	+33.9	+44.8	+56.7	+56.7	+44.3	+33.9	+25.5	+18.9	+13.8	+9.8	+6.7	+4.6	+2.4	+0.6	−0.5					
						$Z = Z_1 + Z_2$	−0.5	+32	+66	+99	+133	+168	+204	+240	+279	+319	+361	+405	−291	−272	−251	−228	−202	−176	−148	−120	−90	−61	−31	−0.5					

Stress 23-22 $= \mp 0.08163\, m + \eta_x$
$= (-1)\,[\pm 0.08163\, m - \eta_x]$
$= (-1)\,[Z_0 + Z_1 + Z_2]$.

TABLE 16.—Determination of Influence Ordinates.—For Verticals. Live Load per Panel = 510 000 lb. = 1 000 P. All Stresses Given in Units of 1 000 lb.

+ Denotes tension.
− Denotes compression.

Panel Points.—All Values in These Columns Are Influence Ordinates and Are Expressed in Thousandths of a Unit.

Member.	(1) X, in feet.	(2) X', in feet.	(3) $\frac{x}{y}$	(4) $\frac{x'}{y}$	(5) $\frac{y}{r}$	(6)	0	2	4	6	8	10	12	14	16	18	20	22	22	20	18	16	14	12	10	8	6	4	2	0	(31) Influence Area, $Z = \Sigma z$. Positive. +	(32) Negative. −	(33) Total.	(34) Live-Load Stress $= P \frac{y}{r} \Sigma z$. +	(35) −	
Influence ordinate. Hor. reaction	....	....	....	....	....	Z_0	+4	+111	+213	+311	+410	+502	+588	+665	+730	+783	+820	+841	+841	+820	+783	+730	+665	+588	+502	+410	+311	+213	+111	+4	....	....	....	....	....	
0-1	42.50	935.00	−1.159	−25.500	0.863	Z_1	0	−1 109	−1 058	−1 008	−958	−907	−857	−806	−756	−706	−655	−605	−554	−504	−454	−403	−353	−302	−252	−202	−151	−101	−50	0						
					Above floor.	$Z = Z_0 + Z_1$	+4	−998	−845	−697	−548	−405	−269	−141	−26	+77	+165	+236	+287	+316	+329	+327	+312	+286	+250	+208	+160	+112	+61	+4	3.119	3.916	−0.797	1 372	1 722	Above floor.
					Below floor.	$Z = Z_0 + Z_1$	−1 155																								3.119	4.783	−1.664	1 372	2 105	Below floor.
2-3	244.74	732.76	−1.253	−3.752	+0.966	Z_1	0	−163	−1 144	−1 090	−1 085	−981	−926	−872	−817	−763	−708	−654	−599	−545	−490	−436	−382	−327	−273	−218	−164	−109	−54	0						
						$Z = Z_0 + Z_1$	+4	−52	−931	−779	−625	−479	−338	−207	−87	+20	+112	+187	+242	+275	+293	+294	+283	+261	+229	+192	+147	+104	+57	+4	2.691	3.488	−0.797	1 325	1 718	Above floor.
						$Z = Z_0 + Z_1$		−1 088																							2.691	4.524	−1.833	1 325	2 228	Below floor.
4-5	321.98	655.52	−1.357	−2.703	+1.001	Z_1	0	−120	−240	−1 180	−1 121	−1 062	−1 008	−944	−885	−826	−767	−709	−649	−590	−531	−472	−413	−354	−295	−236	−177	−118	−59	0						
						$Z = Z_0 + Z_1$	+4	−9	−27	−869	−711	−560	−415	−279	−155	−43	+53	+132	+192	+230	+252	+258	+252	+234	+207	+174	+184	+95	+52	+4	2.258	3.055	−0.797	1 153	1 560	Above floor.
						$Z = Z_0 + Z_1$			−1 026																						2.258	4.054	−1.796	1 153	2 070	Below floor.
6-7	481.77	495.73	−1.495	−1.538	+0.910	Z_1	0	−67	−133	−201	−1 235	−1 170	−1 105	−1 040	−975	−910	−845	−780	−715	−650	−585	−520	−455	−390	−325	−260	−195	−130	−65	0	1.928	2.725	−0.797	894	1 264	
						$Z = Z_0 + Z_1$	+4	+44	+80	+110	−825	−668	−517	−375	−245	−127	−25	+61	+126	+170	+198	+210	+210	+198	+177	+150	+116	+83	+46	+4						
8-9	719.61	257.89	−1.680	−0.602	+0.780	Z_1	0	−26	−52	−79	−105	−1 315	−1 242	−1 169	−1 096	−1 022	−949	−877	−804	−730	−657	−584	−511	−438	−365	−292	−219	−146	−73	0	1.824	2.621	−0.797	725	1 042	
						$Z = Z_0 + Z_1$	+4	+85	+161	+232	+305	−813	−654	−504	−366	−239	−129	−36	+37	+90	+126	+146	+154	+150	+137	+118	+92	+67	+38	+4						
10-11	825.18	152.32	−1.883	−0.348	+0.715	Z_1	0	−15	−30	−45	−60	−76	−1 392	−1 310	−1 228	−1 146	−1 064	−982	−900	−819	−737	−655	−573	−491	−409	−327	−246	−164	−82	0	1.818	2.615	−0.797	663	954	
						$Z = Z_0 + Z_1$	+4	+96	+183	+266	+350	+426	−804	−645	−498	−363	−244	−141	−59	+1	+46	+75	+92	+97	+93	+83	+65	+49	+29	+4						
12-13	987.16	9.66	−2.160	+0.021	+0.624	Z_1	0	+1	+2	+3	+4	+5	+6	−1 502	−1 408	−1 314	−1 221	−1 127	−1 033	−939	−845	−751	−657	−563	−470	−376	−282	−188	−94	0	2.155	2.952	−0.797	687	940	
						$Z = Z_0 + Z_1$	+4	+112	+215	+314	+414	+507	+594	−837	−678	−531	−401	−286	−192	−119	−62	−21	+8	+25	+32	+34	+29	+25	+17	+4						
14-15	1 291.22	313.72	−2.591	+0.629	+0.502	Z_1	0	+27	+55	+82	+109	+137	+164	+192	−1 690	−1 577	−1 464	−1 352	−1 239	−1 126	−1 014	−901	−788	−676	−563	−450	−338	−225	−113	0	3.344	4.141	−0.797	855	1 059	
						$Z = Z_0 + Z_1$	+4	+138	+268	+393	+519	+639	+752	+857	−960	−794	−644	−511	−398	−306	−231	−171	−123	−88	−61	−40	−27	−12	−2	+4						
16-17	2 166.72	1 189.22	−3.440	+1.888	+0.345	Z_1	0	+82	+164	+246	+328	+410	+493	+575	+657	−2 094	−1 944	−1 795	−1 645	−1 496	−1 346	−1 197	−1 047	−897	−748	−598	−449	−299	−150	0	6.152	6.949	−0.797	1 082	1 222	
						$Z = Z_0 + Z_1$	+4	+193	+377	+557	+738	+912	+1 081	+1 240	+1 387	−1 311	−1 124	−954	−804	−676	−563	−467	−382	−309	−246	−188	−138	−86	−39	+4						
18-19	80 066	81 043	−6.452	+6.530	+0.154	Z_1	0	+284	+568	+852	+1 136	+1 420	+1 704	+1 988	+2 271	+2 555	−3 647	−3 366	−3 086	−2 805	−2 525	−2 244	−1 964	−1 683	−1 403	−1 122	−842	−561	−281	0	16.330	17.127	−0.797	1 285	1 348	
						$Z = Z_0 + Z_1$	+4	+395	+781	+1 163	+1 546	+1 922	+2 292	+2 653	+3 001	+3 338	−2 827	−2 525	−2 245	−1 985	−1 742	−1 514	−1 310	−1 095	−901	−712	−531	−348	−170	+4						
20-21	1 131.54	2 109.04	+11.964	−22.298	−0.061	Z_1	0	−969	−1 934	−2 909	−3 878	−4 848	−5 817	−6 787	−7 756	−8 725	−9 695	+6 242	+5 721	+5 201	+4 681	+4 161	+3 663	+3 121	+2 600	+2 080	+1 560	+1 040	+520	0	45.419	46.216	−0.797	1 432	1 408	
						$Z = Z_0 + Z_1$	+4	−858	−1 726	−2 598	−3 468	−4 346	−5 229	−6 122	−7 026	−7 942	−8 875	+7 083	+6 562	+6 021	+5 464	+4 891	+4 328	+3 709	+3 102	+2 490	+1 871	+1 253	+631	+4						
22-23	302.21	1 279.71	+1.374	−5.817	−0.286	Z_1	0	−177	−354	−531	−707	−884	−1 061	−1 238	−1 415	−1 592	−1 768	−1 945	−179	−163	−147.0	−130	−114	−984	−82	−65.0	−49.0	−33	−16.0	0	4.196	6.033	−1.837	879	612	
						Z_2	+1.2	−1.4	−5.8	−11.1	−16.1	−23.6	−33.2	−45.4	−61.8	−81.5	−106.7	−136.4	−136.4	−106.7	−81.5	−61.3	−45.4	−33.2	−23.6	−16.1	−11.1	−5.8	−1.4	+1.2						
						$Z = Z_0 + Z_1 + Z_2$	+5	−67	−147	−231	−313	−406	−506	−618	−746	−891	−1 055	−1 240	+526	+550	+554	+539	+506	+457	+396	+329	+251	+174	+94	+5						

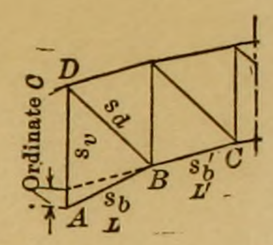
Stress (22-23) $= -0.2858\,(\eta_H - 0.17684\, m + 2.405\, \eta_x)$.
Stress (22-23)' $= -0.2858\,(\eta_H - 0.016305\, m + 2.405\, \eta_x)$
$= -0.2858\,(Z_0 + Z_1 + Z_2)$.

Member.	$\frac{\lambda}{\pi}$	$\frac{\nu}{\pi}$	$\frac{\mu}{\pi}$	M_1, lbs. feet.	M_2, lbs. feet.	Influence ordinate. Hor. reaction.
(a)	(2)	(4)	(3)	(1)	(2)	
	Below roof.					
$Z_0 + Z_1 = Z$	0.208	− 2.128	+ 1.128	+ 32,000	+ 32,000	1.0
$Z_0 + Z_2 = Z$	0.208	− 3.125	+ 2.125	+ 59,375	+ 59,375	1.9
$Z_0 + Z_3 = Z$	0.208	− 4.122	+ 3.122	+ 87,500	+ 87,500	2.8
$Z_0 + Z_4 = Z$	0.208	− 5.119	+ 4.119	+ 115,000	+ 115,000	3.7
$Z_0 + Z_5 = Z$	0.208	− 6.116	+ 5.116	+ 142,812	+ 142,812	4.6
$Z_0 + Z_6 = Z$	0.208	− 7.113	+ 6.113	+ 170,625	+ 170,625	5.5
$Z_0 + Z_7 = Z$	0.208	− 8.110	+ 7.110	+ 198,437	+ 198,437	6.4
$Z_0 + Z_8 = Z$	0.208	− 9.107	+ 8.107	+ 226,250	+ 226,250	7.3
$Z_0 + Z_9 = Z$	0.208	− 10.104	+ 9.104	+ 254,062	+ 254,062	8.2
$Z_0 + Z_{10} = Z$	0.208	− 11.101	+ 10.101	+ 281,875	+ 281,875	9.1
$Z_0 + Z_{11} = Z$	0.208	− 12.098	+ 11.098	+ 309,687	+ 309,687	10.0
$Z_0 + Z_{12} = Z$	0.208	− 13.095	+ 12.095	+ 337,500	+ 337,500	10.9
$Z_0 + Z_{13} = Z$	0.208	− 14.092	+ 13.092	+ 365,312	+ 365,312	11.8
$Z_0 + Z_{14} = Z$	0.208	− 15.089	+ 14.089	+ 393,125	+ 393,125	12.7
$Z_0 + Z_{15} = Z$	0.208	− 16.086	+ 15.086	+ 420,937	+ 420,937	13.6
$Z_0 + Z_{16} = Z$	0.208	− 17.083	+ 16.083	+ 448,750	+ 448,750	14.5
$Z_0 + Z_{17} = Z$	0.208	− 18.080	+ 17.080	+ 476,562	+ 476,562	15.4
$Z_0 + Z_{18} = Z$	0.208	− 19.077	+ 18.077	+ 504,375	+ 504,375	16.3
$Z_0 + Z_{19} = Z$	0.208	− 20.074	+ 19.074	+ 532,187	+ 532,187	17.2
$Z_0 + Z_{20} = Z$	0.208	− 21.071	+ 20.071	+ 560,000	+ 560,000	18.1
$Z_0 + Z_{21} = Z$	0.208	− 22.068	+ 21.068	+ 587,812	+ 587,812	19.0
$Z_0 + Z_{22} = Z$	0.208	− 23.065	+ 22.065	+ 615,625	+ 615,625	19.9
$Z_0 + Z_{23} = Z$	0.208	− 24.062	+ 23.062	+ 643,437	+ 643,437	20.8
$Z_0 + Z_{24} = Z$	0.208	− 25.059	+ 24.059	+ 671,250	+ 671,250	21.7
$Z_0 + Z_{25} = Z$	0.208	− 26.056	+ 25.056	+ 699,062	+ 699,062	22.6
$Z_0 + Z_{26} = Z$	0.208	− 27.053	+ 26.053	+ 726,875	+ 726,875	23.5
$Z_0 + Z_{27} = Z$	0.208	− 28.050	+ 27.050	+ 754,687	+ 754,687	24.4
$Z_0 + Z_{28} = Z$	0.208	− 29.047	+ 28.047	+ 782,500	+ 782,500	25.3
$Z_0 + Z_{29} = Z$	0.208	− 30.044	+ 29.044	+ 810,312	+ 810,312	26.2
$Z_0 + Z_{30} = Z$	0.208	− 31.041	+ 30.041	+ 838,125	+ 838,125	27.1
$Z_0 + Z_{31} = Z$	0.208	− 32.038	+ 31.038	+ 865,937	+ 865,937	28.0
$Z_0 + Z_{32} = Z$	0.208	− 33.035	+ 32.035	+ 893,750	+ 893,750	28.9
$Z_0 + Z_{33} = Z$	0.208	− 34.032	+ 33.032	+ 921,562	+ 921,562	29.8
$Z_0 + Z_{34} = Z$	0.208	− 35.029	+ 34.029	+ 949,375	+ 949,375	30.7
$Z_0 + Z_{35} = Z$	0.208	− 36.026	+ 35.026	+ 977,187	+ 977,187	31.6
$Z_0 + Z_{36} = Z$	0.208	− 37.023	+ 36.023	+ 1,005,000	+ 1,005,000	32.5
$Z_0 + Z_{37} = Z$	0.208	− 38.020	+ 37.020	+ 1,032,812	+ 1,032,812	33.4
$Z_0 + Z_{38} = Z$	0.208	− 39.017	+ 38.017	+ 1,060,625	+ 1,060,625	34.3
$Z_0 + Z_{39} = Z$	0.208	− 40.014	+ 39.014	+ 1,088,437	+ 1,088,437	35.2
$Z_0 + Z_{40} = Z$	0.208	− 41.011	+ 40.011	+ 1,116,250	+ 1,116,250	36.1
$Z_0 + Z_{41} = Z$	0.208	− 42.008	+ 41.008	+ 1,144,062	+ 1,144,062	37.0
$Z_0 + Z_{42} = Z$	0.208	− 43.005	+ 42.005	+ 1,171,875	+ 1,171,875	37.9
$Z_0 + Z_{43} = Z$	0.208	− 44.002	+ 43.002	+ 1,199,687	+ 1,199,687	38.8
$Z_0 + Z_{44} = Z$	0.208	− 44.999	+ 43.999	+ 1,227,500	+ 1,227,500	39.7
$Z_0 + Z_{45} = Z$	0.208	− 45.996	+ 44.996	+ 1,255,312	+ 1,255,312	40.6
$Z_0 + Z_{46} = Z$	0.208	− 46.993	+ 45.993	+ 1,283,125	+ 1,283,125	41.5
$Z_0 + Z_{47} = Z$	0.208	− 47.990	+ 46.990	+ 1,310,937	+ 1,310,937	42.4
$Z_0 + Z_{48} = Z$	0.208	− 48.987	+ 47.987	+ 1,338,750	+ 1,338,750	43.3
$Z_0 + Z_{49} = Z$	0.208	− 49.984	+ 48.984	+ 1,366,562	+ 1,366,562	44.2
$Z_0 + Z_{50} = Z$	0.208	− 50.981	+ 49.981	+ 1,394,375	+ 1,394,375	45.1
$Z_0 + Z_{51} = Z$	0.208	− 51.978	+ 50.978	+ 1,422,187	+ 1,422,187	46.0
$Z_0 + Z_{52} = Z$	0.208	− 52.975	+ 51.975	+ 1,450,000	+ 1,450,000	46.9
$Z_0 + Z_{53} = Z$	0.208	− 53.972	+ 52.972	+ 1,477,812	+ 1,477,812	47.8
$Z_0 + Z_{54} = Z$	0.208	− 54.969	+ 53.969	+ 1,505,625	+ 1,505,625	48.7
$Z_0 + Z_{55} = Z$	0.208	− 55.966	+ 54.966	+ 1,533,437	+ 1,533,437	49.6
$Z_0 + Z_{56} = Z$	0.208	− 56.963	+ 55.963	+ 1,561,250	+ 1,561,250	50.5
$Z_0 + Z_{57} = Z$	0.208	− 57.960	+ 56.960	+ 1,589,062	+ 1,589,062	51.4
$Z_0 + Z_{58} = Z$	0.208	− 58.957	+ 57.957	+ 1,616,875	+ 1,616,875	52.3
$Z_0 + Z_{59} = Z$	0.208	− 59.954	+ 58.954	+ 1,644,687	+ 1,644,687	53.2
$Z_0 + Z_{60} = Z$	0.208	− 60.951	+ 59.951	+ 1,672,500	+ 1,672,500	54.1
$Z_0 + Z_{61} = Z$	0.208	− 61.948	+ 60.948	+ 1,700,312	+ 1,700,312	55.0
$Z_0 + Z_{62} = Z$	0.208	− 62.945	+ 61.945	+ 1,728,125	+ 1,728,125	55.9
$Z_0 + Z_{63} = Z$	0.208	− 63.942	+ 62.942	+ 1,755,937	+ 1,755,937	56.8
$Z_0 + Z_{64} = Z$	0.208	− 64.939	+ 63.939	+ 1,783,750	+ 1,783,750	57.7
$Z_0 + Z_{65} = Z$	0.208	− 65.936	+ 64.936	+ 1,811,562	+ 1,811,562	58.6
$Z_0 + Z_{66} = Z$	0.208	− 66.933	+ 65.933	+ 1,839,375	+ 1,839,375	59.5
$Z_0 + Z_{67} = Z$	0.208	− 67.930	+ 66.930	+ 1,867,187	+ 1,867,187	60.4
$Z_0 + Z_{68} = Z$	0.208	− 68.927	+ 67.927	+ 1,895,000	+ 1,895,000	61.3
$Z_0 + Z_{69} = Z$	0.208	− 69.924	+ 68.924	+ 1,922,812	+ 1,922,812	62.2
$Z_0 + Z_{70} = Z$	0.208	− 70.921	+ 69.921	+ 1,950,625	+ 1,950,625	63.1
$Z_0 + Z_{71} = Z$	0.208	− 71.918	+ 70.918	+ 1,978,437	+ 1,978,437	64.0
$Z_0 + Z_{72} = Z$	0.208	− 72.915	+ 71.915	+ 2,006,250	+ 2,006,250	64.9
$Z_0 + Z_{73} = Z$	0.208	− 73.912	+ 72.912	+ 2,034,062	+ 2,034,062	65.8
$Z_0 + Z_{74} = Z$	0.208	− 74.909	+ 73.909	+ 2,061,875	+ 2,061,875	66.7
$Z_0 + Z_{75} = Z$	0.208	− 75.906	+ 74.906	+ 2,089,687	+ 2,089,687	67.6
$Z_0 + Z_{76} = Z$	0.208	− 76.903	+ 75.903	+ 2,117,500	+ 2,117,500	68.5
$Z_0 + Z_{77} = Z$	0.208	− 77.900	+ 76.900	+ 2,145,312	+ 2,145,312	69.4
$Z_0 + Z_{78} = Z$	0.208	− 78.897	+ 77.897	+ 2,173,125	+ 2,173,125	70.3
$Z_0 + Z_{79} = Z$	0.208	− 79.894	+ 78.894	+ 2,200,937	+ 2,200,937	71.2
$Z_0 + Z_{80} = Z$	0.208	− 80.891	+ 79.891	+ 2,228,750	+ 2,228,750	72.1
$Z_0 + Z_{81} = Z$	0.208	− 81.888	+ 80.888	+ 2,256,562	+ 2,256,562	73.0
$Z_0 + Z_{82} = Z$	0.208	− 82.885	+ 81.885	+ 2,284,375	+ 2,284,375	73.9
$Z_0 + Z_{83} = Z$	0.208	− 83.882	+ 82.882	+ 2,312,187	+ 2,312,187	74.8
$Z_0 + Z_{84} = Z$	0.208	− 84.879	+ 83.879	+ 2,340,000	+ 2,340,000	75.7
$Z_0 + Z_{85} = Z$	0.208	− 85.876	+ 84.876	+ 2,367,812	+ 2,367,812	76.6
$Z_0 + Z_{86} = Z$	0.208	− 86.873	+ 85.873	+ 2,395,625	+ 2,395,625	77.5
$Z_0 + Z_{87} = Z$	0.208	− 87.870	+ 86.870	+ 2,423,437	+ 2,423,437	78.4
$Z_0 + Z_{88} = Z$	0.208	− 88.867	+ 87.867	+ 2,451,250	+ 2,451,250	79.3
$Z_0 + Z_{89} = Z$	0.208	− 89.864	+ 88.864	+ 2,479,062	+ 2,479,062	80.2
$Z_0 + Z_{90} = Z$	0.208	− 90.861	+ 89.861	+ 2,506,875	+ 2,506,875	81.1
$Z_0 + Z_{91} = Z$	0.208	− 91.858	+ 90.858	+ 2,534,687	+ 2,534,687	82.0
$Z_0 + Z_{92} = Z$	0.208	− 92.855	+ 91.855	+ 2,562,500	+ 2,562,500	82.9
$Z_0 + Z_{93} = Z$	0.208	− 93.852	+ 92.852	+ 2,590,312	+ 2,590,312	83.8
$Z_0 + Z_{94} = Z$	0.208	− 94.849	+ 93.849	+ 2,618,125	+ 2,618,125	84.7
$Z_0 + Z_{95} = Z$	0.208	− 95.846	+ 94.846	+ 2,645,937	+ 2,645,937	85.6
$Z_0 + Z_{96} = Z$	0.208	− 96.843	+ 95.843	+ 2,673,750	+ 2,673,750	86.5
$Z_0 + Z_{97} = Z$	0.208	− 97.840	+ 96.840	+ 2,701,562	+ 2,701,562	87.4
$Z_0 + Z_{98} = Z$	0.208	− 98.837	+ 97.837	+ 2,729,375	+ 2,729,375	88.3
$Z_0 + Z_{99} = Z$	0.208	− 99.834	+ 98.834	+ 2,757,187	+ 2,757,187	89.2
$Z_0 + Z_{100} = Z$	0.208	− 100.831	+ 99.831	+ 2,785,000	+ 2,785,000	90.1
$Z_0 + Z_{101} = Z$	0.208	− 101.828	+ 100.828	+ 2,812,812	+ 2,812,812	91.0
$Z_0 + Z_{102} = Z$	0.208	− 102.825	+ 101.825	+ 2,840,625	+ 2,840,625	91.9
$Z_0 + Z_{103} = Z$	0.208	− 103.822	+ 102.822	+ 2,868,437	+ 2,868,437	92.8
$Z_0 + Z_{104} = Z$	0.208	− 104.819	+ 103.819	+ 2,896,250	+ 2,896,250	93.7
$Z_0 + Z_{105} = Z$	0.208	− 105.816	+ 104.816	+ 2,924,062	+ 2,924,062	94.6
$Z_0 + Z_{106} = Z$	0.208	− 106.813	+ 105.813	+ 2,951,875	+ 2,951,875	95.5
$Z_0 + Z_{107} = Z$	0.208	− 107.810	+ 106.810	+ 2,979,687	+ 2,979,687	96.4
$Z_0 + Z_{108} = Z$	0.208	− 108.807	+ 107.807	+ 3,007,500	+ 3,007,500	97.3
$Z_0 + Z_{109} = Z$	0.208	− 109.804	+ 108.804	+ 3,035,312	+ 3,035,312	98.2
$Z_0 + Z_{110} = Z$	0.208	− 110.801	+ 109.801	+ 3,063,125	+ 3,063,125	99.1
$Z_0 + Z_{111} = Z$	0.208	− 111.798	+ 110.798	+ 3,090,937	+ 3,090,937	100.0
$Z_0 + Z_{112} = Z$	0.208	− 112.795	+ 111.795	+ 3,118,750	+ 3,118,750	100.9
$Z_0 + Z_{113} = Z$	0.208	− 113.792	+ 112.792	+ 3,146,562	+ 3,146,562	101.8
$Z_0 + Z_{114} = Z$	0.208	− 114.789	+ 113.789	+ 3,174,375	+ 3,174,375	102.7
$Z_0 + Z_{115} = Z$	0.208	− 115.786	+ 114.786	+ 3,202,187	+ 3,202,187	103.6
$Z_0 + Z_{116} = Z$	0.208	− 116.783	+ 115.783	+ 3,230,000	+ 3,230,000	104.5
$Z_0 + Z_{117} = Z$	0.208	− 117.780	+ 116.780	+ 3,257,812	+ 3,257,812	105.4
$Z_0 + Z_{118} = Z$	0.208	− 118.777	+ 117.777	+ 3,285,625	+ 3,285,625	106.3
$Z_0 + Z_{119} = Z$	0.208	− 119.774	+ 118.774	+ 3,313,437	+ 3,313,437	107.2
$Z_0 + Z_{120} = Z$	0.208	− 120.771	+ 119.771	+ 3,341,250	+ 3,341,250	108.1
$Z_0 + Z_{121} = Z$	0.208	− 121.768	+ 120.768	+ 3,369,062	+ 3,369,062	109.0
$Z_0 + Z_{122} = Z$	0.208	− 122.765	+ 121.765	+ 3,396,875	+ 3,396,875	109.9
$Z_0 + Z_{123} = Z$	0.208	− 123.762	+ 122.762	+ 3,424,687	+ 3,424,687	110.8
$Z_0 + Z_{124} = Z$	0.208	− 124.759	+ 123.759	+ 3,452,500	+ 3,452,500	111.7
$Z_0 + Z_{125} = Z$	0.208	− 125.756	+ 124.756	+ 3,480,312	+ 3,480,312	112.6
$Z_0 + Z_{126} = Z$	0.208	− 126.753	+ 125.753	+ 3,508,125	+ 3,508,125	113.5
$Z_0 + Z_{127} = Z$	0.208	− 127.750	+ 126.750	+ 3,535,937	+ 3,535,937	114.4
$Z_0 + Z_{128} = Z$	0.208	− 128.747	+ 127.747	+ 3,563,750	+ 3,563,750	115.3
$Z_0 + Z_{129} = Z$	0.208	− 129.744	+ 128.744	+ 3,591,562	+ 3,591,562	116.2
$Z_0 + Z_{130} = Z$	0.208	− 130.741	+ 129.741	+ 3,619,375	+ 3,619,375	117.1
$Z_0 + Z_{131} = Z$	0.208	− 131.738	+ 130.738	+ 3,647,187	+ 3,647,187	118.0
$Z_0 + Z_{132} = Z$	0.208	− 132.735	+ 131.735	+ 3,675,000	+ 3,675,000	118.9
$Z_0 + Z_{133} = Z$	0.208	− 133.732	+ 132.732	+ 3,702,812	+ 3,702,812	119.8
$Z_0 + Z_{134} = Z$	0.208	− 134.729	+ 133.729	+ 3,730,625	+ 3,730,625	120.7
$Z_0 + Z_{135} = Z$	0.208	− 135.726	+ 134.726	+ 3,758,437	+ 3,758,437	121.6
$Z_0 + Z_{136} = Z$	0.208	− 136.723	+ 135.723	+ 3,786,250	+ 3,786,250	122.5
$Z_0 + Z_{137} = Z$	0.208	− 137.720	+ 136.720	+ 3,814,062	+ 3,814,062	123.4
$Z_0 + Z_{138} = Z$	0.208	− 138.717	+ 137.717	+ 3,841,875	+ 3,841,875	124.3
$Z_0 + Z_{139} = Z$	0.208	− 139.714	+ 138.714	+ 3,869,687	+ 3,869,687	125.2
$Z_0 + Z_{140} = Z$	0.208	− 140.711	+ 139.711	+ 3,897,500	+ 3,897,500	126.1
$Z_0 + Z_{141} = Z$	0.208	− 141.708	+ 140.708	+ 3,925,312	+ 3,925,312	127.0
$Z_0 + Z_{142} = Z$	0.208	− 142.705	+ 141.705	+ 3,953,125	+ 3,953,125	127.9
$Z_0 + Z_{143} = Z$	0.208	− 143.702	+ 142.702	+ 3,980,937	+ 3,980,937	128.8
$Z_0 + Z_{144} = Z$	0.208	− 144.699	+ 143.699	+ 4,008,750	+ 4,008,750	129.7
$Z_0 + Z_{145} = Z$	0.208	− 145.696	+ 144.696	+ 4,036,562	+ 4,036,562	130.6
$Z_0 + Z_{146} = Z$	0.208	− 146.693	+ 145.693	+ 4,064,375	+ 4,064,375	131.5
$Z_0 + Z_{147} = Z$	0.208	− 147.690	+ 146.690	+ 4,092,187	+ 4,092,187	132.4
$Z_0 + Z_{148} = Z$	0.208	− 148.687	+ 147.687	+ 4,120,000	+ 4,120,000	133.3
$Z_0 + Z_{149} = Z$	0.208	− 149.684	+ 148.684	+ 4,147,812	+ 4,147,812	134.2
$Z_0 + Z_{150} = Z$	0.208	− 150.681	+ 149.681	+ 4,175,625	+ 4,175,625	135.1
$Z_0 + Z_{151} = Z$	0.208	− 151.678	+ 150.678	+ 4,203,437	+ 4,203,437	136.0
$Z_0 + Z_{152} = Z$	0.208	− 152.675	+ 151.675	+ 4,231,250	+ 4,231,250	136.9
$Z_0 + Z_{153} = Z$	0.208	− 153.672	+ 152.672	+ 4,259,062	+ 4,259,062	137.8
$Z_0 + Z_{154} = Z$	0.208	− 154.669	+ 153.669	+ 4,286,875	+ 4,286,875	138.7
$Z_0 + Z_{155} = Z$	0.208	− 155.666	+ 154.666	+ 4,314,687	+ 4,314,687	139.6
$Z_0 + Z_{156} = Z$	0.208	− 156.663	+ 155.663	+ 4,342,500	+ 4,342,500	140.5
$Z_0 + Z_{157} = Z$	0.208	− 157.660	+ 156.660	+ 4,370,312	+ 4,370,312	141.4
$Z_0 + Z_{158} = Z$	0.208	− 158.657	+ 157.657	+ 4,398,125	+ 4,398,125	142.3
$Z_0 + Z_{159} = Z$	0.208	− 159.654	+ 158.654	+ 4,425,937	+ 4,425,937	143.2
$Z_0 + Z_{160} = Z$	0.208	− 160.651	+ 159.651	+ 4,453,750	+ 4,453,750	144.1
$Z_0 + Z_{161} = Z$	0.208	− 161.648	+ 160.648	+ 4,481,562	+ 4,481,562	145.0
$Z_0 + Z_{162} = Z$	0.208	− 162.645	+ 161.645	+ 4,509,375	+ 4,509,375	145.9
$Z_0 + Z_{163} = Z$	0.208	− 163.642	+ 162.642	+ 4,537,187	+ 4,537,187	146.8
$Z_0 + Z_{164} = Z$	0.208	− 164.639	+ 163.639	+ 4,565,000	+ 4,565,000	147.7
$Z_0 + Z_{165} = Z$	0.208	− 165.636	+ 164.636	+ 4,592,812	+ 4,592,812	148.6
$Z_0 + Z_{166} = Z$	0.208	− 166.633	+ 165.633	+ 4,620,625	+ 4,620,625	149.5
$Z_0 + Z_{167} = Z$	0.208	− 167.630	+ 166.630	+ 4,648,437	+ 4,648,437	150.4
$Z_0 + Z_{168} = Z$	0.208	− 168.627	+ 167.627	+ 4,676,250	+ 4,676,250	151.3
$Z_0 + Z_{169} = Z$	0.208	− 169.624	+ 168.624	+ 4,704,062	+ 4,704,062	152.2
$Z_0 + Z_{170} = Z$	0.208	− 170.621	+ 169.621	+ 4,731,875	+ 4,731,875	153.1
$Z_0 + Z_{171} = Z$	0.208	− 171.618	+ 170.618	+ 4,759,687	+ 4,759,687	154.0
$Z_0 + Z_{172} = Z$	0.208	− 172.615	+ 171.615	+ 4,787,500	+ 4,787,500	154.9
$Z_0 + Z_{173} = Z$	0.208	− 173.612	+ 172.612	+ 4,815,312	+ 4,815,312	155.8
$Z_0 + Z_{174} = Z$	0.208	− 174.609	+ 173.609	+ 4,843,125	+ 4,843,125	156.7
$Z_0 + Z_{175} = Z$	0.208	− 175.606	+ 174.606	+ 4,870,937	+ 4,870,937	157.6
$Z_0 + Z_{176} = Z$	0.208	− 176.603	+ 175.603	+ 4,898,750	+ 4,898,750	158.5
$Z_0 + Z_{177} = Z$	0.208	− 177.600	+ 176.600	+ 4,926,562	+ 4,926,562	159.4
$Z_0 + Z_{178} = Z$	0.208	− 178.597	+ 177.597	+ 4,954,375	+ 4,954,375	160.3
$Z_0 + Z_{179} = Z$	0.208	− 179.594	+ 178.594	+ 4,982,187	+ 4,982,187	161.2
$Z_0 + Z_{180} = Z$	0.208	− 180.591	+ 179.591	+ 5,010,000	+ 5,010,000	162.1
$Z_0 + Z_{181} = Z$	0.208	− 181.588	+ 180.588	+ 5,037,812	+ 5,037,812	163.0
$Z_0 + Z_{182} = Z$	0.208	− 182.585	+ 181.585	+ 5,065,625	+ 5,065,625	163.9
$Z_0 + Z_{183} = Z$	0.208	− 183.582	+ 182.582	+ 5,093,437	+ 5,093,437	164.8
$Z_0 + Z_{184} = Z$	0.208	− 184.579	+ 183.579	+ 5,121,250	+ 5,121,250	165.7
$Z_0 + Z_{185} = Z$	0.208	− 185.576	+ 184.576	+ 5,149,062	+ 5,149,062	166.6
$Z_0 + Z_{186} = Z$	0.208	− 186.573	+ 185.573	+ 5,176,875	+ 5,176,875	167.5
$Z_0 + Z_{187} = Z$	0.208	− 187.570	+ 186.570	+ 5,204,687	+ 5,204,687	168.4
$Z_0 + Z_{188} = Z$	0.208	− 188.567	+ 187.567	+ 5,232,500	+ 5,232,500	169.3
$Z_0 + Z_{189} = Z$	0.208	− 189.564	+ 188.564	+ 5,260,312	+ 5,260,312	170.2
$Z_0 + Z_{190} = Z$	0.208	− 190.561	+ 189.561	+ 5,288,125	+ 5,288,125	171.1
$Z_0 + Z_{191} = Z$	0.208	− 191.558	+ 190.558	+ 5,315,937	+ 5,315,937	172.0
$Z_0 + Z_{192} = Z$	0.208	− 192.555	+ 191.555	+ 5,343,750	+ 5,343,750	172.9
$Z_0 + Z_{193} = Z$	0.208	− 193.552	+ 192.552	+ 5,371,562	+ 5,371,562	173.8
$Z_0 + Z_{194} = Z$	0.208	− 194.549	+ 193.549	+ 5,399,375	+ 5,399,375	174.7
$Z_0 + Z_{195} = Z$	0.208	− 195.546	+ 194.546	+ 5,427,187	+ 5,427,187	175.6
$Z_0 + Z_{196} = Z$	0.208	− 196.543	+ 195.543	+ 5,455,000	+ 5,455,000	176.5
$Z_0 + Z_{197} = Z$	0.208	− 197.540	+ 196.540	+ 5,482,812	+ 5,482,812	177.4
$Z_0 + Z_{198} = Z$	0.208	− 198.537	+ 197.537	+ 5,510,625	+ 5,510,625	178.3
$Z_0 + Z_{199} = Z$	0.208	− 199.534	+ 198.534	+ 5,538,437	+ 5,538,437	179.2
$Z_0 + Z_{200} = Z$	0.208	− 200.531	+ 199.531	+ 5,566,250	+ 5,566,250	180.1
$Z_0 + Z_{201} = Z$	0.208	− 201.528	+ 200.528	+ 5,594,062	+ 5,594,062	181.0
$Z_0 + Z_{202} = Z$	0.208	− 202.525	+ 201.525	+ 5,621,875	+ 5,621,875	181.9
$Z_0 + Z_{203} = Z$	0.208	− 203.522	+ 202.522	+ 5,649,687	+ 5,649,687	182.8
$Z_0 + Z_{204} = Z$	0.208	− 204.519	+ 203.519	+ 5,677,500	+ 5,677,500	183.7
$Z_0 + Z_{205} = Z$	0.208	− 205.516	+ 204.516	+ 5,705,312	+ 5,705,312	184.6
$Z_0 + Z_{206} = Z$	0.208	− 206.513	+ 205.513	+ 5,733,125	+ 5,733,125	185.5
$Z_0 + Z_{207} = Z$	0.208	− 207.510	+ 206.510	+ 5,760,937	+ 5,760,937	186.4
$Z_0 + Z_{208} = Z$	0.208	− 208.507	+ 207.507	+ 5,788,750	+ 5,788,750	187.3
$Z_0 + Z_{209} = Z$	0.208	− 209.504	+ 208.504	+ 5,816,562	+ 5,816,562	188.2
$Z_0 + Z_{210} = Z$	0.208	− 210.501	+ 209.501	+ 5,844,375	+ 5,844,375	189.1
$Z_0 + Z_{211} = Z$	0.208	− 211.498	+ 210.498	+ 5,872,187	+ 5,872,187	190.0
$Z_0 + Z_{212} = Z$	0.208	− 212.495	+ 211.495	+ 5,900,000	+ 5,900,000	190.9
$Z_0 + Z_{213} = Z$	0.208	− 213.492	+ 212.492	+ 5,927,812	+ 5,927,812	191.8
$Z_0 + Z_{214} = Z$	0.208	− 214.489	+ 213.489	+ 5,955,625	+ 5,955,625	192.7
$Z_0 + Z_{215} = Z$	0.208	− 215.486	+ 214.486	+ 5,983,437	+ 5,983,437	193.6
$Z_0 + Z_{216} = Z$	0.208	− 216.483	+ 215.483	+ 6,011,250	+ 6,011,250	194.5
$Z_0 + Z_{217} = Z$	0.208	− 217.480	+ 216.480	+ 6,039,062	+ 6,039,062	195.4
$Z_0 + Z_{218} = Z$	0.208	− 218.477	+ 217.477	+ 6,066,875	+ 6,066,875	196.3
$Z_0 + Z_{219} = Z$	0.208	− 219.474	+ 218.474	+ 6,094,687	+ 6,094,687	197.2
$Z_0 + Z_{220} = Z$	0.208	− 220.471	+ 219.471	+ 6,122,500	+ 6,122,500	198.1
$Z_0 + Z_{221} = Z$	0.208	− 221.468	+ 220.468	+ 6,150,312	+ 6,150,312	199.0
$Z_0 + Z_{222} = Z$	0.208	− 222.465	+ 221.465	+ 6,178,125	+ 6,178,125	199.9
$Z_0 + Z_{223} = Z$	0.208	− 223.462	+ 222.462	+ 6,205,937	+ 6,205,937	200.8
$Z_0 + Z_{224} = Z$	0.208	− 224.459	+ 223.459	+ 6,233,750	+ 6,233,750	201.7
$Z_0 + Z_{225} = Z$	0.208	− 225.456	+ 224.456	+ 6,261,562	+ 6,261,562	202.6
$Z_0 + Z_{226} = Z$	0.208	− 226.453	+ 225.453	+ 6,289,375	+ 6,289,375	203.5
$Z_0 + Z_{227} = Z$	0.208	− 227.450	+ 226.450	+ 6,317,187	+ 6,317,187	204.4
$Z_0 + Z_{228} = Z$	0.208	− 228.447	+ 227.447	+ 6,345,000	+ 6,345,000	205.3
$Z_0 + Z_{229} = Z$	0.208	− 229.444	+ 228.444	+ 6,372,812	+ 6,372,812	206.2
$Z_0 + Z_{230} = Z$	0.208	− 230.441	+ 229.441	+ 6,400,625	+ 6,400,625	207.1
$Z_0 + Z_{231} = Z$	0.208	− 231.438	+ 230.438	+ 6,428,437	+ 6,428,437	208.0
$Z_0 + Z_{232} = Z$	0.208	− 232.435	+ 231.435	+ 6,456,250	+ 6,456,250	208.9
$Z_0 + Z_{233} = Z$	0.208	− 233.432	+ 232.432	+ 6,484,062	+ 6,484,062	209.8
$Z_0 + Z_{234} = Z$	0.208	− 234.429	+ 233.429	+ 6,511,875	+ 6,511,875	210.7
$Z_0 + Z_{235} = Z$	0.208	− 235.426	+ 234.426	+ 6,539,687	+ 6,539,687	211.6
$Z_0 + Z_{236} = Z$	0.208	− 236.423	+ 235.423	+ 6,567,500	+ 6,567,500	212.5
$Z_0 + Z_{237} = Z$	0.208	− 237.420	+ 236.420	+ 6,595,312	+ 6,595,312	213.4
$Z_0 + Z_{238} = Z$	0.208	− 238.417	+ 237.417	+ 6,623,125	+ 6,623,125	214.3
$Z_0 + Z_{239} = Z$	0.208	− 239.414	+ 238.414	+ 6,650,937	+ 6,650,937	215.2
$Z_0 + Z_{240} = Z$	0.208	− 240.411	+ 239.411	+ 6,678,750	+ 6,678,750	216.1
$Z_0 + Z_{241} = Z$	0.208	− 241.408	+ 240.408	+ 6,706,562	+ 6,706,562	217.0
$Z_0 + Z_{242} = Z$	0.208	− 242.405	+ 241.405	+ 6,734,375	+ 6,734,375	217.9
$Z_0 + Z_{243} = Z$	0.208	− 243.402	+ 242.402	+ 6,762,187	+ 6,762,187	218.8
$Z_0 + Z_{244} = Z$	0.208	− 244.399	+ 243.399	+ 6,790,000	+ 6,790,000	219.7
$Z_0 + Z_{245} = Z$	0.208	− 245.396	+ 244.396	+ 6,817,812	+ 6,817,812	220.6
$Z_0 + Z_{246} = Z$	0.208	− 246.393	+ 245.393	+ 6,845,625	+ 6,845,625	221.5

TABLE 17.—Determination of Dead-Load Stresses for Case II.

Panel Length, $\lambda = 42.5$ ft. All Loads and Stresses Given in Units of 1 000 Lb.

Panel Points.	(1) 0-1	(2) 2-3	(3) 4-5	(4) 6-7	(5) 8-9	(6) 10-11	(7) 12-13	(8) 14-15	(9) 16-17	(10) 18-19	(11) 20-21	(12) 22-23
Loads at top chord	+ 188	+ 187	+ 174	+ 157	+ 160	+ 155	+ 135	+ 140	+ 140	+ 140	+ 137	+ 130
Loads at floor height	+ 308	+ 261	+ 234									
Loads at bottom chord	+ 324	− 143.23	− 196.23	− 42.23	− 92.23	− 97.23	− 143.23	− 164.23	− 165.23	− 175.23	− 182.23	− 170.23
Total	+ 820	+ 304.77	+ 211.77	+ 114.77	+ 67.77	+ 57.77	− 8.23	− 24.23	− 25.23	− 35.23	− 45.23	− 40.23
Shear		+ 578.47	+ 273.70	+ 61.93	− 52.84	− 120.61	− 178.38	− 170.15	− 145.92	− 120.69	− 85.46	− 40.23
Moment = M	0	+ 578	+ 852	+ 914	+ 861	+ 741	+ 562	+ 392	+ 246	+ 126	+ 40	0
Top Chord.	1-3	3-5	5-7	7-9	9-11	11-13	13-15	15-17	17-19	19-21	21-23	23-23A
Stress, Case II, $-\frac{M\lambda}{r}$	− 223	− 408	− 523	− 558	− 529	− 441	− 336	− 228	− 124	− 41	0	0
Bottom Chord.	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-22A
Stress, Case II, $\frac{M\lambda}{r}$	0	+ 276	+ 477	+ 573	+ 587	+ 549	+ 452	+ 341	+ 229	+ 125	+ 41	0
$\frac{L}{s_b} = \frac{\text{Bottom chord stress}}{\text{Length of member, in feet}}$	0	+ 5.12	+ 9.17	+ 11.41	+ 12.10	+ 11.69	+ 9.89	+ 7.65	+ 5.25	+ 2.90	+ 0.97	0
$\frac{L'}{s_{b'}} - \frac{L}{s_b}$	+ 5.12	+ 4.05	+ 2.24	+ 0.69	− 0.41	− 1.80	− 2.24	− 2.40	− 2.35	− 1.93	− 0.97	0
Diagonal.	1-2	3-4	5-6	7-8	9-10	11-12	13-14	15-16	17-18	19-20	21-22	23-22
Stress = $s_d\left[\frac{L'}{s_{b'}} - \frac{L}{s_b}\right]$	+ 572	+ 365	+ 170	+ 47	− 26	− 109	− 131	− 137	− 132	− 109	− 55	0
Vertical.	0-1	2-3	4-5	6-7	8-9	10-11	12-13	14-15	16-17	18-19	20-21	22-23
$-s_v\left[\frac{L'}{s_{b'}} - \frac{L}{s_b}\right]$	Reaction for Case II = 1 400	− 716.80	− 457.37	− 208.17	− 55.20	+ 29.18	+ 114.17	+ 127.23	+ 123.05	+ 110.10	+ 84.04	+ 40.09
$C\frac{L'}{s_{b'}}$		+ 17.07	+ 30.56	+ 38.02	+ 40.29	+ 38.96	+ 32.96	+ 25.50	+ 17.50	+ 9.67	+ 3.23	0
Lower panel point concentration	− 324	− 143.23	− 196.23	− 42.23	− 92.23	− 97.23	− 143.23	− 164.23	− 165.23	− 175.23	− 182.23	− 170.23
Stress in vertical below floor	− 1 076	− 843	− 623	− 212	− 107	− 29	+ 4	− 12	− 25	− 55	− 95	− 130
Stress in vertical above floor	− 768	− 582	− 389									



Length of $A\,B = s_b$
Length of $B\,C = s_{b'}$
Length of $D\,B = s_d$
Length of $A\,D = s_v$
Stress in $A\,B = L$
Stress in $B\,C = L'$
Ordinate $C = 3\frac{1}{3}$ ft.

To simplify the calculation, these ordinates were assumed as multiplied by the value, $\frac{r}{y}$; in other words, the ordinates, $A_1\ A_2$ and $B_1\ B_2$, were made equal to $\frac{x}{y}$ and $\frac{x'}{y}$, respectively (Fig. 55). The intermediate ordinates, Z_1, of the lines, $A_1\ C_2$ and $C_2\ B_1$, were then obtained by simple proportion from the ordinates, $A_1\ A_2$ and $B_1\ B_2$. (Tables 13, 14, 15, and 7.) These ordinates, Z_1, were then added algebraically to the influence ordinates, Z_0, for the horizontal reaction (polygon, $A_1\ C_1\ B_1$), in order to get the influence ordinates, Z, for the two-hinged arch condition.

To obtain any stress, the sum of the influence ordinates, Z, finally had to be multiplied by the coefficient, $\frac{y}{r}$.

3.—Live-Load Stresses.

The assumed live load is 6 000 lb. per lin. ft. of track, or 12 000 lb. per lin. ft. of truss, which gives a full panel load per truss of

$$12\,000 \times 42.5 \text{ ft.} = 510\,000 \text{ lb.}$$

To determine the maximum live-load stress in any member, the influence ordinates of the same sign were added (Columns 31 and 32, Tables 13, 14, 15, and 16), proper corrections being made for partial panel loads at the end vertical and at panel points adjacent to the zero points of the influence line.

The sum of the influence ordinates was then multiplied by the coefficient, $\frac{y}{r}$, multiplied by the panel load, 510 000 lb. (Columns 34 and 35, Tables 13, 14, 15, and 16).

4.—Dead-Load Stresses.

The arch was erected so as to act as three-hinged for all the dead load except the concrete and timber flooring, ballast, and tracks, which were placed after the trusses had been converted into two-hinged arches.

For the three-hinged arch, the stresses were determined as follows:

First, the horizontal reaction was determined for the actual panel concentrations. Then, a uniform load was determined which would cause the same horizontal reaction. For this uniform load (16 000 lb. per lin. ft., or 680 000 lb. per panel per truss), the stresses were determined in the bottom chord members (Case I). As the bottom chord panel points are on a parabola, no other members are stressed for this case, and the stress in any bottom chord is equal to the horizontal reaction multiplied by the ratio between the length of the member and the panel length.

TABLE 18—DEAD-

All Stresses Given in Units of 1 000 lb.

Bottom Chord.	0-2	2-4	4-6	6-8
Case I	— 11 457	— 11 023	— 10 616	— 10 239
Case II	0	+ 276	+ 477	+ 573
Total case, I + II	— 11 457	— 10 747	— 10 139	— 9 666
Case III	— 5 008	— 4 712	— 4 406	— 4 101
Total case, I + II + III	16 465	— 15 459	— 14 545	— 13 767
Top Chord.	1-3	3-5	5-7	7-9
Case I	0	0	0	0
Case II	— 223	— 408	— 523	— 558
Total case, I + II	— 223	— 408	— 523	— 558
Case III	— 84	— 199	— 340	— 488
Total case, I + II + III	— 307	— 607	— 863	— 1 046
Diagonals.	1-2	3-4	5-6	7-8
Case I	0	0	0	0
Case II	+ 572	+ 365	+ 170	+ 47
Total case, I + II	+ 572	+ 365	+ 170	+ 47
Case III	+ 216	+ 230	+ 224	+ 215
Total case, I + II + III	+ 788	+ 595	+ 394	+ 262
Verticals.	0-1 Below floor.	2-3 Below floor	4-5 Below floor	6-7
Case I	0	0	0	0
Case II	— 1 076	— 843	— 623	— 212
Total case, I + II	— 1 076	— 843	— 623	— 212
Case III	— 374	— 561	— 570	— 230
Total case, I + II + III	— 1 450	— 1 403	— 1 193	— 442
Verticals.	Above floor	Above floor	Above floor	
Case II	— 768	— 582	— 389	.....
Total case, I + II	— 768	— 582	— 389	.....
Case III	— 218	— 244	— 253	.....
Total case, I + II + III	— 986	— 826	— 642	.....

Horizontal Reaction for Case I = 8 672 900 lb. Bottom Chord Stress for Case I = Horizontal

Load Stresses.

8–10	10–12	12–14	14–16	16–18	18–20	20–22	22–22′
— 9 896	— 9 585	— 9 315	— 9 089	— 8 910	— 8 779	— 8 700	— 8 673
+ 587	+ 549	+ 452	+ 341	+ 229	+ 125	+ 41	0
— 9 309	— 9 036	— 8 863	— 8 748	— 8 681	— 8 654	— 8 659	— 8 673
— 3 812	— 3 532	— 3 259	— 3 008	— 2 785	— 2 604	— 2 481	— 2 305
— 13 121	— 12 568	— 12 122	— 11 756	— 11 466	— 11 258	— 11 140	10 978

9–11	11–13	13–15	15–17	17–19	19–21	21–23	23–23′
0	0	0	0	0	0	0	0
— 529	— 441	— 336	— 228	— 124	— 41	0	0
— 529	— 441	— 336	— 228	— 124	— 41	0	0
— 636	— 793	— 951	— 1 103	— 1 234	— 1 332	— 1 384	— 1 287
— 1 165	— 1 234	— 1 287	— 1 331	— 1 358	— 1 373	— 1 384	— 1 287

9–10	11–12	13–14	15–16	17–18	19–20	21–22	23–22′
0	0	0	0	0	0	0	0
— 26	— 109	— 131	— 137	— 132	— 109	— 55	0
— 26	— 109	— 131	— 137	— 132	— 109	— 55	0
+ 222	+ 227	+ 225	+ 213	+ 185	+ 140	+ 77	— 138
+ 196	+ 118	+ 94	+ 76	+ 53	+ 31	+ 22	— 138

8–9	10–11	12–13	14–15	16–17	18–19	20–21	22–23
0	0	0	0	0	0	0	0
— 107	— 29	+ 4	— 12	— 25	— 55	— 95	— 130
— 107	— 29	+ 4	— 12	— 25	— 55	— 95	— 130
— 197	— 181	— 158	— 127	— 87	— 39	+ 15	+ 166
— 304	— 210	— 154	— 139	— 112	— 94	— 80	+ 36
.....	.....	.....	.....	.....	.....	.....	.....
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Reaction $\times \frac{\text{Length of Member}}{\text{Panel Length}}$. + Denotes Tension. — Denotes Compression.

Then the stresses were determined for the difference between the actual and the uniform load (Case II). As, for both the actual and the uniform loads, the horizontal reactions are the same, that is, their difference is zero, it was possible to determine the stresses for Case II as for a simple span. The stresses in the chord members were determined from the bending moments. The stresses in the web members were then obtained by resolving stresses at the lower panel points. With the notations given in the diagram on Table 17 (Plate XLIII) we have, stress in diagonal $= s_d \left(\frac{L'}{s_b'} - \frac{L}{s_b}\right)$, stress in vertical below floor $= - s_v \left(\frac{L'}{s_b'} - \frac{L}{s_b}\right) + C \frac{L'}{s_b'}$ + panel load at bottom chord.

The stress in the vertical above the floor is found by deducting the panel load at the floor height from the stress below the floor.

For the two-hinged arch (Case III), the stresses were determined from the influence ordinates in a manner similar to that described for live loads. For panel concentrations, see Plate XXVIII.

Finally, the stresses for Cases I, II, and III were combined to obtain the total dead-load stresses (Table 18).

5.—Temperature Stresses.

Assuming the arch bearings as free to move longitudinally, a horizontal force of unity applied at each hinge causes a change in the span length equal to $\Sigma \Delta l$. A change in temperature of $t = 72°$ Fahr. causes a change in the span length equal to $\varepsilon t l = \frac{1}{150\ 000} \times 72 \times 977.5 \times 12 = 5.63$ in.

The horizontal reaction due to a change of temperature of 72° Fahr., therefore, is

$$H_t = \frac{\varepsilon t l}{\Sigma \Delta l} = 215\ 460 \text{ lb.}$$

The temperature stress in any member is then found as the product of H_t with the corresponding value, $S_1 = \frac{y}{r}$ (Table 19).

TABLE 19.—Temperature Stresses for a Variation of ± 72° Fahr.

Stresses Given in Units of 1 000 lb.

Top Chord.	1-3	3-5	5-7	7-9	9-11	11-13	13-15	15-17	17-19	19-21	21-23	23-23′
Temperature stress.....	± 72	± 170	± 291	± 416	± 543	± 677	± 811	± 940	± 1 052	± 1 136	± 1 180	± 1 157
Bottom Chord.	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-22′
Temperature stress.....	∓ 285	∓ 363	∓ 463	∓ 573	∓ 683	∓ 802	∓ 925	∓ 1 049	∓ 1 166	∓ 1 269	∓ 1 349	∓ 1 416
Diagonal.	1-2	3-4	5-6	7-8	9-10	11-12	13-14	15-16	17-18	19-20	21-22	23-22
Temperature stress.....	∓ 184	∓ 196	∓ 192	∓ 183	∓ 190	∓ 194	∓ 192	∓ 181	∓ 158	∓ 119	∓ 66	± 29
Vertical.	0-1	2-3	4-5	6-7	8-9	10-11	12-13	14-15	16-17	18-19	20-21	22-23
Temperature stress.....	± 186	± 208	± 216	± 196	± 168	± 154	± 134	± 108	± 74	± 33	∓ 13	∓ 81

Horizontal reaction, H_t (for ± 72° Fahr.) = 215.460.
Stress in member = horizontal reaction × S_1.
For values of S_1, see Tables 7 and 8.

+ Denotes tension.
— Denotes compression.

APPENDIX B

Financing, and Franchise of the New York Connecting Railroad Company.

A General Statement, Furnished by the Railroad Company.

The scope and cost of this extraordinary work and the great possibilities for transportation service which it opens up, justify a short record for public information as to the principal facts concerning the company under whose powers it was built, its incorporation, and its financing.

The railroad was originally conceived in 1892 by Oliver Barnes and Gustav Lindenthal, Members, Am. Soc. C. E., on practically the same location as that on which it was built.

The Certificate of Incorporation of The New York Connecting Railroad Company was filed and recorded in the office of the Secretary of State of the State of New York on April 21st, 1892, and provided for a steam railroad of about 10 miles in length, the termini of which were to be "in Westchester County, east of the Bronx River, and in the City of Brooklyn."

Among the incorporators were: Oliver W. Barnes, Frank M. Clute, Alfred P. Boller, Charles Macdonald, and Thomas S. King; the capital stock was placed at one thousand shares, par $100 000, all preferred stock.

In April, 1902, The Pennsylvania Railroad Company completed the purchase of the entire outstanding stock of the Connecting Company, and, in accordance with a prior understanding with The New York, New Haven and Hartford Railroad Company, a short time thereafter, sold one-half of this stock to that Company.

On June 11th, 1903, application for the franchise required from the City of New York was made by the Company to the Board of Rapid Transit Railroad Commissioners for that City, predecessor of the present Public Service Commission for the First District of the State of New York, and a tentative franchise was granted by that Board on June 23d, 1904, subject to the approval of the Board of Aldermen and the Mayor. After consideration by the Board of Aldermen for nearly a year, however, the proposed franchise was, on April 18th, 1905, returned to the Board of Rapid Transit Railroad Commissioners "disapproved."

On November 17th, 1905, the application to the Board of Rapid Transit Railroad Commissioners for a franchise was renewed, and, as a result of negotiations extending over a year, the franchise now held

by the Company, dated February 14th, 1907, was granted. This Certificate was accepted by the Company under date of February 28th of that year; was approved by the Board of Estimate and Apportionment of the City on March 8th (by amendments to the City Charter and Rapid Transit Act of the State, the duty and power to confirm franchises of this character had been transferred since the previous application from the Board of Aldermen to the Board of Estimate and Apportionment); was approved by the Mayor on March 14th, and, later, the Certificate was filed in the office of the Secretary of State of New York and in the offices of the Clerks of the Counties of New York, Queens, and Kings.

In addition to fixing the center line of the proposed railroad, the franchise, among other requirements, prescribed that the consent of the owners of one-half in value of the property bounded on the portions of streets crossed by the line, to the construction and operation thereof, should be obtained within one year from the acceptance of the franchise by the Company; that construction was to commence within 3 months after filing said consents with the Rapid Transit Board; and that construction was to be completed and the railroad in operation within 5 years after the commencement of construction. Provision was made in each case for extensions of time under certain conditions. The motive power prescribed to be used was steam, with the right to the Railroad Company to substitute electricity therefor, and, in addition, the Rapid Transit Board reserved the right, in the event of the use of steam constituting a nuisance or becoming dangerous to residents along the route, to require a change to electricity, or other motive power not less convenient to the public, within a period not to be less than 3 years after notice by the Board. The franchise fixes the rentals to be paid by the Company for the first period of 25 years from 2 years after obtaining the required consent of property owners (except for the use of Wards and Randalls Islands, in which case the rental was payable from the date of first occupation) and provides for a readjustment thereof at the end of said period and every 25 years thereafter. Said payments (as amended by order of the Public Service Commission in connection with the shortening of the franchise route of the Company from Knickerbocker Avenue to Fresh Pond Junction, hereinafter dealt with) are as follows: (*a*) For the right to construct and operate across streets and other public property other than Wards and Randalls Islands, $23 925 per annum for the first 10 years of said 25-year period, and $47 850 for the next 15 years; (*b*) $100 per annum for the right to cross the East River between bulkhead lines; and (*c*) for the use of ground on Wards and Randalls Islands occupied, permanently and temporarily, during the

period of construction, by the abutments, piers, and other supports of the bridges and elevated structures, and for the use of portions of overhead space above the islands occupied by such bridges or elevated structures or for any other purpose, such annual payments as agreed upon by the Company and the Board of Commissioners of the Sinking Fund of the City, or other authorities in control of the islands. In addition to said rentals, the Company was compelled to file with the Comptroller of the City, within 60 days of the approval of the franchise by the Mayor, a bond in the sum of $50 000, executed by it and by The Pennsylvania Railroad Company and The New York, New Haven and Hartford Railroad Company, as security for the performance by the Railroad Company of the terms and conditions of the franchise, especially with respect to the annual payments to the City, and had to pay a bonus of $110 000 to the City within 60 days after the necessary consents of property owners had been obtained and before construction work was commenced.

On April 16th, 1907, the stockholders approved of an increase in the capital stock from $100 000, Preferred, to $3 000 000, Common, the holders of the Preferred stock having agreed to exchange their stock, par for par, for Common stock. The increase was approved by the Board of Railroad Commissioners on May 31st, and, as of June 1st, the exchange of Preferred stock was made and $2 816 400 of the Common stock was issued, half and half, to the Pennsylvania Railroad Company and The New York, New Haven and Hartford Railroad Company, to take up a like amount of advances to the Connecting Company. Subsequently, the remainder of the authorized stock was issued, the entire $3 000 000 thereof being held by The Pennsylvania Railroad Company and The New York, New Haven and Hartford Railroad Company in one-half proportions, all of the stock having been fully paid for at par.

The necessary consents of property holders to the construction of the line were filed with the Public Service Commission for the First District of the State of New York on July 20th, 1910, the said Commission having succeeded to the rights and powers of the Board of Rapid Transit Commissioners under the law. The original period fixed by the franchise in which to obtain said consents expired on February 28th, 1908, but, by certificate of the Commission dated February 3d, 1908, and March 5th, 1910, the time was extended to August 31st, 1910.

The period prescribed in the franchise for the completion of the railroad, including the Hell Gate Bridge, expired on October 20th, 1915, but the time was extended by the Commission to December 31st, 1917, and the same date was fixed by General and Special Laws of the State of New York.

The railroad as constructed conforms to the route prescribed in the franchise, except for a slight change of alignment between the north side of Wards Island and 132d Street, in the Borough of The Bronx, a change in grade between Broadway and Calamus Avenue, in the Borough of Queens, and a change in the southern terminus of the line from the Brooklyn Borough line to Fresh Pond Junction. The two changes first mentioned were approved by the Public Service Commission on July 19th, 1912, and maps covering the revisions were filed in New York and Kings Counties on July 23d, 1912, and in Queens County on July 24th, 1912; and the shortening of the franchise route to Fresh Pond Junction was approved by said Commission on June 7th, 1915; approved by the Board of Estimate and Apportionment on July 29th; by the Mayor on July 30th; and a map showing the modification was filed in the office of the Secretary of State of New York on August 13th, 1915, and also in the offices of the Clerks of the Counties of New York, Queens, Kings, and The Bronx. The approval of the War Department to the plans for the Hell Gate, Little Hell Gate, and Bronx Kill Bridges was granted by Certificate dated June 22d, 1906, and the changes in the plans for the two last mentioned bridges necessitated by the change in the route of the road across Randalls Island, hereinbefore mentioned, were approved by Certificate of April 4th, 1912.

In accordance with the requirements of the Franchise, the Company, on May 11th, 1907, submitted the plans for the Hell Gate Bridge to the Municipal Art Commission of the City of New York for its approval. By certificate of June 27th of that year, such approval was deferred, the Commission objecting to the proposed decorative treatment of the towers of the bridge. On May 29th, 1911, the plans were re-submitted to the Commission, the architectural features of the towers having been altered to meet the views of that body, and, by resolution of June 13th, 1911, the necessary approval was granted, a certified copy of which was, on August 7th, 1911, forwarded to the Public Service Commission for file, and receipt was acknowledged by it under date of August 8th.

The cost of constructing The New York Connecting Railroad, over and above the $3 000 000 received from the sale of capital stock, was financed through the issue and sale of bonds covered by a first mortgage, dated May 31st, 1913, executed to the Guaranty Trust Company of New York, Trustee, for $30 000 000, which was approved by the Public Service Commission by Order dated November 14th, 1913. Of these bonds, $24 000 000 have been issued to this date and about $1 500 000 additional will be issued; all are guaranteed, principal and interest, by The Pennsylvania Railroad Company and The New York, New Haven and Hartford Railroad Company.

The present Directors of the Company are:

SAMUEL REA, W. W. ATTERBURY, GEORGE D. DIXON, W. H. MYERS, and A. J. COUNTY.	Representing The Pennsylvania Railroad Company.
HOWARD ELLIOTT, B. CAMPBELL, E. G. BUCKLAND, J. M. TOMLINSON, and A. R. WHALEY.	Representing The New York, New Haven and Hartford Railroad Company.

Mr. Samuel Rea (President of The Pennsylvania Railroad Company) is President of The New York Connecting Railroad Company; Mr. Howard Elliott (President of The New York, New Haven and Hartford Railroad Company) is Vice-President; and Mr. A. J. County (Vice-President of The Pennsylvania Railroad Company) is Assistant to the President of The New York Connecting Railroad.

DISCUSSION

W. H. BREITHAUPT,* M. AM. SOC. C. E. (by letter).—This magnificent structure, well described as of imposing magnitude, adds but another, if not the greatest, to the long list of achievements of its noted designer and chief engineer. The bridge, as likewise the whole New York Connecting Railroad of which it forms a part, will rank among the great things done in American engineering. Mr. Breithaupt.

Mr. Ammann's paper is a valuable addition to the literature of bridge engineering in its record and analysis of the bold design as a whole, and the clear-cut account of details and new departures, among which may be mentioned the 2-in. web plates and make-up of the main compression members, the large rivet sizes, the main arch bottom chord joint detail, the braking resistance, etc.

In the face of such excellence of design and execution, it appears graceless to criticize; however, a few remarks on the general design may be in place.

The dignified and beautiful abutment towers of the main arch are wholly admirable, more especially when compared with the earlier design, Fig. 8, with its suspended sign-board keystone, its meaningless surface elaborations, its flaring base, its lack of form as a whole.

For the Little Hell Gate Crossing, deck-truss, parallel-chord spans, as at first designed, but ruled out by the War Department on account of navigation requirements, would have made the most fitting structure. Arches are not discussed. It is not clear why a two-hinged, spandrel-braced arch design, or two-hinged, braced, double-rib arches, should not have been used. A low-spring, high versed-sine arch design would have effected a considerable saving in masonry with little or no addition in superstructure weight. Aside from the floor support struts, high at the ends, the superstructure frame could have been of less weight. Erection could have been balanced outward from the piers—tied from end abutments—the material floated to position underneath and hoisted, with only a few bents under end-panel points each way, thus effecting a large saving in expensive falsework. The skew interference could have been taken care of as well as, or better than, with the bowstring trusses. The bracing could have been better. The bowstring spans have no lateral bracing along the bottom chords; the stiff bracing between the trusses, although without bottom transverse struts, is, in the description of the main arch bracing, rightly deprecated as tending to distortion under one-sided loading. Expansion would have been taken care of separately for each span. The relative movement of $\pm$ 6 in. at the center pier of the bowstring spans, as described, is unduly large. With arches, the high clearance for navigation would have been at the center of the span, with plenty of room for lateral

* Kitchener, Ont., Canada.

Mr. Breithaupt. movement of tow, instead of danger of crowding against the piers, as with the present spans. On the other hand, the present spans, for the greater part of their length, give fair clearance for ordinary navigation. For navigation, in fact, as well as for general appearance, three arches for the crossing would have been the preferable design. Lastly, there is the reason, the more deserving of consideration in so monumental and visually prominent a structure, of harmony of general design. The inverted arch effect of the bowstring girders, so near the main arch, involves something in the nature of mental gymnastics.

The gate-post design of the towers marking the ends of the Little Hell Gate Crossing is not entirely satisfactory, either in photograph or to the eye *en plein*. A conceivable purpose would have been as shelters and material or implement stores for watchmen, sentries, or workmen, and a design, lower while retaining massiveness, would have been in keeping.

Mr. Moisseiff. Leon S. Moisseiff,* M. Am. Soc. C. E.—This paper is of unusual importance in the literature of bridge building, and deserves attention and study. It occurred, under a fortuitous constellation, that a first-class railroad, provided with ample financial resources, undertook to build a short connecting line in the metropolis of the Western hemisphere. It, of course, decided to build the new line in a first-class manner; and, as part of the line happened to require the erection of a long-span bridge capable of carrying a very heavy traffic, the management of the road, in its wisdom, selected a first-class bridge engineer with a broad vision and decided scientific leanings. Adding the opportunity of ample time to conceive, to plan, to compare, to design, and to re-design, and again adding to the plans and specifications the resources in mind and matter of one of the greatest bridge manufacturing concerns of the world, the resulting product is such that American engineers may be proud of the achievement. The longest arch bridge in the world, having a span of nearly a thousand feet, has been planned and built so as to produce an imposing and pleasing structure, well proportioned in outlines as well as in details.

The paper is full in its description and, as it takes up successively the consideration and treatment given to the design in general and to its details, numerous points are brought out worthy of commendation and discussion. Many of the vital points of bridge engineering are touched, too many indeed to be given the discussion they deserve.

The comparative designs of the several types of bridges, all with a central span of 850 ft., are most instructive. The stiffened suspension type with eye-bar chains has shown off remarkably well, for a span comparatively small for a suspension bridge, such bridges being best adapted for the longest spans. The good showing made by this type is

* New York City.

due to the eye-bar design. If built, it would have resulted in a fine and economic bridge. Somehow, from reading the paper, one obtains the impression that if the difficulties encountered in one of the abutment foundations had been known at the time of selecting the type, the suspension bridge with a longer span, and not the arch, would have been chosen. Mr. Moisseiff.

The difficulties encountered in the foundation of the Wards Island abutment were unusual, and, if known, the foundations would certainly have been avoided. The appearance of the masonry towers flanking the steel arch is good, but their use, to restrict the size of the foundation and by their weight to obtain a steep resultant to pass within the middle third of the base area, can hardly be claimed to be economical.

Very properly the bridge was erected as a three-hinged arch for most of the dead load and was made two-hinged for moving load and temperature only. In connection with this the author remarks: "This, however, is a convenience in erection rather than advantage as regards stress action." This statement is true where the abutments are absolutely unyielding, but, where any limited settlement may be apprehended, erecting the arch first as three-hinged has a more substantial advantage than mere convenience of erection. It has the purpose of eliminating stresses caused by yielding, which stresses may become uncomfortably large. In that case it is good policy to leave the arch three-hinged for at least a year, so that all probable settlement and yielding may have taken place and be eliminated, and, after that time, when no further yielding is observed, to make the arch two-hinged in order to obtain the advantage of increased stiffness.

Making the bottom chord a massive rib sustaining much of the dead load is excellent designing and testifies to the engineer's clear conception of the stiffening function of the truss members of a braced abutment without straining the remaining truss members. This is good mechanics as well as economic design.

The floor lateral truss and the braking girders are additional instances of well-considered design.

The use of 2-in. plates in compression members is extraordinary. The claim made that metal of such thickness showed in tests the same elasticity and ultimate resistance as $\frac{1}{2}$-in. material is contrary to general experience. If the manufacturers were able to roll, of the same material, 2-in. plates of strength equal to the usual structural sizes, it would prove of great interest to the engineering world. Hitherto it has been held that the heavier plate receives less work and shows considerable decrease in strength. Should this claim be substantiated, the use of stitch rivets would decrease considerably, and heavy plates would come into use extensively for columns. This is a matter of much practical importance, and deserves a full discussion by the steel makers.

Mr. Moisseiff.

The "Rules of Design" laid down by Mr. Lindenthal are so interesting that the speaker regrets that they have not been appended in full to the paper.

The three-face joint of the bottom chord is a novel feature in compression chord bearing, and is a move in the right direction to reduce secondary stresses caused by erection deflections. High edge stresses are thereby avoided. The speaker, however, is of the opinion that the concentration of stress on the middle portion should be taken care of in the allowable unit stress. In other words, the computed excess stress should be subtracted from the specified unit stress for the compression members.

The principle adopted by the designer for loads and unit stresses is the right one for long-span bridges. To take into consideration all possible forces and causes, including even an allowance for secondary stresses, and then to fix a proportionately high unit stress is a step in the direction of scientific approximation of actual conditions in the bridge. Of course, it will require much study and consideration to arrive at a final judgment as to the proper unit stress, but, considering all factors, a nearer approach to true conditions will be attained. At the initiative of Mr. Lindenthal, this principle has been partly applied to the Queensboro' Bridge and entirely to the Manhattan Bridge, and now has been adhered to in the Hell Gate Arch. He deserves credit here for his clearness of vision.

A step out of the ordinary is the proportioning of compression members on the basis of net area, deducting the rivet holes. Good reasons for the unusual procedure are brought forward in the paper. The matter, however, is of such extraordinary importance, affecting the design and the economy of all steel structures, that it requires careful research and exhaustive testing to adjust and co-ordinate present standards and specifications. The speaker hopes that the Society's Special Committee on Steel Columns and Struts will give the matter the consideration it deserves.

The speaker has attempted to touch only a few of the most important points in the paper. Many other interesting innovations which it contains are deserving of full discussion.

The paper is well presented, and full of information, and Mr. Ammann deserves the thanks of the Profession for his work.

The Hell Gate Arch Bridge is an excellent example of what engineering genius can accomplish if the project is entrusted to one mind to plan and direct, unhampered by red tape and lay commissioners. The bridge reflects credit on the Engineering Profession, and much praise is due to Mr. Lindenthal and his able associates and co-workers, as well as to the engineers of the American Bridge Company. They have done well.

Mr. Wagner.

SAMUEL T. WAGNER,* M. AM. SOC. C. E. (by letter).—The thanks of the Profession are due to Mr. Ammann for the admirable manner in which he has presented the data of the conception, design, and execution of this most monumental structure. It is possible to find answers to nearly all the questions which arise in its careful reading. This is quite unusual in a work of this magnitude and character.

The use of plates 2 in. thick in built-up members is so out of ordinary practice as to attract special attention. It is hoped that, in closing, Mr. Ammann will see fit to give some of the actual tests of this material. The writer's experience has been that such thicknesses are likely to produce material which is not wholly reliable. Years ago, plates 1 in. in thickness of satisfactory quality were difficult to obtain, especially before the use of open-hearth steel was general. If Mr. Ammann could give the details of the manufacture of these plates, it would be specially interesting. If it is possible to obtain such sections of good quality, even at slightly increased cost, it will open the way for many details which have not been considered possible up to the present time.

The details of the latticing are interesting. There are many advantages in the use of stiff latticing on members of any considerable size. Many members are designed with such light and inadequate lattice bars that even the most careful handling in the shop and during erection results in damage to them; and, frequently, splendid work in the shop is spoiled before it is placed in the structure. The spacing and general arrangement of the latticing are also to be commended.

The facing of the ends of such large members is a work that requires unusual care in the shop. There can be nothing more important than proper bearing in compression members, and the machine work required, in the writer's opinion, is one of the most difficult parts of the shop fabrication that has to be done on such a structure. It would appear, from the stress measurements made on the completed structure and given in the paper† by Mr. Steinman, that this part of the fabrication is specially commendable on the part of the manufacturers. The details of the joints in the bottom chord are most unusual and interesting, and indicate the great care which was taken in planning them. The results undoubtedly confirm the wisdom of the design.

The method of drilling the main members by punching a limited number of holes, using tack-bolts, and drilling the majority of the holes through the solid, is good practice. There is no doubt in the writer's mind, judging from his past experience, that this is the proper way to do work of this character, not only from the standpoint of doing good

* Philadelphia, Pa.

† *Transactions*, Am. Soc. C. E., Vol. LXXXII, p. 1040.

Mr. Wagner.

work, but also of doing it in the most economical and workmanlike manner. It is specially true for the higher carbon steel which was used.

The assembling of the arch in sections is also another most excellent detail of the fabrication. It would be interesting to know whether this was a requirement of the specifications. For arch work, it is doubtful whether such details could have been fabricated satisfactorily in any other way. The writer's first experience on work of this character was on a very small scale in connection with the three-hinged arch trusses of the train-shed of the Reading Terminal in Philadelphia.* In this case, the complete half arches were assembled in the shop. If this had not been done, there surely would have been bad workmanship and difficulty in the erection. It is a great comfort to any manufacturer to know that the joints thus assembled are sure to match. This is especially true in the case of the Hell Gate Arch, with its difficulties in erection.

The use of ballasted floor construction is to be commended, although its cost is greater than the other types referred to in the paper as having been considered. The reasons given for its adoption are well expressed, namely, "more uniform roadbed, less noise and impact, smaller cost of maintenance, and greater safety in case of derailment or fire." Although it seems to be difficult to show in actual figures the desirability of such construction from a financial standpoint, there can be no doubt that under the conditions in this case the additional cost was fully warranted. The advantage of having a standard track in place of special ties is important. Experience in viaduct construction in which a solid floor was used has shown conclusively that it is the best known detail for the prevention of noise. In a number of lawsuits, in which damage from noise has been claimed, no attempt has ever been made to produce proof. It is rather unusual at this time to see a floor of this type designed so that there is sufficient steel to carry the loads without any allowance for the use of the embedding concrete. It is a detail, however, of which the writer approves, especially in such a thin floor. He would have gone one step further, however, in that the floor would have been water-proofed in order to give additional durability to the construction, and it is believed that the additional cost would have been fully justified. If, by some means, water gets through the concrete and to the beams, a condition is created which is very unpleasant to contemplate. After traffic has once been placed on a structure, the difficulties of making any repairs to the floor are very serious.

The officials of the New York Connecting Railroad are to be congratulated on having the foresight and financial courage to design and construct this bridge for four tracks instead of two. It is a monument to all concerned.

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* *Transactions*, Am. Soc. C. E., Vol. XXXIV, p. 181.

Mr. Fowler.

CHARLES EVAN FOWLER,* M. AM. SOC. C. E.—The speaker's acquaintance with Mr. Lindenthal and his work began about 30 years ago, and the genius displayed in his designs for the Seventh Avenue and Smithfield Street Bridges, in Pittsburgh, led one to expect the design by him of some great structure like the Hell Gate Arch. The speaker hopes that Mr. Lindenthal may live to see an even greater culmination of his career, by the building of the great suspension span proposed for the North River.

The Seventh Street Bridge, in Pittsburgh, built in 1884, is a two-span, eye-bar, suspension bridge, with braced parallel chains, and was a milestone in the design of suspension bridges. The two main spans are 330 ft. each, and the side spans 165 ft. each, with a width of 42 ft.

The Smithfield Street Bridge, which is still in use, is a very striking structure, consisting of two Pauli truss spans, the longest and heaviest of this type in America.

That the younger members of the Profession may not forget to have pride in American bridge engineers, it is well to call to mind the work of the Dean of this branch of the Profession, the late C. Shaler Smith, M. Am. Soc. C. E. His Kentucky River High Bridge was a wonderful structure, with three 375-ft. spans, 275 ft. above the river, a continuous-span bridge, which, by the cutting of the chords to fix the points of contraflexure, became one of the first great cantilever bridges. The four river spans of his Lachine Bridge, over the St. Lawrence River, were at the same time an example of the most scientific bridge engineering and an altogether striking and beautiful structure. The main bridge had two 408-ft spans, with two 269-ft. side spans.

When this structure required rebuilding, within the last few years, in order to carry modern heavy loading, it was with regret that we saw that its outlines were not preserved. When Mr. Lindenthal was called on to replace High Bridge with a heavier structure, a few years ago, the speaker, for one, felt that a vote of thanks was due to him for preserving the original outline. His work on the bridges of New York City, although, unfortunately, hampered in many ways, is reflected in the completed Hell Gate Bridge, which is the first wholly creditable structure to be built here since the completion of the old Roebling Bridge.

To the speaker the outline of the structure is most pleasing. This is due not only to the smooth curve of the arch proper, but mostly to the reverse curve of the top chords near the towers. The combination resembles, and is as pleasing as, a similar combination of curves in the "Camelback Bridge" in the Imperial Palace grounds at Pekin, China. To the speaker, the latter is the most pleasing of Oriental bridges.

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* New York City.

Mr. Fowler.

The sickle-shaped truss design, which was abandoned for the Hell Gate Arch, for reasons having to do mainly with erection, could hardly have been more pleasing in its architectural appearance.

The towers as built are plain and simple in design, but are in every way appropriate and in harmony with the great arch. The speaker's reason for calling attention to this is the hope that some artistic person in the New York City government or the Art Commission may some day succeed in getting the parapet, or "attic story" as it might be termed architecturally, added to the towers of the Roebling Bridge, as originally planned.

Many great bridges have been marred architecturally by the appearance of the approaches, but when one visits this structure at Hell Gate and views the great masonry approach piers in perspective, their pleasing and appropriate character is fully appreciated. The whole structure may be said to be the most complete lesson in bridge esthetics in America, and one of the best in the world. The proper consideration of simplicity, symmetry, harmony, and proportion in bridge design was first enunciated by the speaker in "Engineering Studies", and, as it has been elaborated in many subsequent writings and discussions, he will take time only to repeat that the failure to regard any one of these four fundamentals must lead in some degree to failure.

The Dusseldorf and Bonn Bridges are both notable examples, which have been referred to by Mr. Ammann, but the Bonn design, especially when the bridge is seen in perspective, comes nearest to fulfilling all requirements, including one other, that of an unequal number of spans, the main span of 614 ft. and the side spans of 307 ft. There are so many notable arch bridges in France, both of masonry and steel, that it would be presumptuous in a limited discussion to do more than call to the attention of designers this great field of study. The great arch span at Oporto, with both upper and lower roadway, is perhaps the most striking of any of the examples mentioned by Mr. Ammann.

The great 840-ft. Clifton Arch at Niagara is one of the best examples in America, but it is with regret that one views the very inappropriate approach spans of the wonderful Grand Trunk Arch, its near-by neighbor. The Eads Bridge, across the Mississippi at St. Louis, with its three great steel arch spans of about 500 ft. each, with simple, appropriate, and well-designed masonry approaches, answers all the basic requirements of an artistic design.

There are five designs of the Quebec crossing which should be preserved in the final monograph of that great bridge. First, the cantilever design of many years ago, by Brunles and Light, a 1 442-ft. span for which much of the credit was due to Professor Fidler. Second, the 1 800-ft. cantilever which failed, due to neglect to observe fundamental features of design for compression members, which had been discussed previously by the speaker's one-time assistant, the late Pro-

Mr. Fowler.

PROPOSED LAYOUT FOR A
CANTILEVER BRIDGE OVER SAN FRANCISCO BAY.

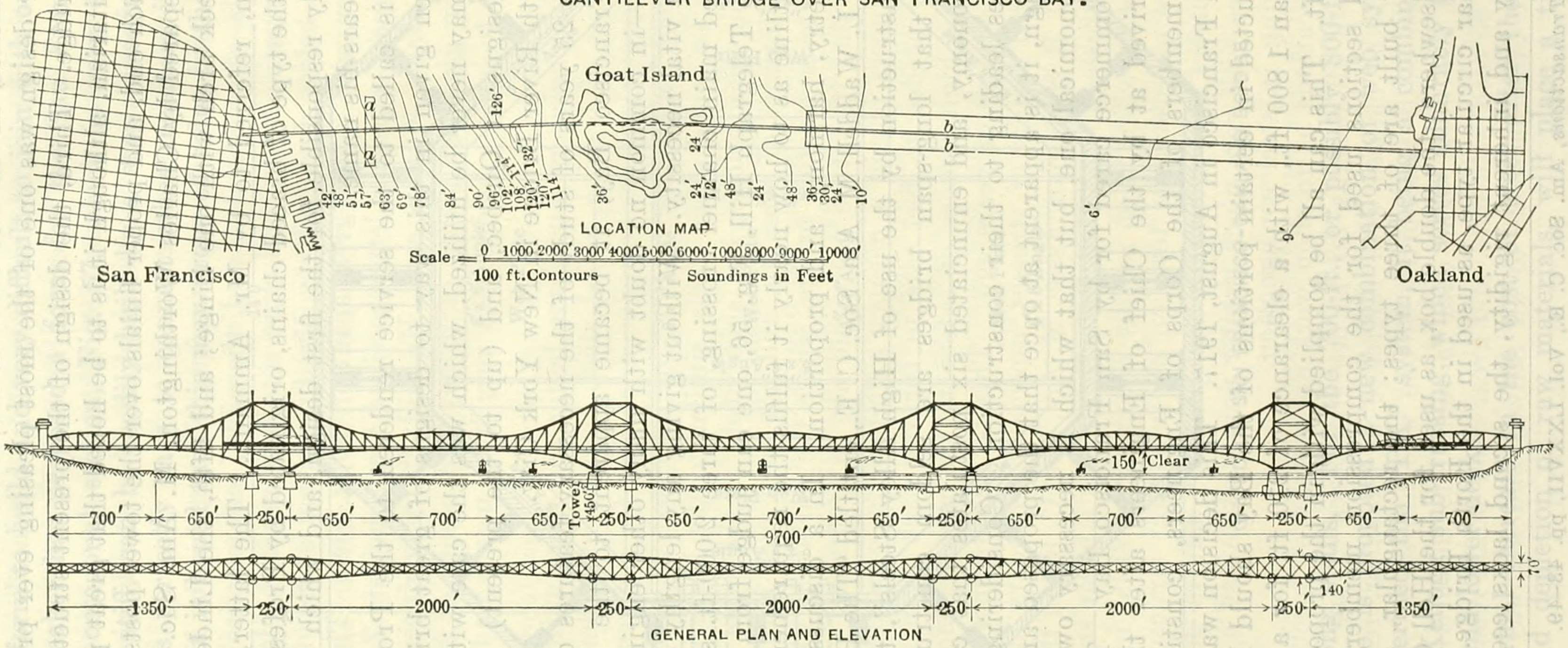

FIG. 56.

Mr. Fowler.

fessor A. H. Heller, in his "Stresses in Structures". The outline of this cantilever design was one of the most pleasing ever prepared for the Quebec Bridge. Third, the design of the present structure, which is strictly utilitarian, although it is to be hoped that great pylons will be added at each end, and proper finials over the tower posts. Fourth, the design prepared by Charles Worthington, M. Am. Soc. C. E., for an 1 800-ft. deck arch with one hinge; and fifth, the Lindenthal suspension design, referred to by Mr. Ammann. The latter, curiously enough, is of the type of braced chains, originated by Professor Fidler, who was partly responsible for the first design, and which suspension type usually bears his name.

Attention is called to the service rendered to the Profession by the study often given in this way to designs of great bridges, even though they may never be utilized, which was the case with three of the notable designs at Quebec, and (up to the present) is true as regards a North River Bridge in New York City.

After some 25 years of study of the necessary features of a bridge across San Francisco Bay, it became apparent to the speaker, a few years ago—in common no doubt with many other engineers—that a bridge was a vital necessity. Without giving any lengthy description of the proposed main channel crossing, of three 2 000-ft. spans from Goat Island to Telegraph Hill, Fig. 56, one can judge from an inspection of the outline as to how nearly it fulfils the requirements of simplicity, symmetry, harmony, and proportion. In a discussion of the paper by J. A. L. Waddell, M. Am. Soc. C. E., entitled "The Possibilities in Bridge Construction by the use of High-Alloy Steels", the speaker pointed out* that long-span bridges are seldom constructed from motives of economy, and enunciated six postulates that covered the basic principles leading to their construction. Considering this San Francisco design, it is apparent at once that the proposed arrangement is not the economical one, but that which is necessary owing to the great ocean commerce cared for by San Francisco Bay. This is the conclusion arrived at by the Chief of Engineers after the hearing before three members of the Corps of Engineers, constituting the Board, at San Francisco in August, 1917. This decision was, that any bridge constructed in certain portions of the Bay should have spans of not less than 1 800 ft., with a clearance of 210 ft. for a horizontal width of 800 ft. This can all be complied with in the proposed design.

The chord sections used for the compression members of great spans already built are of three types: the rectangular, as used at Quebec and elsewhere; the double box, as used for the Hell Gate Arch; and the cellular circular type, as used in the Forth Bridge. The first lacks economy and inherent rigidity, the second lacks economy, but

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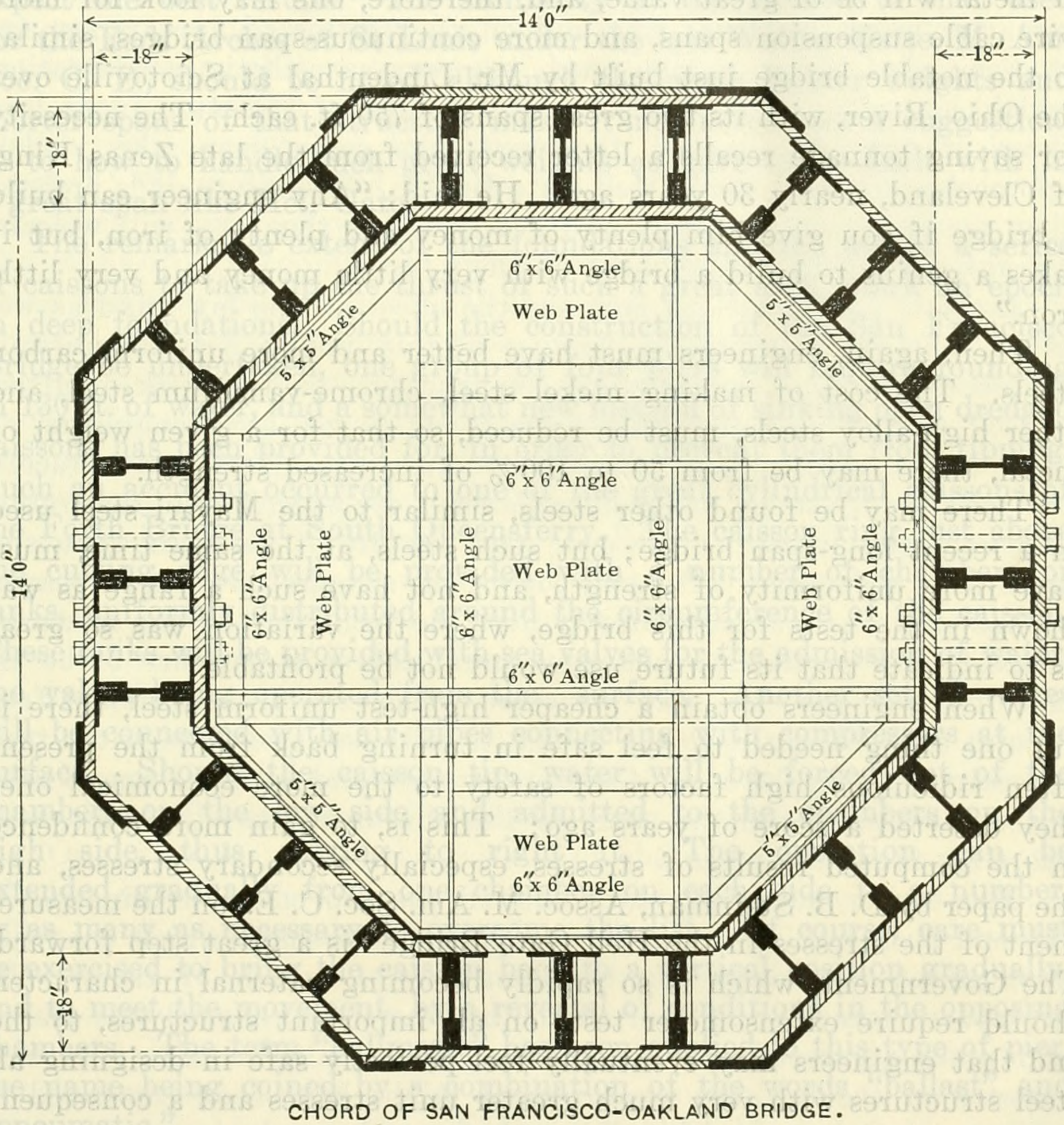
* *Transactions*, Am. Soc. C. E., Vol. LXXVII, pp. 43–49.

the third seems to lack nothing, theoretically, but practically is a very expensive form to build and very difficult to join with other members. Mr. Fowler.

The speaker has proposed a form for the San Francisco Bridge, a cellular octagonal section, Fig. 57, which is very economical, is inherently stiff, can be readily built by a shop in segments, and to which it is perfectly easy to connect other members in an efficient manner. It is hoped that this may prove of value in the construction



CHORD OF SAN FRANCISCO-OAKLAND BRIDGE.

FIG. 57.

of many long spans. For the 700-ft., or possibly 800-ft., suspended spans, there is proposed a new combination of web systems, which has been termed the "PK" system, it being a combination of the Petit and Kellogg sub-trussing, which serves admirably to carry both the upper and lower decks.

The normal increase in the cost of steel after the war, owing to the rapid depletion of raw materials, will place a time limit on

Mr. Fowler.

the period within which great spans may be constructed, and, in a conversation with Mr. Lindenthal recently, he pointed out that this period would not be likely to extend beyond 30 years, or, say, 1950. Therefore it behooves engineers, not only to be more conservative in the use of steel, but also to be less prodigal; they should plan in every way to extend the time of depletion.

The use of such types of structures as will require the least weight of metal will be of great value, and, therefore, one may look for more wire cable suspension spans, and more continuous-span bridges, similar to the notable bridge just built by Mr. Lindenthal at Sciotoville over the Ohio River, with its two great spans of 750 ft. each. The necessity for saving tonnage recalls a letter received from the late Zenas King, of Cleveland, nearly 30 years ago. He said: "Any engineer can build a bridge if you give him plenty of money and plenty of iron, but it takes a genius to build a bridge with very little money and very little iron."

Then, again, engineers must have better and more uniform carbon steels. The cost of making nickel steel, chrome-vanadium steel, and other high-alloy steels, must be reduced, so that for a given weight of metal, there may be from 50 to 100% of increased strength.

There may be found other steels, similar to the Mayari steel used in a recent long-span bridge; but such steels, at the same time, must have more uniformity of strength, and not have such a range as was shown in the tests for this bridge, where the variation was so great as to indicate that its future use would not be profitable.

When engineers obtain a cheaper high-test uniform steel, there is but one thing needed to feel safe in turning back from the present often ridiculous high factors of safety to the more economical ones they deserted a score of years ago: This is, to gain more confidence in the computed results of stresses, especially secondary stresses, and the paper by D. B. Steinman, Assoc. M. Am. Soc. C. E., on the measurement of the stresses in the Hell Gate Bridge* is a great step forward. The Government, which is so rapidly becoming paternal in character, should require extensometer tests on all important structures, to the end that engineers may eventually feel perfectly safe in designing all steel structures with very much greater unit stresses and a consequent enormous saving in steel consumption.

The specifications as to shop work are worthy of special study in connection with all designs for future structures of any importance whatever, and the clauses with reference to punching, reaming, and drilling are especially noteworthy to one who has spent years in the fabrication of steelwork. The data in regard to the driving of long rivets should be read with care, and great good would result from the application of these methods to all railway and highway bridges of

* *Transactions*, Am. Soc. C. E., Vol. LXXXII, p. 1040.

Mr. Fowler.

importance. Unless the engineer has had occasion to rebuild some old bridges, he cannot realize how criminally careless the average bridge erector is in driving all rivets with a long grip.

This subject is of such great interest that it is impossible to close without congratulating all those having to do with the erection of the Hell Gate Arch on the unusual success of all the operations, although the care exercised in planning every step would lead one to expect the best results. The similarity of the method to that used for the Eads Arches at St Louis under the late Walter Katte, M. Am. Soc. C. E., should be noted, although the much lighter weights and shorter spans of that structure make it no more than a suggestion, as to how to handle such great weights as have to be dealt with in a great span like Hell Gate.

The remarkable extent of the foundations and the use of a series of caissons to take up the thrust of such a great arch, mark an epoch in deep foundations. Should the construction of the San Francisco Bridge be undertaken, one group of four piers will require founding in 130 ft. of water, and a somewhat new method of sinking open dredged caissons has been provided for, in order to prevent them from tipping. Such an accident occurred to one of the great cylindrical caissons of the Forth Bridge at South Queensferry. The caisson ring just above the cutting edge will be provided with a number of chambers or tanks, uniformly distributed around the circumference of the caisson. These tanks will be provided with sea valves for the admission of water, the valves being operated from the surface. Another set of valves will be connected with air pipes connecting with compressors at the surface. Should the caisson tip, water will be forced out of the chambers on the low side and admitted to the chambers on the high side, thus serving to right it. The operation can be extended gradually from one chamber on each side to a number, or as many as necessary to overcome the list. Of course, care must be exercised to bring the caisson back to a vertical position gradually, and to meet the movement by a reversal of conditions in the opposing chambers. The term "ballmatic" has been applied to this type of pier, the name being coined by a combination of the words "ballast" and "pneumatic."

The speaker realizes that papers having such a wide scope as these, describing this great bridge, and the stress measurements of the members, will call forth a very extended discussion, and that lack of space will require their curtailment. Thus, many features that could be profitably considered will have to be omitted, and the speaker will close in the hope that others will cover many other points of value, and of vital importance for consideration, in the design of future long spans.

Mr. Quimby.

HENRY H. QUIMBY,* M. AM. SOC. C. E.—In the presence of the magnificent achievement of this imposing monument, the product of the very highest grade of technical and practical skill in the world, through years of study under apparently ideal conditions, with *carte blanche* as to time and cost, and free play for fancy and cultivated taste, one hesitates to introduce a note of comment that may even seem like criticism. In the published writings on the work, however, in the paper as presented, and in Mr. Lindenthal's supplemental oral remarks, much emphasis was laid on the matter of appearance, and it seems that it was not so much economy as esthetic considerations which controlled in the selection of the type of structure in at least three of the divisions of the work—the main span, the viaduct approaches, and the Little Hell Gate Bridge. Wherefore, it is probable that the designers, masterly and successful as they are, will welcome a frank expression of judgment on their taste.

Esthetic considerations embrace the effect on the lay mind as well as on that of professionals; because of this, public Art Commissions are created to pass on technical construction—on the work of artists as well as that of mechanics—of architects as well as of engineers. It is desirable, therefore, to make a structure appear not only graceful but satisfyingly stable to the miscellaneous eye as well as to the trained and understanding scientific eye. One feature of the main span suggests these reflections. The end post of the spandrel bracing in bearing on the end pin is almost directly over the front face of the abutment support, and to the miscellaneous mind this post appears to be the end of the bridge—the span structure. The effect, when viewed from a point at right angles to the bridge, is to make it seem as if the truss is resting just on the edge of the bridge seat, and with the considerable space between it and the shaft of the abutment pier, the truss looks short for the span opening.

The earlier published plans showed the top chord terminating at the end post, leaving a gap between it and the shaft, which condition accentuated the impression of scantiness of length of truss. The subsequent extension of that chord into the shaft relieved this feeling and much improved the appearance. As a false member was thus inserted at the top, the improvement might have been continued by adding a panel, or a false diagonal, from the end pin to the shaft, but as the end post has no necessary relation to the end pin in its present location—which evidently was fixed to secure adequate distribution of bearing on the skewback—it would seem to have been better to reduce the gap by moving the post 3 or 4 ft. closer to the shaft, so that its direction would intersect with the direction of the thrust of the end lower chord member at a point nearer the skewback, and then give it a separate pin. This would not complicate the determination

* Philadelphia, Pa.

of stresses, and it would give the structure the appearance to everybody of adequate length and bearing on the bridge seat. The length of each panel and the theoretical length of the span would thereby be increased very slightly, but the angle of thrust is so steep that the stresses in the arch would probably not be increased sufficiently to constitute an objection to the change. Mr. Quimby.

A view of the approach piers during their construction, and before any of the steelwork was on them, gave some the impression of disproportionate massiveness compared with the spaces between them, and this feeling still continues with the superstructure in place. Would not such height and volume of masonry, in the case of the higher piers, more economically and more pleasingly support longer and heavier spans? We are not told whether the pier spacing was determined by economy of proportion between superstructure and substructure, or by right-of-way conditions. The exhibit of alternative designs with steel bents in itself justifies the type adopted and used, for the rocker bents—absolutely without a base in the side elevation—would give a very unpleasant feeling of instability. The ordinary public observer would recognize at once that somewhere something must be provided to make up for the palpable deficiency in longitudinal stability of the individual bents, and the Art Commission's objection to the steel designs shown is, of course, clear to all; but, was a trial made of a design of tapering steel towers—steel bents with buttresses—each stable in itself against longitudinal as well as transverse forces, and, if so, how did it compare in cost, including the capitalized cost of maintenance? The paper gives the reasons for not adopting concrete arches for the approaches, and the soundness of such reasons will probably not be questioned, but the appropriateness of that type of construction there has probably occurred to almost every interested observer.

One other feature of the work is of character and proportions that do not accord with the taste of some students of bridge architecture. This is the bridge over Little Hell Gate. The inverted bowstring trusses introduce a note that, although not really discordant, seems very outstanding, and, in dipping down so close to the water, they give the impression of superstructure depth disproportionate to the whole-structure height. The depth of construction at mid-span is quite appreciably more than half of the whole height of the structure above the water. The paper states that the trusses were made of the bowstring type so as to give additional clear height near the piers for vessels to pass—a somewhat unusual provision for navigation which generally prefers to give channel piers a wide berth. As the foundation here was on rock at a reasonable depth, it would seem that arches of steel, or even of concrete, might have been adopted as consistent and pleasing, and less obstructive to navigation. Was an arch design investigated, and, if so, how did it compare with the bowstring?

Mr. Quimby.

The design of the Bronx Kill Bridge, with inclined end posts at the abutments and vertical ones at the pier, suggests the case of more than one bridge where an original single span has been converted into two spans by building a pier under its middle after it had become inadequate. Presumably, the heels or counterweight trusses (the design of which is not found in the paper) will, when placed, remove the present disparity, but, as that may not be for many years, would it not have been worth while to make the trusses symmetrical in outline during the interim by inclining the pier ends, which is a common practice for the outer ends of bascule leaves.

The paper is an unusually satisfying one, both in the fact that it appears while the public and the professional interest in the remarkable feat is still fresh, and in that it discusses so freely the reasons for the various features of the design. The oral presentation of the subject by the author was also exceptionally felicitous, summarizing and supplementing the paper rather than repeating it by reading word for word, as is too often done with preprinted papers.

Mr. Seaman.

Henry B. Seaman,* M. Am. Soc. C. E. (by letter).—This description of the largest, the most scientific, and, it is believed, the most artistic bridge yet constructed, has been so complete that anything added might seem to be a presumption. It is probably the first large construction wherein was adopted the new method of proportioning, by which a static unit strain is allowed and the excess due to the action of the live load is provided for entirely by an impact formula. This method of proportioning is the most rational and the most scientific which has yet been used, and receives its ultimate interpretation in the present structure. After investigating the subject very thoroughly, while acting as Consulting Engineer to the Department of Bridges, New York City, the writer, later, took occasion to present the result of such study to the Society in 1912.† It seems to be but a progressive step from that which was first instituted some thirty years ago by the late Charles C. Schneider, Past-President, Am. Soc. C. E.

The description of the higher scientific characteristics of this structure have been thoroughly outlined by Mr. Ammann, but it might not be amiss, in a short space, to mention the methods adopted in erecting the tall masonry piers of the approach viaduct.

The foundations of these piers were not unusual in any way, but, being of reinforced concrete, of an extreme height of about 125 ft., and in a continued series about 90 ft. apart, the method of constructing them rapidly necessitated some new thought, or, as might be better expressed, new application of old principles.

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* Washington, D. C.

† "Specifications for the Design of Bridges and Subways", *Transactions,* Am. Soc. C. E., Vol. LXXV, p. 313.

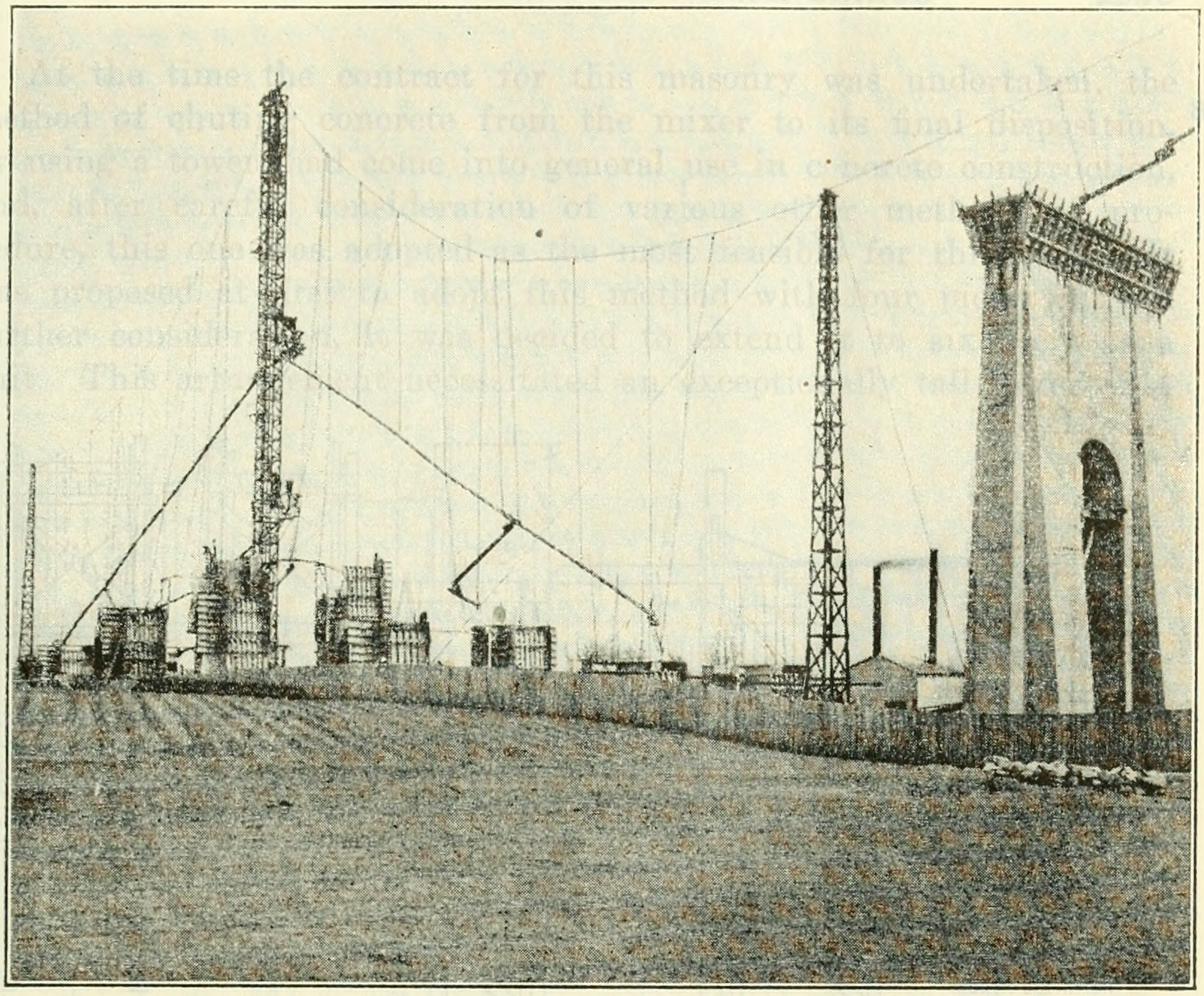


FIG. 58.—GENERAL ARRANGEMENT FOR CHUTING CONCRETE.

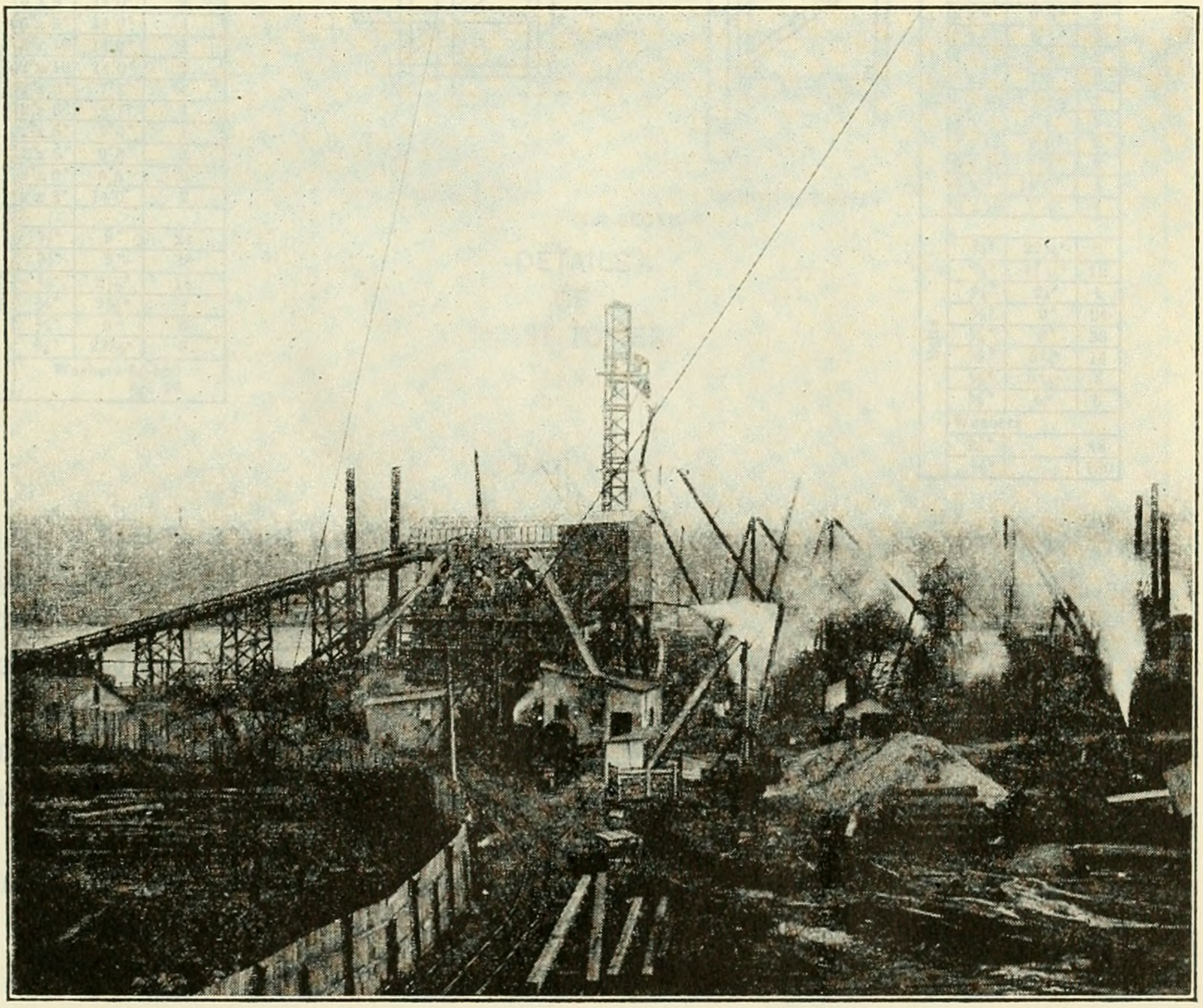


FIG. 59.—WARDS ISLAND UNLOADING PLANT.

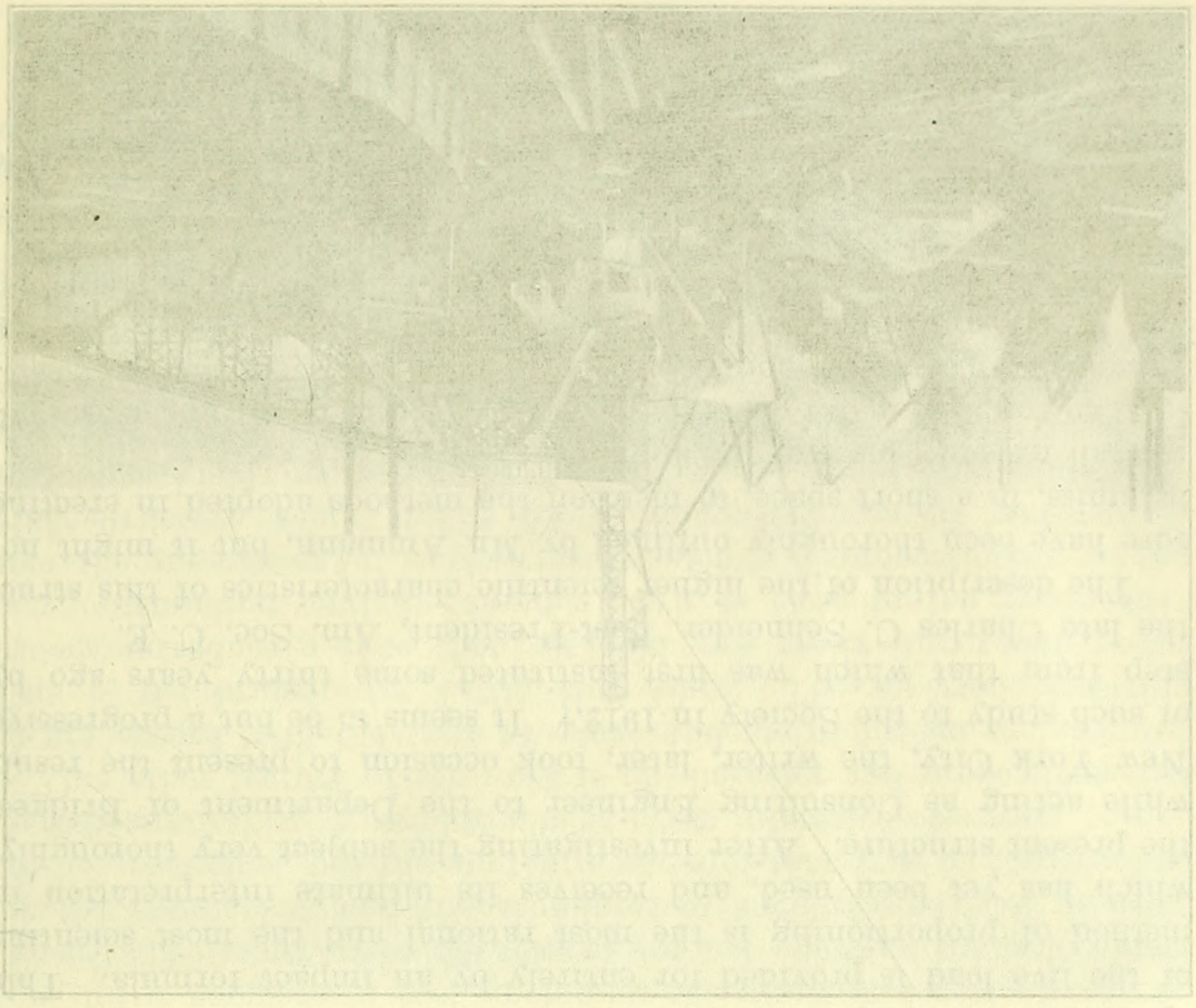
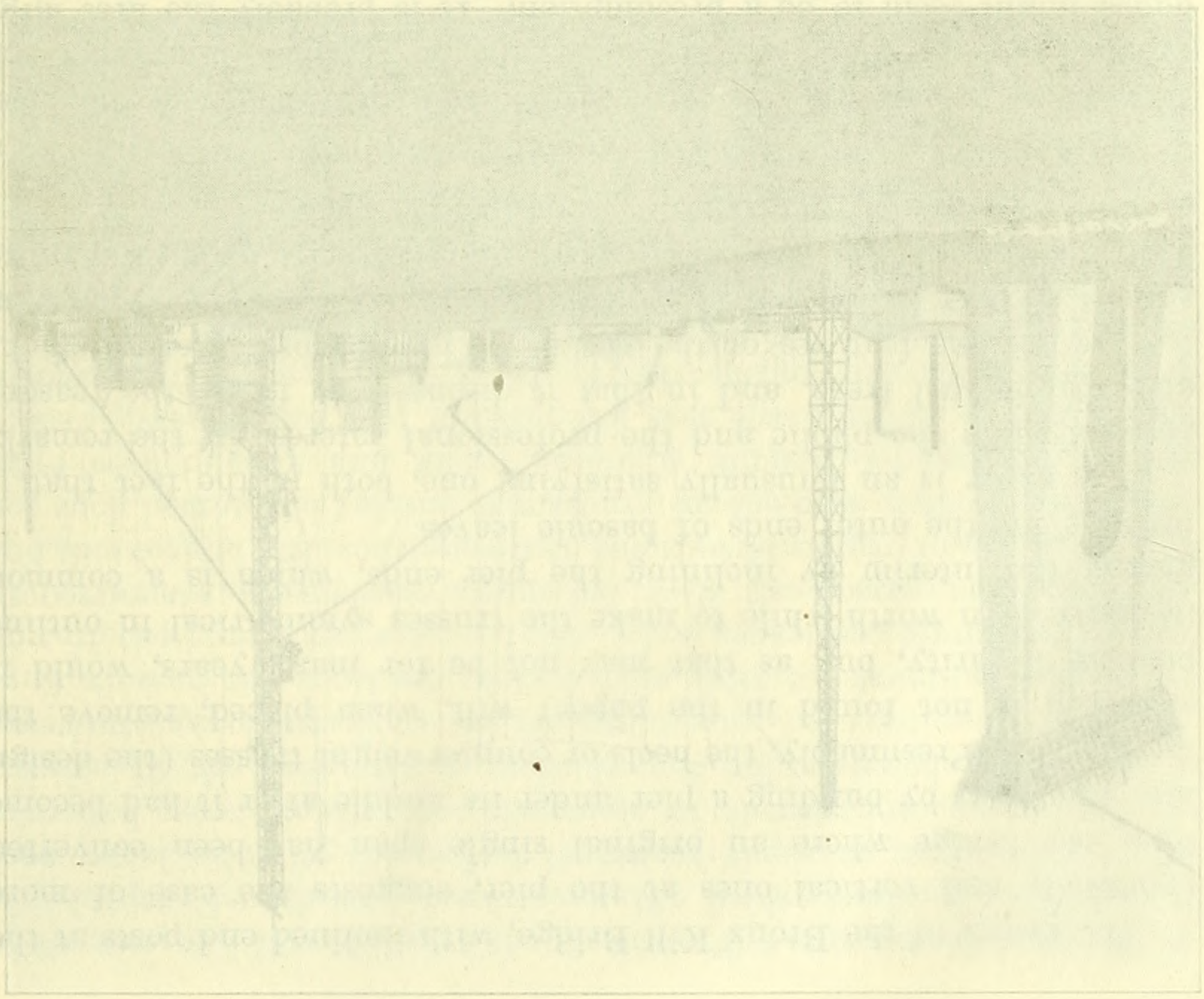
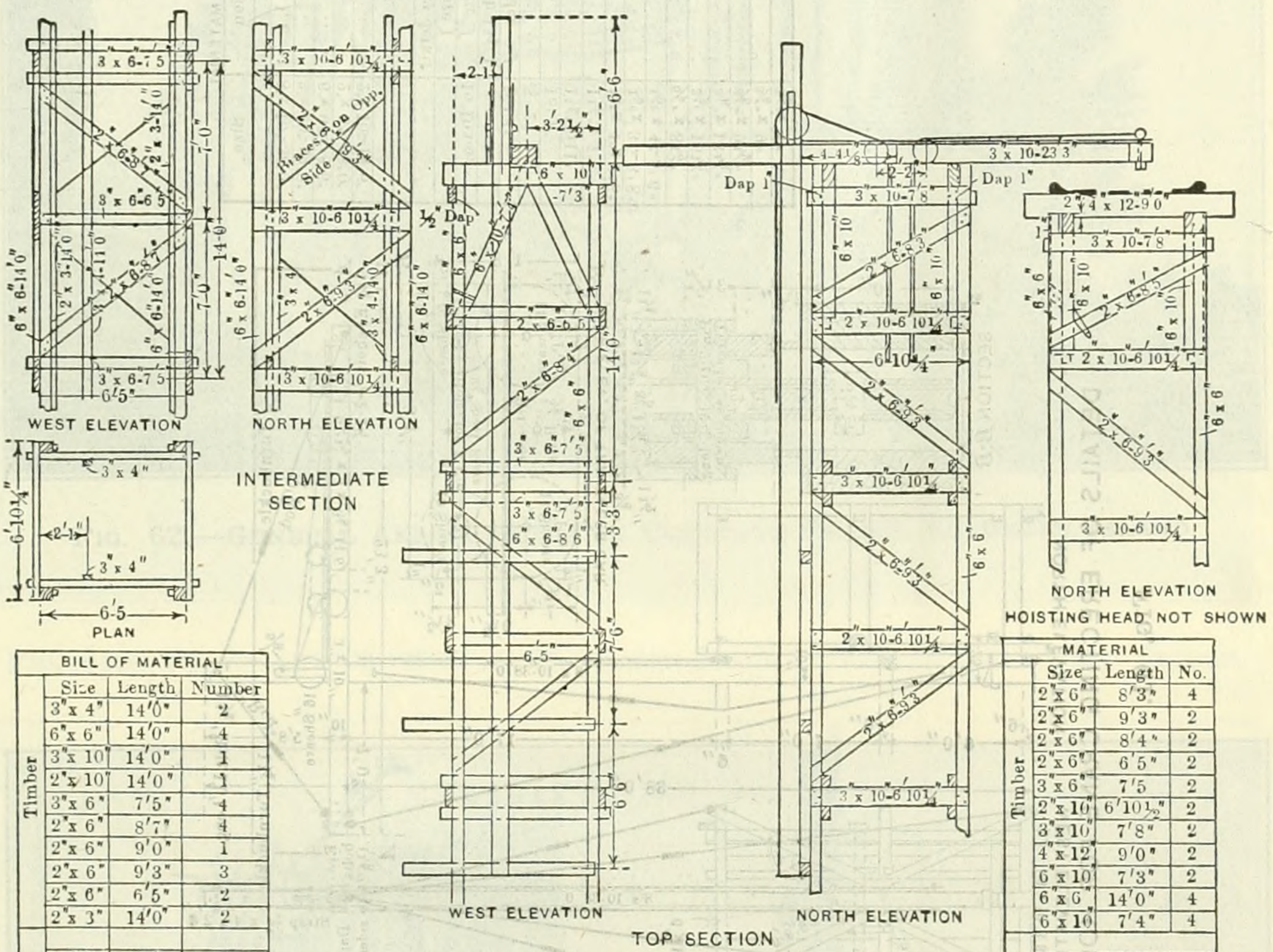


FIG. 28.—GENERAL ARRANGEMENT FOR CASTING CONCRETE.



Mr. Seaman.

At the time the contract for this masonry was undertaken, the method of chuting concrete from the mixer to its final disposition, by using a tower, had come into general use in concrete construction, and, after careful consideration of various other methods of procedure, this one was adopted as the most feasible for this work. It was proposed at first to adopt this method with four piers, but, on further consideration, it was decided to extend it to six piers as a unit. This arrangement necessitated an exceptionally tall tower (214



BILL OF MATERIAL

	Size	Length	Number
Timber	3"x 4"	14′0"	2
	6"x 6"	14′0"	4
	3"x 10"	14′0"	1
	2"x 10"	14′0"	1
	3"x 6"	7′5"	4
	2"x 6"	8′7"	4
	2"x 6"	9′0"	1
	2"x 6"	9′3"	3
	2"x 6"	6′5"	2
	2"x 3"	14′0"	2
Bolts			
	⅝"	9"	24
	⅝"	8"	38
	⅝"	8½"	14
	⅝"	9½"	8
	⅝"	6"	6
	¾"	22½"	8
	Washers ⅝"-180 ¾"-16		

MATERIAL

	Size	Length	No.
Timber	2"x 6"	8′3"	4
	2"x 6"	9′3"	2
	2"x 6"	8′4"	2
	2"x 6"	6′5"	2
	3"x 6"	7′5	2
	2"x 10"	6′10½"	2
	3"x 10"	7′8"	2
	4"x 12"	9′0"	2
	6"x 10"	7′3"	2
	6"x 6"	14′0"	4
	6"x 10"	7′4"	4
Bolts			
	¾"	22½"	8
	¾"	11½"	12
	¾"	21"	4
	⅝"	9"	24
	⅝"	8"	38
	⅝"	8½"	14
	⅝"	9½"	8
	⅝"	6"	6
	Washers		
	¾"		48
	⅝"		180

DETAILS OF HOIST TOWER

FIG. 60.

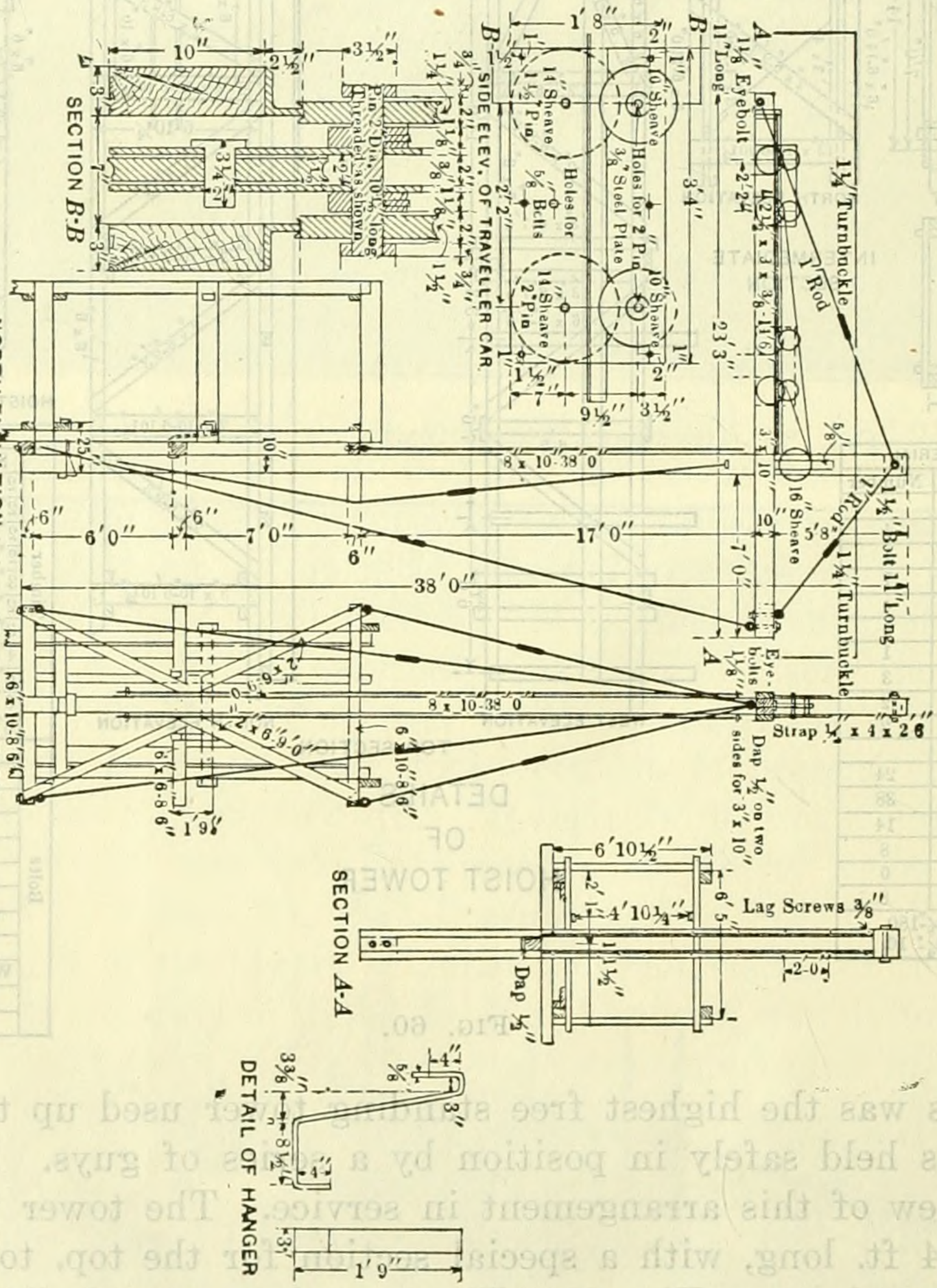
ft.). This was the highest free standing tower used up to that time, and it was held safely in position by a series of guys. Fig. 58 is a general view of this arrangement in service. The tower was built in sections 14 ft. long, with a special section for the top, to provide for the working sheaves, Fig. 60. These sections were built up with an erecting crane, Fig. 61, which was carried up with the tower as the erection proceeded, and was used as part of it while in service.

The planning of the construction of this masonry, which was in charge of the writer as Consulting Engineer for the contractor, was

Mr. Seaman.

BILL OF MATERIAL

No. of Pieces	Description	Size
1	Timber	
2	Yellow Pine L.L.	8"x 10"-38'0"
3	" " "	3"x 10"-24'0"
6	" " "	6"x 6"- 8'6"
2	" " "	2"x 6"- 8'0"
2	" " "	2"x 10"-14'6"
	Iron —	
1	Trolley as per detail	
1	Sheave Bronze } Bushed 2"Pin }	10"Diam.
2	1"Rods, with 1¼" Turnbuckles }	16'8"
1	" " " "	8'2"
2	" " " "	18'7"
3	Eyebolt	1⅛"x 11"
1	"	1⅛"x 7"
2	Straps	⅝"x 4"x 2'6"
2	"	⅜"x 3"-3'8"
1	"	⅝"x 4"-5'6"
14	Bolts	⅝"x 8½"
6	"	⅜"x 10½"
12	"	⅜"x 13½"
3	"	⅜"x 9½"
6	"	⅝"x 9"
16	Lag Screws	⅜"x 4"



DETAILS OF ERECTING CRANE FOR HOIST TOWER

FIG. 61.

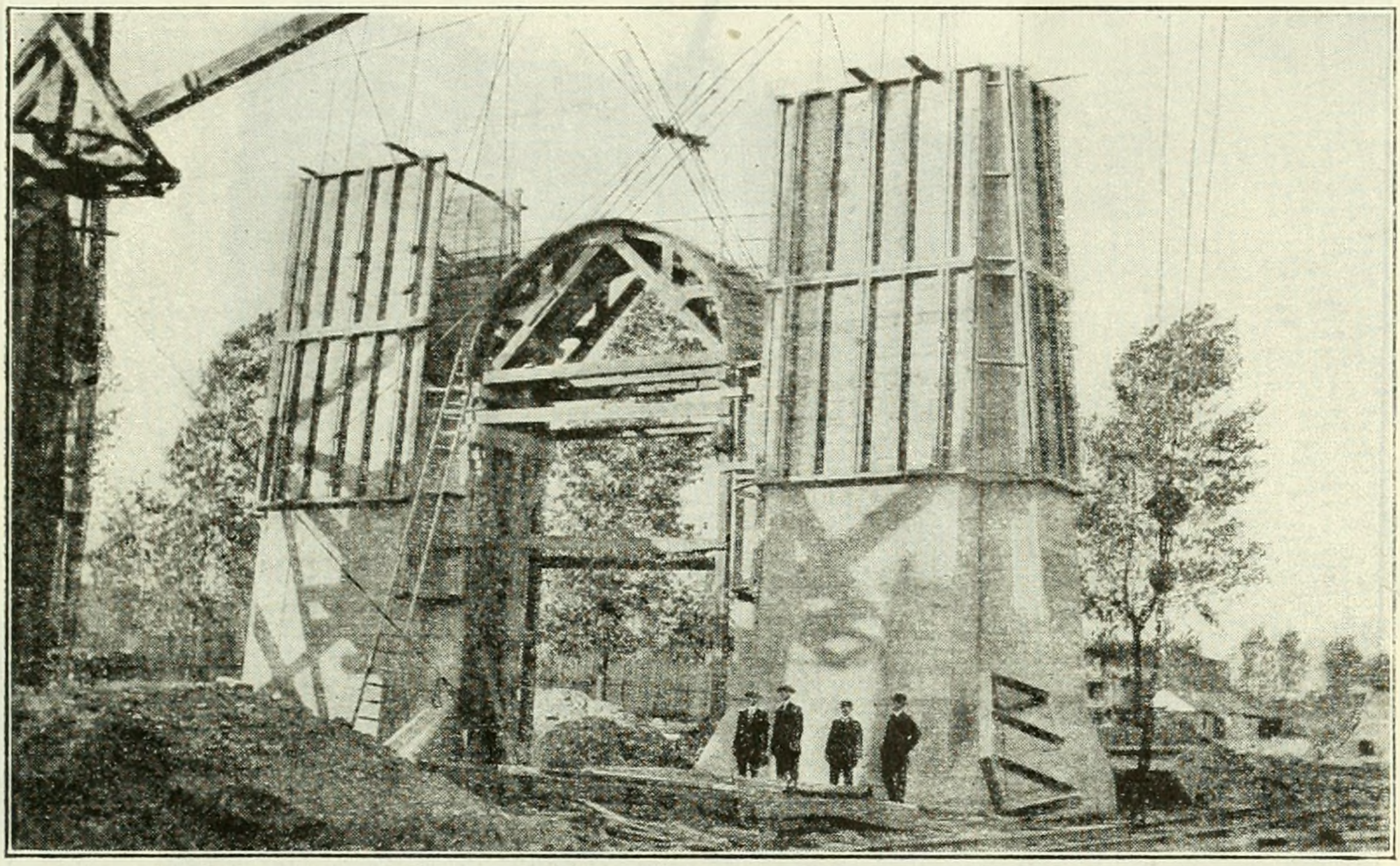


FIG. 62.—GENERAL ARRANGEMENT OF CONCRETE FORMS, RANDALLS ISLAND.

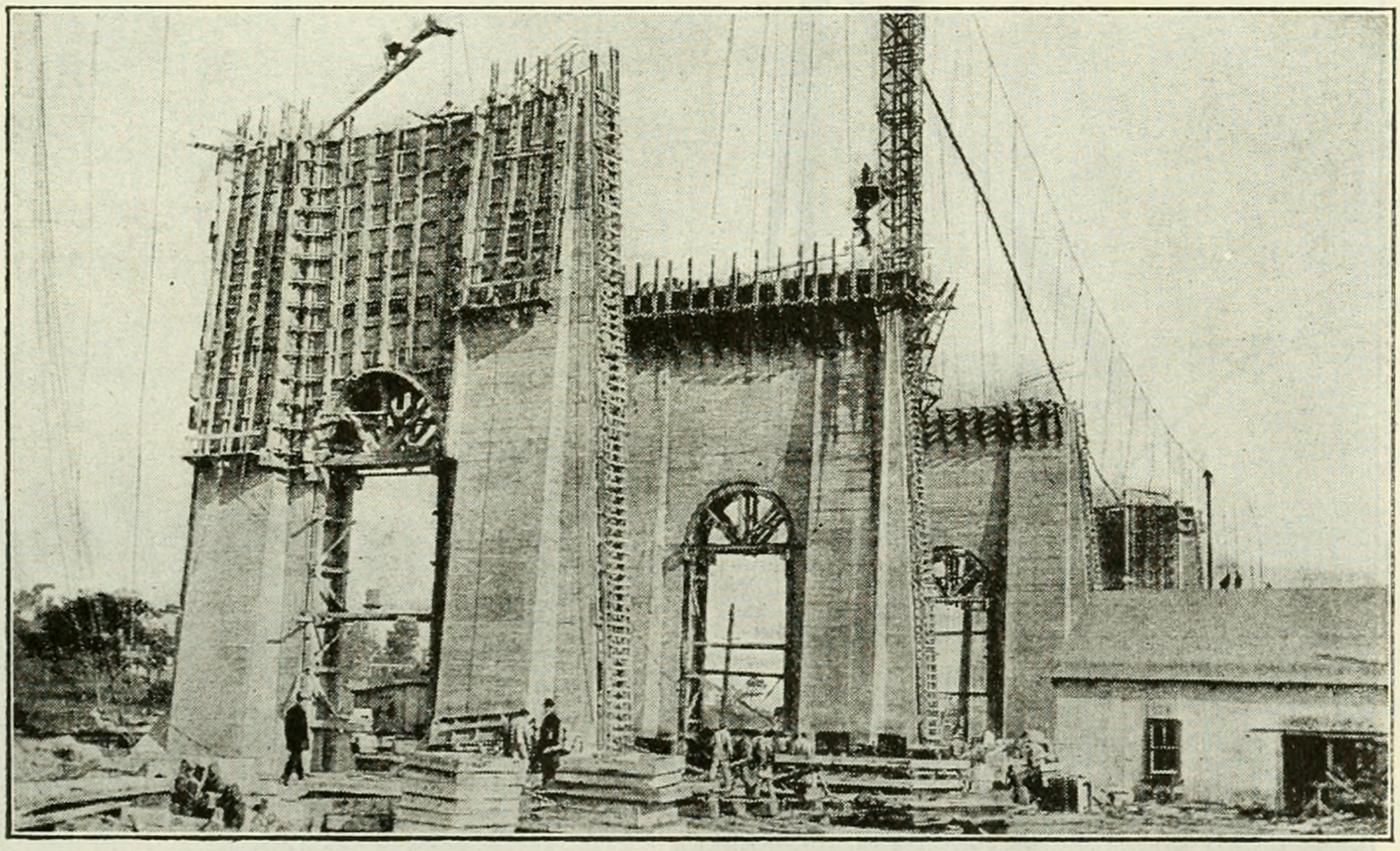


FIG. 63.—CONSTRUCTION OF CONCRETE PIERS ON LONG ISLAND.

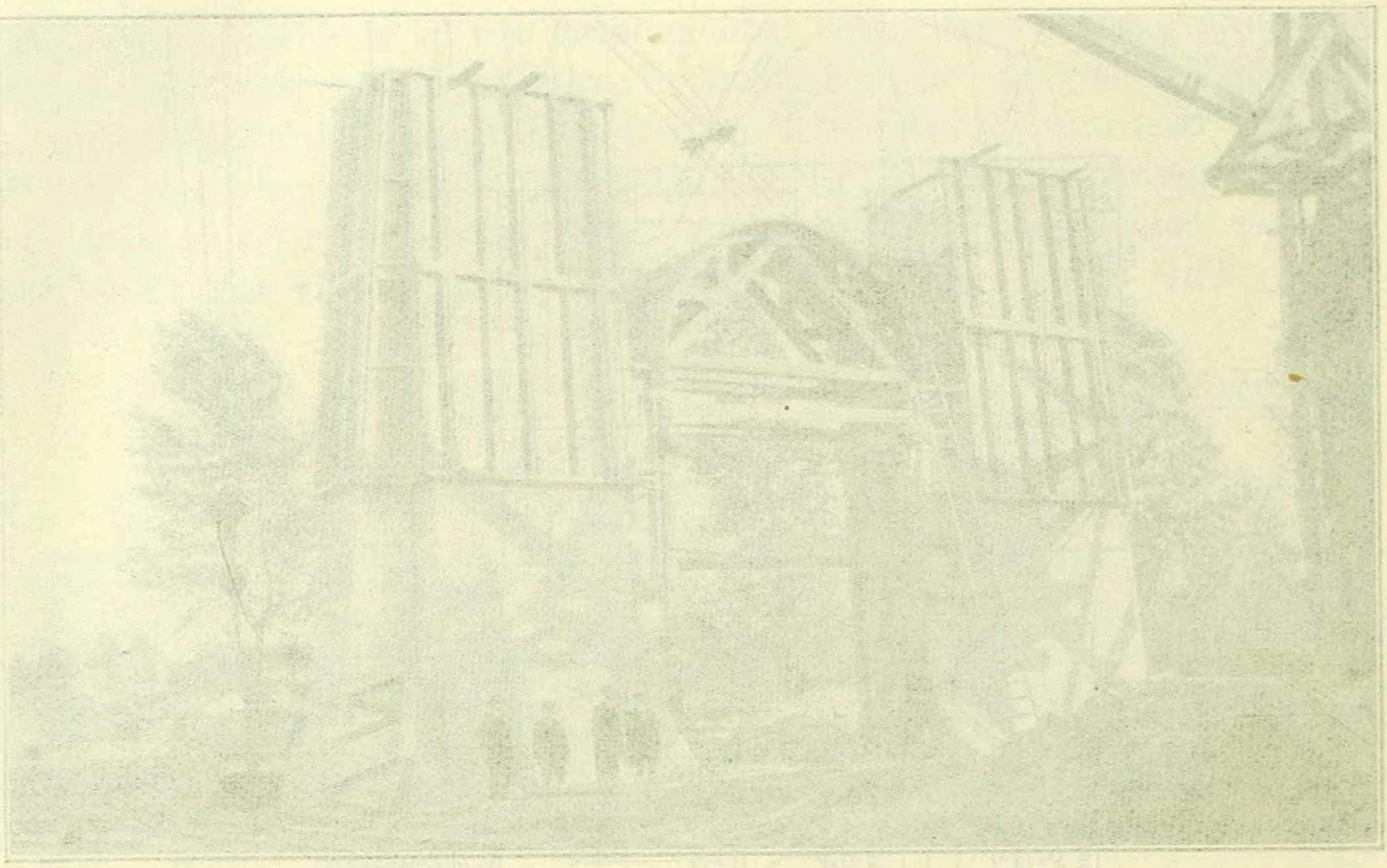


FIG. 62.—GENERAL ARRANGEMENT OF CONCRETE PIERS, RANDALLS ISLAND.

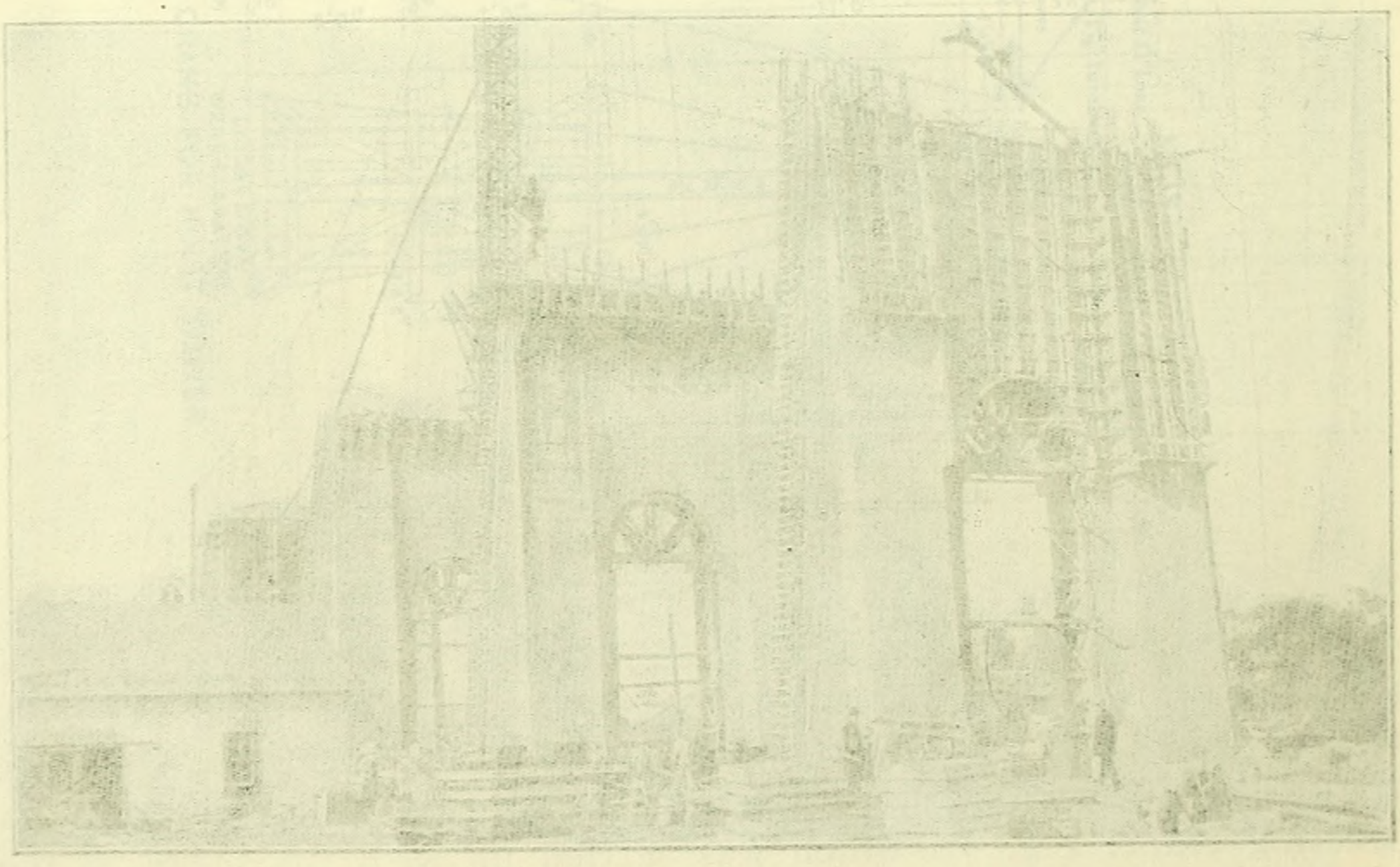


FIG. 63.—CONSTRUCTION OF CONCRETE PIERS ON LONG ISLAND.

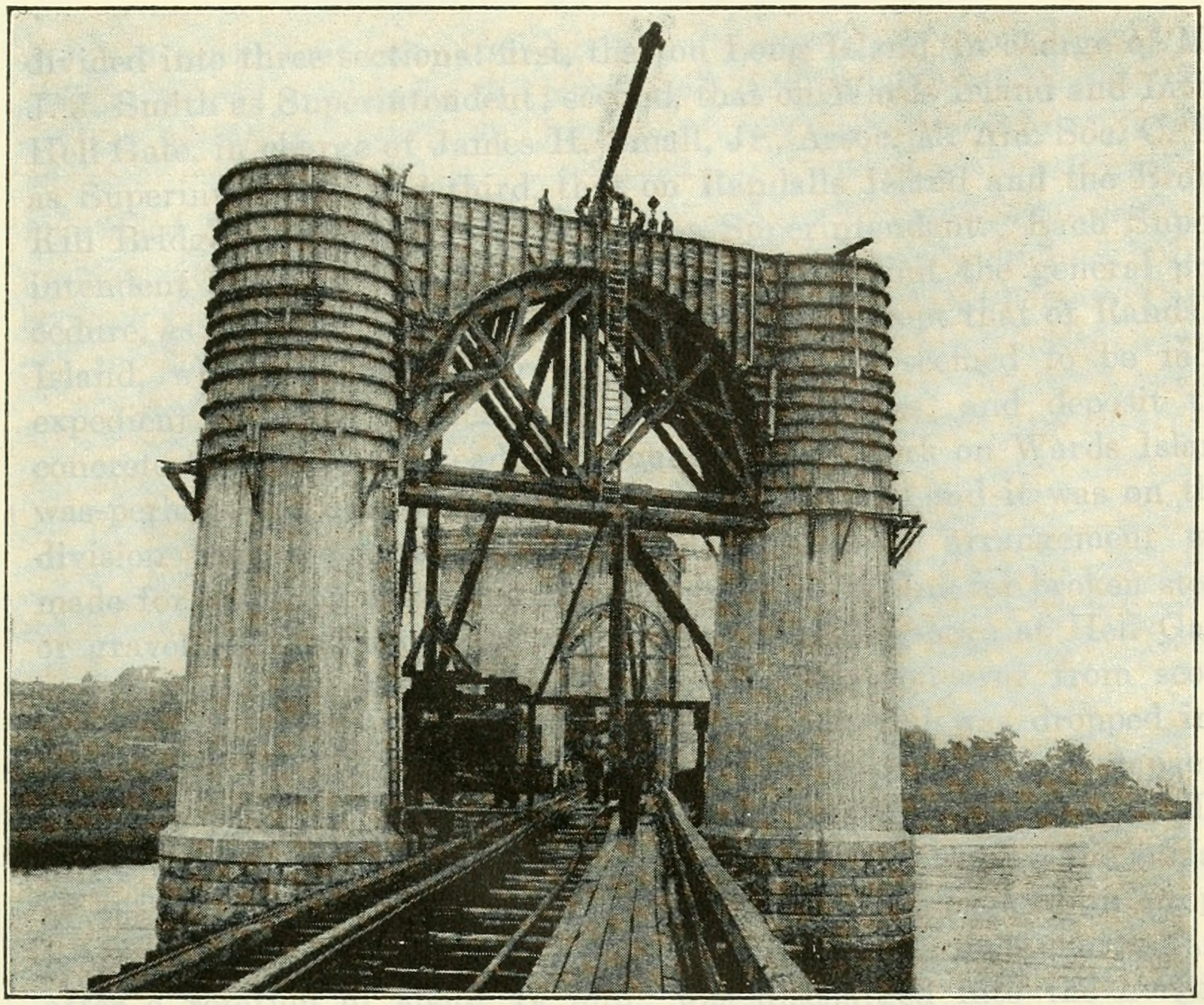


FIG. 64.—FORMS AND CENTERS IN PLACE FOR PIERS OF LITTLE HELL GATE BRIDGE.

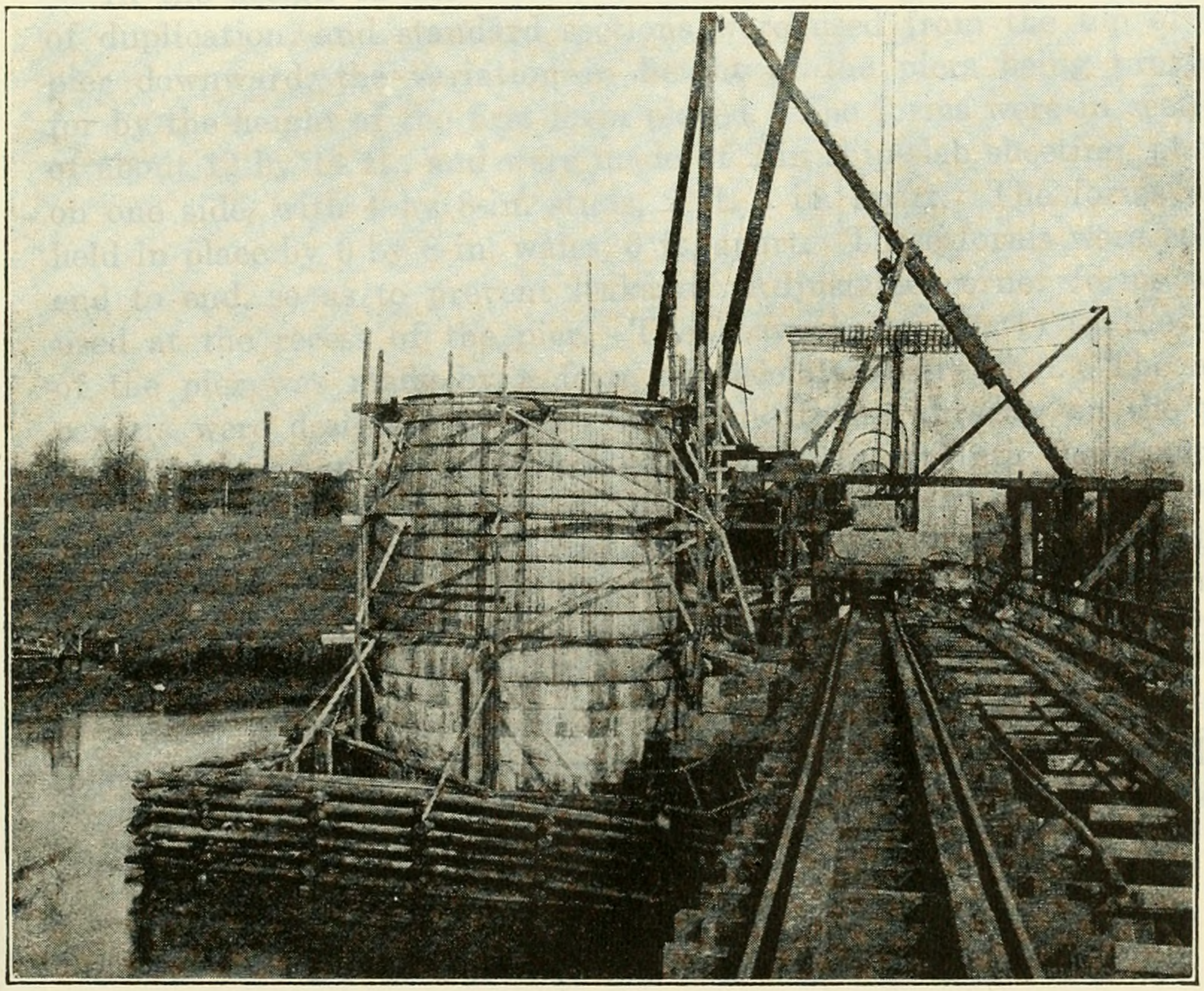


FIG. 65.—PIERS FOR LITTLE HELL GATE BRIDGE PROTECTED BY HEAVY CRIB COFFER-DAMS.

FIG. 63.—PIERS FOR GALLOPING HELL GATE BRIDGE PROTECTED BY HEAVY CRIB
COLLEEN-DAVIS.

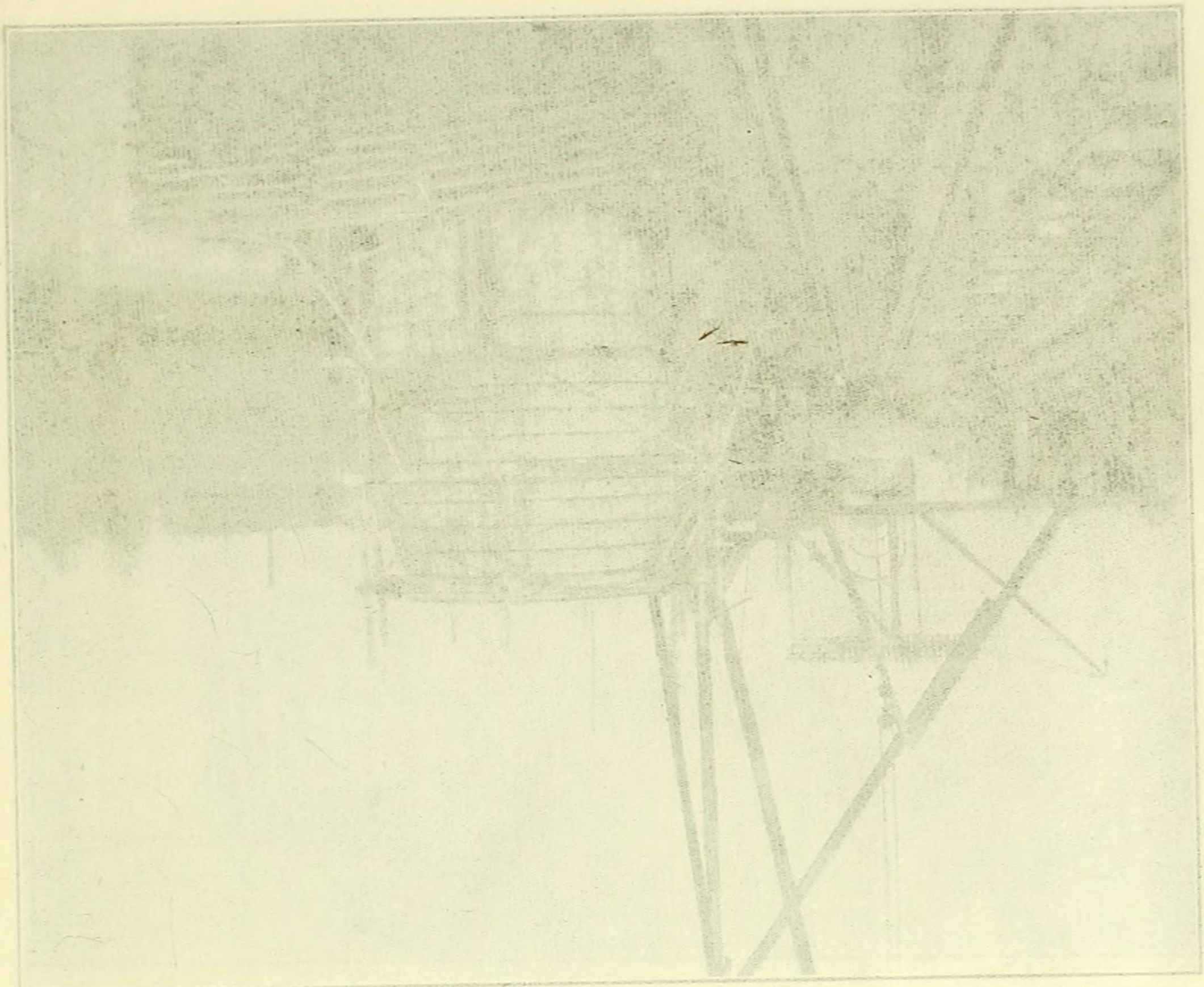
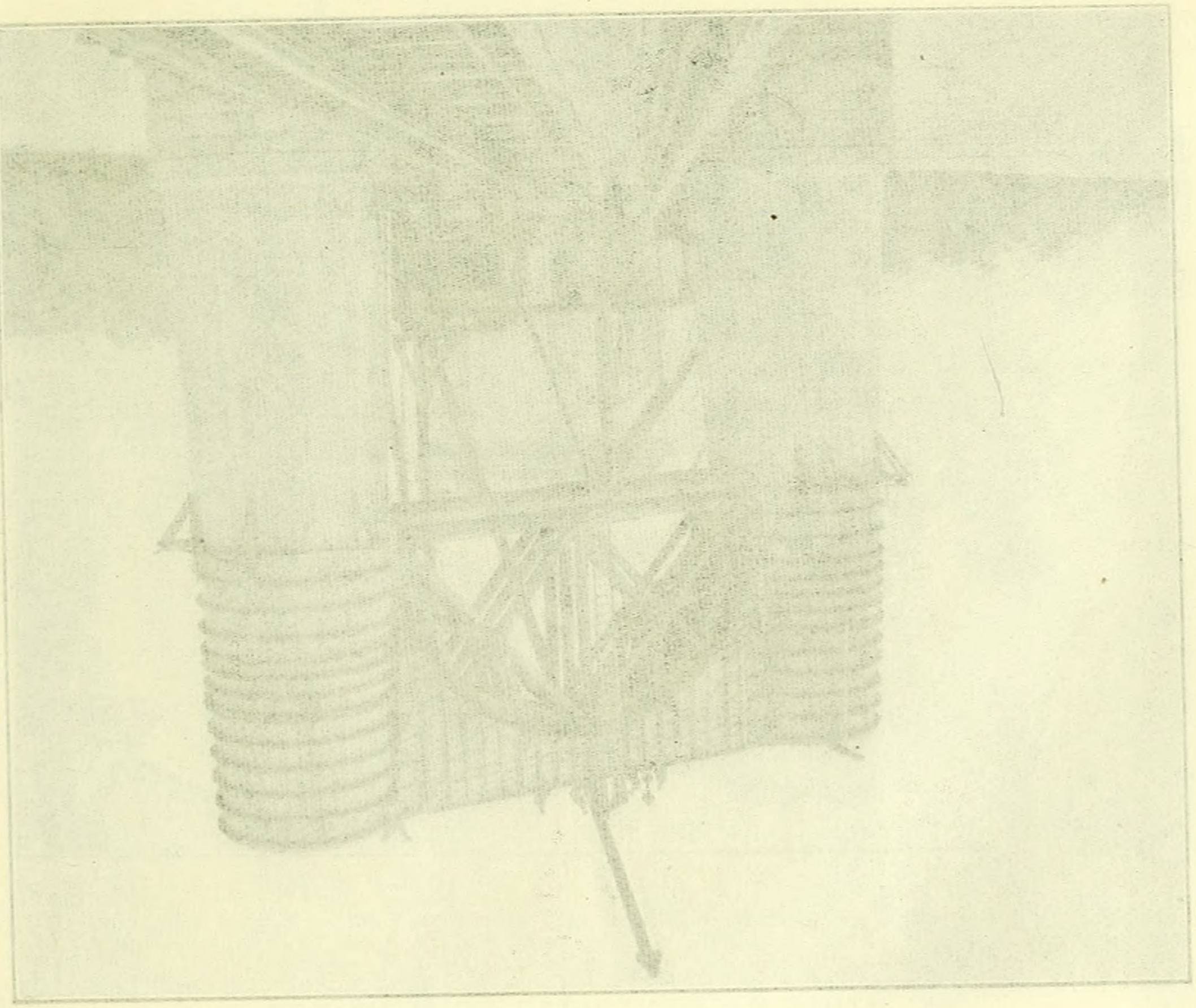


FIG. 64.—FORMS AND CENTERS IN PLACE FOR PIERS OF GALLOPING HELL GATE BRIDGE.



divided into three sections: first, that on Long Island, in charge of Mr. J. J. Smith as Superintendent; second, that on Wards Island and Little Hell Gate, in charge of James H. Small, Jr., Assoc. M. Am. Soc. C. E., as Superintendent; and third, that on Randalls Island and the Bronx Kill Bridge, in charge of Mr. Little as Superintendent. Each Superintendent arranged the details of his own work, but the general procedure, as outlined, was adopted on all sections, except that of Randalls Island, where the piers were lowest. There it seemed to be most expedient to place derricks on elevated platforms, and deposit the concrete by buckets, instead of by chutes. The work on Wards Island was perhaps the most carefully prepared in layout, and it was on this division that the most complete and satisfactory arrangement was made for handling the materials. A large set of bins for broken stone or gravel, sand, and cement was erected near the shore at Hell Gate, and the material was carried by a 20-in. belt conveyor from scows to the bins, Fig. 59. From these bins the material was dropped into cars especially constructed for the purpose, each having a capacity of about 12 cu. yd., and by this means conveyed to the mixer. The mixer was placed so that the charging hopper was set below the surface of the earth, thus permitting the material to be dropped in proper proportions directly into the hopper. The mixer was then charged and, after a rotation of about 1½ min., was discharged into the hoisting bucket and lifted to the chutes. Mr. Seaman.

In the design of the forms, attention was paid to the importance of duplication, and standard sections were used from the top of the pier downward, the variation in height of the piers being provided for by the height of the first form placed. The forms were in sections of about 12 by 14 ft., and were made of 2-in ship-slab sheeting, planed on one side, with 4 by 8-in. studs, 2 ft. 6 in. apart. The forms were held in place by 6 by 8-in. wales, 6 ft. apart. These forms were bolted end to end, so as to prevent leakage. Adjustable corner forms were used at the recess of the pier. The heavy batter (1:1) at the base of the pier was made by a form of special construction. The arch centers were designed so as to throw the thrust directly on the pier as a buttress, and were supported by timbers set into the concrete. These timbers were arranged so that they could be unbolted and withdrawn without injury. Considerable difficulty was experienced in removing these arch centers, and, on a repetition of the work, a modification would probably be devised by which the form could be hinged at the top, and thus made collapsible.

Owing to the great weight of the forms, a special crab-derrick was placed on top of the concrete and raised as the pier progressed.

Figs. 62 and 63 show the general arrangement of this work on Randalls Island and Long Island, respectively.

Mr. Seaman.

Fig. 64 shows the construction of the forms and centers for the piers of Little Hell Gate Bridge. The forms are practically hooped barrels, the first form being of variable height, as already described for the rectangular piers.

The foundations of these piers are on rock, about 12 ft. below low water, and in a current running 7 miles per hour. The construction was protected by heavy crib coffer-dams (Fig. 65). The pit was pumped dry and the bottom leveled off. Forms were then set inside for the

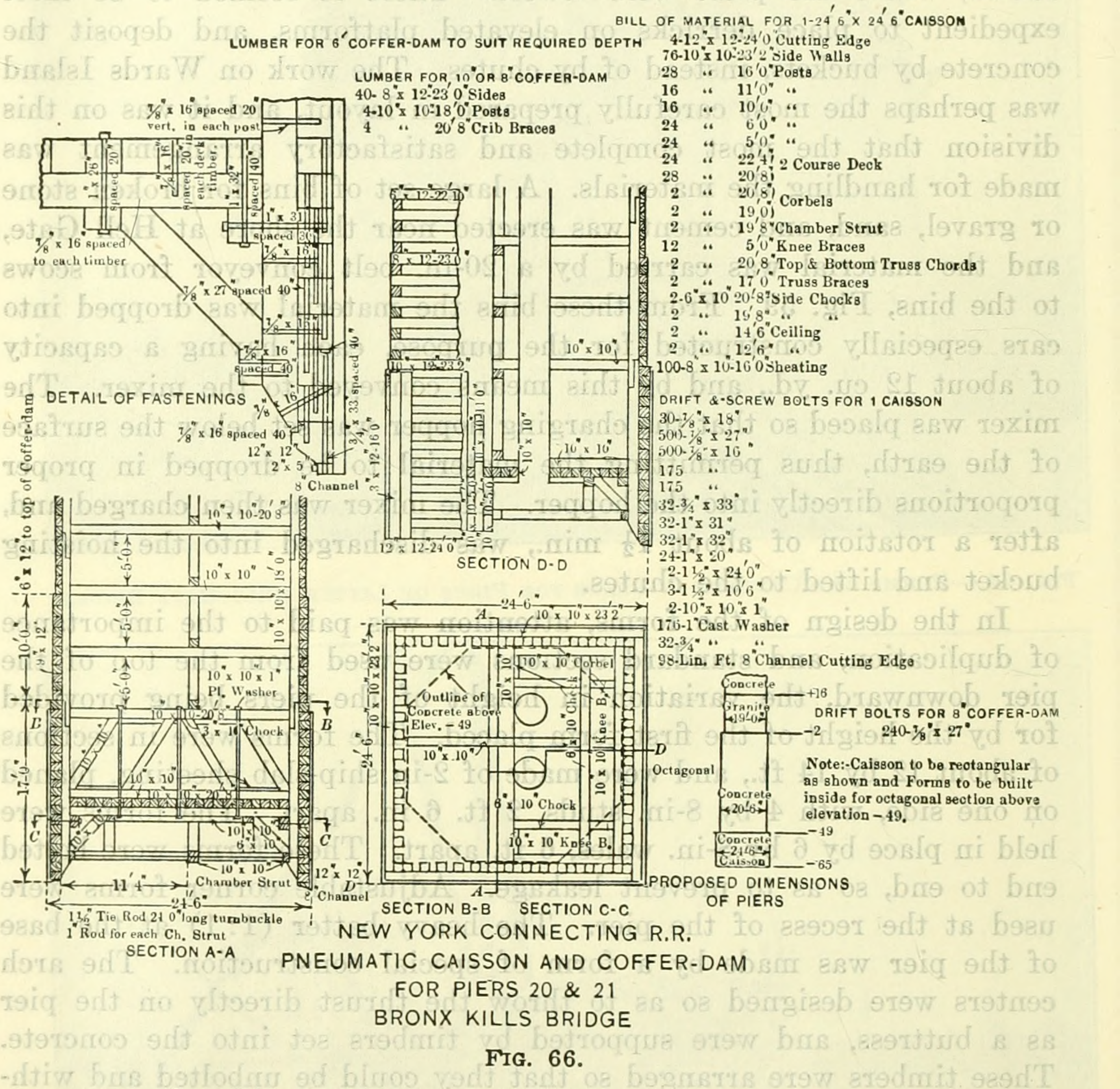


FIG. 66.

concrete. On this concrete foundation, masonry was laid, about 12 ft. high, so as to protect the pier at the water surface. On this masonry the concrete shafts were then constructed. A trestle of simple pile construction, three piles per bent, with 12 by 12-in. caps and 12 by 12-in. stringers, was used at this river work.

The construction of the tower of the Wards Island foundation for the main arch span has already been described by Mr. Ammann. The preliminary studies of this work on the part of the contractor

contemplated sinking these various cylindrical and rectangular concrete caissons by open dredging. This process proceeded with eminent satisfaction as to accuracy to a depth of about 30 ft. below ground, when it was believed by the engineers that greater progress could be accomplished by the use of compressed air. As such work, however, had not been contemplated by the contractor, it was taken over by the Railroad Company. Mr. Seaman.

The foundations of the Bronx Kill Bridge were sub-contracted to the Arthur McMullen Company. Fig. 66 shows the caissons used by that firm. The deepest part of the foundations for these piers was 91 ft. below high water.

GUSTAV LINDENTHAL,* M. AM. SOC. C. E. (by letter).—Mr. Breithaupt believes that for Little Hell Gate either spandrel-braced or double-ribbed arches should have been used, in place of the inverted bow-string type. Mr. Lindenthal.

There is no navigation on this short stream, except that of small pleasure launches and sailing boats. The clear height above the water was of less importance, in the opinion of the Government engineers, than the obstruction from piers. It is expected that this channel (which connects the East River with the Harlem River) will be deepened, not for purposes of navigation, but for a more ample flow of the tides through it. Long piers in the direction of the currents were regarded as offering more obstruction to the tidal flow than the column piers as arranged. Metal arches in place of bow-string girders would have required heavier column piers to take up the resultant between loaded and unloaded adjacent spans. There would have been, not only no saving in the total cost of this crossing, but more obstruction to the current.

For an arrangement of four continuous spandrel-braced arches, delivering only vertical pressure on the river piers and subject to heavy temperature stresses, violent reversion stresses, and large secondary stresses, the relation of live load to dead load was not favorable. It is not an arrangement suitable to a railroad bridge.

At first, parallel-chord, riveted trusses were considered, but most of the members were so large as to require cross-sections with four webs. That is very objectionable in tension chords. The inverted bow-string girder, with the tension chord composed of eye-bars, made a more satisfactory structural arrangement. The web stresses in this form of truss are small, and the details for them simple. The eye-bars are the heaviest ever forged—16 by $2\frac{1}{4}$ in., with 16-in. pins.

The superstructure receives its lateral rigidity through an extra heavy and rigid wind and vibration truss in the plane of the top chords. It is sufficient to transmit the wind pressure, which acts on the bottom

* New York City.

Mr. Lindenthal.

chord at each vertical web member, by a stiff diagonal to the wind truss above; therefore, no bracing is required in the bottom chord. The correctness of this detail is proved, also, by the fact that the bridge is laterally quite rigid under trains. The rigidity of the superstructure is further enhanced by the fact that its center line of gravity is about 15 ft. below the points of support on top of the piers and abutments. It would be in stable equilibrium, therefore, even without any lateral bracing below the wind truss.

It will be noticed, in the silhouette of the bridge, that its floor-table has the same thickness (10 ft.) as the plate-girder approaches. The harmony of design is thus preserved. That uniformly deep floor-table forms a continuous heavy line on top of the masonry piers; it appears on this structure, at a distance, as if supported on slim catenary chains, but, in fact, these chains each form the heaviest eye-bar chord member in any existing, simple-span, truss bridge.

The masonry tower piers at each end of Little Hell Gate form a sufficient break to mark the longer spans. These towers are hollow, and contain emergency stairways. Their form, it is true, could have been somewhat improved, had they been studied in a model, as was done in the case of the granite towers for the large arch span.

The writer would have preferred to make provision from the start for a second deck throughout for a boulevard over the railroad tracks, which he foresaw would be wanted in the future for highway traffic between the Bronx and Long Island; but his efforts at that time to get the public and city authorities interested in a boulevard were not successful, and were also not favored by the Railroad Company. The time will come, however, when such a boulevard may be added. It will be cheaper to do that than to build a separate parallel structure for that purpose.

Contrary to Mr. Quimby's opinion, the span length of the concrete viaduct is not too short. As stated in the paper, the most economical length was chosen, except where affected by the location of the streets. By examining the costs of masonry and steelwork given, Mr. Quimby can easily convince himself of this fact.

An important reason for adopting the concrete piers was that objection was made by the authorities of Wards and Randalls Islands to the steel columns, because they feared that inmates of the municipal institutions on those islands would climb them and make their escape. It was insisted that the design adopted should prevent this.

Mr. Quimby's idea as to what should have been the arrangement of the end panel in the large arch is based on an erroneous assumption. Assuming that the top chord of the arch would be anchored in the towers, making of the arch a type with fixed ends and end moments, a diagonal member to the end of the top chord would be entirely superfluous, and would appear so to the technically trained eye, as the

shearing stress would be taken care of by the masonry. The anchoring of the top chord, however, would have required much heavier stone towers, and of different form, in order to take up the horizontal anchorage stresses at the top. Mr. Lindenthal.

The top chord was prolonged into the masonry simply for convenience of inspection. There is a little stairway hidden in the end of the deep top chord over which one is enabled to enter from the interior of the tower and go out on the chord. The objection of the Municipal Art Commission to the first design was merely against the too ornate form of the towers of moulded concrete. The form of towers, therefore, was simplified, and granite was chosen for the exterior faces.

CLEMENT E. CHASE,* JUN. AM. SOC. C. E. (by letter).—It is largely through innovations introduced into the design of large bridges, such as the Hell Gate Arch, that structural engineering advances. Precedents launched with such prestige may, in fact, become established in practice without being subjected to such critical examination of their merits as should be the case. An important new feature of the Hell Gate design, quite likely to be of wide influence, is the use of single, unusually heavy plates for the webs of the main chords. Mr. Chase.

Mr. Ammann states:

"It is frequently maintained that better material is obtained in the thinner than in the thicker plates. This was not substantiated by the great number of specimen tests made for the Hell Gate Bridge from material varying from $\frac{1}{2}$ in. to 2 in. in thickness. In general, the thick material showed as high elastic properties and ultimate strength as the thinner material rolled from the same heat."

From this the impression might be gathered that, in some unexplained way, the mills which produced the steel for the Hell Gate Arch had overcome the difficulties which had hitherto prevented the thickness of material being increased beyond average gauges ($\frac{3}{8}$ to $\frac{3}{4}$ in.) except at the expense of quality. This decrease in quality with increase in thickness has not only been the commonly accepted belief of those whose experience has been greatest in testing steel, but there is on record ample experimental evidence to support this view, for example, Table 16 in the paper by Henry S. Prichard, M. Am. Soc. C. E., entitled "The Effects of Straining Structural Steel and Wrought Iron."†

It would seem that Mr. Ammann relies on the yield point and ultimate strength showing of the specimen tests (and these, presumably, the inspector's mill tests) to support the belief that the thick plates were obtained at no sacrifice of quality. However, neither of these values, as commercially determined, can be accepted as more than

* Poughkeepsie, N. Y.

† *Transactions*, Am. Soc. C. E., Vol. LXXX, p. 1541.

Mr. Chase.

a fractional part of the measure of quality. For its complete determination, the actual elastic behavior, such as is presented to the eye in an accurate stress-strain curve, and the ductility, at least, must be known. Quality for structural uses should also take account—though possibly in lesser degree—of shock resistance and endurance under repeated loadings.

The increasing inferiority in ductility with increasing thickness is usually taken note of in specifications, to which the Hell Gate requirements (Table 1) are no exception.

There is no mystery in the inverse relation of thickness and quality of rolled steel; the necessities of commercial mill practice are sufficient explanation. Usually, plates or shapes of a wide range in weight per foot are rolled from ingots of the same cross-section, which causes the ratio of reduction to be less as the thickness of the product increases. Aside from this, the construction of the mills in use, which were designed almost without exception for lighter sections, demands that, when heavy products are being rolled, the steel must be finished hotter. Even if finished at the same temperature, a thick piece will cool more slowly than a thin one. The result, considered in terms of micro-structure, will be a coarser-grained material. It is well established that the larger the grain size of steel, the lower will be the stress at which deviation from elastic behavior occurs, the lower the elongation and reduction of area at rupture, the lower the shock resistance and the endurance under repeated loads.

The yield point and ultimate strength determinations of the mill tests, on which Mr. Ammann based his statement, have their usefulness in guarding against serious variations of a standard product from normal, but, as experimental proof of the adequacy of a new product for a new use, they would have but little weight. There are several reasons for this: they are made hurriedly, in commercial routine, which must disregard the niceties of testing; the samples selected are not representative of all parts of the material; and, as explained before, these characteristics are themselves a very incomplete statement of quality.

Ordinary mill tensile tests are completed at a rate of from thirty to fifty per hour, and, even under the severest specifications in use, they are made much too quickly to have any scientific standing. It is not the writer's understanding that the Hell Gate specimens were tested with any refinements beyond those customary in inspecting material for other high-grade bridgework.

The misrepresentative character of mill tests is a matter of common knowledge. In comparing heats rolled into similar products, this may be overlooked, for the specimens are taken from the same locality in the product each time, and the tests are comparable, as far as they go. To judge of the full strength of the material over its entire

cross-section, they would be inadequate. For instance, in the case of universal mill plate, tests are cut next to the rolled edge. By reason of the amount of work received, the lower working temperature, the radiation from three sides in cooling, and the distance from the more or less segregated axis of the plate, this part of the plate is ordinarily much superior to any other. It will show a higher elastic ratio and higher ductility than the center, and the thicker the plate the greater the difference between the showing of the edge and what would be the showing of the center. For sheared plate, the circumstances are much the same, except that the actual ragged edge is first removed by shearing. Mr. Chase.

That the yield point and ultimate strength (even if accurately determined and representative) do not alone tell the whole story of the structural usefulness of metal has, for columns, been strikingly pointed out in the recent final report of the Society's Special Committee on Steel Columns and Struts.

It is stated therein that:

"Neither the 'yield points' as indicated by the drop of the beam, nor the ultimate strengths, bear any consistent relation to the U. L. P.'s, or ultimate strengths, of the full-size columns."

The point to which is ascribed the greatest importance, the "Useful Limit Point", can only be located when the critical portion of the stress-strain curve is known with precision. Barring accidental blows, which would call ductility and shock resistance into play, or unlikely repetition of over-loads to the extent of testing repeated stress endurance, it is this curve which determines structural fitness. This is as true of tension members as of the compression members with which the report deals.

Bearing directly on the subject under discussion, the Committee finds that the useful limit of a column's unit strength decreases as the thickness of the component material increases.* However, the Committee's tests did not include columns of equal cross-section, made up in the one case of single, thick plates, and in the other of several thinner plates, stitch-riveted together, and this leaves a loophole for arguments such as Mr. Ammann's on page 891. Referring to the Committee's test, he says:

"If the heavy columns had been built up of several thin plates, tack-riveted together, their compressive strength would probably have been even less."

Although, possibly, some weakness is introduced in this way, the writer feels that it is of far smaller degree than the influence of thickness of material in reducing the limit of structural usefulness

* The reader is referred to Tables 22 and 23 of the Final Report of the Special Committee on Steel Columns and Struts, *Proceedings*, Am. Soc. C. E., December, 1917.

Mr. Chase. under load. It is rather strange that, with all the large-sized column tests which have recently been made, there should have been none to throw light on this point directly. The designers of the new Quebec Bridge evidently held one view, the designer of the Hell Gate Arch, the other; and yet only a few, relatively inexpensive tests would be required to settle the matter. The question is of fundamental importance, and should be removed from the field of speculation and controversy.

Mr. Ammann. O. H. AMMANN,* M. AM. SOC. C. E. (by letter).—It is gratifying to note that the design of the Hell Gate Bridge structure as a whole as well as its general structural and architectural features are approved without exception by those who discussed this paper.

Some of the criticism on the general design of the Hell Gate Arch, the Little Hell Gate Bridge and on certain features of the approach viaducts has been answered by the designer himself, and it remains only to reply briefly to the discussion of various special features and details.

Selection of Type of Bridge.—Mr. Moisseiff is of the impression that the suspension type of bridge with a longer span would have been selected had the unfavorable foundation conditions on the Wards Island side been better known. As stated in the paper, the arch was selected in preference to the suspension type principally on account of the local conditions, which precluded long shore spans. Whether a suspension bridge with a shorter span would have been more economical under the soil conditions as actually found could be determined only by a complete design, but it is unquestionable that a suspension bridge with a longer span than the arch would have been more costly, in spite of cheaper foundations.

Had the unfavorable soil conditions been fully known earlier, a relocation of the line might have been closely investigated, but it is doubtful whether any saving could have been effected, because any other location would have meant a longer span.

Towers and Foundations.—The writer agrees with Mr. Moisseiff that towers are not an economical feature if intended for the exclusive purpose of keeping the resultant reaction within the middle third of the foundation area, because, if it is a mere question of sufficient weight of masonry, such weight can be provided more cheaply by extending the foundation in the direction of the axis of the bridge. Where it is a question of providing towers for architectural reasons, however, such towers constitute at the same time a considerable saving in the foundation masonry and are then not entirely for architectural purposes.

* South Amboy, N. J.

Bottom Chord Section.—Mr. Fowler comments on the section of the bottom chord as lacking in economy, when compared with the cellular circular type. This is true as far as economy of metal is concerned, but practical difficulties in making the circular section offset the economy. The form of section which Mr. Fowler proposes for the bottom chord of a cantilever bridge of 2 000-ft. span is an excellent substitute for the circular one, and would be particularly suited also for the main rib of a great arch. It is compact, and has no outstanding flanges or wide thin webs, which are always a source of local buckling; and the effective section is well distributed at equidistant points from the center of gravity. However, on account of its many pockets and oblique angles, which make fabrication expensive, it can hardly be recommended for sections of less than 1 000 sq. in. of effective area, especially when the chord is to be tapered, as was found desirable in the case of the bottom chord of the Hell Gate Bridge. Mr. Ammann.

The Three-Faced Joint.—The comment on this joint of the bottom chord has been favorable. Mr. Moisseiff would deduct the excess stress, caused by the bearing over the middle third of the joint, from the specified unit stress for compression members. This does not appear to be necessary, and would offset the economy of the design. The bearing stress at the middle of the joint can reach the yield point without harm, as the metal thus strained is well confined on all sides within metal which is strained to less than the permissible limit, and to which it imparts its excess stress within a short distance of the joint. The three-faced joint approaches the condition of a roller, or ball bearing, for which the maximum bearing stress is allowed to run at least 50% higher than the average stress of a compression member.

Thick vs. Thin Plates.—One of the features most widely commented on is the extraordinary thickness of 2 in. adopted for the webs of the heavy compression chords. It is variously stated that experience has shown that thick metal is commonly not as strong and elastic as thin metal rolled from the same material.

The designer fully recognized this fact, and for this reason allowed a wider variation for the ultimate strength of hard steel, of which the 2-in. plates are made, than is allowed on ordinary structural steel (see foot-note below Table 1). It was expected that the thicker material would generally be below the "desired" strength or nearer the minimum, and the thinner one nearer the upper limit; also, a graduated lower limit was set for the elongation of thick material (see foot-note below Table 1).

The final specifications were drawn after thorough consultation with the manufacturers, whose obligation it was to produce, and who did produce, the desired qualities.

Mr. Ammann.

In spite of the expected slightly inferior physical qualities, the designer considered thick plates for compression members superior to thin plates, which are weakened by many stitch-rivets and the possibility of local buckling between the rivets.

The expectation of lower results for thick plates, however, was not borne out by the results. The individual test reports are not readily available at the present time, also space would not permit of their complete reproduction, but the fact is, as stated in the paper, that the tests of thick metal were generally as favorable as those of thin metal from the same heat. The only exception was the elongation, which ran slightly lower in the thick metal, but not to the extent allowed by the specifications.

The question as to the superiority of thick plates over stitch-riveted plates for compression members can be convincingly solved only by a series of systematic tests. In the absence of such tests, only judgment can govern; it would seem to the writer that it is sounder judgment to build up a member of solid metal unweakened by forceful injuries. Mr. Chase seems to take into consideration only the direct strains. How important the shearing stresses are in compression members is now well recognized with respect to the latticing of columns. That a lamellar bar is stronger in that respect than a solid bar does not seem plausible.

There is also a practical question to be considered. Rivets are apt to be less perfect the longer their grip and the greater the number of plates to be riveted together, and, therefore, it would seem that it is especially desirable to avoid riveting in large members, as far as possible.

Mr. Chase appears to assume that the writer disregarded the ductility of the steel, in comparing the test results of thin and thick metal. On the contrary, he took the elongation and reduction fully into consideration. These qualities in connection with the elastic limit and ultimate strength give the engineer a fair measure of the quality of the stress for practical purposes, without going into the theoretical niceties of stress-strain diagrams. Where the number of tests is as great as in the case of the Hell Gate Bridge, the fact that they are not made with extreme refinement has little bearing on the relative quality of thin and thick metal. Contrary to Mr. Chase's understanding, the tests were conducted with more than ordinary care and watchfulness on the part of the inspector. The testing machine was not allowed to run with a greater speed than 2 in. per min., and the objectionable features of commercial mill testing, as mentioned by Mr. Chase, were avoided. More specimens were made from each heat than in ordinary practice, and the specimens were cut both parallel and at right angles to the direction of rolling, and from both the edge and center of the plate.

Mr. Ammann.

The writer cannot see what shock resistance and endurance under repeated loads have to do with the question under consideration.

Assembling of Trusses.—In reply to Mr. Wagner's inquiry, relative to the assembling of the arch trusses, the writer wishes to state that the specifications prescribed complete assembling of the truss. On request, the Bridge Company was subsequently allowed to assemble each section of four panels, the last panel being reassembled to the next section, thus insuring accuracy for all connections.

Floor Construction.—The writer cannot agree with Mr. Wagner that the floor construction would have been made more durable by water-proofing, by which presumably is meant a water-proof covering of the concrete. The cost of a good water-proof covering is out of proportion to the advantage gained, but even the best water-proofing known cannot be as durable as the concrete base itself. If it becomes defective, it is likely to cause more trouble than if it had not been used. In the case of the Hell Gate Bridge and approaches, the concrete slab has been made water-proof by carefully selecting the materials and determining the proper proportion to secure the greatest density, by carefully troweling the surface to a smooth finish before the concrete had set, by using sufficient steel reinforcement to prevent cracking, and by giving the surface sufficient slope and providing frequent and large drain holes to give the water a free run-off.

In concluding, the writer heartily thanks all who have taken the pains to discuss this paper for their courteous expressions of approval of the manner in which this rather complex subject has been presented. He earnestly hopes that the discussion on some of the disputed questions may be an inducement for further investigation.

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Paper No. 1418

STRESS MEASUREMENTS ON THE HELL GATE ARCH BRIDGE*

By D. B. Steinman, Assoc. M. Am. Soc. C. E.†

With Discussion by Messrs. H. J. Bingham Powell, J. A. L. Waddell, F. H. Frankland, L. A. Waterbury, F. D. Hughes, A. H. Fuller, James E. Howard, Isidore Delson, Gustav Lindenthal, John I. Parcel, H. M. MacKay, F. E. Turneaure, Henry S. Jacoby, O. H. Ammann, C. A. Randorf, David A. Molitor, and D. B. Steinman.

Synopsis.

The magnitude of the Hell Gate Arch Bridge and its interesting construction features suggested the desirability of utilizing the bridge as an instrument for scientific research by conducting a series of stress measurements extending through all the different stages of erection until the structure was completed. As a result of these observations, the Hell Gate Arch is probably the only structure ever built in which the true stress conditions are known from experimental determination.

In order to give this investigation its proper setting and perspective, there is presented, as an introduction, a résumé of previously published measurements of stresses in bridges, with a brief statement of the most significant results obtained. From these findings, particularly with reference to secondary stresses, conclusions are drawn by which to improve the design of railway viaducts and bridges. The record, on the whole, indicates the scarcity of such observations in the past, and serves to emphasize the value and importance of undertaking

* Presented at the meeting of November 21st, 1917.

† Now M. Am. Soc. C. E.

more investigations of this character in order to confirm or correct the results of theoretical analysis. This experimental verification of stress conditions is particularly desirable in fields previously unexplored, or wherever there are special features which may produce uncertain variations from calculated conditions.

The original objects in view in undertaking these stress measurements were: to follow up the stresses in arch and back-stays as critical erection stages were approached; to check and control certain critical operations in the erection; to check the stresses in the completed indeterminate structure, in order to detect any variation from assumed conditions; and to determine the true secondary stresses for comparison with the calculated values.

The paper includes a brief description of the instruments, the method of operation, and the precautions required for accurate work. This is followed by a full presentation of the results, including a tabulated comparison of the calculated and measured primary and secondary stresses.

From the results of the measurements and comparisons, conclusions are deduced on the following subjects:

1.—The precision attainable with the instrument, and the reliability of this method of measuring stresses;

2.—The probable error of calculated stresses, indicating the futility of excessive precision in their computation;

3.—The extreme variations of fiber stress in a member, representing the combined effect of all known and unknown secondary strains;

4.—Safe working values for the design of bridge members, leaving a margin below the elastic limit for extreme variations from calculated stresses;

5.—Relative values of calculated and measured secondary stresses (it is shown that the latter are generally lower);

6.—The effect of the erection operations on the secondary stresses;

7.—The efficacy of the three-faced joints in the lower chord (Fig. 6), which were provided in order to produce a hinge action at the panel points during erection and to concentrate a larger part of the direct stress in the middle third of the cross-section, it having been demonstrated that this novel splicing feature has accomplished the desired objects, thereby reducing the secondary and direct stresses in the outer fibers;

8.—The re-distribution of stresses and the release of secondary strains at a splice during the replacing of drift-pins by rivets;

9.—The comparative freedom of the arch truss from secondary stresses; and

10.—The final extreme fiber stresses as compared with the calculated values.

Special acknowledgment is due to Gustav Lindenthal, M. Am. Soc. C. E., the Chief Engineer of the bridge, who undertook to make these measurements in furtherance of engineering science.

Résumé of Previously Published Stress Measurements.

Since the early days of bridge building, when empirical methods of design prevailed, theoretical analysis has made rapid advances, and has now far outstripped experimental verification. There is a growing movement, however, as shown by the modern testing of large compression members, and in load tests and strain measurements on bridges, to supplement the results of analysis with experimental observation in order to confirm or correct the former, or to reveal unsuspected conditions.

The first measurements of the actual stresses in trusses were made by Professor Fränkel* in 1883. He applied his extensometer to the experimental determination of secondary stresses, and his results produced such uneasiness among German engineers that some of them abandoned the riveted truss in favor of pin connections.

In 1899 Mesnager published an account of the measurement of stresses in a bridge of 180 ft. span, on the Orléans Railway in France, with a discussion† of the results. It was a ten-panel, Pratt truss bridge carrying a single track. The stresses were measured only in the web members, and for a train load of two locomotives and four cars, covering the entire span. The extensometers, devised by M. Rabut, were applied at four points of the cross-section near each end of a member. The results showed the average stresses in the members to be in fair agreement with the calculated primary stresses. There were considerable differences, however, between the stresses on opposite sides of a member. In the posts, secondary stresses were found amounting to 200% of the direct stress, involving a complete reversal

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* Versuche mit dem Dehnungszeichner", *Der Civilingenieur,* 1883.

† "Les Fatigues Réelles et Fatigues Calculées dans un Pont à Grandes Mailles", *Annales des Ponts et Chaussées,* 1899, II.

of stress on one side of the member. In the diagonals the greatest secondary stresses were 45% of the direct stress. The foregoing large secondary stresses do not form the basis for any general conclusions, as they were mainly due to peculiarities in the design of the structure.

Mesnager did not undertake a computation of the secondary stresses in the test bridge, so that no comparison of actual with calculated values is afforded.

In 1901 M. Rabut described* a series of stress measurements which had been made on the bridges of the Orléans Railway. The experiments covered small plate-girder and pony-truss spans. In some plate girders, in reference to which apprehension had been aroused by excessive calculated stresses, the measured stresses were found to be considerably lower. Plate girders supporting longitudinal ties and rails were found to act as beams with fixed ends, with a large reduction of stress at the center, and reversed stresses at the ends. In pony trusses, secondary stresses as high as 30% appeared on the outside of the top chords, caused by the trusses bending inward near the center of the bridge as a result of the floor-beam deflections. The bottom chords of through bridges showed smaller stresses than the top chords, on account of the former being partly relieved by the stringers and the bottom laterals taking part in the elongation. All floor-beams were found to act nearly as simple beams, the end constraint being negligible. Stringers acted as simple beams for a load on one side of the floor-beam, and as beams with fixed ends for symmetrical loading.

The results of these experiments demonstrated the necessity of detailing stringers for full negative bending moments where they frame into floor-beams; the desirability of using deep stringers to avoid high torsional stresses in the floor-beams from one-sided loading; and the importance of using deep floor-beams and verticals comparatively slender in the transverse direction in order to minimize the bending stresses arising from the floor-beam deflections.

In 1905 and 1906, W. Gehler conducted a series of tests and measurements† on a railway bridge of 128 ft. span at Elsterwerda, Saxony. It was a ten-panel, Pratt truss, skew-bridge; and the loading

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* "Conference sur l'Experimentation des Ponts", *Annales des Ponts et Chaussées*, 1901, III.

† "Nebenspannungen eiserner Fachwerkbrücken", Berlin, 1910.

consisted of two tank cars placed near the middle of the span. The vertical deflections at the lower panel points were measured with Fränkel-Leuner deflectometers, and the rotation angles of the same points were observed with extreme care by using highly sensitive spirit levels. The measurements agreed closely with the values previously calculated, except for some small variations which were traced to the stiffness of the track and the effect of the skew arrangement of the trusses. The results, on the whole, afforded a valuable proof of the remarkably close agreement attainable between theoretical computations and actual conditions in a bridge structure.

Gehler had computed the secondary stresses in the trusses from the theoretical vertical and angular deflections of the lower panel points; and he regarded this check on these deflections as a verification of the calculated secondary stresses. He recommends this procedure as an experimental method for determining the actual secondary stresses in a truss.

To make the foregoing investigation complete, some direct stress measurements were made with Fränkel-Leuner extensometers,* which give automatic graphic records of the varying stresses. On account of the unsatisfactory precision of these instruments, the results were so erratic that Gehler concluded that extensometers are not suitable for the measurement of secondary stresses.

In 1907-09 a sub-committee of the American Railway Engineering Association, consisting of F. E. Turneaure, M. Am. Soc. C. E., C. L. Crandall, M. Am. Soc. C. E., the late C. H. Cartlidge, M. Am. Soc. C. E., and the late C. C. Schneider, Past-President, Am. Soc. C. E., conducted a series of tests on a large number of plate-girder and truss bridges, ranging in span from 30 to 440 ft. They used special test trains, and measured the deflection at the center of the span and the strains in the various kinds of members. The object of the tests was, not to determine secondary stresses, nor to compare calculated and measured primary stresses, but to ascertain the relative amounts of the resulting deflections and strains for various speeds of the moving loads, in order to establish the proper provision to be made for impact stresses. The results and conclusions are contained in the report of the Committee.†

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* *Der Civilingenieur*, 1882, p. 200.

† *Bulletin No. 125*, Am. Ry. Eng. Assoc., July, 1910.

In 1911 the Sub-Committee of the American Railway Engineering Association made a theoretical and experimental study of secondary stresses in truss bridges. In several bridges, of both riveted and pin-connected types, the secondary stresses at selected points were measured with extensometers. In the case of a 105-ft. pony Warren truss, a comparison was made of the calculated and measured secondary stresses in the members. The results are published in the report* of the Committee.

With the exception of the 105-ft. span last mentioned, the writer knows of no published comparison between measured and calculated secondary stresses.

Gehler's work was only a partial check on the calculations of secondary stresses, as he merely measured functions, namely, panel point deflections and rotations, from which the actual secondary stresses had to be computed.

All the published stress measurements were made for loads on existing structures. They afford no information as to the strains produced by dead load or the effect of the erection on the secondary stresses.

None of the published measurements was made on indeterminate structures or on any structure exceeding 440 ft. in span.

Reasons for Taking the Measurements.

The magnitude of the Hell Gate Arch Bridge and its unusual structural features suggested to Mr. Lindenthal the desirability of utilizing the structure for scientific research. By preparing the members for extensometer measurements and taking initial readings before erection, the huge arch was converted into an instrument for the experimental study of the true stress conditions in a structure.

The trusses of the bridge are doubly indeterminate: they are two-hinged arches, and they have redundant diagonals in the center panel. It has often been advanced, as an objection to the use of indeterminate bridge types, that the computations are highly theoretical and the actual stress conditions are uncertain. It is hardly necessary to remark that this argument should have little weight under modern methods of design and construction. As an answer to such objections in the future, it appeared to be of interest to demonstrate, by measure-

* *Bulletin No. 163,* January, 1914.

ment, the identity of calculated and actual stresses in the finished structure.

Another possible purpose of extensometer measurements is to supply an added safeguard for a structure during erection by measuring the stresses at critical points. The stresses in the various members of the arch and in the back-stays were measured in the successive stages as the trusses were built out from shore, panel by panel, to their junction at mid-span. When any of the stresses were nearing their maximum values, they were closely watched until the critical stage was passed.

In one instance, one of the eye-bars in the forward stay was found to have a stress 55% higher than the average stress in the group of eye-bars. This appeared in an early stage of the erection, when the stresses were low. As the erection progressed, however, and as was to be expected, the difference in stress gradually diminished until, in the last stage, all the eye-bars were found to be uniformly strained to nearly 20 000 lb. per sq. in.

Another application of the extensometer was to provide a check in some of the critical operations in the erection of the structure. After seven panels had been erected, the hydraulic jacks at the tops of the erection towers were operated to put the forward stay in tension and release the lower stay. The completion of this operation was checked by measurements of the stresses in both stays. When this jacking was performed on the Wards Island side, a small compression was found in the lower stay, indicating that the jacking had gone too far; the jacks were then lowered slightly until the stress was reduced to zero.

After the trusses met at mid-span, the lowering of the jacks to close the arch was closely followed by measurement of the diminishing stress in the forward stay. Measurements were also made in the central chord member, in order to make sure that there was no eccentricity of bearing at the junction of the two half-arches.

The final operation consisted in closing the top chord at the crown in order to convert the structure into a two-hinged arch. A large bolt, connecting across the joint, was provided in order to hold the members together until the drilling of holes and riveting could be completed. This bolt had to be adjusted, by a set of nuts, to an initial condition corresponding to zero stress at 60° Fahr. The exten-

someter was used to control this operation and to check the subsequent stresses in the top-chord member.

The final object of the extensometer measurements was to provide a comparison between the calculated and the actual secondary stresses in the structure. In addition to the general scientific value of such a comparison, there were special conditions which called for a determination of the true stresses. These conditions were the cantilever method of erection, the use of drift-pins for temporary connection and their subsequent replacement by rivets, the three-faced butt-joints in the lower chord, and the unprecedented dimensions and form of cross-section. These features, separately and combined, modify the secondary stresses materially, and render it extremely difficult, if not impossible, to arrive at the true secondary stresses by calculation.

The Instrument and the Method of its Operation.

The instrument used for the stress measurements on the Hell Gate Arch Bridge (Fig. 1) was a 20-in. strain gauge, designed by Mr. James E. Howard. This instrument is essentially a micrometer caliper. The measuring points, made of hard steel and conical in form, are attached to the barrel and the rod, respectively; and the distance between the two points is measured by a micrometer contact screw at one end of the barrel. This screw reads, by a circumferential vernier, to 0.0001 in. It is provided with two milled heads having a spring ratchet between them which slips when contact is made. Some operators prefer to use the inside milled head, relying on their sensitiveness of touch to detect the instant of contact; others use the outside or ratchet head. The barrel of the instrument is covered with leather for mechanical and thermal protection.

Accompanying the instrument is a rectangular steel bar used as a reference or comparison bar (Fig. 2). It is provided with two center holes or gauge points 20 in. apart. Whenever a reading is taken on the member, a comparison reading is taken on the bar; the difference between these two readings is the measurement. This method of operation eliminates the effects of personal equation, as the same observer takes the readings on the bar and on the member. It also dispenses with the necessity of knowing the temperature of the instrument, as the unknown effect of this temperature is eliminated in subtracting the member reading from the bar reading.

No expansion correction is necessary if the reference bar and the member have the same temperature; and, for this reason, the bar is allowed to rest on the member in order to equalize the temperatures before the extensometer is applied. Nevertheless, two thermometers are provided, one permanently inserted in the face of the reference bar and the other to be placed between the gauge points on the member being measured. When the measurement is taken, the two thermometer readings are observed and recorded, so that a correction may be made for any difference between them.

There is also furnished with the instrument a steel trammel bar (Fig. 3) holding two prick punches 20 in. apart. This is used for laying out the holes in the members to be tested.

The holes in the steel are drilled with a hand ratchet drill provided with combination bits, consisting of drill and countersink. The size of the drill is No. 57, and the countersink is 60 degrees. After drilling a hole, the edge between the countersink and cylindrical shaft is removed with a center punch which has been ground to an included angle of 55 degrees. With the tap of a hammer, the punch leaves a small conical seat to receive the measuring points of the extensometer which are also ground to 55 degrees. This special form of recessed gauge mark, with its combination of three surfaces, has a number of advantages: The extreme tips of the measuring points are not used, thus eliminating the errors which would result from the unavoidable wear of these tips; the contact is on an area, instead of a point or line, thereby reducing the wear of the measuring points and the errors from compression of the bearing surface; the bearing area is depressed below the surface of the steel, so that it is better protected from surface dirt and mechanical injury. The 60° countersink helps in guiding the measuring points to their seat. The cylindrical shaft will hold small particles of dust or grit remaining at the bottom of the hole without interfering with the precision of the measurement.

Oil, vaseline, or ivory black paint are used for protecting, filling, or covering the holes, depending on the interval between successive measurements. A pointed aluminum rod, a small cedarwood stick, and some absorbent cotton are used for cleaning the holes before taking any readings.

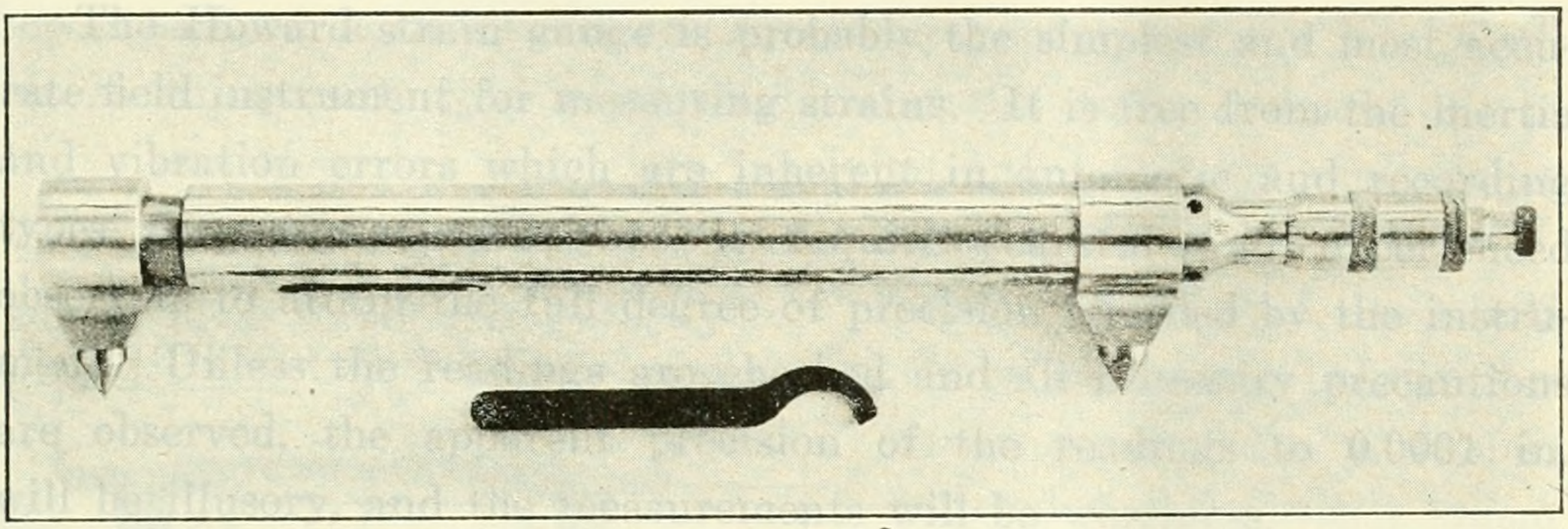


FIG. 1.—HOWARD EXTENSOMETER.

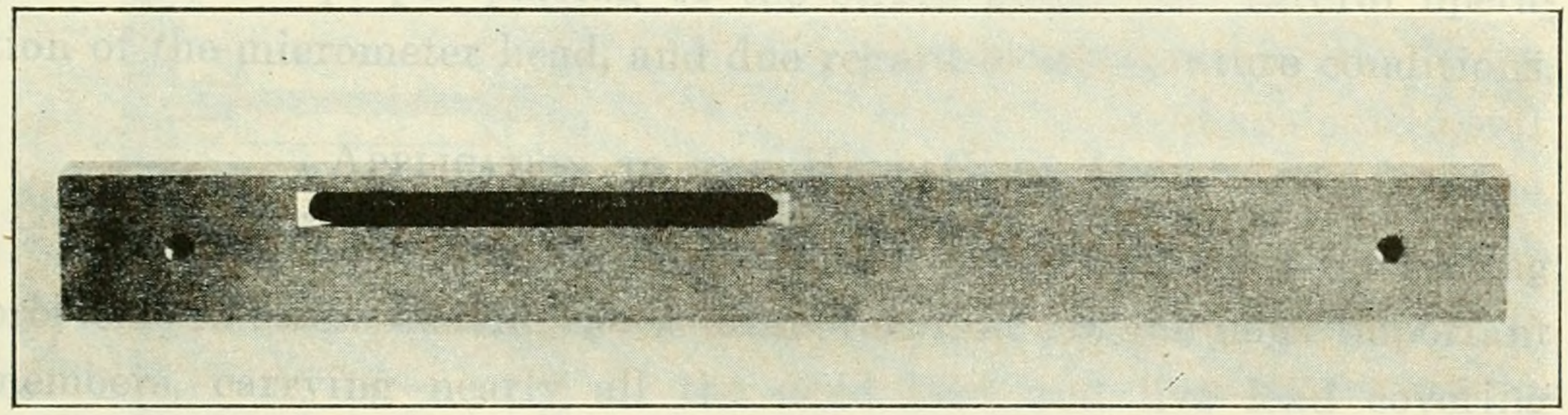


FIG. 2.—COMPARISON BAR.

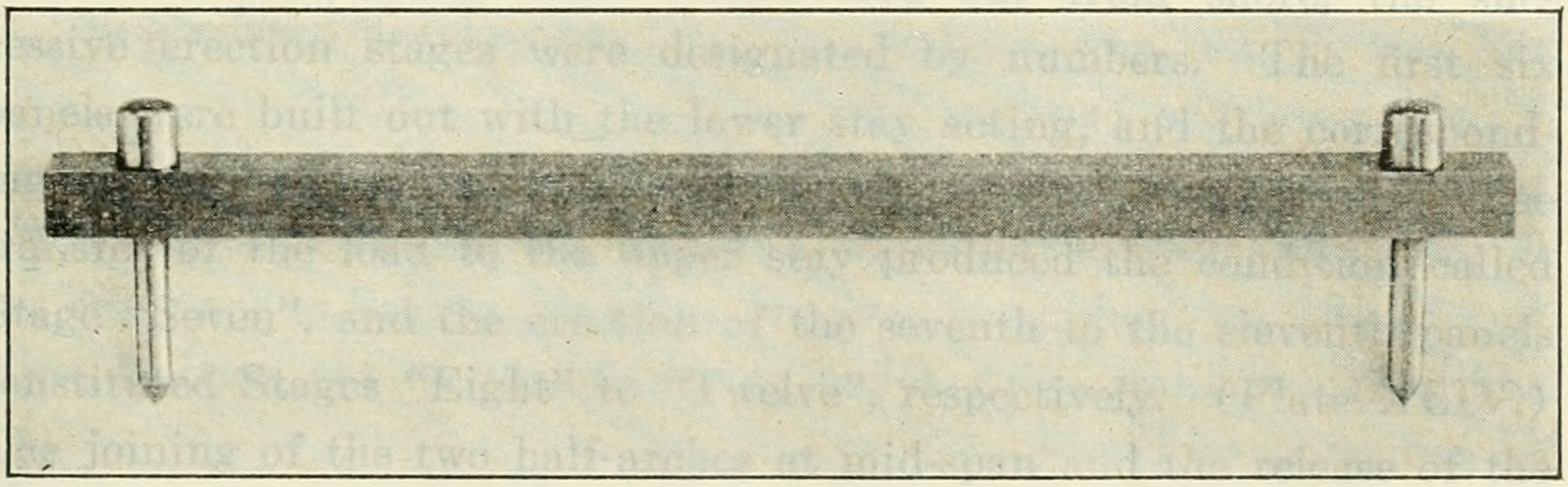


FIG. 3.—TRAMMEL BAR.

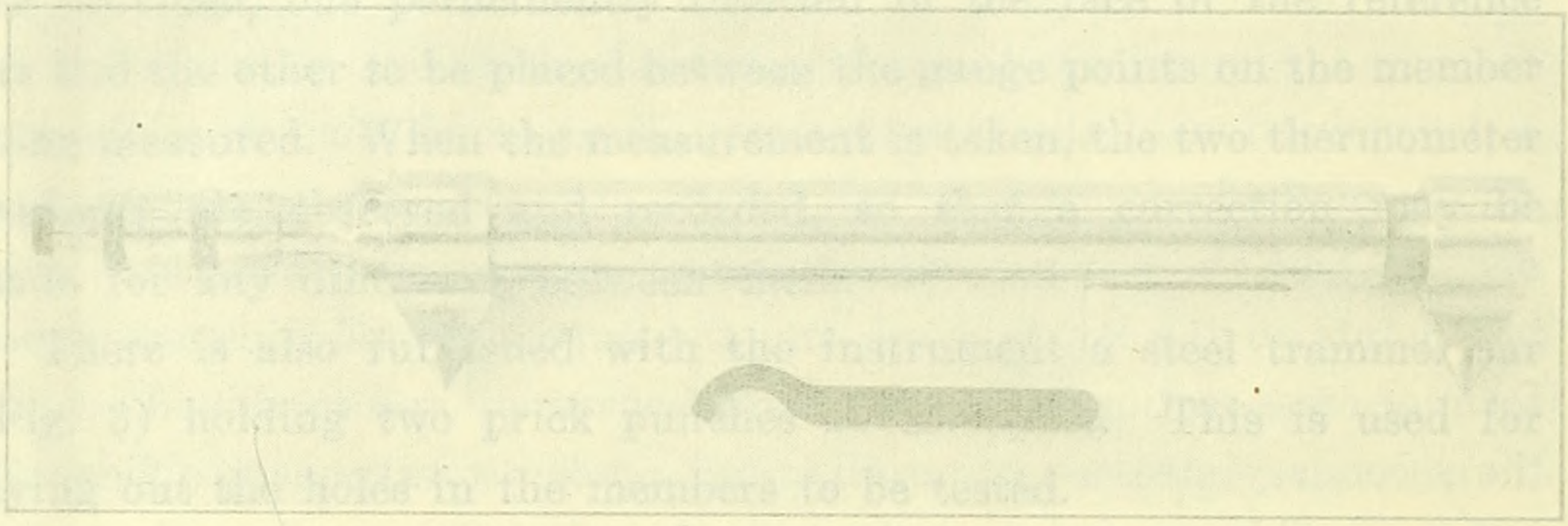


FIG. 1.—HOWARD EXTENSOMETER.

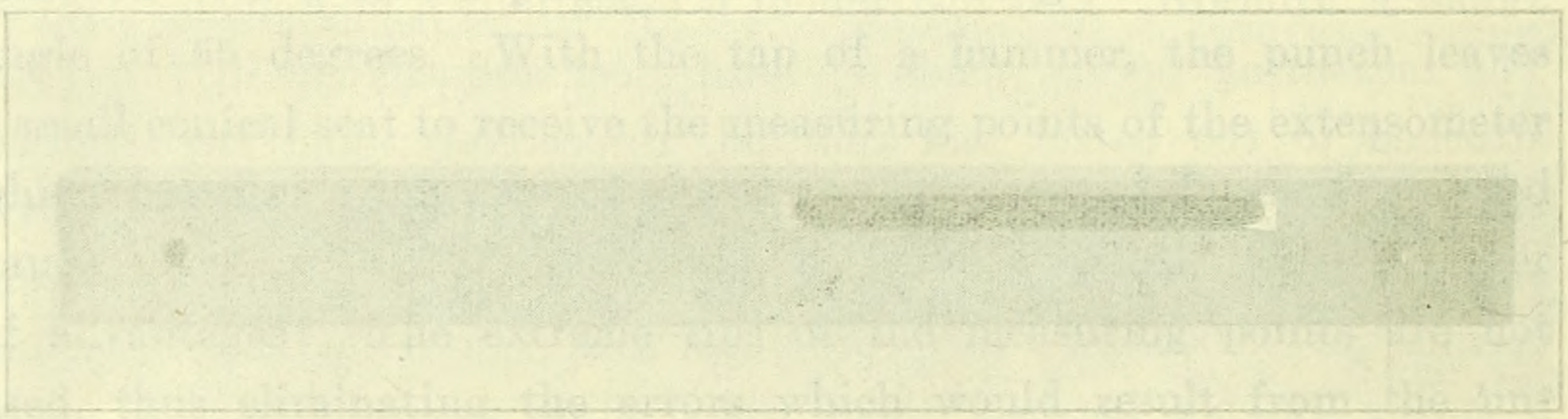


FIG. 2.—COMPARISON BAR.

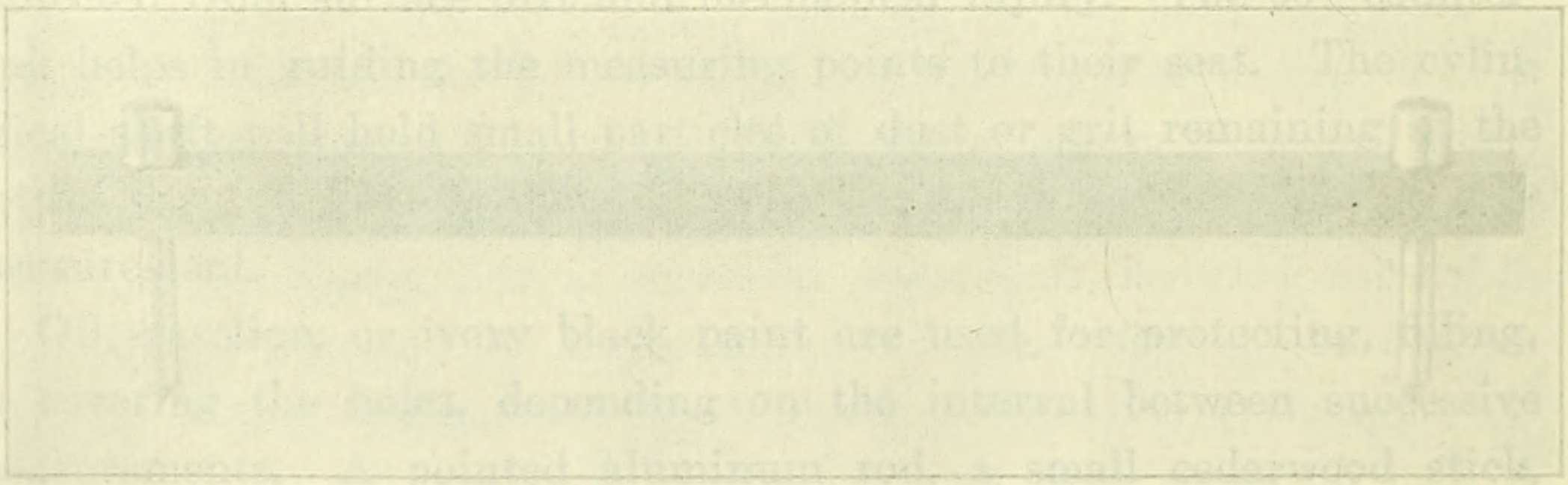


FIG. 3.—TEMPER BAR.

For the best operation of the instrument, two men are required: one to set the measuring points in the holes and hold the instrument square against the member; the other to operate the micrometer and take the readings.

The Howard strain gauge is probably the simplest and most accurate field instrument for measuring strains. It is free from the inertia and vibration errors which are inherent in automatic and recording types of extensometers; but it requires careful and experienced observers to attain the full degree of precision afforded by the instrument. Unless the readings are checked and all necessary precautions are observed, the apparent precision of the readings to 0.0001 in. will be illusory, and the measurements will be worthless.

Without mentioning all the precautions, most of which will readily suggest themselves to any one who has worked with instruments of precision, it may be stated in general that in order to get good results from the measurements, everything depends on the cleanliness of the holes, the proper placing of the strain gauge, the careful operation of the micrometer head, and due regard to temperature conditions.

Application to the Hell Gate Arch.

In the Hell Gate Arch, the lower chords present the most interesting problems in stress distribution. Moreover, they are the most important members, carrying nearly all the dead load and live load covering the whole span. For this reason, together with the restricted time afforded between erection stages to take more stress measurements, it was decided to confine the observations, in general, to the lower chord members.

As the half-arches were cantilevered out from shore, the successive erection stages were designated by numbers. The first six panels were built out with the lower stay acting, and the corresponding erection stages are numbered "One" to "Six", respectively. The transfer of the load to the upper stay produced the condition called Stage "Seven", and the erection of the seventh to the eleventh panels constituted Stages "Eight" to "Twelve", respectively. (Plate XLIV.) The joining of the two half-arches at mid-span and the release of the back-stays brought the trusses to the three-hinged condition, which was designated as "Stage 3-*H*." Then followed the erection of the hangers, floor-beams, stringers, floor, and track, and the closing of

the crown-hinge, bringing the arch to the final or two-hinged condition known as "Stage 2-*H*."

To secure full information as to the distribution of stress in any member, it is necessary to take measurements at not less than three, and preferably four, points of the cross-section near each end of the member. The intensity of stress at any other point of the member can then be found by planar and linear interpolation.

The lower chords of the Hell Gate Arch have a double rectangular section (Fig. 4), consisting of two compartments separated by a horizontal diaphragm. On account of the mass of metal concentrated at and near this diaphragm, measurements at the four extreme corners of the section would not be fairly representative of the conditions throughout the entire area. Any difference of temperature between the horizontal diaphragm and the outside webs tends to produce internal stresses in the member, resulting in a difference between the stress in the diaphragm and the average stress throughout the rest of the cross-section. Furthermore, the beveling of the outer thirds of the webs (Fig. 6), produces a concentration of stress in the middle third of the joint. For these reasons it was judged necessary to take six readings at each cross-section, instead of four; the two additional readings being taken at mid-height in the vertical webs, as indicated in Fig. 4.

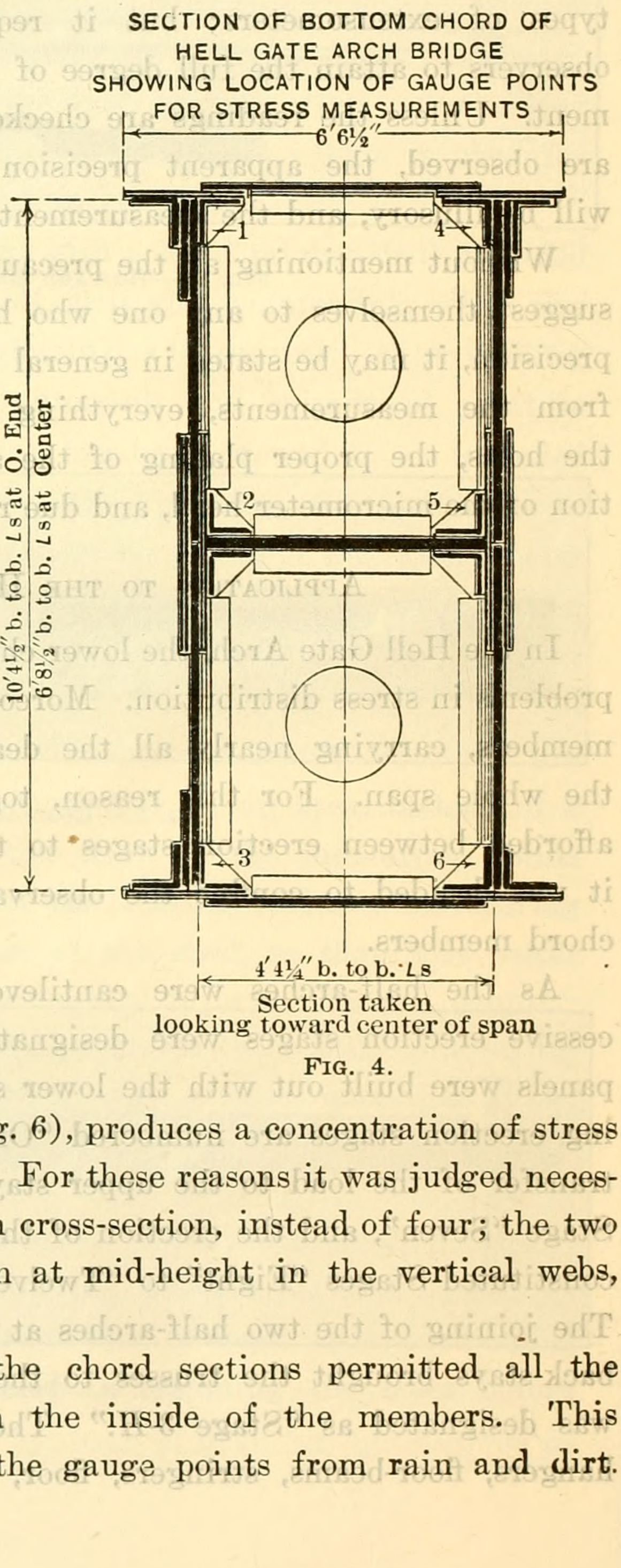


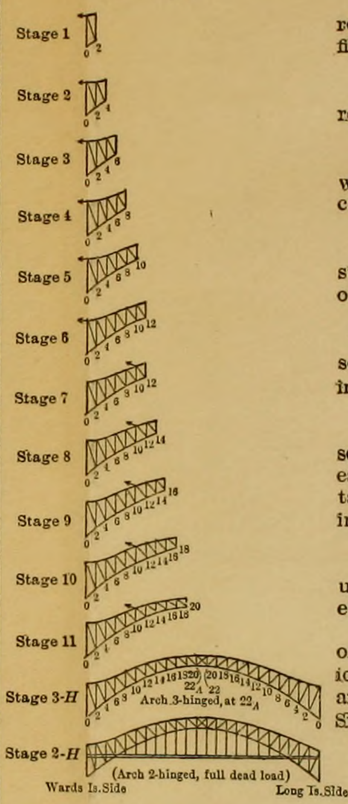
FIG. 4.

The large dimensions of the chord sections permitted all the measurements to be taken on the inside of the members. This afforded better protection for the gauge points from rain and dirt.

MEMBER		0–2				2–4				4–6				6–8				8–10				10–12			
Erection Stage	TRUSS	Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary.	
		Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum
1	S. Truss, Long Is. Side	220	400	1260	122																				
	N. Truss, Long Is. Side	—	—																						
	N. Truss, Wards Is. Side	—	—																						
2	S. Truss, Long Is. Side	695	925	320	24	—	—	—	—																
	N. Truss, Long Is. Side	695	1125	320	90	—	—	—	—																
	N. Truss, Wards Is. Side	609	925	320	93	447	700	835	70																
3	S. Truss, Long Is. Side	—	—	—	—	828	610	225	184	—	—	—	—												
	N. Truss, Long Is. Side	—	—	—	—	828	860	225	214	—	—	—	—												
	N. Truss, Wards Is. Side	1660	1040	102	7	828	800	225	56	480	440	385	77												
4	S. Truss, Long Is. Side	2010	1890	58	10	1475	1590	85	61	800	890	101	97	232	475	235	118								
	N. Truss, Long Is. Side	2010	1540	58	131	1475	1475	85	66	800	1150	101	65	—	—	—	—								
	N. Truss, Wards Is. Side	2010	1860	58	10	1820	2040	85	46	1070	950	101	71	—	—	—	—								
5	S. Truss, Long Is. Side	3190	2925	28	26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—				
	N. Truss, Long Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—				
	N. Truss, Wards Is. Side	2660	2425	28	8	2220	2140	58	58	1440	1460	76	75	585	425	140	115	290	250	—	—				
6	S. Truss, Long Is. Side	—	—	—	—	3520	3465	30	11	2630	2550	51	28	1600	2210	105	44	940	1290	161	125	282	725	479	236
	N. Truss, Long Is. Side	3650	3110	6	68	3520	3300	30	34	2440	2590	51	54	1600	1375	105	67	900	1040	161	108	282	625	479	72
	N. Truss, Wards Is. Side	3580	3260	6	12	3237	3040	30	60	2354	2160	51	50	1379	1275	105	80	720	690	161	142	—	—	—	—
7	S. Truss, Long Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	N. Truss, Long Is. Side	2080	—	—	—	1450	—	—	—	725	—	—	—	587	—	—	—	150	—	—	—	282	650	53	6
	N. Truss, Wards Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	S. Truss, Long Is. Side	2570	1525	95	37	2000	1890	104	58	1310	1175	102	96	830	1090	235	86	815	1090	300	131	940	1040	109	130
	N. Truss, Long Is. Side	2570	1540	95	81	2090	1910	104	110	1310	1075	102	45	830	440	235	136	815	1000	300	98	948	960	109	82
	N. Truss, Wards Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	S. Truss, Long Is. Side	—	—	—	—	2570	2375	74	51	—	—	—	—	—	—	—	—	1700	1550	132	78	—	—	—	—
	N. Truss, Long Is. Side	3060	1775	72	55	—	—	—	—	1920	1125	58	47	1560	1100	113	17	—	—	—	—	1890	—	—	—
	N. Truss, Wards Is. Side	3110	2675	72	30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10	S. Truss, Long Is. Side	3640	2825	53	8	—	—	—	—	2820	2275	32	30	2410	2450	65	26	—	—	—	—	—	—	—	—
	N. Truss, Long Is. Side	—	—	—	—	3280	2640	53	63	—	—	—	—	—	—	—	—	2760	2860	106	70	3060	—	106	—
	N. Truss, Wards Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
11	S. Truss, Long Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4470	3990	97	38
	N. Truss, Long Is. Side	4300	—	—	—	4100	—	—	—	3720	—	—	—	3390	—	—	—	4000	—	—	—	—	—	—	—
	N. Truss, Wards Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8H	S. Truss, Long Is. Side	7200	5910	12	—	7130	6850	9	—	6330	5775	9	—	5520	5610	11	—	5770	5790	12	—	5560	5640	18	—
	N. Truss, Long Is. Side	6340	5075	12	—	6050	5510	9	—	5520	5150	9	—	4730	4540	11	—	5160	5510	12	—	5150	5040	18	—
	N. Truss, Wards Is. Side	5900	5525	12	—	5800	5825	9	—	5300	5190	9	—	4650	4310	11	—	4920	4775	12	—	5080	5040	18	—
2H	S. Truss, Long Is. Side	—	—	—	—					—	—	—	—					—	—	—	—				
	N. Truss, Long Is. Side	11890	10150	2	23					11500	10990	5	13					11290	11225	4	13				
	N. Truss, Wards Is. Side	11890	11450	2	3					11500	11650	5	11					11290	10575	4	7				

MEMBER		12–14				14–16				16–18				18–20				20–22				22–22_A			
Erection Stage	TRUSS	Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary	
		Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum
8	S. Truss, Long Is. Side	310	690	339	152																				
	N. Truss, Long Is. Side	310	690	339	158																				
	N. Truss, Wards Is. Side	—	—	—	—																				
9	S. Truss, Long Is. Side	—	—	—	—	—	—	—	—																
	N. Truss, Long Is. Side	1030	—	—	—	338	—	—	—																
	N. Truss, Wards Is. Side	—	—	—	—	—	—	—	—																
10	S. Truss, Long Is. Side	—	—	—	—	—	—	—	—	—	—	—	—												
	N. Truss, Long Is. Side	2060	—	77	—	1130	—	83	—	375	—	165	—												
	N. Truss, Wards Is. Side	—	—	—	—	—	—	—	—	—	—	—	—												
11	S. Truss, Long Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
	N. Truss, Long Is. Side	3340	—	—	—	2240	—	—	—	1250	—	—	—	405	—	—	—								
	N. Truss, Wards Is. Side	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
8H	S. Truss, Long Is. Side	5360	5560	24	—	5660	5770	26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	N. Truss, Long Is. Side	5300	6050	24	—	5580	5990	26	—	6000	6590	32	—	6470	6910	37	—	6720	7240	38	—	7150	7675	48	—
	N. Truss, Wards Is. Side	5330	6090	24	—	5550	6060	26	—	6030	6260	32	—	6430	6760	34	—	6750	7290	48	—	—	—	—	—
2H	S. Truss, Long Is. Side	—	—	—	—					—	—	—	—					—	—	—	—	—	—	—	—
	N. Truss, Long Is. Side	11240	11640	6	19					11610	11725	7	15					11980	12175	11	16	11820	11775	6	27
	N. Truss, Wards Is. Side	11240	11015	6	6					11610	11315	7	7					—	—	—	—	—	—	—	—

Erection Stages:



NOTES

All Primary Stresses are compression, and are given in pounds per square inch.

All Secondary Stresses are either tension or compression, representing equal values of opposite sign in the top and bottom fibers of a section.

All Secondary Stresses are given in percentages of the corresponding Primary Stresses.

Each Calculated Primary Stress is calculated for the actual weight and condition of structure and travelers at the time the corresponding stress was measured.

Each Measured Primary Stress is the average of twelve strain measurements, six points being observed at each end of each member.

Each Calculated Secondary Stress is the larger of the two secondary stresses calculated for the two ends of each member in each erection stage.

Each Measured Secondary Stress is the larger of the two secondary stresses found for the two ends of each member in each erection stage, the secondary stress at each end being obtained as one-half the difference between the average stresses in the top and bottom fibers of the cross-section.

For Stage 3-H, the Calculated Secondary Stresses were figured for the theoretical stage with no floor-beams or hangers erected.

As the field measurements were taken during the erection of floor-beams and hangers, with varying positions of the erection travelers, the Measured Secondary Stresses for this stage are not properly comparable with the Calculated Secondary Stresses and are therefore omitted from the tabulation.

EXTENSOMETER MEASUREMENTS OF STRESSES IN BOTTOM-CHORD MEMBERS OF HELL GATE ARCH BRIDGE.

MEMBER		0–2				2–4				4–6			
	TRUSS	Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary	
		Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum
1	1	280	100	1250	125								
	2	—	—	—	—								
	3	—	—	—	—								
2	4	805	925	320	75	—	—	—	—				
	5	805	1125	320	90	—	—	—	—				
	6	605	925	320	85	415	700	825	70				
3	7	—	—	—	—	828	810	725	185	—	—	—	—
	8	—	—	—	—	828	800	725	215	—	—	—	—
	9	1860	1810	102	7	828	800	725	36	410	150	410	385
4	10	3010	1850	86	10	1475	1300	1475	61	800	800	2500	101
	11	3010	1810	86	131	1475	1175	1475	85	800	800	1175	101
	12	3010	1860	58	10	1850	3010	1850	85	1070	970	970	101
5	13	—	—	—	—	—	—	—	—	—	—	—	—
	14	—	—	—	—	—	—	—	—	—	—	—	—
	15	3060	2165	85	8	2220	2120	85	58	1410	1160	1410	76
6	16	—	—	—	—	—	—	—	—	—	—	—	—
	17	—	—	—	—	—	—	—	—	—	—	—	—
	18	3010	3110	6	85	3220	3120	85	30	2410	2080	2410	31
	19	3080	3200	6	12	3220	3020	85	30	2410	2080	2410	31
7	20	—	—	—	—	—	—	—	—	—	—	—	—
	21	—	—	—	—	—	—	—	—	—	—	—	—
	22	—	—	—	—	—	—	—	—	—	—	—	—
8	23	2870	1053	85	37	2000	1800	2000	101	1410	1115	1410	102
	24	2910	1340	85	81	2000	1010	2000	104	1310	1025	1310	108
	25	—	—	—	—	—	—	—	—	—	—	—	—
9	26	—	—	—	—	—	—	—	—	—	—	—	—
	27	3000	1715	72	93	—	—	—	—	—	—	—	—
	28	3110	2015	72	70	—	—	—	—	—	—	—	—
10	29	3040	2060	85	8	—	—	—	—	—	—	—	—
	30	—	—	—	—	—	—	—	—	—	—	—	—
	31	—	—	—	—	—	—	—	—	—	—	—	—
11	32	—	—	—	—	—	—	—	—	—	—	—	—
	33	—	—	—	—	—	—	—	—	—	—	—	—
	34	—	—	—	—	—	—	—	—	—	—	—	—
12	35	—	—	—	—	—	—	—	—	—	—	—	—
	36	—	—	—	—	—	—	—	—	—	—	—	—
	37	—	—	—	—	—	—	—	—	—	—	—	—
13	38	—	—	—	—	—	—	—	—	—	—	—	—
	39	—	—	—	—	—	—	—	—	—	—	—	—
	40	—	—	—	—	—	—	—	—	—	—	—	—
14	41	—	—	—	—	—	—	—	—	—	—	—	—
	42	—	—	—	—	—	—	—	—	—	—	—	—
	43	—	—	—	—	—	—	—	—	—	—	—	—
15	44	—	—	—	—	—	—	—	—	—	—	—	—
	45	—	—	—	—	—	—	—	—	—	—	—	—
	46	—	—	—	—	—	—	—	—	—	—	—	—
16	47	—	—	—	—	—	—	—	—	—	—	—	—
	48	—	—	—	—	—	—	—	—	—	—	—	—
	49	—	—	—	—	—	—	—	—	—	—	—	—

MEMBER		12–14				14–16				16–18			
	TRUSS	Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary		Primary Stress		Secondary Stress in % of Primary	
		Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum	Calculated	Measured	Calculated Maximum	Measured Maximum
1	1	310	990	329	102								
	2	310	990	329	158								
	3	—	—	—	—								
2	4	—	—	—	—	—	—	—	—				
	5	—	—	—	—	—	—	—	—				
	6	—	—	—	—	—	—	—	—	—	—	—	—
3	7	—	—	—	—	—	—	—	—	—	—	—	—
	8	—	—	—	—	—	—	—	—	—	—	—	—
	9	—	—	—	—	—	—	—	—	—	—	—	—
4	10	—	—	—	—	—	—	—	—	—	—	—	—
	11	—	—	—	—	—	—	—	—	—	—	—	—
	12	—	—	—	—	—	—	—	—	—	—	—	—
5	13	—	—	—	—	—	—	—	—	—	—	—	—
	14	—	—	—	—	—	—	—	—	—	—	—	—
	15	—	—	—	—	—	—	—	—	—	—	—	—
6	16	—	—	—	—	—	—	—	—	—	—	—	—
	17	—	—	—	—	—	—	—	—	—	—	—	—
	18	—	—	—	—	—	—	—	—	—	—	—	—
	19	—	—	—	—	—	—	—	—	—	—	—	—
7	20	—	—	—	—	—	—	—	—	—	—	—	—
	21	—	—	—	—	—	—	—	—	—	—	—	—
	22	—	—	—	—	—	—	—	—	—	—	—	—
8	23	—	—	—	—	—	—	—	—	—	—	—	—
	24	—	—	—	—	—	—	—	—	—	—	—	—
	25	—	—	—	—	—	—	—	—	—	—	—	—
9	26	—	—	—	—	—	—	—	—	—	—	—	—
	27	—	—	—	—	—	—	—	—	—	—	—	—
	28	—	—	—	—	—	—	—	—	—	—	—	—
10	29	—	—	—	—	—	—	—	—	—	—	—	—
	30	—	—	—	—	—	—	—	—	—	—	—	—
	31	—	—	—	—	—	—	—	—	—	—	—	—
11	32	—	—	—	—	—	—	—	—	—	—	—	—
	33	—	—	—	—	—	—	—	—	—	—	—	—
	34	—	—	—	—	—	—	—	—	—	—	—	—
12	35	—	—	—	—	—	—	—	—	—	—	—	—
	36	—	—	—	—	—	—	—	—	—	—	—	—
	37	—	—	—	—	—	—	—	—	—	—	—	—
13	38	—	—	—	—	—	—	—	—	—	—	—	—
	39	—	—	—	—	—	—	—	—	—	—	—	—
	40	—	—	—	—	—	—	—	—	—	—	—	—
14	41	—	—	—	—	—	—	—	—	—	—	—	—
	42	—	—	—	—	—	—	—	—	—	—	—	—
	43	—	—	—	—	—	—	—	—	—	—	—	—
15	44	—	—	—	—	—	—	—	—	—	—	—	—
	45	—	—	—	—	—	—	—	—	—	—	—	—
	46	—	—	—	—	—	—	—	—	—	—	—	—
16	47	—	—	—	—	—	—	—	—	—	—	—	—
	48	—	—	—	—	—	—	—	—	—	—	—	—
	49	—	—	—	—	—	—	—	—	—	—	—	—

Incidentally, it secured better protection for the observers from falling drift-pins and rivets.

The gauge points were drilled and the initial readings taken while the members were still on the dock or on the car-floats. The points were located in the vertical legs of the 8 by 8-in. flange angles connecting the vertical and horizontal webs, and as near the panel points as possible, but not within gusset or splice-plates. There were thus twelve gauge points in each member, each gauge point consisting of two holes 20 in. apart. The exact location of each gauge point was noted for permanent record.

For designation, the gauge points were numbered from 1 to 6 at each section, and to these figures was prefixed the number of the member. Thus, "hole 681" denotes the upper, left-hand gauge point near panel point 6 of member 6-8; similarly, "hole 864" denotes the upper right-hand gauge point (No. 4) near panel point 8 of the same member.

The number of the hole and that of the erection stage thus suffice to identify any extensometer reading.

Because of the more rapid erection on the Wards Island side, one half-truss on that side had to be omitted from the programme of measurements. The remaining three sets of measurements, however, are sufficient to furnish information as to any variations from the symmetrical disposition of stresses about the longitudinal and transverse center lines of the bridge.

There were thus twelve pairs of holes in each of thirty-five chord members to be drilled; and stress readings were to be taken at these points in fourteen different stages. This meant a total of more than 3 000 extensometer measurements in the regular schedule, besides special measurements in the back-stays and the eye-bar connection links.

In some of the later stages, time did not permit all the chord stresses to be measured; and, therefore, alternate members were omitted.

On account of the difficulty of traveling through the members with the instruments, involving crawling through numerous diaphragm holes, much time was consumed in going to and from the points of measurement. Consequently, it was seldom possible to measure more than two members in a day.

TABLE 1.—TYPICAL FIELD RECORD SHEET.

RECORD OF EXTENSOMETER READINGS AT HELL GATE ARCH BRIDGE.

Location: Long Island End, Member 0-2, South Truss, and Member 2-4, North Truss.

Date: Sept. 15, 1915. Time: 8.30 A. M. to 4.15 P. M. Weather: Clear.

Observers: O. H. S. Koch and A. V. O'Donnell.

No.	Temperature, Degrees Fahr.			Reading.		Difference in Length between Bar and Member.		Length corrected to 60° Fahr.	Location on member where measurements were taken.
	Air.	Bar.	Member.	Bar.	Member.	Measured.	Corrected for temp.		
South Truss, Member 0-2.									
2025	.....	78.5	78	9983	9877	—0106	—0105	19.9895	025
2026	.....	80	78	9978	9850	—0128	—0125	19.9875	024
2027	.....	80	78.5	9977	9855	—0122	—0120	19.9880	021
2028	.....	80	79.5	9977	0139	+0162	+0163	20.0163	022
2029	.....	81	78	9978	9898	—0080	—0076	19.9924	205
2030	.....	81	79	9978	9935	—0043	—0040	19.9960	204
2031	.....	82	80	9978	9739	—0239	—0236	19.9764	201
2032	.....	82	80	9978	9878	—0100	—0097	19.9903	202
2033	.....	83	81	9981	9896	—0085	—0082	19.9918	023
2034	.....	83	79.5	9980	9966	—0014	—0009	19.9991	026
2035	.....	83	80	9980	9892	—0088	—0084	19.9916	206
2036	84	83.5	84	9980	9930	—0050	—0051	19.9949	203
North Truss, Member 2-4.									
2037	.....	85.5	83	9982	9952	—0030	—0027	19.9973	245
2038	.....	85.5	87	9982	9975	—0007	—0009	19.9991	244
2039	.....	86	86	9982	9995	+0013	+0013	20.0013	242
2040	.....	86	88	9981	9950	—0031	—0034	19.9966	241
2041	.....	86.5	85.5	9982	9990	+0008	+0009	20.0009	422
2042	.....	86.5	89	9981	9919	—0062	—0065	19.9935	421
2043	.....	87	83.5	9981	0013	+0032	+0037	20.0037	425
2044	.....	87	87.5	9981	0340	+0359	+0358	20.0358	424
2045	.....	87	86	9983	9991	+0008	+0009	20.0009	246
2046	.....	87	90.5	9982	0045	+0063	+0058	20.0058	243
2047	.....	87.5	87	9981	9974	—0007	—0006	19.9994	423
2048	86	87.5	86	9981	0008	+0027	+0029	20.0029	426

NOTES.—No. = serial number of observation. Under temperature: air = atmospheric temp., bar = temp. of std. reference bar, member = temp. of member or metal. Under reading: bar = strain gauge measurement of std. reference bar, member = strain gauge measurement of member or metal.

REMARKS.—Erection Stage No. 10, traveler still at 15–17. Wind: southwest, mild.

Signed, OTTO H. S. KOCH.

Table 1 shows a typical field-office record sheet. It also serves to indicate the arrangement in the field notebook, as the record sheet is practically a typewritten copy of the field entries.

From the record sheets the measurements were entered and computed on office cards, illustrated by Tables 2, 3, and 4. One of these cards was provided for each member, with spaces for entering the

observed strains in each erection stage. Following the numbers or marks of the gauge points there is a column headed "Initial" in which are recorded the lengths between gauge points before the member is stressed. The standard length of 20 in. is understood to be added to each of these figures. In the remaining columns the figures denote the actual strain, that is the increase or decrease from the initial gauge length for the different erection stages. Each unit of strain (0.0001 in.) represents a fiber stress of 150 lb. per sq. in.; hence the sum of the six strains at a section, multiplied by 25, gives the average unit stress in the member. The calculated stresses, inserted for comparison with the measured stresses, were computed from the shipping weights of the erected members, with an allowance for erection material.

The extreme strain readings observed for each member, multiplied by 150, give the minimum and maximum fiber stresses in the member.

From the cards the summary table (Plate XLIV) was compiled, giving a comparison of the calculated and the observed stresses in the various erection stages.

The secondary stresses were computed from the strain measurements, as follows: The average of the observed strains in Holes 1 and 4 at any section gave the upper fiber stress; the average for Holes 3 and 6 gave the bottom fiber stress; one-half the difference between these two extreme fiber stresses gave the secondary stress. This result divided by the average stress in the member gave the secondary stress as a percentage of the primary stress.

Results of the Stress Measurements.

A tabulation of the results of the stress measurements on the Hell Gate Arch during erection is shown on Plate XLIV. The notes and data on that plate render it self-explanatory.

The results of the observations have been studied from various angles, in order to determine the following relations:

1.—Comparison of the measured average stresses at the two ends of each member, as an index of the precision of the method;

2.—Comparison of measured with calculated primary stresses;

3.—Comparison of extreme secondary stresses with primary stresses, in order to establish empirical relations between the two;

TABLE 2.—Typical Office Record Card.

Extensometer Measurements of Erection Stresses, Hell Gate Arch.

Wards Island End. North Truss. Member 0-2.

	Hole.	Initial.	Stage 1.	Stage 2.	Stage 3.	Stage 4.	Stage 5.	Stage 6.	Stage 7.	Stage 8.	Stage 9.	Stage 10.	Stage 11.	3-*H*.	2-*H*.
Lower end.	021	+ 0020	.....	— 10	— 12	— 16	— 23	— 26	.....	.....	— 24	.....	.....	— 39	— 77
	022	+ 0013	.....	0	— 7	— 13	— 15	— 17	.....	.....	— 16	.....	.....	— 34	— 83
	023	— 0007	.....	+ 1	— 12	— 13	— 20	— 27	.....	.....	— 13	.....	.....	— 36	— 73
	024	+ 0011	.....	— 13	— 5	— 12	— 16	— 19	.....	.....	— 23	.....	.....	— 38	— 60
	025	+ 0048	.....	— 12	— 3	— 15	— 14	— 16	.....	.....	— 19	.....	.....	— 38	— 76
	026	— 0031	.....	— 1	— 4	— 12	— 14	— 24	.....	.....	— 17	.....	.....	— 33	— 67
	Total.....	.....	.....	— 35	— 43	— 81	— 102	— 129	.....	.....	— 112	.....	.....	— 218	— 436
	Aver. Stress..	.....	.....	— 875	—1 075	— 2 025	— 2 550	— 3 225	.....	.....	— 2 800	.....	.....	— 5 450	— 10 900
Upper end.	201	+ 0002	.....	— 3	— 7	— 9	— 18	— 15	.....	.....	— 13	.....	.....	— 29	— 61
	202	— 0001	.....	— 11	— 13	— 13	— 22	— 25	.....	.....	— 17	.....	.....	— 35	— 96
	203	+ 0002	.....	+ 2	— 7	— 11	— 17	— 24	.....	.....	— 8	.....	.....	— 34	— 75
	204	+ 0008	.....	— 18	— 6	— 16	— 14	— 23	.....	.....	— 31	.....	.....	— 49	— 72
	205	+ 0025	.....	— 5	— 1	— 10	— 10	— 21	.....	.....	— 18	.....	.....	— 40	— 108
	206	+ 0011	.....	— 4	— 6	— 9	— 11	— 24	.....	.....	— 15	.....	.....	— 37	— 68
	Total.....	.....	.....	— 39	— 40	— 68	— 92	— 132	.....	.....	— 102	.....	.....	— 224	— 480
	Aver. stress..	.....	.....	— 975	—1 000	— 1 700	— 2 300	— 3 300	.....	.....	— 2 550	.....	.....	— 5 600	— 12 000
	Calc. stress..	.....	.....	— 609	— 1 660	— 2 010	— 2 660	— 3 580	.....	.....	— 3 110	.....	.....	— 5 900	— 11 890
	Min. stress...	.....	.....	+ 300	— 150	— 1 350	— 1 500	— 2 250	.....	.....	— 1 200	.....	.....	— 4 350	— 9 000
	Max. stress..	.....	.....	— 2 700	— 1 950	— 2 400	— 3 450	— 4 050	.....	.....	— 4 650	.....	.....	— 7 350	— 16 200

TABLE 3.—TYPICAL OFFICE RECORD CARD.

EXTENSOMETER MEASUREMENTS OF ERECTION STRESSES, HELL GATE ARCH.

Long Island End. North Truss. Member 2-4.

Hole.		Initial.	Stage 1.	Stage 2.	Stage 3.	Stage 4.	Stage 5.	Stage 6.	Stage 7.	Stage 8.	Stage 9.	Stage 10.	Stage 11.	3-*H*.	2-*H*.
Lower end.	241	— 0017	.. ..	— 3	— 5	..	— 9	..	— 8	..	— 17	..	— 32	....
	242	+ 0024	.. ..	— 4	— 2	..	— 16	..	— 5	..	— 11	..	— 31	....
	243	+ 0075	.. ..	— 2	— 6	..	— 27	..	— 10	..	— 17	..	— 34	....
	244	+ 0015	.. ..	— 12	— 12	..	— 22	..	— 22	..	— 24	..	— 38	....
	245	— 0012	.. ..	— 9	— 14	..	— 22	..	— 11	..	— 15	..	— 36	....
	246	+ 0032	.. ..	— 10	— 25		— 34	..	— 16	..	— 23	..	— 47	....
Total.....		.....	.. ..	— 40	— 64	..	— 130	..	— 72	..	— 107	..	— 218	....
Aver. stress...		.....	.. ..	— 1 000	— 1 600	..	— 3 250	..	— 1 800	..	— 2 675	..	— 5 450	....
Upper end.	421	— 0036	.. ..	— 18	— 11	..	— 28	..	— 28	..	— 29	..	— 44	....
	422	+ 0023	.. ..	+ 1	— 4	..	— 18	..	— 7	..	— 14	..	— 33	....
	423	— 0001	.. ..	+ 12	— 1	..	— 16	..	— 2	..	— 5	..	— 28	....
	424	+ 0386	.. ..	— 16	— 18	..	— 29	..	— 31	..	— 28	..	— 45	....
	425	+ 0057	.. ..	— 12	— 18	..	— 23	..	— 12	..	— 20	..	— 42	....
	426	+ 0037	.. ..	+ 4	— 2	..	— 20	..	— 1	..	— 8	..	— 33	....
Total.....		.....	.. ..	— 29	— 54	..	— 134	..	— 81	..	— 104	..	— 225	....
Aver. stress...		.....	.. ..	— 725	—1 350	..	— 3 350	..	— 2 025	..	— 2 600	..	— 5 625	....
Calc. stress...		.....	.. ..	— 828	—1 475	..	— 3 520	..	— 2 000	..	— 3 280	..	— 6 050	....
Min. stress....		.....	.. ..	+1 800	— 150	..	— 1 350	..	— 150	..	— 750	..	— 4 200	....
Max. stress...		.....	.. ..	—2 700	—3 750	..	— 5 100	..	— 4 650	..	— 4 350	..	— 7 050	....

4.—Comparison of calculated with measured secondary stresses, in order to ascertain the effects of special features of fabrication and erection on these stresses; and

5.—Comparison of calculated with measured extreme fiber stresses, in order to determine the resultant effect of all variations in primary and secondary stress.

COMPARISON OF MEASUREMENTS AT THE TWO ENDS OF EACH MEMBER.

By averaging the six measurements near each end of a member, two values are obtained for the average intensity of stress in the member. The difference between these two values, provided there is

TABLE 4.—Typical Office Record Card.

Extensometer Measurements of Erection Stresses, Hell Gate Arch.

Wards Island End. North Truss. Member 4-6.

Hole.		Initial.	Stage 1.	Stage 2.	Stage 3.	Stage 4.	Stage 5.	Stage 6.	Stage 7.	Stage 8.	Stage 9.	Stage 10.	Stage 11.	3–*H*.	2–*H*.
Lower end.	461	— 0078	..	..	— 12	— 11	— 17	— 15	..	..	..	..	..	— 37	— 62
	462	+ 0015	..	..	— 7	— 6	— 17	— 13	..	..	..	..	..	— 35	— 75
	463	— 0037	..	..	— 14	— 12	— 14	— 27	..	..	..	..	..	— 43	— 76
	464	— 0016	..	..	+ 9	+ 4	+ 5	— 1	..	..	..	..	..	— 20	— 51
	465	— 0024	..	..	+ 4	— 5	— 11	— 14	..	..	..	..	..	— 35	— 76
	466	— 0068	..	..	+ 2	— 9	— 5	— 18	..	..	..	..	..	— 38	— 70
Total.....		.....	..	..	— 18	— 39	— 59	— 88	..	..	..	..	..	— 208	— 410
Aver. stress..		.....	..	..	— 450	— 975	—1 475	—2 200	..	..	..	..	..	— 5 200	— 10 250
Upper end.	641	— 0032	..	..	— 4	— 14	— 21	— 17	..	..	..	..	..	— 44	— 70
	642	— 0012	..	..	— 5	— 8	— 14	— 17	..	..	..	..	..	— 42	— 90
	643	+ 0023	..	..	— 5	— 3	— 1	— 15	..	..	..	..	..	— 32	— 69
	644	— 0004	..	..	— 2	— 4	— 10	— 8	..	..	..	..	..	— 28	— 81
	645	— 0367	..	..	— 1	— 11	— 11	— 15	..	..	..	..	..	— 34	— 139
	646	+ 0040	..	..	0	+ 3	— 1	— 13	..	..	..	..	..	— 27	— 73
Total.....		.....	..	..	— 17	— 37	— 58	— 85	..	..	..	..	..	— 207	— 522
Aver. stress..		.....	..	..	— 425	— 925	—1 450	—2 125	..	..	..	..	..	— 5 175	— 13 050
Calc. stress..		.....	..	..	— 480	—1 070	—1 440	—2 354	..	..	..	..	..	— 5 300	— 11 500
Min. stress...		.....	..	..	+1 350	+ 600	+ 750	— 150	..	..	..	..	..	— 3 000	— 7 650
Max. stress...		.....	..	..	—2 100	—2 100	—3 150	—4 050	..	..	..	..	..	— 6 600	— 20 850

no inequality of respective cross-sections, is an index of the precision of the observations. The average value of this difference for all the lower chord measurements during erection was 140, and the greatest value was 500 lb. per sq. in. Consequently, by the theory of errors, the result obtained by averaging the two end stresses has an average probable error of 47 and a maximum probable error of 170 lb. per sq. in.

The foregoing figures represent the combined effect of all personal and instrumental errors in taking the observations. A very small error in each of the instrumental readings will account for these results. Thus, an inaccuracy of 0.0002 in. in each micrometer reading,

and of 1° Fahr. in each temperature reading, will produce the foregoing average probable error, even if all inaccuracies compensate according to the theory of probabilities. A very small fraction of the same inaccuracies will suffice to account for the observed discrepancies, if the inaccuracies do not compensate fully according to the probability theory. As the least reading of the micrometer is 0.0001 and of the thermometer is 1° Fahr., the results given indicate that the work, as a whole, was carefully executed, and that the utmost precision afforded by the method was actually attained.

Another fact to be observed is that the magnitude of the differences between the end measurements has no connection with the magnitude of the respective stresses. This method of measuring the stresses makes the errors independent of the intensity of the stress. No matter how small the stress, the result, as just shown, may be in error by as much as 170 lb. per sq. in. These considerations indicate the unsuitableness of this method of stress measurement for the smaller stresses, as the results would be too erratic. Although the stresses measured on the Hell Gate Bridge ranged from 400 to 20 000, the writer would not recommend the use of the extensometer for stresses of less than 1 000 lb. per sq. in.

The foregoing discussion of results does not include the final stage (2-*H*). In that stage, in the members near the ends of the span, the upper end in every case was found to present a larger average stress than did the lower end. The differences ranged from 450 to 2 800 lb. per sq. in. This anomalous result appears to arise from some unexplained disturbance of stress distribution, and does not represent any error of observation.

Comparison of Measured with Calculated Stresses.

A comparison of measured with calculated stresses discloses a greater discrepancy than is found between the measurements at the two ends of a member. This is due to other factors, besides instrumental and personal inaccuracies, entering into consideration.

The results of the comparison, including all measurements on chords, eye-bars, and back-stays, are as follows:

For stresses in the ranges 0-1 000, 1 000-3 000, 3 000-5 000, 5 000-10 000, and 10 000-20 000, the average percentage discrepancies between the calculated and measured stresses were 50, 17, 13, 8, and 6%, respec-

tively. These percentages indicate that the method is of little value for checking calculated stresses of less than 1 000 lb. per sq. in., but is of increasing value as the intensity of stress increases.

The average discrepancies in the foregoing ranges were 223, 313, 498, 525, and 829 lb. per sq. in., respectively, and may be expressed very closely by the formula,

$$\left.\begin{array}{l}\text{Average discrepancy between}\\ \text{calculated and measured stress}\end{array}\right\} = 5\% \text{ of the stress} + 200.$$

The greatest discrepancies in the foregoing ranges were 535, 1 045, 1 285, 1 290, and 1 740, respectively, and may be expressed approximately by the formula,

$$\left.\begin{array}{l}\text{Maximum discrepancy between}\\ \text{calculated and measured stress}\end{array}\right\} = 5\% \text{ of the stress} + 1\,000.$$

It will be observed that these formulas contain both a percentage factor and an absolute term, indicating that the discrepancy between calculation and measurement is the resultant of two groups of errors, one dependent on, and the other independent of, the magnitude of the stress.

The absolute term in these formulas (average 200, maximum 1 000 lb. per sq. in.) may be assumed to represent the actual inaccuracy of the measurements, or the total effect of personal, instrumental, and physical errors.

The most important of the physical errors are those due to thermal effects. An error of 1° Fahr. in determining the temperature of the steel at the point of measurement, or a change of 1° Fahr. in the temperature of the instrument during any observation, produces an error of 195 lb. per sq. in. in the resulting stress. The first error named is not necessarily due to incorrect reading of the thermometer, but may be caused by a difference in temperature between the inside and the outside of the thick webs composing the members. Such errors, as a rule, are not compensating; they would naturally occur in the same direction at both ends of a member, and, therefore, would not appear in the difference between the two end measurements.

Other errors arise from differences in temperature between different points of the large chord sections. Thus, on a sunny day, the outside webs and covers may be hotter than the middle diaphragm. The resulting differences of expansion produce a redistribution of

the stress in the cross-section. Although the actual average stress throughout the entire section must remain unchanged, the measured average stress will be affected by these internal temperature strains, as the readings are taken at a limited number of points in the cross-section.

It is difficult to estimate the total probable error due to these effects of temperature. Nevertheless, from a study of the results of the measurements, it appears that the effect of the combined temperature errors, together with any other unascertainable physical effects, is to increase the probable error of the stresses, as previously determined, from 47 (average) and 170 (maximum) to 200 (average) and 1 000 (maximum).

The foregoing discussion should serve to emphasize the importance of careful attention to temperature conditions in taking observations; and it explains why cloudy days should be selected in order to obtain the most accurate results.

After deducting all personal, instrumental, and temperature errors, represented by the absolute term in these formulas, there still remains a discrepancy between the calculated and the measured stresses represented by the percentage term (5%). This covers all percentage errors in the measured and the calculated stresses, that is, all errors which are proportional to the magnitude of the stress.

In the measured stresses, the only percentage error arises from variations in the value of the modulus of elasticity from the assumed value of 30 000 000. Such variation does not affect the accuracy of the measurements, but simply the conversion of the measured strains into intensities of stress. The resulting stress is then affected by a percentage error equal to the percentage variation in the value of E, and this may amount to $\pm$ 3 per cent.

In the calculated stresses, all errors are percentage errors. They are caused principally by variations in the loading and cross-sections from the values assumed. In the assumed loads there is a probable error of about $\pm$ 3%, on account of the uncertainty in allowing for the weight of traveler, track, rivets, pins, staging, etc. In the cross-sections there is a probable error of $\pm$ 2% on account of the difference between actual and theoretical sections.

The combined effect of the foregoing errors in assuming the loading, cross-sections, and value of E, will account for the percentage

term ($\pm$ 5%) in the discrepancy between calculated and measured stresses.

The major part of this discrepancy of 5% arises from the errors in the calculated stress. This demonstrates the futility of excessive refinement in the calculation of stresses.

Comparison of Extreme Secondary Stresses with Primary Stresses.

The object of this comparison was to establish a relation between the average stresses and the extreme variations from these average stresses in the members of the structure. The information derived from such comparisons should be of value in guiding the selection and specification of working stresses for bridge materials. For this purpose it appeared desirable to get the extreme variation of fiber stress in each member, without regard to the cause or causes producing it. Such secondary stresses would include, not only the stress from bending of the members in the plane of the truss, as usually computed, but also any horizontal or lateral bending stresses from wind and transverse strains, effect of shop inaccuracies, internal temperature strains, lack of uniformity of material, and any other possible causes.

As a fair measure of this extreme secondary stress, including the resultant effect of all possible contributing elements, the difference was taken between the average of the twelve measurements in a member and that one of the twelve measurements departing most widely from the average.

A comparison of the secondary stresses thus obtained with the corresponding primary (or average) stresses yielded the following results:

For primary stresses in the ranges 0-1 000, 1 000-2 000, 2 000-3 000, 3 000-5 000, 5 000-7 000, and 7 000-8 000, the percentages of extreme secondary stress averaged 268, 148, 71, 59, 37, and 32, respectively. This illustrates the diminishing relative importance of secondary stresses with increasing primary stress.

Although the percentages diminish in the foregoing series of ranges, the absolute amounts of the secondary stress show a small increase, averaging 1 580 lb. per sq. in. in the lowest range and 2 370 lb. per sq. in. in the highest range.

In the one hundred cases represented in the foregoing results, the greatest extreme secondary stress that appears is 3 700 lb. per sq. in.

Plotting the results with primary and extreme secondary stresses, respectively, as co-ordinates, nearly all the observations were found to be included in a belt between two lines having a 12% slope. From this may be deduced the relation,

$$\left.\begin{matrix}\text{Extreme secondary}\\ \text{stress in a member}\end{matrix}\right\} = 12\% \text{ of primary stress} + K,$$

where K varies from 600 to 2 600, with an average value of 1 600 lb. per sq. in.

The form of this expression, a percentage term plus an absolute term, indicates that the measured secondary stresses include effects proportional to the direct stress in the member as well as effects independent of that stress. The latter effects, constituting the major portion of the measured secondary stresses, are produced by such causes as the bending of a member due to its own weight, wind and lateral strains, internal temperature strains, erection strains, etc.

The proportional component or percentage term in the formula represents the secondary stresses resulting from the direct primary strains. It amounts to only 12% of the primary stresses in the case of the Hell Gate chords. It is interesting to note that this component of the secondary stress is smaller than the combined effect of the other contributions which are generally omitted from consideration in the computation of secondary stresses.

Combining the results of this and the preceding comparison, we may establish certain conclusions for guidance in specifying extreme working stresses for bridge members. As the maximum discrepancy between calculated and measured stress is 5% + 1 000, and as a possible extreme value of the secondary stress is 12% + 2 600, it appears that the sum of these two variations should be left as a necessary margin between the elastic limit of the material and the maximum calculated primary stress. As the Hell Gate chords had certain special features tending to reduce the secondary stresses, it is probable that the 12% factor is not typical, and that about 20% would be a better allowance for most bridges. Hence, about 25% + 4 000 should be deducted from the minimum elastic limit of the material in order to obtain the safe working stress for bridge members.

It is recognized, of course, that a rule of this form can properly apply only to structures in which the secondary stresses have normal

or average values. Where unusually large secondaries may be anticipated, these should receive special investigation and attention.

Comparison Between Calculated and Measured Secondary Stresses.

In order to be comparable with the calculated secondary stresses due to bending of the members in the plane of the truss, the "measured secondary stresses" had to be defined as one-half the difference between the average measured stresses in the top and bottom fibers of the cross-section. Any lateral variation in the stress had to be ignored.

The "calculated secondary stresses" were obtained by analytical methods for Stages 2, 4, 6, 8, 10, 3-*H*, and 2-*H*, and by interpolation for the intermediate stages.

Both the measured and the calculated values were reduced to percentages of the corresponding primary stresses and recorded in the table, Plate XLIV.

It should be noted that the high percentages always occur with low primary stresses. The law of variation is asymptotic.

The average of the calculated secondary stresses, for all bottom chord members and all erection stages, was about 1 050 lb. per sq. in. The average of all measured secondary stresses was only 700 lb. per sq. in.

The highest calculated secondary stress in any stage was 2 920; and the corresponding measured stress was 1 990 lb. per sq. in.

A comparison of individual values of calculated and of measured secondary stresses is not very illuminating, as any systematic variations are more or less obscured by the effects of erratic readings and other disturbing factors.

More instructive results are obtained by averaging the individual readings in groups so as to eliminate accidental variations and disclose the true relations between the calculated and the measured values.

If the secondary stresses are grouped and averaged in ranges of percentages, the following comparison is obtained:

In the ranges,

0-20%, 20-40%, 40-100%, 100-300%, and more than 300%,

the average calculated secondary stresses were:

6%, 30%, 69%, 143%, and 473%,

and the corresponding average measured secondary stresses were:

26%, 28%, 46%, 93%, and 110%, respectively.

From these figures, the following relations are evident:

1.—Except for the smaller percentage of secondary stresses, the measured values are consistently lower than the calculated values.

2.—As the percentage of calculated stresses increases, the percentage of measured values also increases, but not as rapidly.

3.—For the highest percentages of secondary stresses, the measured values amount to only a small fraction of the calculated values.

The lower values of the measured secondary stresses are partly due, in the case of the Hell Gate Arch, to the three-faced joints in the bottom chords; but, to a large extent, the effect is to be ascribed to a readjustment of strains tending to relieve the higher secondary stresses. As this effect obtains in all structures, it may be concluded that the actual secondary stresses are generally lower than the calculated values.

These results may be expressed by an approximate empirical relation between the calculated and the measured percentages of secondary stress. If the various observations are plotted with these percentages as co-ordinates, all but the extreme points are found to be grouped along a straight line of the formula,

$$\text{Percentage of measured secondary stress} = \frac{1}{2}\,(\text{percentage of calculated secondary stress}) + 15.$$

This expression may be interpreted as follows:

1.—The factor, $\frac{1}{2}$, was the average ratio of actual to calculated stresses, and represents the reduction of the bending strains by the yielding of the joints and the internal readjustment of the structure to relieve the secondary stresses. This factor would probably have a higher value in other structures.

2.—The absolute term, 15%, represents additional secondary strains due to factors not included in the computations, such as inaccuracies of fabrication, effects of temperature, dead weight of members, eccentric bearings, etc. This term would probably have a lower value in other structures.

For the most instructive comparison, it is necessary to average the calculated and measured values in groups corresponding to the successive erection stages. This yields the results shown in Table 5.

TABLE 5.

Stage.	Average primary stress.	Average Secondary Stress.	
		Calculated.	Measured.
2	612	1 220	485
4	1 370	703	547
6	2 040	847	796
8	1 254	1 335	711
10	2 393	1 629	725
3-*H*	5 820	557	832
2-*H*	11 620	513	1 140 (296)

The values in Table 5 are plotted in graphs in Fig. 5. Three curves are shown: primary stresses, calculated secondary stresses, and measured secondary stresses.

The curve of primary stresses shows a large drop from Stage 6 to Stage 8, representing the reduction in stresses when the load was transferred from the lower to the upper back-stay.

The curve of calculated secondary stresses is rather high in the first two stages. This is simply because at these stages only one or two panels were erected, and these end panels usually have higher secondary stresses than the intermediate panels of the span. From Stage 4 to Stage 12, the calculated secondary stresses increase continuously with the increasing deflections. At Stage 3-*H* there is a sudden drop, as the 3-hinged condition has very small deflections and, consequently, low secondary stresses.

The measured secondary stresses for the first few stages are very considerably below the calculated values. This proves that the three-faced joints (Fig. 6) acted partly as hinges which relieved the secondary stresses. As the direct stress increased, however, the joints were compressed, the bearing area was enlarged, and the drift-pins in the outer thirds of the joint began to take stress. This accounts for the increasing slope of the curve as Stage 6 is approached, and the diminishing difference between calculated and measured values. From Stage 6 to Stage 8 there is a decline in the measured secondary stresses due to the temporary release in the direct compression at the joints, combined with a reversal of flexure at some of the joints during this transition. Beyond Stage 8 the measured secondary stresses increase continuously.

The final drop in the calculated stress curve is not duplicated in the curve of measured secondary stresses. In the former curve, the

strains for Stages 12 and 3-*H* are the results of independent computations for two entirely distinct erection conditions, and this accounts for the break in the curve. In the measured secondary stresses, how-

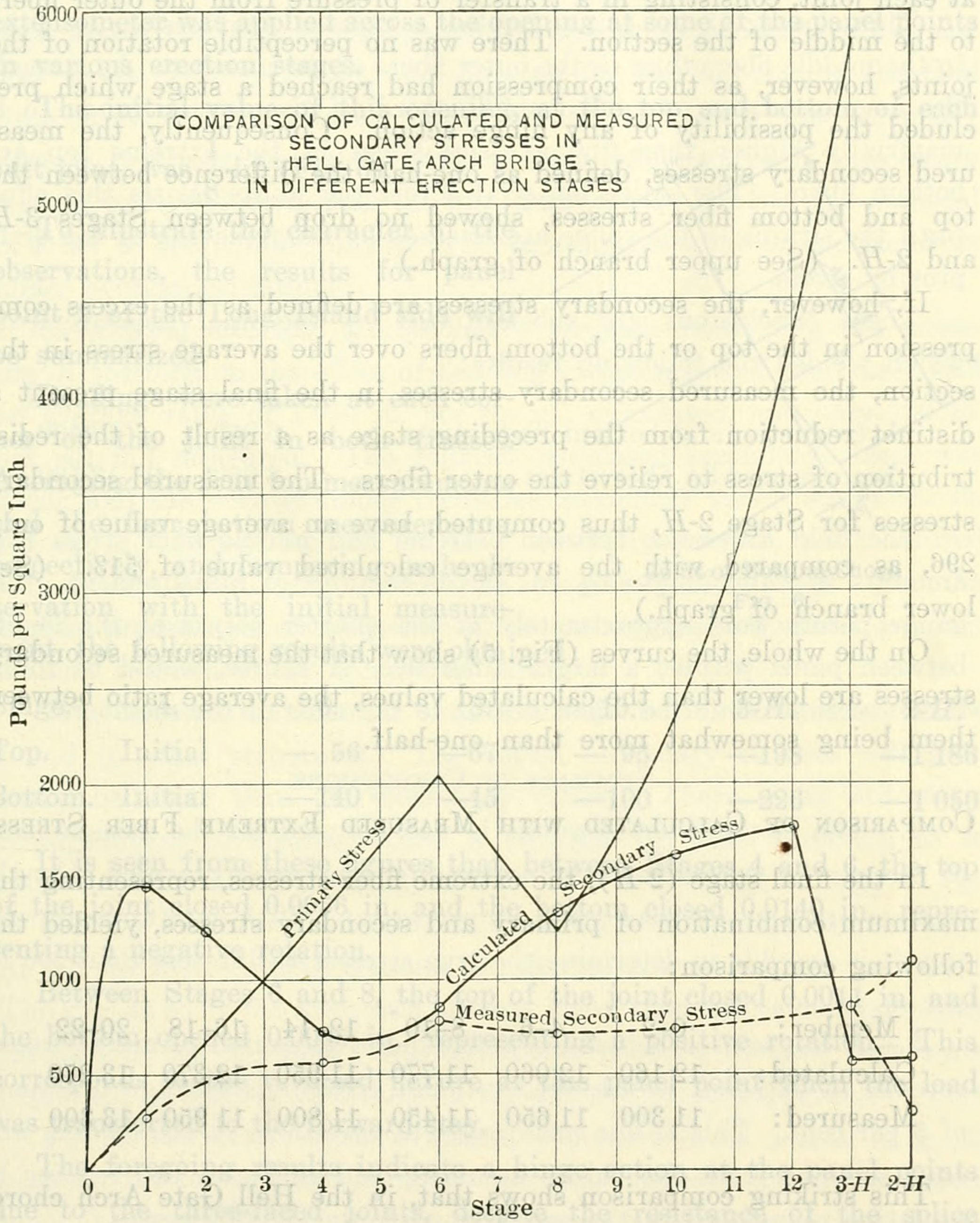


FIG. 5.

ever, there is a continuity of operation, and therefore of stress conditions. The joints are fixed under the large direct compressions, so that some of the secondary stresses induced under the preceding stages persist in the structure when Stages 3-*H* and 2-*H* are reached.

The stresses in Stage 3-*H* were measured before the joints were riveted. During the operation of riveting, as the drift-pins, one by one, were replaced by rivets, there was a gradual redistribution of stresses at each joint, consisting in a transfer of pressure from the outer fibers to the middle of the section. There was no perceptible rotation of the joints, however, as their compression had reached a stage which precluded the possibility of any hinge action. Consequently, the measured secondary stresses, defined as one-half the difference between the top and bottom fiber stresses, showed no drop between Stages 3-*H* and 2-*H*. (See upper branch of graph.)

If, however, the secondary stresses are defined as the excess compression in the top or the bottom fibers over the average stress in the section, the measured secondary stresses in the final stage present a distinct reduction from the preceding stage as a result of the redistribution of stress to relieve the outer fibers. The measured secondary stresses for Stage 2-*H*, thus computed, have an average value of only 296, as compared with the average calculated value of 513. (See lower branch of graph.)

On the whole, the curves (Fig. 5) show that the measured secondary stresses are lower than the calculated values, the average ratio between them being somewhat more than one-half.

Comparison of Calculated with Measured Extreme Fiber Stress.

In the final stage (2-*H*), the extreme fiber stresses, representing the maximum combination of primary and secondary stresses, yielded the following comparison:

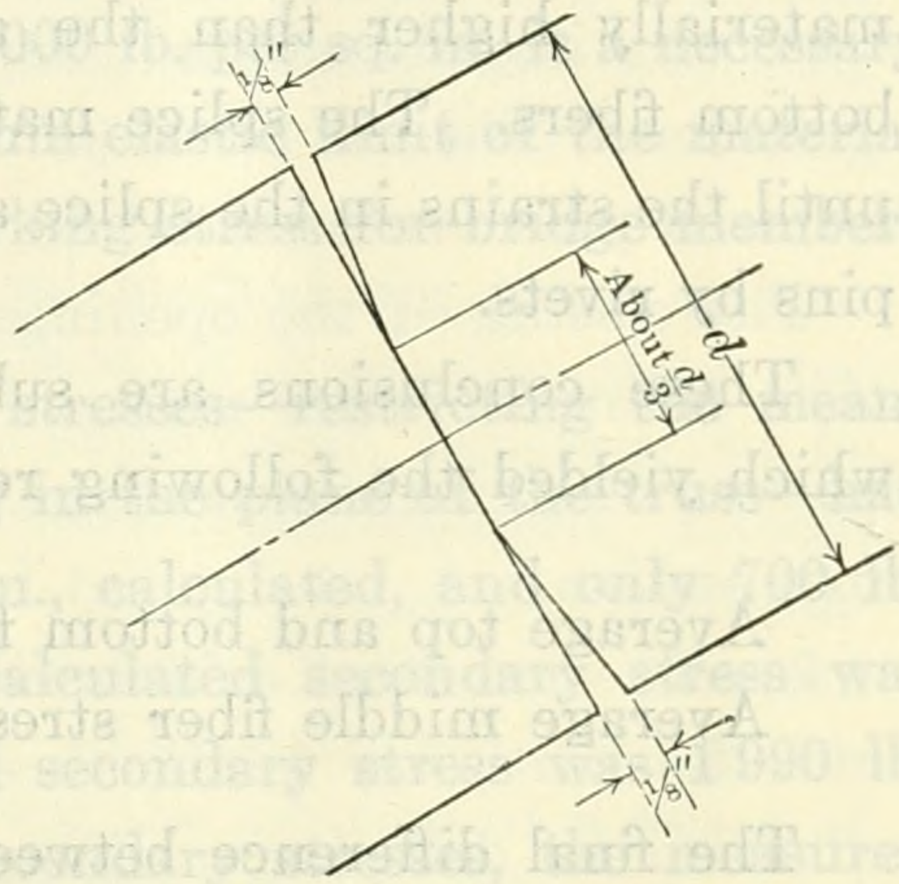
Member:	0–2	4–6	8–10	12–14	16–18	20–22
Calculated:	12 160	12 060	11 770	11 950	12 370	13 270
Measured:	11 300	11 650	11 450	11 800	11 950	13 300

This striking comparison shows that, in the Hell Gate Arch chord members, the factors tending to increase the calculated extreme fiber stresses and those tending to reduce them very nearly balance each other. Were it not for the three-faced butt joints and the deferred riveting, the measured extreme stresses would have exceeded the calculated values.

Other Applications of the Extensometer.

In order to observe the movements at the lower chord splices, the extensometer was applied across the opening at some of the panel points in various erection stages.

The initial value of this opening, at the top and bottom of each butt joint, was $\frac{1}{8}$ in. (Fig. 6).



Joint of Bottom Chord

Fig. 6.

To illustrate the character of the observations, the results for panel point 2 of the Long Island side will be summarized.

Readings were taken at each corner of the joint in both trusses. Averaging the four top measurements and the four bottom measurements, respectively, and comparing each observation with the initial measurement, the following results were obtained:

Stage.		4.	6.	8.	10.	3-*H*.	2-*H*.
Top.	Initial	— 56	—67	— 95	—198	—1 186	
Bottom.	Initial	—140	—45	—100	—226	—1 050	

It is seen from these figures that, between Stages 4 and 6, the top of the joint closed 0.0056 in. and the bottom closed 0.0140 in., representing a negative rotation.

Between Stages 6 and 8, the top of the joint closed 0.0011 in. and the bottom opened 0.0095 in., representing a positive rotation. This corresponds to the reversed flexure at this panel point when the load was transferred to the forward stay.

The foregoing results indicate a hinge action at the panel points due to the three-faced joints, despite the resistance of the splice material.

The closing of the joints, at top and bottom, took place mainly between Stages 3-*H* and 2-*H*, during which stages the splices were riveted, which indicates a gradual re-adjustment of stress at the joint as the drift-pins were replaced one by one by rivets.

Similar results were obtained at the other panel points observed.

In every case there was a movement of nearly $\frac{1}{8}$ in. from the initial measurement, indicating practically a complete closing of the joint.

This closing of the joints implies a greater compression, by $\frac{1}{8}$ in., in the middle fibers than in the outer fibers of the member. As a result, the average stress over the middle third of the section tends to be materially higher than the average stress in the extreme top and bottom fibers. The splice material across the joints resists this effect until the strains in the splice are released by the replacing of the drift-pins by rivets.

These conclusions are substantiated by the final measurements, which yielded the following results:

	Stage 3–*H*.	Stage 2–*H*.
Average top and bottom fiber stress.....	6 050	10 400
Average middle fiber stress.....	6 050	13 100

The final difference between extreme and middle fiber stress thus amounts to 2 700 lb. per sq. in. at the sections of measurement. The gauge points were approximately at the quarter points of the length between panel points; a larger difference, of course, would be found if the sections could be taken nearer to the ends of the members.

Summary of Conclusions.

1.—The Howard strain gauge is well adapted for the measurement of stresses in steel structures under quiescent load, provided that the stresses are not less than 1 000 lb. per sq. in. With careful manipulation it is possible to determine the true stresses within less than 200 lb. per sq. in.

2.—A comparison between calculated and measured primary stresses reveals, in addition to the observational errors, an average difference of 5 per cent. This is due principally to variations in the loading and cross-sections from the values assumed, and indicates the futility of excessive refinement in the ordinary computation of stresses.

3.—The extreme variation of fiber stress from the average stress in a member was found to range from about 1 600 lb. per sq. in. with the lowest primary stresses to about 2 500 with the highest primary stresses. A part of this variation represented the secondary stress from vertical bending; the greater part, however, was due to the effects of lateral bending, shop inaccuracies, temperature strains, splice details,

non-uniform material, and other causes which are omitted from consideration in the computation of secondary stresses.

4.—Combining the maximum discrepancy between calculated and measured stress (5% + 1 000) with the maximum variation of extreme fiber stress (12% + 2 600), and allowing for the fact that in the Hell Gate Arch the three-faced joints tended to reduce the secondary stresses, it appears that about 25% + 4 000 lb. per sq. in. is a necessary margin to be deducted from the minimum elastic limit of the material in order to obtain the limiting safe working stress for bridge members under average conditions.

5.—During erection, the secondary stresses—restricting the meaning of the term to the effect of bending in the plane of the truss—had an average value of 1 050 lb. per sq. in., calculated, and only 700 lb. per sq. in., measured. The highest calculated secondary stress was 2 920, and the corresponding measured secondary stress was 1 990 lb. per sq. in. Except for the smallest secondary stresses, the measured values were consistently lower than the calculated values. For the highest percentages of secondary stresses, the measured values are only a small fraction of the calculated values.

It is believed that similar results, though not as marked, would be found in other structures. The actual secondary stresses will generally be lower than the calculated values. There is an automatic re-adjustment of strains within a structure in such direction as to relieve the secondary stresses.

6.—The variations of calculated and measured secondary stresses in the successive erection stages are plotted for comparison in Fig. 5. The graphs show the measured stresses lower than the calculated values throughout the erection. The differences between the two curves are explained by special conditions in the erection of the structure.

7.—In the early erection stages, the three-faced joints (Fig. 6) between the lower chord members acted as hinges to permit a certain amount of rotation, so as to ease the secondary stresses. In the succeeding stages, as the direct stress increased, the joints became compressed, and the rotation was restricted. Between Stages 6 and 8, when there was a decrease in the direct compression, rotation occurred again, and the secondary stresses were partly released.

Extensometer measurements across the splice openings confirmed the above-described hinge action of the joints.

Another object of the three-faced joints was to produce a concentration of pressure in the middle third of the section, accompanied by a reduction of direct stress in the outer fibers. This distribution of stress, with the largest intensities in the middle third, was confirmed by actual measurement in the final stage.

8.—Measurements across the splice openings (Fig. 6) before and after riveting showed a complete closing of the joints during this operation. This is proof of the release of strains at a joint when the drift-pins are replaced by rivets.

The closing of the openings represents a desired transfer of initial stress from the splice material to the butt joint; it is also visible evidence of the greater concentration of stress in the middle third of the cross-section.

9.—The curve of calculated secondary stresses shows a large drop from the cantilever to the three-hinged stage, on account of the large reduction in deflections. This indicates the comparative freedom of the arch type from secondary stresses.

10.—In the final stage, the stresses in the extreme fibers, representing the maximum combined effect of primary and secondary stresses, show a remarkable agreement with the calculated values of the same stresses.

Acknowledgments.

Acknowledgments, in addition to those already given, are due to Mr. Frank E. Berry and to Theodore Belzner, Assoc. Am. Soc. C. E., for instruction and suggestions in the use of the extensometer; to Messrs. Brown and Sharpe, manufacturers of the instrument, for courtesies extended; to O. H. S. Koch, Jun. Am. Soc. C. E., who, with assistants, took all the measurements, and on whose painstaking thoroughness their value depended; to Mr. F. de Schauensee, who calculated the secondary stresses and assisted in reducing the observations; and to O. H. Ammann, M. Am. Soc. C. E., for suggestions during the prosecution of the investigation.

APPENDIX

SECONDARY STRESSES IN ARCH TRUSSES OF HELL GATE BRIDGE.

In view of the unusual size and form of the trusses of the Hell Gate Arch Bridge and for comparison with stress measurements, a complete analysis was made of the secondary stresses from dead load, live load, and temperature changes, in the finished structure, as well as during the various stages of erection.

The writer selected Winkler's analytical method, for these computations, in preference to the graphic method of Professor Mohr, because the greater precision and ease of supervision appeared to be sufficient compensation for the slight increase in time required. Except for some changes in the arrangement and tabulation of the computations, the procedure given in Johnson, Bryan, and Turneaure's "Modern Framed Structures" was closely followed. The calculation of secondary stresses for each load condition involves the solution of a series of simultaneous equations equal in number to the total number of panel points in the truss, and the expediting of the entire computation depends largely on the method selected for solving these equations. The quickest and best results were obtained with a modification of the method of successive approximations described by F. E. Turneaure, M. Am. Soc. C. E.* The modification consisted in substituting, in the first and each successive solution, the new values of the unknowns as far as already obtained, instead of using the values from the preceding approximation.

The computation of the secondary stresses from dead load on the three-hinged arch were simplified by treating the two halves of the arch, up to the temporary crown-hinge, as separate frames. The effect of friction at the hinges was neglected.

For the dead-load secondary stresses in the two-hinged arch, because of symmetry, only one-half of the truss had to be computed. The load for this case consisted of the concrete floor and tracks, which were added after the center top chord was connected.

In computing the secondary stress for live load, one-half of the arch was considered loaded, as this load produces nearly the maximum primary stress in most members. A simple reversal of the diagram and algebraic addition of the two sets of stresses gives the secondary stresses for live load covering the full span.

The results of the secondary stress computations for the Hell Gate Arch are recorded on Plates XLV, XLVI, and XLVII. These plates also contain all data necessary for reproducing the computations.

* *Engineering News*, September 5th, 1912.

The diagrams give, for each designated condition of loading, the secondary stresses at both ends of each member. A double sign prefixed to each figure indicates the kind of stress in the top and bottom fibers of the section, respectively. In addition, the primary stress for the same condition of loading is marked on each member in parentheses.

Another feature in these diagrams is the graphic representation, for the individual members, of the deformations corresponding to the secondary stresses. These deformations are shown exaggerated to an arbitrary relative but non-proportional scale. After the curves are drawn they serve principally as a general indication of relative distortions. In addition, the sharpness of curvature at any point represents the intensity of bending moment; the points of contraflexure mark points of zero secondary stress; the direction of curvature gives the signs of the secondary stress; the configuration of curves meeting at any panel point determines the direction of deflection of that point; and, in general, the deformation curves afford a visual check on the correctness of the work.

Results of the Secondary Stress Calculations.

An inspection of the secondary stresses recorded on Plate XLV yields the following facts:

The largest dead-load secondary stresses, in pounds per square inch, are as follows:

Lower Chord.	Upper Chord.	Diagonals.	Verticals.
1 290 in 20-22	2 468 in 1-3	4 950 in 22-*M*	2 186 in 20-21
1 032 in 18-20	961 in 21-23	1 376 in 3-4	1 671 in 18-19
		1 316 in 1-2	
All the other stresses are below			
800	800	1 300	1 600

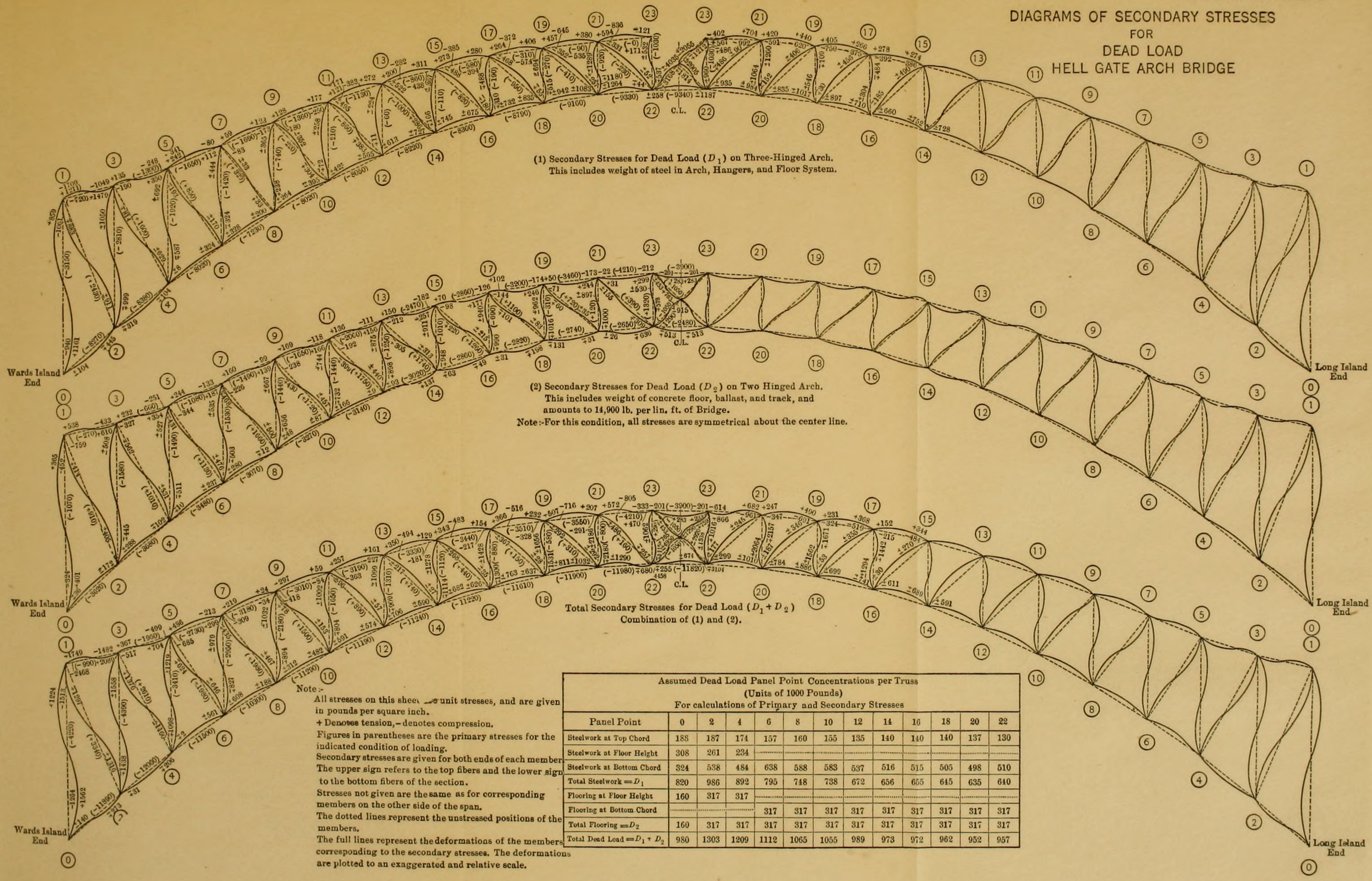
It will be observed that the largest secondary stresses in each class of members occur in the end panels and crown panels of the span. This effect is accounted for by the large concentrations of stress at Panel Points 0 and 22 of the three-hinged arch. It will generally be found that the largest secondary stresses in any structure occur where there is an interruption in the continuity of the truss configuration, or in the uniformity of the loading conditions.

Plate XLV also affords a comparison of the secondary stresses in a three-hinged and a two-hinged arch. The former are generally the larger.

In the outstanding dead-load secondary stresses it will be found that the major contributions to the total stress come from the three-hinged condition. This effect is most marked in the panels near the temporary crown-hinge, as shown by Table 6.

PLATE XLV.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LXXXII, No. 1418.
STEINMAN ON
STRESS MEASUREMENTS,
HELL GATE ARCH BRIDGE.

DIAGRAMS OF SECONDARY STRESSES FOR DEAD LOAD HELL GATE ARCH BRIDGE



(1) Secondary Stresses for Dead Load (D_1) on Three-Hinged Arch.
This includes weight of steel in Arch, Hangers, and Floor System.

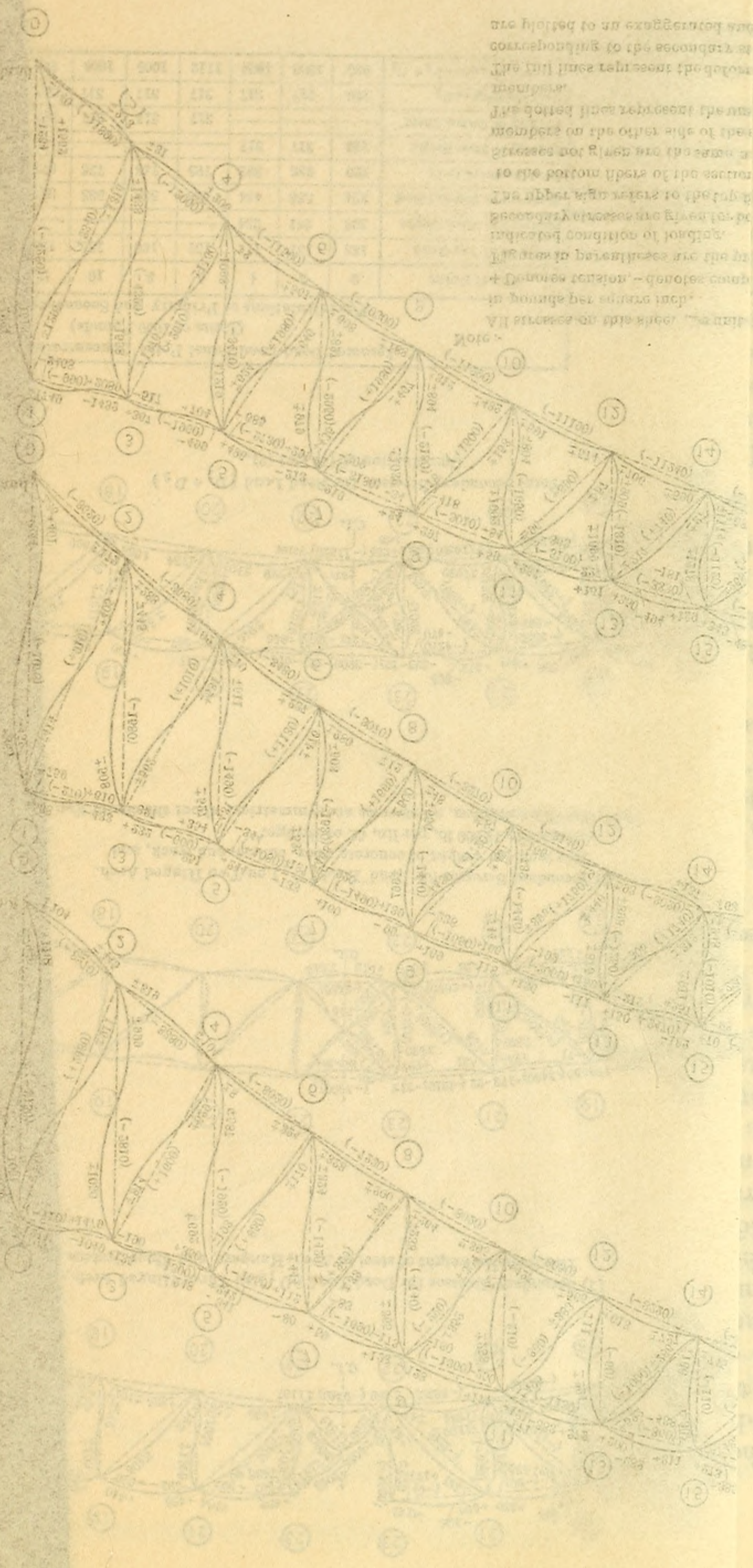
(2) Secondary Stresses for Dead Load (D_2) on Two Hinged Arch.
This includes weight of concrete floor, ballast, and track, and amounts to 14,900 lb. per lin. ft. of Bridge.
Note:-For this condition, all stresses are symmetrical about the center line.

Total Secondary Stresses for Dead Load ($D_1 + D_2$)
Combination of (1) and (2).

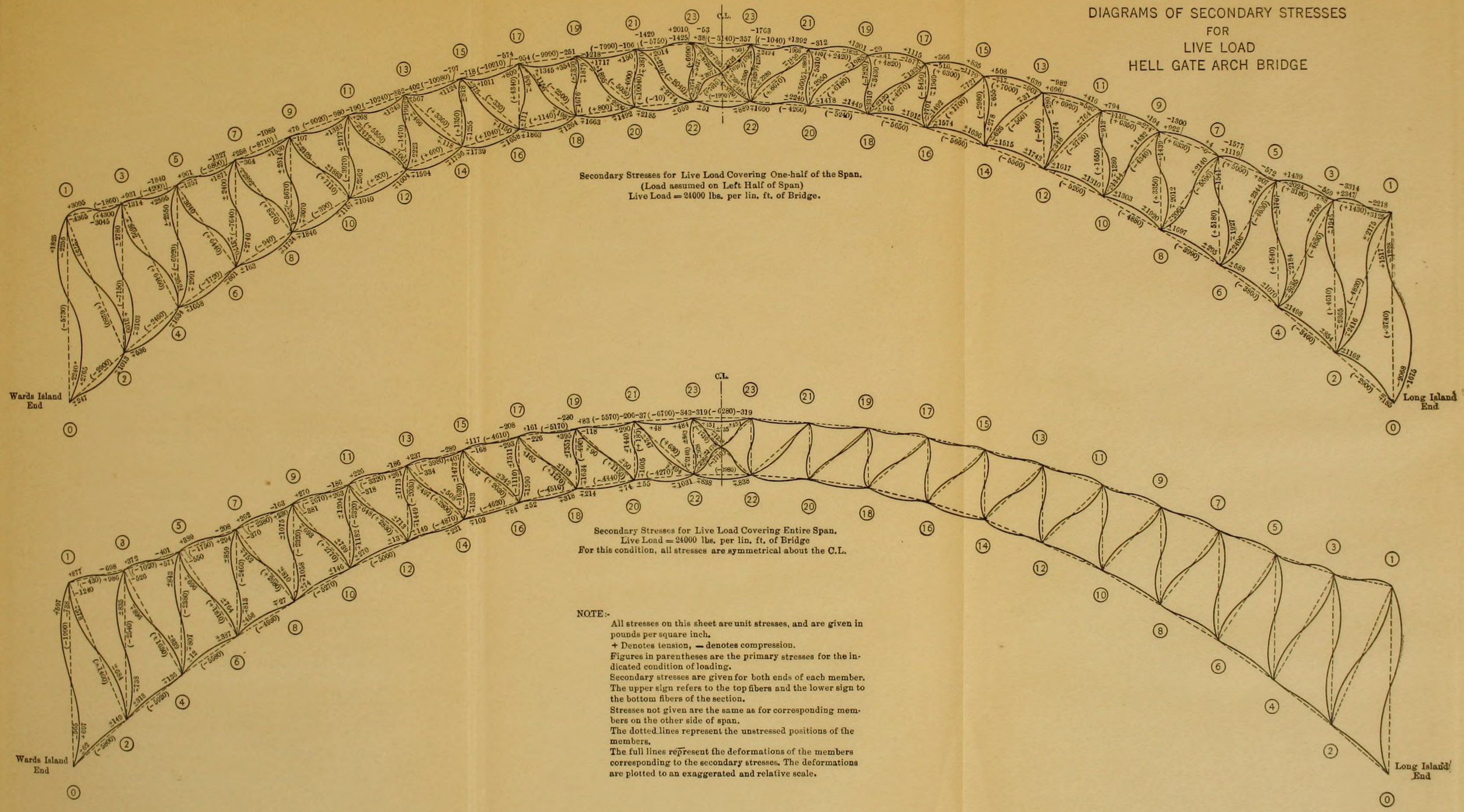
Note:-
All stresses on this sheet are unit stresses, and are given in pounds per square inch.
+ Denotes tension, – denotes compression.
Figures in parentheses are the primary stresses for the indicated condition of loading.
Secondary stresses are given for both ends of each member. The upper sign refers to the top fibers and the lower sign to the bottom fibers of the section.
Stresses not given are the same as for corresponding members on the other side of the span.
The dotted lines represent the unstressed positions of the members.
The full lines represent the deformations of the members corresponding to the secondary stresses. The deformations are plotted to an exaggerated and relative scale.

Assumed Dead Load Panel Point Concentrations per Truss
(Units of 1000 Pounds)
For calculations of Primary and Secondary Stresses

Panel Point	0	2	4	6	8	10	12	14	16	18	20	22
Steelwork at Top Chord	188	187	174	157	160	155	135	140	140	140	137	130
Steelwork at Floor Height	308	261	234	.....	.....	.....	.....	.....	.....	.....	.....	.....
Steelwork at Bottom Chord	324	538	484	638	588	583	537	516	515	505	498	510
Total Steelwork $= D_1$	820	986	892	795	748	738	672	656	655	645	635	640
Flooring at Floor Height	160	317	317	.....	.....	.....	.....	.....	.....	.....	.....	.....
Flooring at Bottom Chord	.....	.....	.....	317	317	317	317	317	317	317	317	317
Total Flooring $= D_2$	160	317	317	317	317	317	317	317	317	317	317	317
Total Dead Load $= D_1 + D_2$	980	1303	1209	1112	1065	1055	989	973	972	962	952	957



DIAGRAMS OF SECONDARY STRESSES FOR LIVE LOAD HELL GATE ARCH BRIDGE



Secondary Stresses for Live Load Covering One-half of the Span.
(Load assumed on Left Half of Span)
Live Load = 24000 lbs. per lin. ft. of Bridge.

Secondary Stresses for Live Load Covering Entire Span.
Live Load = 24000 lbs. per lin. ft. of Bridge
For this condition, all stresses are symmetrical about the C.L.

NOTE:-

All stresses on this sheet are unit stresses, and are given in pounds per square inch.

+ Denotes tension, — denotes compression.

Figures in parentheses are the primary stresses for the indicated condition of loading.

Secondary stresses are given for both ends of each member. The upper sign refers to the top fibers and the lower sign to the bottom fibers of the section.

Stresses not given are the same as for corresponding members on the other side of span.

The dotted lines represent the unstressed positions of the members.

The full lines represent the deformations of the members corresponding to the secondary stresses. The deformations are plotted to an exaggerated and relative scale.

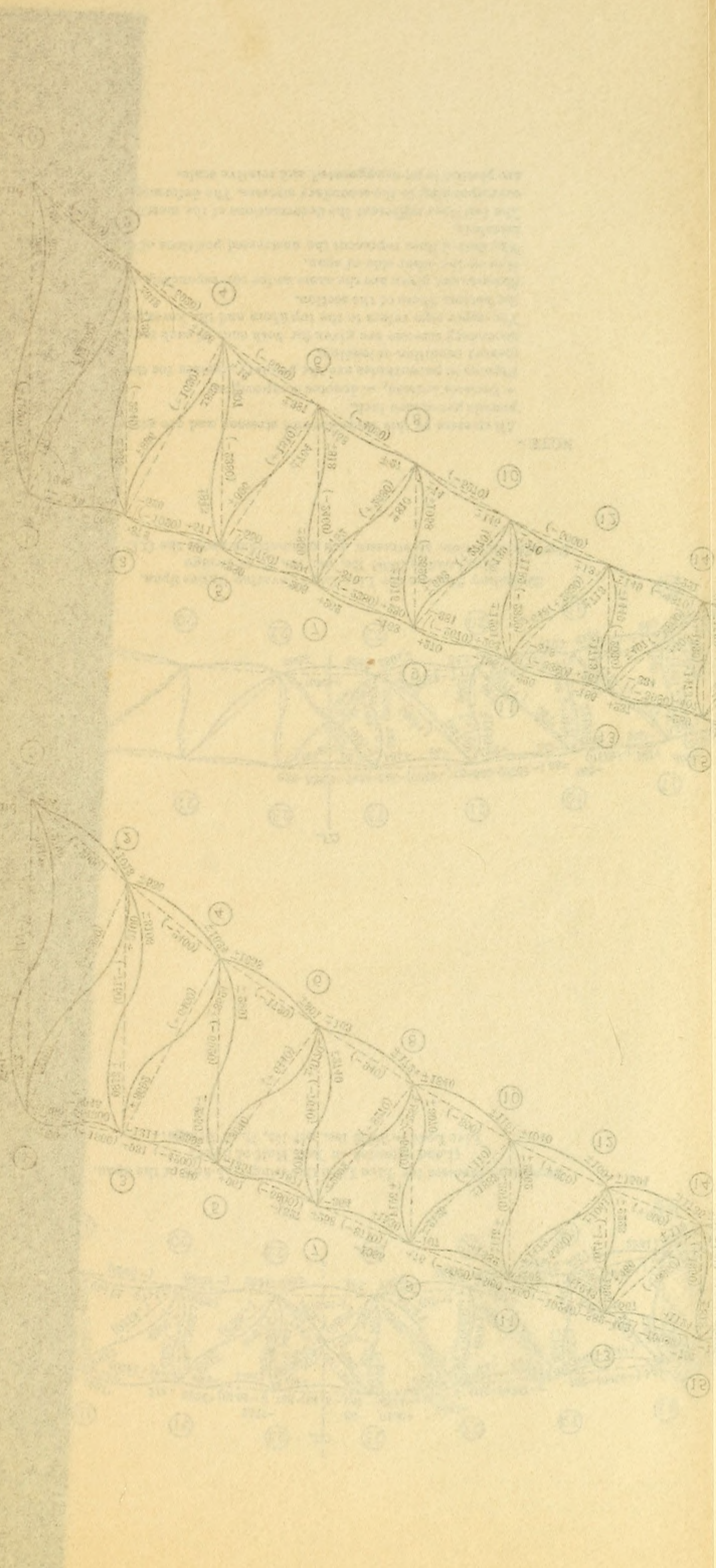
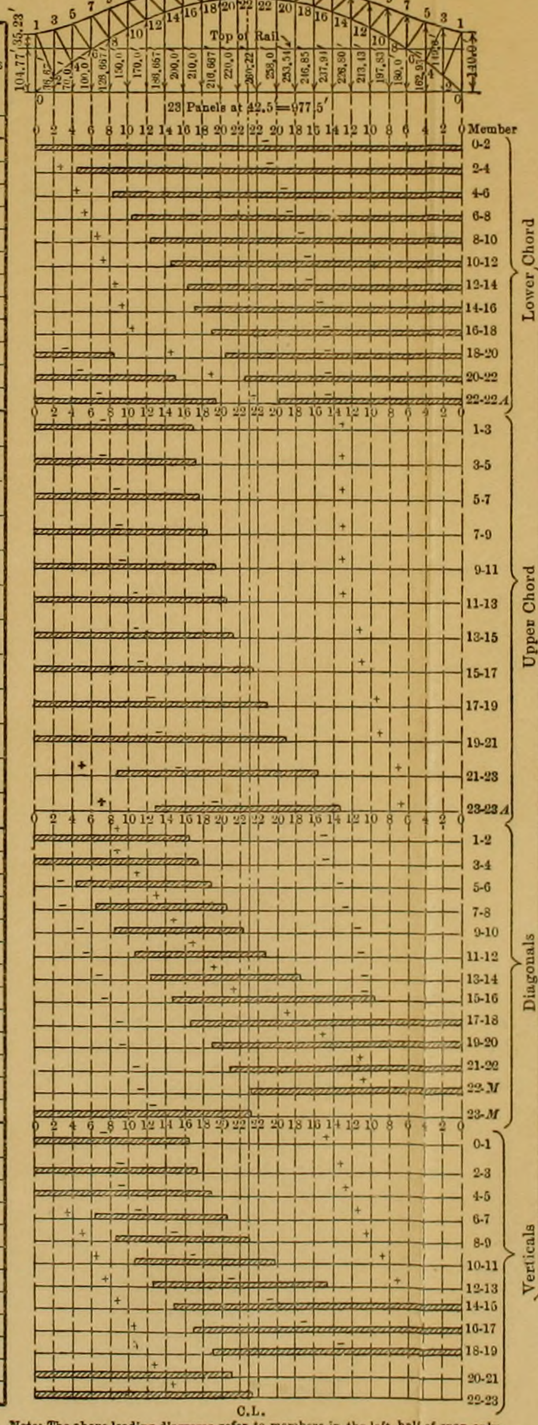


TABLE OF SECONDARY STRESSES HELL GATE ARCH BRIDGE.

	Member	Constants: Length	Cross Section	Moment of Inertia	Extreme Fiber Distance	Secondary Stress From: ① Dead Load — Wards I. Side	① Dead Load — Long I. Side	② Live Load Covering One-half Span — Loaded Side	② Live Load Covering One-half Span — Unloaded Side	③ Live Load Covering Whole Span	④ Approximate Secondary Stress Occurring Simultaneously with Max. Live Load Stress	⑤ Total ① + ④ — Wards I. Side	⑤ Total ① + ④ — Long I. Side	Primary Stress Corresponding To ①	④	⑤	Per cent Secondary Stress of Primary Stress ①	④	⑤
		In.	In.²	In.⁴	In.														
Lower Chord	0-2	674	1384	2319000	63.6	± 140	∓ 273	± 140	∓ 273	± 247	∓1013	∓ 185	±1162	± 62	± 149	± 202	∓ 124	± 202	∓ 124	-11890	- 5800	-17690	2	3	1
	2-4	648	1281	1928000	61.4	∓ 31	∓ 206	∓ 31	∓ 206	∓ 536	∓1634	± 854	±1498	± 318	± 136	± 382	± 60	± 351	∓ 146	-12060	- 6050	-18110	2	6	2
	4-6	624	1265	1857000	59.6	∓ 2	± 561	∓ 2	± 561	∓1058	∓ 201	±1070	± 588	± 12	± 387	± 300	± 442	± 298	±1003	-11500	- 6070	-17570	5	7	6
	6-8	602	1337	1943000	58.4	± 608	± 188	± 608	± 188	± 163	∓1724	± 295	±1697	± 458	∓ 27	± 396	± 628	±1004	± 816	-10300	- 5760	-16060	6	11	6
	8-10	582	1163	1461000	55.4	± 312	± 482	± 312	± 482	∓1846	∓1157	±1920	±1303	± 74	± 146	± 931	± 682	±1243	±1164	-11290	- 6570	-17860	4	14	7
	10-12	564	1121	1250000	52.6	± 591	± 574	± 591	± 574	∓1040	∓1604	±1310	±1617	± 270	± 13	± 828	± 874	±1419	±1448	-11190	- 6680	-17870	5	13	8
	12-14	548	1080	1050000	49.4	± 706	± 590	± 706	± 591	∓1594	∓1736	±1743	±1515	± 149	∓ 221	±1104	± 818	±1810	±1408	-11240	- 6670	-17910	6	17	10
	14-16	535	1046	971000	47.6	± 682	± 626	± 689	± 611	∓1739	∓1658	±1635	±1574	∓ 103	∓ 84	±1037	±1004	±1719	±1630	-11220	- 6410	-17630	6	16	10
	16-18	524	988	867000	45.5	± 763	± 637	± 741	± 690	∓1863	∓1264	±1915	± 946	± 52	∓ 318	±1345	± 550	±2108	±1196	-11610	- 6080	-17690	7	22	12
	18-20	516	946	764000	43.4	± 811	±1032	± 886	± 784	∓1663	∓1492	±1449	±1418	∓ 214	∓ 74	± 532	± 595	±1343	±1627	-11900	- 5460	-17360	9	11	9
	20-22	512	929	699000	41.9	±1290	∓ 680	±1010	± 299	∓2185	± 659	±2240	∓1690	± 55	∓1031	± 580	∓1189	±1870	∓1869	-11980	- 4920	-16900	11	24	11
	22-22A	510	929	699000	41.9	∓ 255	± 674	± 674	∓ 255	± 51	∓ 889	∓ 889	± 51	∓ 838	∓ 838	± 846	∓ 695	∓1101	∓ 22	-11820	- 4920	-16740	5	17	7
Upper Chord	1-3	523	315	98000	19.5 27.5	+ 1749 - 2468	- 1482 + 2089	+ 1749 - 2468	- 1482 + 2089	+ 3095 - 4365	- 3045 + 4300	- 2218 + 3125	+ 2347 - 3314	+ 877 - 1240	- 698 + 986	+ 2215 - 3130	- 2130 + 3080	+ 3964 - 5598	- 3662 + 5169	+ 3964 - 5598	- 3662 + 5169	- 990	- 2114	- 3104	177 250	105 148	128 180
	3-5	535	315	98000	19.5 27.5	+ 367 - 517	- 499 + 704	+ 367 - 517	- 499 + 704	+ 931 - 1314	- 1840 + 2595	- 559 + 788	+ 1439 - 2024	+ 372 - 526	- 401 + 571	+ 695 - 980	- 1371 + 1931	+ 1062 - 1497	- 1870 + 2635	+ 1062 - 1497	- 1870 + 2635	- 1960	- 4690	- 6650	25 36	29 41	28 40
	5-7	550	315	98000	19.5 27.5	+ 486 - 685	- 213 + 299	+ 486 - 685	- 213 + 299	+ 961 - 1357	- 1327 + 1871	- 572 + 807	+ 1119 - 1577	+ 389 - 550	- 208 + 294	+ 746 - 1051	- 1030 + 1452	+ 1232 - 1736	- 1243 + 1751	+ 1232 - 1736	- 1243 + 1751	- 2730	- 7410	-10140	18 25	14 20	12 17
	7-9	553	330	110000	19.5 27.5	+ 219 - 309	+ 24 - 34	+ 219 - 309	+ 24 - 34	+ 258 - 364	- 1085 + 1530	+ 4 - 6	+ 922 - 1300	+ 262 - 370	- 163 + 230	+ 210 - 295	- 881 + 1241	+ 429 - 604	- 857 + 1207	+ 429 - 604	- 857 + 1207	- 3180	- 9330	-12510	7 10	9 13	7 10
	9-11	543	386	110000	19.5 27.5	+ 297 - 418	+ 59 - 84	+ 297 - 418	+ 59 - 84	+ 76 - 107	- 980 + 1382	+ 194 - 274	+ 794 - 1119	+ 270 - 381	- 186 + 263	+ 64 - 90	- 828 + 1169	+ 361 - 508	- 769 + 1085	+ 361 - 508	- 769 + 1085	- 3010	- 9430	-12440	10 14	9 12	6 9
	11-13	535	386	110000	19.5 27.5	+ 257 - 363	+ 161 - 227	+ 257 - 363	+ 161 - 227	- 190 + 268	- 882 +1243	+ 416 - 586	+ 696 - 982	+ 226 - 318	- 186 + 261	- 170 + 240	- 788 +1111	+ 87 - 123	- 627 + 884	+ 87 - 123	- 627 + 884	- 3190	-10460	-13650	8 11	8 11	5 6
	13-15	527	386	110000	19.5 27.5	+ 350 - 494	+ 129 - 181	+ 350 - 494	+ 129 - 181	- 402 + 567	- 707 +1124	+ 639 - 901	+ 508 - 717	+ 237 - 334	- 289 + 407	- 378 + 534	- 750 +1059	- 28 + 40	- 621 + 878	- 28 + 40	- 621 + 878	- 3330	-11040	-14370	11 15	7 10	4 6
	15-17	521	386	110000	19.5 27.5	+ 343 - 483	+ 154 - 217	+ 344 - 484	+ 152 - 215	- 718 +1011	- 574 + 809	+ 835 - 1179	+ 366 - 516	+ 117 - 168	- 208 + 293	- 718 +1011	- 574 + 809	- 375 + 528	- 420 + 592	- 374 + 527	- 422 + 594	- 3440	-10900	-14340	10 14	7 9	3 4
	17-19	516	386	110000	19.5 27.5	+ 366 - 516	+ 232 - 328	+ 368 - 519	+ 231 - 324	- 954 +1345	- 251 + 354	+1115 -1571	- 29 + 41	+ 161 - 226	- 280 + 395	- 385 +1247	- 269 + 378	- 519 + 731	- 37 + 50	- 517 + 728	- 38 + 54	- 3510	-10180	-13690	10 15	9 12	4 5
	19-21	51.	386	110000	19.5 27.5	+ 507 - 716	+ 207 - 291	+ 490 - 691	+ 247 - 347	-1218 +1717	+ 106 - 150	+1301 -1835	- 312 + 440	+ 83 - 118	- 206 + 290	-1052 +1497	+ 69 - 97	- 555 + 781	+ 276 - 388	- 572 + 806	- 316 + 444	- 3550	- 8890	-12440	14 20	12 17	5 6
	21-23	511	330	110000	19.5 27.5	+ 572 - 805	- 333 + 470	+ 682 - 961	- 614 + 866	-1429 +2014	+1425 -2010	+1392 -1966	-1768 +2494	- 37 + 48	- 343 + 434	- 404 + 570	+ 283 - 400	+ 168 - 235	- 50 + 70	+ 278 - 391	+ 331 - 446	- 4210	- 8880	-13090	16 23	5 6	3 4
	23-23A	510	330	110000	19.5 27.5	- 201 + 283	- 201 + 283	- 201 + 283	- 201 + 283	+ 38 - 53	- 357 + 504	- 357 + 504	+ 38 - 53	- 319 + 451	- 319 + 451	- 137 + 194	- 137 + 194	- 338 + 477	- 338 + 477	- 338 + 477	- 338 + 447	- 3900	- 8140	-12040	5 7	2 2	3 4
Diagonals	1-2	1340	236	80000	27	∓1297	± 1316	∓1297	± 1316	∓2753	± 3100	± 2175	∓2416	∓ 578	± 684	∓1970	± 2220	∓3267	± 3536	∓3267	± 3536	+ 3340	+ 7140	+10480	39	31	34
	3-4	1083	228	76000	27	∓1376	± 1160	∓1376	± 1160	∓3692	± 3524	± 2796	∓2685	∓ 896	± 839	∓2790	± 2665	∓4166	± 3825	∓4166	± 3825	+ 2610	+ 7100	+ 9710	53	39	43
	5-6	911	202	52000	24	∓ 624	± 646	∓ 624	± 646	∓3040	± 3170	± 2344	∓2406	∓ 696	± 764	∓1983	± 2070	∓2607	± 2716	∓2607	± 2716	+ 1980	+ 6910	+ 8890	33	30	30
	7-8	818	130	28500	21	∓ 435	± 467	∓ 435	± 467	∓2892	± 2874	± 2140	∓2064	∓ 752	± 810	∓1805	± 1792	∓2240	± 2259	∓2240	± 2259	+ 1980	+ 9180	+11160	24	20	20
	9-10	768	130	28500	21	∓ 78	± 153	∓ 78	± 153	∓2125	± 1883	± 1432	∓1144	∓ 693	± 739	∓1300	± 1150	∓1378	± 1303	∓1378	± 1303	+ 1500	+ 8640	+10140	10	15	14
	11-12	729	130	28500	21	± 96	± 57	± 96	± 57	∓1412	± 1061	± 764	∓ 348	∓ 648	± 713	∓ 881	± 663	∓ 785	± 720	∓ 785	± 720	+ 890	+ 8310	+ 9200	11	11	9
	13-14	705	130	28500	21	± 217	± 27	± 217	± 27	∓ 466	∓ 118	∓ 31	± 626	∓ 497	± 508	∓ 214	± 102	± 3	± 129	± 3	± 129	+ 740	+ 8420	+ 9160	29	3	1
	15-16	684	169	30700	21	± 268	± 35	± 270	± 30	± 375	∓1150	∓ 727	± 1492	∓ 352	± 342	∓ 318	± 530	∓ 50	± 565	∓ 48	± 560	+ 440	+ 7190	+ 7630	61	7	7
	17-18	675	169	30700	21	± 367	± 28	± 355	± 53	±1155	∓1937	∓1320	± 2120	∓ 165	± 133	∓1039	± 1639	∓ 672	±1667	∓ 684	±1692	+ 150	+ 8800	+ 8950	240	19	19
	19-20	675	196	36000	21	± 292	± 292	± 346	± 187	±1895	∓2900	∓1985	± 2950	∓ 90	± 50	∓1658	± 2450	∓1366	± 2742	∓1312	± 2637	+ 310	+ 9010	+ 9320	112	27	29
	21-22	684	196	36000	21	± 436	∓ 257	± 245	± 177	±2107	∓2774	∓1860	± 2280	± 247	∓ 494	∓1683	± 2046	∓1197	±1789	∓1438	± 2223	+ 100	+10020	+10180	304	20	22
	22-M	351	130	28400	21	∓4458	±4950	∓3104	± 2920	∓ 204	± 2074	∓1884	∓ 598	∓2088	±1476	∓1884	∓ 598	∓6342	±4352	∓4988	± 2323	- 1050	- 8720	- 9770	471	22	65
	23-M	351	130	28400	21	± 453	∓ 765	∓ 759	±1290	± 2527	∓1954	∓1792	± 716	± 735	∓1238	± 2527	∓1954	± 2980	∓2719	±1768	∓ 664	- 1050	- 8720	- 9770	123	29	31
Verticals	0-1	1680	312*	110000*	27.5 34	- 1264 + 1562	+ 1224 - 1513	- 1264 + 1562	+ 1224 - 1513	- 2240 + 2765	+ 1825 - 2255	+ 1075 - 2068	- 1228 + 1517	- 565 + 697	+ 597 - 738	- 1709 + 2105	+ 1390 - 1720	- 2973 + 3667	+ 2614 - 3233	- 2973 + 3667	+ 2614 - 3233	- 4220*	- 6400*	-10620*	39 37	27 33	28 35
	2-3	1355	250*	94000*	27	∓1438	± 1558	∓1438	± 1558	∓3103	± 2780	± 2365	∓1945	∓ 738	± 835	∓2380	± 2128	∓3818	± 3686	∓3818	± 3686	- 4300	- 7760	-12150	35	31	31
	4-5	1115	235*	79000*	27	∓1098	± 1219	∓1098	± 1219	∓2901	± 2550	± 2184	∓1707	∓ 807	± 843	∓2442	± 2082	∓3540	± 3301	∓3540	± 3301	- 3410	- 7420	-10830	36	33	33
	6-7	960	151	36000	21	∓ 827	± 979	∓ 827	± 979	∓2740	± 2400	± 1927	∓1541	∓ 813	± 859	∓1710	± 1500	∓2537	± 2479	∓2537	± 2479	- 2950	- 8380	-11330	33	20	22
	8-9	854	137	31000	21	∓ 894	±1032	∓ 894	±1032	∓3070	± 2514	± 2012	∓1439	∓1058	±1075	∓1020	± 1572	∓2814	± 2604	∓2814	± 2604	- 2180	- 7620	- 9800	47	25	29
	10-11	761	126	27000	21	∓ 804	±1002	∓ 804	±1002	∓2562	± 2117	±1380	∓ 913	∓1182	±1204	∓1181	± 1006	∓1985	± 2008	∓1985	± 2008	- 1650	- 7570	- 9220	61	16	22
	12-13	682	126	27000	21	∓ 909	±1099	∓ 909	±1099	∓2223	±1993	∓ 774	∓ 280	∓1449	±1713	∓ 738	± 809	∓1647	±1908	∓1647	±1908	- 1310	- 7460	- 8770	84	11	22
	14-15	615	126	27000	21	∓1114	±1278	∓1114	±1278	∓1255	± 818	∓ 278	± 655	∓1533	±1473	∓ 654	± 900	∓1768	±2178	∓1768	±2178	- 1120	- 8410	- 9530	114	11	23
	16-17	562	126	27000	21	∓1300	±1428	∓1294	±1424	± 111	∓ 458	∓1701	±1969	∓1590	±1511	∓1073	±1855	∓2073	±3283	∓2967	±3279	- 880	- 9730	-10610	162	19	31
	18-19	522	126	27000	21	∓1531	±1656	∓1562	±1671	±1676	∓1879	∓3310	±3430	∓1634	±1551	∓3029	∓3113	∓4560	±4769	∓4591	±4784	- 580	-10710	-11290	288	29	42
	20-21	496	126	27000	21	∓2180	±2186	∓2064	±2157	±4000	∓3870	∓5605	±5310	∓1605	±1440	∓3664	∓3540	∓1484	∓1354	±1600	∓1383	- 800	+11380	+10580	273	32	15
	22-23	483	126	27000	21	∓1115	±1062	∓1538	±1016	±4374	∓3427	∓5300	±4290	∓ 926	± 863	±4374	∓3427	±3259	∓2365	±2836	∓2411	+ 290	+ 6990	+ 7280	530	63	45

* Average Value

In units of 1000 lb. { 980 1303 1299 1112 1065 1055 989 973 972 962 952 957 957 962 962 972 973 989 1056 1065 1112 1299 1303 980 — Dead Load Panel Concentrations
510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 510 — Live Load Panel Concentrations Assumed Live Load = 2400 lb. per linear foot of Bridge.



Loading Diagrams for Maximum Primary Stress

Note:-The above loading diagrams refer to members in the left half of span only. For corresponding members in the right half of span, the loading diagrams should be reversed.

NOTES

All stresses on this sheet are unit stresses, and are given in pounds per sq. in. + denotes tension, — denotes compression. Secondary stresses are given for both ends of each member in the order of the designating numbers. The upper sign refers to the top fibers and the lower sign to the bottom fibers of the section. In verticals the fibers toward the springing are considered as the upper fibers and those toward the crown as the lower fibres.

Gross sections were used for figuring unit stresses and strains.

The secondary stresses ① from dead load were obtained by combining the secondary stresses produced by the steel weight acting on the 3-hinged arch with the secondary stresses produced by the weight of floor and track acting on the 2-hinged arch.

The secondary stresses ③ from live load covering the whole span, were obtained by combining the secondary stresses ② in the loaded and unloaded sides produced by live load covering one-half of the span.

The approximate secondary stresses ④ occurring simultaneously with maximum live-load stress were obtained by interpolation between the secondary stresses ② from live load covering one-half of the span and those ③ from live load covering the whole span, in accordance with the loading diagrams appended to the table.

The primary stresses given on this sheet were figured from the official Table of Stresses for the Hell Gate Arch.

The loading diagrams indicate by shaded areas the loading which produces maximum primary stress in each member. The signs (+) or (—) indicate the character of the stress produced by any loads in the shaded or unshaded distances.

| Node | Secondary Stress Effects | | |

TABLE 6.

Member.	Secondary stress from dead load on 3-hinged arch, in pounds per square inch.	Total secondary stress from dead load, in pounds per square inch.
18–20	1 083	1 032
20–22	1 264	1 290
21–23	992	961
22–*M*	4 035	4 950
M–23	2 055	1 290

The loading added in the two-hinged condition, although amounting to nearly one-half of the dead load on the three-hinged arch, does not produce its proportional share of the total secondary stress. In many cases it even helps to neutralize or reduce the secondary stress originating in the three-hinged condition.

An inspection of the secondary stresses on Plate XLVI yields the following facts:

As was to be expected from the respective deformations, the secondary stresses for live load covering the half span are considerably greater than for the live load covering the full span.

With only two or three exceptions in each case, the secondary stresses for full live load are less than 400 in the low chord, 500 in the upper chord, 800 in the diagonals, and 1 600 in the verticals. These low values indicate the eminent freedom of the arch from secondary stresses under full or uniform loading.

The second diagram on Plate XLVI is similar in all respects to the second diagram on Plate XLV, and the stresses are found to be proportional. This affords a check on the correctness of the computations, as the respective stresses were found by entirely different and independent procedures.

Plate XLVII presents a compilation of all the secondary stresses and a comparison with the corresponding primary stresses. The data on this plate are self-explanatory.

The secondary stresses of greatest critical interest are those occurring with the loading that produces the maximum primary stress in each member. The total secondary stresses for such condition, including both the dead and live-load contributions, are tabulated in Column 5. These secondary stresses, with a few exceptions in each group, are found to be less than 1 500 for the lower and upper chord members, less than 3 000 for the diagonals, and less than 3 500 for the verticals.

The last column gives the maximum values of the secondary stresses expressed as percentages of the corresponding primary stresses.

It will be observed that most of the percentages, especially those in the chord members, are small.

In the lower chord, the critical secondary stresses range from 1 to 12% of the maximum primary stresses.

In the upper chord, the first three members have comparatively high percentages of secondary stresses (180, 40, and 17%), but this is due to the low primary stresses (3 104, 6 650, and 10 140, respectively) in these members. There is an excess of section which amply provides for these secondary stresses. The totals of secondary plus primary stress in these three members do not reach 12 000 lb. per sq. in.

In the other upper chord members the critical secondary stresses range from 3 to 10% of the primary stresses.

In the web members the critical secondary stresses range from 1 to 65%, but only a few exceed 30 per cent. There is ample margin of cross-section to take care of these stresses.

The web members, as a rule, have larger secondary stresses than the chord members. This may appear surprising, as the web members are longer and narrower than the chord members, and it is generally stated that the secondary stresses in different members vary inversely as their slenderness ratios. The reason is due to the form of the arch; in the three-hinged condition practically all the dead load is carried by the parabolic bottom chord, and the web members receive large secondary stresses from the compression of the lower chord members before they receive any appreciable primary stress.

Conclusions.

These results obtained in the secondary-stress computations bear out the justification of the type of structure, unit stresses, and method of erection adopted for the Hell Gate Arch Bridge.

Some of the larger secondary stresses are found at the crown of the arch, and appear to be caused by the special arrangement of the center panel and the part which it plays in the erection of the structure as a three-hinged arch. There were sufficient advantages in this construction, however, to outweigh the increase in the local secondary stresses. Nevertheless, the results confirm the usual objections to intersecting web members. On the other hand, they demonstrate the desirability, from the consideration at least of secondary stresses, of reducing the number of hinges in an arch.

The low secondary stresses in the bottom chords are particularly gratifying, as they set at rest any apprehensions aroused by the great depth of these members.

The results, on the whole, show that special methods of assembling and erection, such as were adopted for the Quebec and Sciotoville Bridges, to counteract the effect of secondary stresses, were not required for the Hell Gate Arch Bridge.

DISCUSSION

H. J. Bingham Powell,* M. Am. Soc. C. E. (by letter).—The writer appreciates the care observed to obtain reliable results in taking the stress measurements on the Hell Gate Arch Bridge, but would question the accuracy of the methods used, that is, to a matter of tenths of a thousandth of an inch claimed. As Inspection Officer in Charge of the Department of Gauges and Standards of the British Ministry of Munitions in the United States, he has had, during the last two years, a wide experience of the difficulties that are encountered in taking fine measurements correct to a tenth of a thousandth of an inch, unless the instruments are extremely accurate and are used in uniform conditions of working, temperature, etc. Mr. Powell.

The principle of measurement of the Howard extensometer appears to the writer to be fundamentally unreliable, because of the following objectionable features: The measuring points of the micrometer caliper are conical, and the zero reading for any given temperature is taken from a "comparison bar" with holes. The member to be measured has, similarly, conical holes for the insertion of the micrometer measuring points. The bearing of the measuring points is not on the tips, but on the sides, and that is where the unreliable feature in the measurement enters. If the micrometer is not held so that the points are "square" to the holes, the bearing on the conical sides is very unequal, and, consequently, the measurement is a false one. Further, it is a most difficult matter to get a satisfactory micrometer "feel" with conical points bearing against the inclined sides of the holes. These sources of error present themselves twice: in taking the "zero" reading on the comparison bar, and again when measuring the member. Thus, the combined error may be considerable. Further, the holes in the member are filled with oil, vaseline, or ivory black paint for protection until used. The filling is removed by a pointed aluminum rod, a small cedarwood stick, and some absorbent cotton. No doubt, every care was taken in removing the filling, but in the confined space and under the generally disadvantageous conditions in which the measurements were taken, it would be difficult to obtain the thorough cleaning out of the holes so necessary for any degree of accuracy. Even with the polished surfaces of screw gauges, the writer has found that a film of oil on the thread upsets the measurements entirely; and the conical holes are not only similar in form to the cross-section of the thread of a gauge, but also give a bearing on a surface instead of a line, as in the former case, and thus aggravate the evil of any film of matter present. Also, the holes in the member are of very imperfect finish of surface, and aid the adhesion of such a film. The only way to remove these factors of inaccuracy is to abolish the use of holes and substitute

* London, England.

Mr. Powell. slightly projecting ball-ended cylindrical studs, which present a smooth surface, easily cleaned, and over which very accurate measurements can be made. These studs can be of very small diameter and only project sufficiently for an ordinary micrometer (a micrometer caliper is unnecessary in this case) to have a bearing on the anvil and spindle. The comparison bar of the micrometer would be a plain rod with the ends ground true, and so the zero reading would be a direct, and consequently, exact, one.

Mr. Waddell. J. A. L. WADDELL,* M. AM. SOC. C. E. (by letter).—This paper is not only of great value on account of the important measurements of both direct and secondary stresses which the author records and discusses, but it is also exceedingly interesting because of the historical treatment of the subject of measuring the actual intensities of working stresses in bridge members.

The practice of stating near the beginning of a paper the history of the subject treated is one that is to be highly commended in many cases, for the reason that it places the reader at once *au fait* with the matter under consideration, and thus arouses his interest and induces him to study seriously what follows.

Reliable information concerning the actual intensities of working stresses in main members of bridges, and especially in their connecting details, is still rather meager, notwithstanding the fact that such information would be of inestimable value to all designers of steel bridges. The difficulty is that the experiments necessary to obtain the required knowledge are expensive, and are generally beyond the reach of the individual engineer. Engineering societies and railroad companies are the logical organizations for conducting such experiments; and, recognizing this, the American Railway Engineering Association (probably the most active and progressive of all our technical societies), some years ago combined forces with a number of the prominent railroad companies and, by a well-evolved series of experiments, secured a mass of most valuable information on the subject of impact on bridges and bridge members. A similar series of experiments on actual intensities of secondary stresses in bridge members and their connecting details might well be undertaken by the same combination of forces.

The Hell Gate Arch experiments, incomplete as they are, constitute a fine start on the investigation of stress distribution; but they should be completed, so as to include the effects of live loads. Such a finished series of experiments would extend our knowledge of stress conditions, would determine the efficacy of new structural features, would provide reliable information concerning the effects of various methods of erection, and would develop improvements in both design and construction. Again, in this manner, there could be ascertained

* Kansas City, Mo.

the relative importance of indeterminate stresses, as well as the actual effects of using redundant members. Mr. Waddell.

Although, as just stated, the scope of the Hell Gate Arch investigation was somewhat limited, the results obtained are certainly valuable; and the engineers who evolved and conducted the series of experiments are certainly entitled to the hearty thanks of the entire Engineering Profession.

F. H. Frankland,* Assoc. M. Am. Soc. C. E.† (by letter).—This subject is of special importance and interest to all those engineers engaged or interested in the design or construction of important steel bridges, and the author deserves high commendation for his thorough, painstaking, and masterly treatment of a subject acknowledged to be of the highest importance in the development of the science of bridge designing. Mr. Frankland

In planning the work it is doubtful whether the methods selected and devised could be well improved upon, having in mind the purpose in view. Although the paper represents a tremendous amount of work in the calculation of the secondary stresses in the structure, involving the solution of a large number of simultaneous equations for each condition of loading, and the analytical work required in digesting the results and deducing conclusions, it is remarkable that the work has been presented in such condensed form, and yet is fully adequate for complete understanding. This tendency toward elimination of all extraneous matter in technical papers on special subjects is to be highly commended.

The importance of such stress measurements is so great that it might be safely said that such investigations, in the future, will prove one of the most useful aids in the design of bridges of unprecedented span. The writer is of the opinion that the desirability of more and extended investigations of this nature is clearly indicated; and he begs respectfully to suggest that the Society take under consideration the idea of similar stress measurement investigations on the Quebec and Sciotoville Bridges, and the completion of the measurements on the Hell Gate Bridge (those applying to live load), thus securing exceedingly valuable and complete data on the largest bridges, of the cantilever, continuous truss, and arch types, in existence. Owing to the probable great expense of these proposed investigations, which would be too onerous for any one individual to bear, and to the undoubtedly great value to the Profession, it would appear quite fitting and proper for the American Society of Civil Engineers to undertake this work, under the supervision of a special committee. It may be well to mention, in elaboration of the foregoing suggestion, that the determination of the live-load stresses of the Hell Gate

* Kansas City, Mo.

† Now M. Am. Soc. C. E.

Mr. Frankland. Bridge would complete the measurements for that structure, and that initial stress measurements have already been made for the Sciotoville Bridge, under the direction of Gustav Lindenthal, M. Am. Soc. C. E. The results deduced from stress measurements of three types of long-span bridges would be more valuable and conclusive than those for a single structure.

Mr. Waterbury. L. A. WATERBURY,* M. AM. SOC. C. E. (by letter).—On page 1059 of Mr. Steinman's paper, the following paragraph, which forms the topic of this discussion, appears:

"The foregoing discussion of results does not include the final stage (2-*H*). In that stage, in the members near the ends of the span, the upper end in every case was found to present a larger average stress than did the lower end. The differences ranged from 450 to 2 800 lb. per sq. in. This anomalous result appears to arise from some unexplained disturbance of stress distribution, and does not represent any error of observation."

This interesting condition of observed stress may possibly be due to an irregular distribution of the stresses at the observed sections, which might be due to the order in which the drift-pins and bolts were replaced by rivets. The great size of the members and the great difference between the stress at the center of member 0-2 from that at the top and bottom, for each section, for Stage (2-*H*), suggest that there might easily be considerable irregularity in the distribution of stresses at each section, between the lines of gauged deformations.

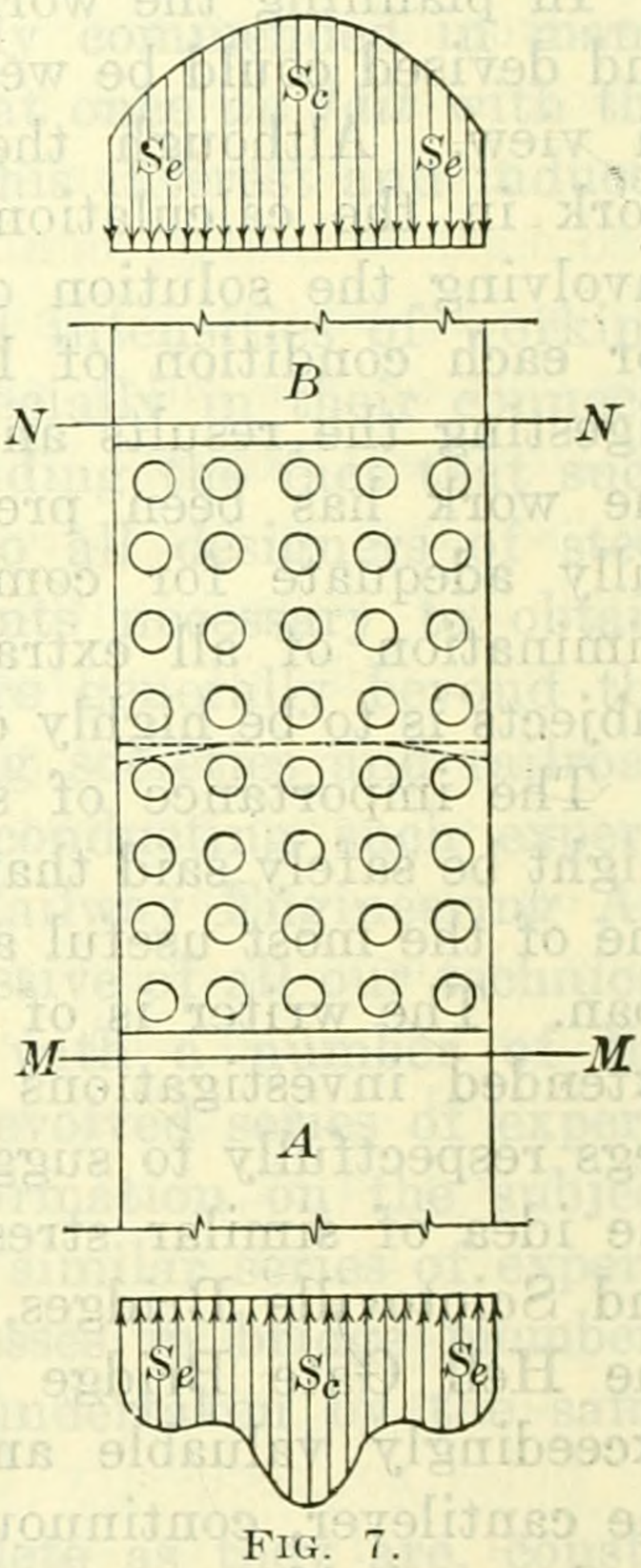


FIG. 7.

To illustrate the principle involved, consider the simple case indicated by the joint shown in Fig. 7, and suppose that the stages of removal of drift-pins and of placing of rivets are as follows: (1) center line of member *A*; (2) outside rows of member *A*; (3) intermediate rows of member *A*; (4) intermediate rows of member *B*; (5) outside rows of member *B*; and (6) center line of member *B*. During each stage a slight closing of the butt joint occurs, which, for joints of the type used in the Hell Gate Arch, develops larger stresses in the middle third of each web than in the outer thirds. At the end of Stage (1), the rivets in place are without shearing stress, but during Stage (2) a slight closing of the joint occurs, producing some stress in the rivets placed during the first stage.

* Nitro, W. Va.

Mr. Waterbury.

During Stage (3), the rivets placed in the second stage receive some stress, and those placed in the first stage receive an additional increment of stress, but the two intermediate rows of member *A* are without stress. During Stage (4) a portion of the load which had been carried by the splice plates, is transferred to the web as the butt joint closes, thus relieving the center and outside rows of member *A* of some stress, but, at the same time, developing some negative shear in the rivets of the two intermediate rows of that member. During Stages (5) and (6), the action on the rivets of member *A* is similar in character to that which occurred during Stage (4). At the end of Stage (6), the center row of rivets of member *B* is without stress, the outside rows of the same member have some stress, and the two intermediate rows have considerable stress. Such a distribution of the load among the rivets of the splice would tend to produce a variation of the stress at Section *M-M* of member *A* of the type indicated by the stress profile, which is shown below the joint in Fig. 7, while the variation at Section *N-N* of member *B* would be more nearly like that indicated by the stress profile shown above the joint. If the deformations are measured along lines coresponding to the stresses, S_c and S_e, of the stress profiles, it is evident that the total stress of member *A* may be equal to the total stress of member *B*, in spite of the fact that all the observed stresses for the first member are greater than those for member *B*. The deformations for member 0-2 for the 2-*H* stage of erection, as stated in Table 2 of the paper, exhibit this character of variation, notwithstanding the fact that the observed values are large enough to be considered fairly reliable.

In attempting to check the average stress at sections where there is an irregularity in the distribution of the load, it should be remembered that, even if readings are observed at enough lines to determine the correct variation in deformation for the section, the average stress will not necessarily be indicated directly by the stress profile, due to the variation in thickness of metal across the section. The correct average stress would be obtained by weighting the stress for an increment length of profile, the weight being proportional to the area of metal over which that particular stress is effective, and by computing the weighted mean of all the stresses shown by the stress profile.

Mr. Hughes.

F. D. Hughes,* M. Am. Soc. C. E. (by letter).—Engineering has been correctly called the creative profession, and yet the engineer, who is most directly connected with production of the world's wealth, such as manufacturing and kindred lines, has very little time to devote to independent investigation of engineering problems, and, for that reason, he, more than any other, owes thanks to the authors of this and similar papers who spend their time in investigation on the frontier

* St. Louis, Mo.

Mr. Hughes.

of engineering thought and experimental research. The average Board of Directors of corporations looks with scant favor on expenditure of either time or money in search of information and formulas in engineering that do not promise an adequate financial return to the corporation which it represents. Therefore, as already stated, this large body of the Engineering Profession, which, for want of a better term, the writer would designate as industrial engineers, owes much to those who take the time and opportunity to investigate, for the benefit of the Profession at large, such problems as were presented in the construction of the Hell Gate Arch Bridge and to give the results to the engineering world. It is permitted to few engineers to be connected with either the design or construction of a structure of this magnitude, and perhaps the conclusions to be reached are not of practical use to a majority of the Profession, and yet the paper, in itself, is a very valuable contribution to engineering literature.

In an experience of some twenty years, mostly devoted to the design and construction of highway bridges, the writer has found a wide diversity of treatment in the consideration of secondary stresses. Such treatment has varied from that degree of refinement which might be termed "painful" in its exactitude, to that in which the stresses were completely ignored. As an instance, he recalls the construction of a 140-ft. span, designed to carry a concrete floor of the Warren subdivided panel type, all members being riveted, in which the wind stress was carried into the lower chord and exact consideration was given to the increment of stress, even to the changing of the size of section in adjoining panels where the live- and dead-load stresses were of equal moment, thus necessitating (if the theory of the design is followed), a field connection between the two panels where practice should have made the two in one single member. In view of the fact that the stringers were rigidly connected to the floor-beam, the writer considered this degree of refinement a waste of material. On the other hand, he has seen numerous structures of highway design, of the pin-connected type, in which the entire lower chord was composed of eye-bars, and in which the first vertical member consisted of two light angles barely sufficient to carry the computed live- and dead-load stresses; and the detail of connecting the floor-beam below the chord, would, by the same analysis of wind stress, cause bending in the vertical member of from 40 000 to 50 000 lb. per sq. in.; and yet these bridges are standing and have been carrying ordinary traffic for years. Between these two extremes lies the most acceptable practice, and the author's conclusion, that any treatment of secondary stresses should be carefully analyzed and the experiments along the line of stress measurements taken with considerable allowance, is very timely.

The lesson to be learned from the result of these measurements, it seems to the writer, is, that all structures which are designed to

carry their own weight during erection, as well as eccentric loading from machinery or other methods of handling, should have some scheme of measurements for checking the secondary stresses. The additional expense would be more than made up by the insured safety of the structure against probable loss of life and limb, and would have prevented numerous disasters which have occurred on structures of this class. Mr. Hughes.

A. H. FULLER,* M. AM. SOC. C. E. (by letter).—The Engineering Profession is certainly indebted to Mr. Lindenthal for developing the erection of the Hell Gate Arch Bridge into a huge, scientific experiment, and to the author for the admirable manner in which this has been carried out and presented. No one who has given any attention to stress measurements can fail to appreciate the precautions taken in this work and the care in carrying them out, and neither can any one who has ever attempted to compute secondary stresses fail to recognize the magnitude of the task along this line. Each of these features could well be considered an accomplishment, and the combination of the two, on a structure of the magnitude and interest of the one under consideration, marks a new achievement in the understanding of the actual behavior of steel structures. Mr. Fuller.

Engineers would not be doing their duty by simply applauding what has been done without making an effort to analyze the results, in a manner that would throw even greater light on the subject, and to remove the last shadow of doubt in regard to the interpretation that may be drawn.

On page 1065, it is stated that the following relations are evident:

"1.—Except for the smaller percentage of secondary stresses, the measured values are consistently lower than the calculated values.

"2.—As the percentage of calculated stresses increases, the percentage of measured values also increases, but not as rapidly.

"3.—For the highest percentages of secondary stresses, the measured values amount to only a small fraction of the calculated values."

On page 1071, in Paragraph 5 of the Summary of Conclusions, the same points are brought out. A probable interpretation from these conclusions is that measured secondary stresses are usually lower than computed ones.

An examination of Plate XLIV discloses the fact that the recorded measured secondary stresses are as great as, or greater (frequently much greater) than, the computed ones in all cases where the primary unit stresses exceed 5 000 lb. per sq. in. and, with one exception, for all cases where the primary unit stresses exceed 3 000 lb. per sq. in., that is, that the measured stress is greater than the computed one whenever the primary stress approaches a maximum working value.

* Easton, Pa.

Mr. Fuller.

Although the author withholds the measured secondary stresses for the stage, 3-*H*, because of the inconsistencies due to added dead load during observation, the primary stresses are given. It seems improbable to the writer that this disturbance could be so great as to render the results valueless, and he would like to ask the author for these results in connection with the points that have just been raised.

On Plate XLVII are given the percentages of computed secondary stresses in all members, and it is of interest to note, as the author has so well brought out, that these are remarkably low in the lower chord members which carry the greater portion of the load. Take Member 4-6, for instance, in which the computed secondary stress due to dead load is 5% of the primary stress. Plate XLIV indicates that the measured secondary stress is 13% in one truss and 11% in another, of the primary stresses, that is, more than double the computed ones. The writer would like to ask what information is available concerning the variation of stresses within the 20-in. gauge line that was used. The intensity is certainly greater toward the end of the member. Measurements that have been taken by the writer on steel and concrete in reinforced concrete structures indicate a very rapid drop in maximum stresses from the point of connection of the member, and suggest that gauge lengths even less than the usual 8 in. are needed to catch the maximum stresses. This point has been brought out much more clearly by Professor McMillan, of the University of Minnesota, in comparisons between 2-in. and 8-in. gauge lengths. It seems, therefore, that the intensity of secondary stresses may be somewhat greater than the averages over 20-in. gauge lengths, and that they are intensified again at the point of decrease in section at the edge of the gusset-plates.

These points can be recognized without questioning the author's general conclusions that the secondary stresses in the structure under consideration are not large enough to cause concern, and that the type of structure is well chosen to keep them within low values. It does not seem to the writer that it is well to minimize these stresses, for that would tend to give unwarranted confidence and possibly result in overlooking secondary stresses and stress measurements in cases where they might otherwise be secured and thus contribute to the store of knowledge.

The writer would like to ask for more information than has been given in the paper, concerning the distribution of stresses in different portions of the members. Easy computations from the data in Table 2 show the average intensity of the stress in the central angles to be about 50% greater than the average intensity at the four extreme angles for the 2-*H*, or last, stage, for the upper end of Member 0-2.

Mr. Fuller.

Similar conclusions may be drawn from Table 4 for the upper end of Member 4-6. The author has well pointed out that the nature of the connections tends to concentrate the stress in the middle portion of the member, but presents only average values, except for the data in Tables 1 to 4. It would be interesting to have any data that may be available for particular locations in regard to the distributions of stress between plates and angles. The writer would not anticipate any great change in work so carefully designed and constructed as this has been, but feels that the information would be desirable.

On page 1051, the author states that the Howard strain gauge is probably the simplest and most accurate field instrument for measuring strains. As the writer has never used this instrument, he is not in the position to question the statement. He is under the impression, however, that more information may be secured from the quicker acting Berry strain gauge for the same expenditure of time and energy. The inference drawn by the writer from the work on the Hell Gate Arch Bridge is that for each load only one measurement was recorded for each gauge length. Possibly this has been verified from several readings and the average recorded. The writer has secured much more consistent results from the Berry instrument by taking three or more sets of readings, with change of dial between each set, and recording all the results, than by taking all the readings at a certain point consecutively and recording an average.

Mr. Howard.

JAMES E. HOWARD,* ESQ. (by letter).—This is a most interesting and instructive paper on the results of measured strains on a structure of magnitude, which, in its conception and execution, occupies the foremost rank in engineering works. The intelligent care which was exercised in acquiring the experimental data, the impartial criticism of this method of measurement, the definition of its proper scope, as well as its limitations, is a source of gratification to the writer, who believes himself to be the one who inaugurated this simple and direct method of examination, a method which is capable of furnishing many useful results which have not yet been placed before the Engineering Profession.

This method of measured strains has been used by the writer in the examination of the dead-load strains in bridges of large span, in the structural members of modern buildings, in the distribution of stresses in the sheets and across the seams in steel boilers, in the investigation of the internal strains in steel rails, and also in the study of thermal effects in street pavements. On dates earlier than the examples just mentioned, the method was used in ascertaining the state of internal strains in steel forgings, in railway axles, in cold-rolled shafting, and

* Washington, D. C.

Mr. Howard.

also the residual strains in steel bars after over-straining loads had been applied and released.

This method of examination was first used by the writer, in 1886 and 1887, in the determination of the internal strains in steel forgings, in lieu of a method used by Gen. Nicholas Kalakousky, of the Russian Artillery, who utilized a microscopic cathetometer for the purpose.

The use of drilled and countersunk holes for defining the gauged lengths enabled such lengths to be established and preserved against accidental injury, even against vicissitudes of street traffic, in the examination of street pavements; in structural members, protection is afforded to the contact surfaces during handling and erection, in addition to which the feature of permanence is present, enabling a re-examination of the gauged lengths after the lapse of time.

A reference bar, having substantially the same coefficient of expansion as that of the work under examination, serves its purpose as a standard of length. The reference bar is kept at the same temperature as the work, whenever it is practicable to do so, at other times a correction for the difference in temperature is applied.

Two types of strain gauges—or transfer instruments, as they might be termed—have been used: one is adjustable to any gauged length from 1 in. upward, 36 in. being considered a practical maximum, the other being represented in the type used by the author. The adjustable gauge was the earlier design. It was intended for use on horizontal surfaces or those nearly so, and is more essentially a laboratory instrument than the type used on the Hell Gate Arch Bridge. The latter admits of being used in different positions.

The subject of measured strains is one which is particularly attractive to the writer, as it appears to open the way to advance knowledge relating to the actual conditions or states of strain which pertain to engineering structures of all kinds. Dead-load strains, equivalent to 20 000 lb. per sq. in., have been observed in bridge members. Temperature differences have been responsible for stresses of several thousand pounds per square inch in large members. In steel rails, wheel pressures have caused the introduction of internal stresses, exceeding 20 000 lb. per sq. in., in the head of the rail next the running surface, with tensile components in the interior of the head, stresses which, singularly, appear to have been overlooked in railway engineering.

It is felt that a most promising opportunity for the acquisition of engineering data has been permitted to pass unimproved, or has been utilized to a limited degree only, in the few examples of measured strains in engineering structures. It is with great pleasure, therefore, that the writer witnesses the presentation of this valuable paper which was provided for by Gustav Lindenthal, M. Am. Soc. C. E., the Chief Engineer of the magnificent Hell Gate Arch Bridge.

ISIDORE DELSON,* ASSOC. M. AM. SOC. C. E.—Several years ago, the Bridge Department of New York City completed the work of strengthening the end spans of the Williamsburg Bridge across the East River. In connection with this work, a great many strain observations were made with the Howard extensometer, in co-operation with the U. S. Bureau of Standards. Mr. Delson.

The speaker desires to describe here especially one series of observations which seem to him of particularly striking interest, inasmuch as they actually assisted in the diagnosis of a perplexing situation, and pointed out the safe method of procedure in a difficult operation.

As indicated in Fig. 8, the end span trusses were supported in the lower chord, at Panel Point *L* 29, by 10-in. pins, by which the reaction was transmitted to the main, that is, suspension, span of the bridge through the panel triangle, *L* 29-*U* 30-*L* 30, containing the tension

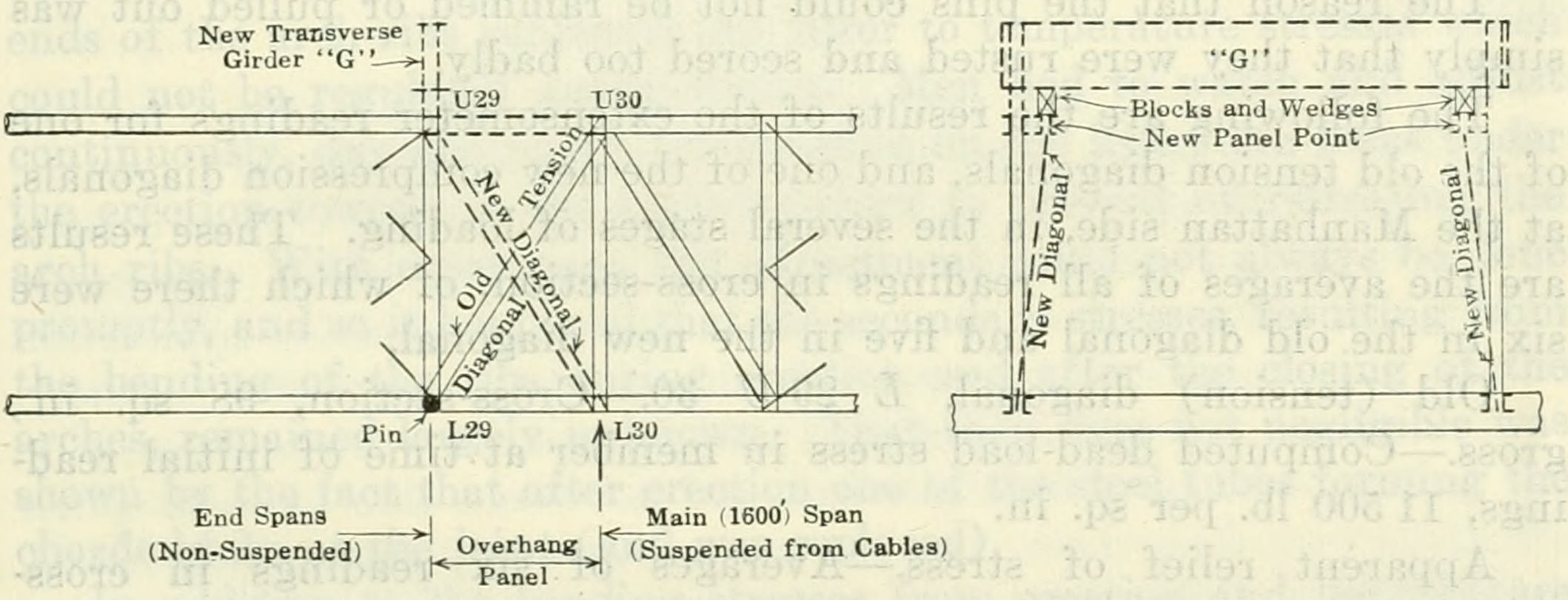


FIG. 8.

diagonal, *L* 29-*U* 30. As a part of the reconstruction, it was required to relieve these pins of their loads, to remove the pins, and then replace them by others of greater diameter.

At each truss a new panel triangle (*U* 29-*L* 30-*U* 30), containing the compression diagonal, *U* 29-*L* 30, was built out from the main span. The vertical end posts, *U* 29-*L* 29, of the end span trusses were connected to a new overhead transverse girder, *G*, which was supported by cast-steel blocks and wedges on the new panel points.

The relief of the pins was effected by wedging up the girder, *G*, from the new points of support, and the end spans were thus made to hang from that girder. As the old tension diagonals were being relieved of tension, the new diagonals were being compressed.

Extensometer points had been established on the old tension diagonal, and, also, on the new members before their erection.

It was determined by computation that the girder, *G* (and the trusses hung from it), would have to be wedged up $\frac{3}{4}$ in., in order to relieve the pins of the dead load re-action. However, when this wedg-

* Stapleton, N. Y.

Mr. Delson.

ing was effected, the pins absolutely refused to budge. The wedging was then increased to 1 in., and still the pins could not be made to move. The contractor apparently saw no other way but to continue this wedging process, and he was permitted to go as high as $1\frac{3}{8}$ in., but without success.

All operations were followed up with extensometer observations on the members affected, but particularly on the diagonals. The results of these observations convinced the engineers of the Department that the contractor was on the wrong tack. Accordingly, the wedging-up process was stopped, and the condition was restored to the first established $\frac{3}{4}$-in. stage. Then the pins were removed by cutting them in halves. On their removal, there was not the slightest sensible dislocation of the members assembled on them, which verified the fact that no more nor less than the actual load on them was taken off.

The reason that the pins could not be rammed or pulled out was simply that they were rusted and scored too badly.

The following are the results of the extensometer readings for one of the old tension diagonals, and one of the new compression diagonals, at the Manhattan side, in the several stages of loading. These results are the averages of all readings in cross-section, of which there were six in the old diagonal and five in the new diagonal.

Old (tension) diagonal, *L* 29-*U* 30.—Cross-section, 98 sq. in., gross.—Computed dead-load stress in member at time of initial readings, 11 500 lb. per sq. in.

Apparent relief of stress.—Averages of six readings in cross-section.

Stage of wedging..	$\frac{3}{4}$ in.	1 in.	$1\frac{3}{8}$ in.	$\frac{3}{4}$ in.	After removal of pin.
"Relief", in pounds per square inch..	11 300	13 600	16 600	11 500	11 600.

New (compression) diagonal, *U* 29-*L* 30.—Cross-section, 148 sq. in.—Computed stress for dead-load reaction, 6 400 lb. per sq. in.

Apparent stresses.—Averages of five readings in cross-section.

Stage of wedging..	$\frac{3}{4}$ in.	1 in.	$1\frac{3}{8}$ in.	$\frac{3}{4}$ in.	After removal of pin.
Compression, in pounds per square inch.	6 500	7 700	10 500	6 650	6 300.

Attention is called, not only to the close agreement of the computed stresses with those disclosed in the $\frac{3}{4}$-in. stage, but also to the fact that, after the pins were removed, there was no change in the stresses of the members, showing that just the exact amount of wedging had been applied.

Mr. Delson.

It should be noted that the series here presented is rather exceptional, and that such good results may not always be expected from the extensometer; but, even where results are not so satisfactory, they, as a rule, would be close enough to help greatly in arriving at a correct judgment of conditions for all practical purposes.

Mr. Lindenthal.

GUSTAV LINDENTHAL,* M. AM. SOC. C. E.—A few words may be added to this able paper as to the speaker's reasons for having the erection stresses measured, as related by the author. He wanted to ascertain what, if any, bending stresses remained in the trusses after erection.

The speaker had in mind that, during the erection of the St. Louis Arch Bridge (built by the late J. B. Eads and Henry Flad, Members, Am. Soc. C. E., in 1870-74), which was the first instance of erecting steel arches by the cantilever method, the actual erection stresses varied considerably from the computed ones. During erection, the fixed ends of the arch ribs subjected the latter to temperature stresses which could not be regulated automatically. Men had to watch and adjust continuously, day and night, the pressure on the hydraulic jacks under the erection towers on the piers in order to prevent overstressing the arch ribs. With every care, the adjustment could not always be done promptly, and so it happened that the secondary stresses, resulting from the bending of the ribs during erection and after the closing of the arches, remained largely unknown. That they were not negligible was shown by the fact that after erection one of the steel tubes forming the chords broke at the joint (and was replaced).

In addition to the bending stresses from erection and temperature changes are the secondary stresses from live load. These stresses must also be quite large in the St. Louis Bridge from the alternating flattening and bulging of the ribs at the quarters. An incidental effect of this is the creeping of the rails (each track is carried by two ribs) in the direction of the traffic to the extent of 12 in. per day. Another is the excessive straining of the cross-diagonals between the ribs, when one pair of ribs flattens and the other pair bulges under meeting trains. That this bridge has stood these extraordinary strains thus far without failure is proof of the excellent quality and elasticity of the crucible steel in the chords. If the strains in them could be measured (which they cannot, because the steel staves are enclosed in an envelope of wrought iron, and are not "get-at-able"), it would be a most instructive guide in other arch designs.

In the design for the Hell Gate Arch, the speaker desired to minimize as much as possible the secondary stresses (from change of form). One way of doing this was by having the trusses stiff.

By making the arch deep at the quarters and three-hinged for dead load (which in this case is 40% greater than the live load) and two-

* New York City.

Mr. Lindenthal.

hinged for live load, it could be arranged to avoid reversal stresses in the bottom chord altogether, and to keep the range (about 8 000 lb.) of unit stresses from minimum to maximum compression to within one-fourth of the elastic limit for compression on the gross net section.

A further means of minimizing the secondary stresses in the bottom chord was by providing against excessive edge pressures at the joints, as described by the author and also by Mr. Ammann.

In this respect, the Hell Gate Bridge is the antithesis of the St. Louis Arch Bridge with its tube sections and sleeve connections which are more subject to large edge pressures than, for instance, the full contact riveted splices in the shallow ribs of the Clifton Arch Bridge at Niagara Falls, which was likewise erected by cantilevering.

The object of the strain measurements, as related by the author, was fully attained. There are no unknown stresses in the Hell Gate Arch structure.

Mr. Parcel.

JOHN I. PARCEL,* ESQ. (by letter).—The writer has read this very able paper with great interest. The introduction of the strain gauge marks a definite epoch in the development of the science of structural design, and the remarkable large-scale experiment which this paper reports can hardly fail to encourage a much more extensive application of this instrument to the practical problems of the structural engineer.

Certain of the author's conclusions on secondary stresses seem to the writer to be open to argument, and he would like to raise a few questions regarding them. As bearing on secondary stresses in general, perhaps the two most important conclusions stated in the paper are:

1.—Actual secondary stresses, in general, will be less than the computed stresses, due largely to a constant automatic re-adjustment of the internal strains tending toward the relief of secondary stresses.

2.—For most bridges it will probably be satisfactory to provide for secondary stresses by an allowance in the specified unit stresses of about 20% of the total primary stress, plus 3 000 lb.

Perhaps the first conclusion is entirely sound, but some question may be raised as to its deduction from the data of this experiment. Fig. 5, as the author explains on page 1066, shows the curve of measured secondary stress approaching the calculated curve continuously from Stage 1 to Stage 6, as the joint condition approaches more nearly the rigidity assumed in the computation of secondary stresses, diverging again as a release of direct compression permits partial hinge action at the joints, and finally crossing above the calculated curve after Stage 12, when the compression is so great that the joints are completely fixed. At the final stage, 2-*H*, when the primary stress

* Associate Professor, Structural Eng., Univ. of Minnesota, Minneapolis, Minn.

Mr. Parcel.

reaches its maximum, the average measured secondary stress is more than twice the computed value. The action of the three-faced butt joints, in concentrating the direct stress near the middle of the section so as practically to nullify the bending effect, shows a remarkably happy solution of the problem in the chords of the Hell Gate Arch, but it remains true that the measured bending stresses are more than double their calculated value. A. H. Fuller, M. Am. Soc. C. E., has pointed out that, for nearly all cases where the primary stress is greater than 3 000 lb. per sq. in., the measured secondary stresses are greater than the corresponding computed stresses. Plate XLIV shows eighteen measurements for secondary stresses where the measured primary is 3 000 lb. or more, and in all but two cases the measured value is equal to or greater than the calculated value. For the final stage, 2-*H*, the average of the computed secondaries is 6% and the average of the corresponding measured values 13 per cent. That these two figures are fairly comparable is indicated by the author's formula (page 1063) where he takes 12% as an average value for secondary stress proper—*i. e.*, that induced by rigidity of joints when the truss distorts in its own plane. The writer cannot help feeling, with Professor Fuller, that this point is of more significance than the author seems to attach to it, and he would like to inquire if there are reasons, other than appear on the surface, for minimizing its importance. If not, the following additional conclusion might fairly be drawn; "for the higher ranges of primary stress, the actual secondary stresses will probably exceed the calculated values by a considerable margin." As in all ordinary cases it is only the secondary stresses that co-exist with the maximum primary stresses that affect the design of the structure, this result would seem more significant practically than the fact that the higher percentages of calculated secondary stress are not borne out by the test.

The second conclusion noted above, regarding an allowance in the specified unit stresses of 20% plus 3 000 to cover secondaries, is of interest in its attempt to distinguish between the two kinds of secondary stress: that which is proportional to the primary, and that which is independent of it, a method which seems quite logical and clearly justified by the data of the test. But it may well be asked here whether any empirical relation based on averages is an adequate means of providing for secondary stresses, even in ordinary structures. Perhaps the author does not mean the formula to be used as a substitute for calculations, but he says, on pages 1063 and 1064:

"It is recognized, of course, that a rule of this form can properly apply only to structures in which the secondary stresses have normal or average values. Where unusually large secondaries may be anticipated, these should be given special investigation and attention."

Mr. Parcel.

This would seem to imply clearly that only the unusual cases need such investigation. To the best of the writer's knowledge, some form of average allowance applied to all members is the common method of providing for secondary stresses in most structural offices, except for very large structures or those of unusual types. None the less, such a practice appears to the writer to involve a serious inconsistency.

Let it be assumed, first, that the theoretical calculation of secondaries gives results even roughly reliable. Now, it is well known that standard types of simple trusses of moderate span will show effective secondary stresses (those occurring simultaneously or nearly so with maximum primaries) ranging from 5% in some members to 50% or more in others. Take, for example, the 396-ft., pin-connected, Petit truss.* This is fairly typical of the commonest type of trusses for spans ranging from 300 to 600 ft. In four of the seven top-chord members the effective secondary is about 15%; in the others it is more than 60 per cent. To say that the average is 35%, and thus design the structure, would seem quite indefensible, in view of the standard of accuracy maintained in the calculation of primary stresses.

If we assume that theoretical computations for secondaries are wholly untrustworthy, and thus justify reliance on an empirical formula for ordinary bridges, how are we to rely on our analysis for the cases where "exceptionally large secondaries may be anticipated", and what basis is there for the elaborate and very expensive correction methods used in the erection of the Quebec and Sciotoville spans, methods which have been referred to in current technical literature as the most remarkable single feature of these great engineering projects?

We need a great many more experimental data to settle the question of actual *versus* calculated secondary stresses, no doubt; but the bulk of the evidence, up to the present, would seem to justify considerable confidence in our theory. Most previous tests, as shown by the author's citations, show fair agreement between the calculated and observed stresses. In how far the large discrepancies in the test under discussion are due to the unique conditions of the structure is uncertain. The author evidently feels that a considerable part of the discrepancy is due to a relief of the secondaries by re-adjustment of internal strain, and the writer does not care to dispute this point; but the relatively close correspondence between the measured and calculated curves at Stages 5 and 6, and at 3-*H* and 2-*H*, where the actual condition of the joints approaches that assumed in the calculations, seems most significant, and until we have more definite experimental evidence that our theoretical calculations are untrustworthy is it not worth while, even

* As analyzed in the *Bulletin* of the American Railway Engineering Association, January, 1914, p. 454.

in structures of ordinary span, to provide, at least, for secondary stresses on the basis of some approximate analysis? Mr. Parcel.

It is stated on page 1064 that a comparison of individual values of calculated and measured secondary stresses is not very illuminating because of the many disturbing factors which would tend to obscure the systematic variations. The writer has had some part in a recent test to determine some of the secondary stresses in the 518-ft. riveted span of the Norfolk and Western Bridge over the Ohio River at Kenova, W. Va. This test, made possible by the generous co-operation of J. E. Crawford, M. Am. Soc. C. E., Chief Engineer of the Norfolk and Western Railway, was in no wise comparable in magnitude to the Hell Gate Arch test, the principal object being to compare the computed with the measured secondary strains under service loading at a few typical joints. The data of the test are not yet completely

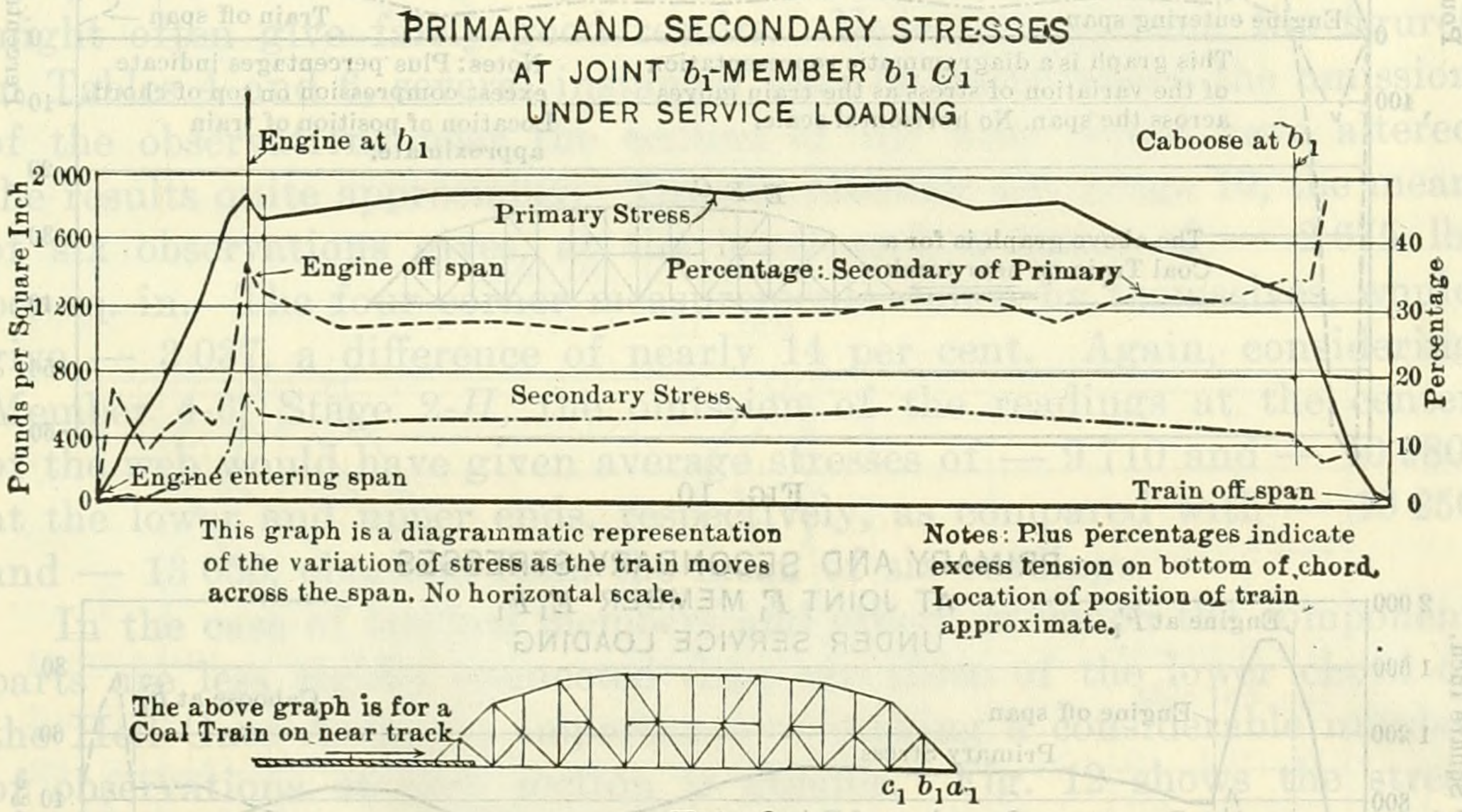


FIG. 9.

analyzed, and any general conclusions deduced would be premature. The graphs of Figs. 9, 10, and 11 are fairly typical, as direct reductions of field data, and the writer has thought they may be of some interest to members of the Society as they stand.

Before making a quantitative comparison with calculated values, the data must be corrected for the location of the instruments, as it was never practically possible to take readings at the extreme edge of the cross-section or the extreme ends of the members. These corrections will generally result in greatly increasing the secondary stress percentages. Before the test was made, the truss was analyzed by joint loads, and influence lines were constructed. These showed that the bending at the lower chord point, b_1, maintained the same direction for all loads, and that the maximum secondary occurred at full loading, though the upper chord points, G and F_1, at bending, reverse during the passing of a single load, and the maximum secondaries occur under

Mr. Parcel.

partial loading. This effect is very clearly brought out, qualitatively, in the curve shown. Several complete sets of readings were taken on each joint tested, and the results were so consistent that, when the data are fully analyzed, it is hoped that a joint-by-joint comparison can be made between measured and computed secondaries, which will be of some interest and value to the Profession.

FIG. 10.

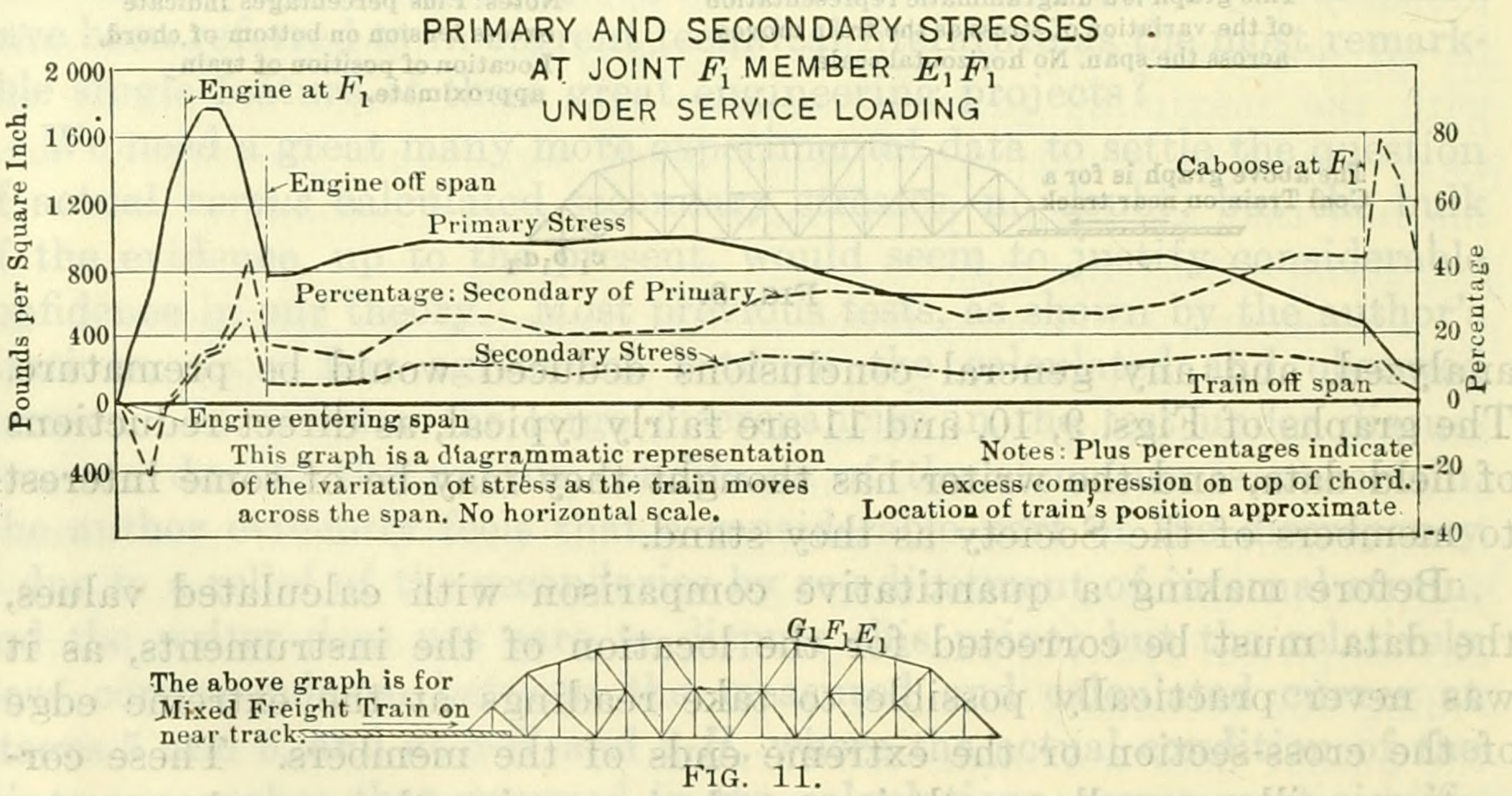


FIG. 11.

Mr. MacKay.

H. M. MACKAY,* M. AM. SOC. C. E. (by letter).—The possibilities of the Howard gauge and other extensometers, as a means of investigating the distribution of stress in structures, do not seem to be as widely appreciated as they should be. The writer, therefore, welcomes this valuable paper, not only for the information it contains, but because

* Montreal, Que., Canada.

it is likely to attract attention to a method of investigation which deserves to be used far more frequently. Mr. MacKay.

The author states:

"To secure full information as to the distribution of stress in any member, it is necessary to take measurements at not less than three, and preferably four, points of the cross-section near each end of the member."

The writer believes that a considerably greater number of points are usually desirable in large members; and he considers the statement: "The intensity of stress at any other point of the member can then be found by planar and linear interpolation," likely to prove very misleading. If literally accepted, the result would often be to cast doubt on this method of investigation. In the case of box members, to be sure, and other members with rigid transverse webs, such interpolation might often give fairly good results. Nevertheless, from the figures in Tables 4 and 6, several instances may be noted where the omission of the observations near the centers of the webs would have altered the results quite appreciably. Thus in Member 4-6, Stage 10, the mean of six observations gives, at the lower end, a stress of — 2 675 lb. per sq. in. The four corner measurements, taken by themselves, would give — 3 037, a difference of nearly 14 per cent. Again, considering Member 4-6, Stage 2-*H*, the omission of the readings at the center of the web would have given average stresses of — 9 710 and — 10 980, at the lower and upper ends, respectively, as compared with — 10 250 and — 13 050, obtained from the mean of six readings.

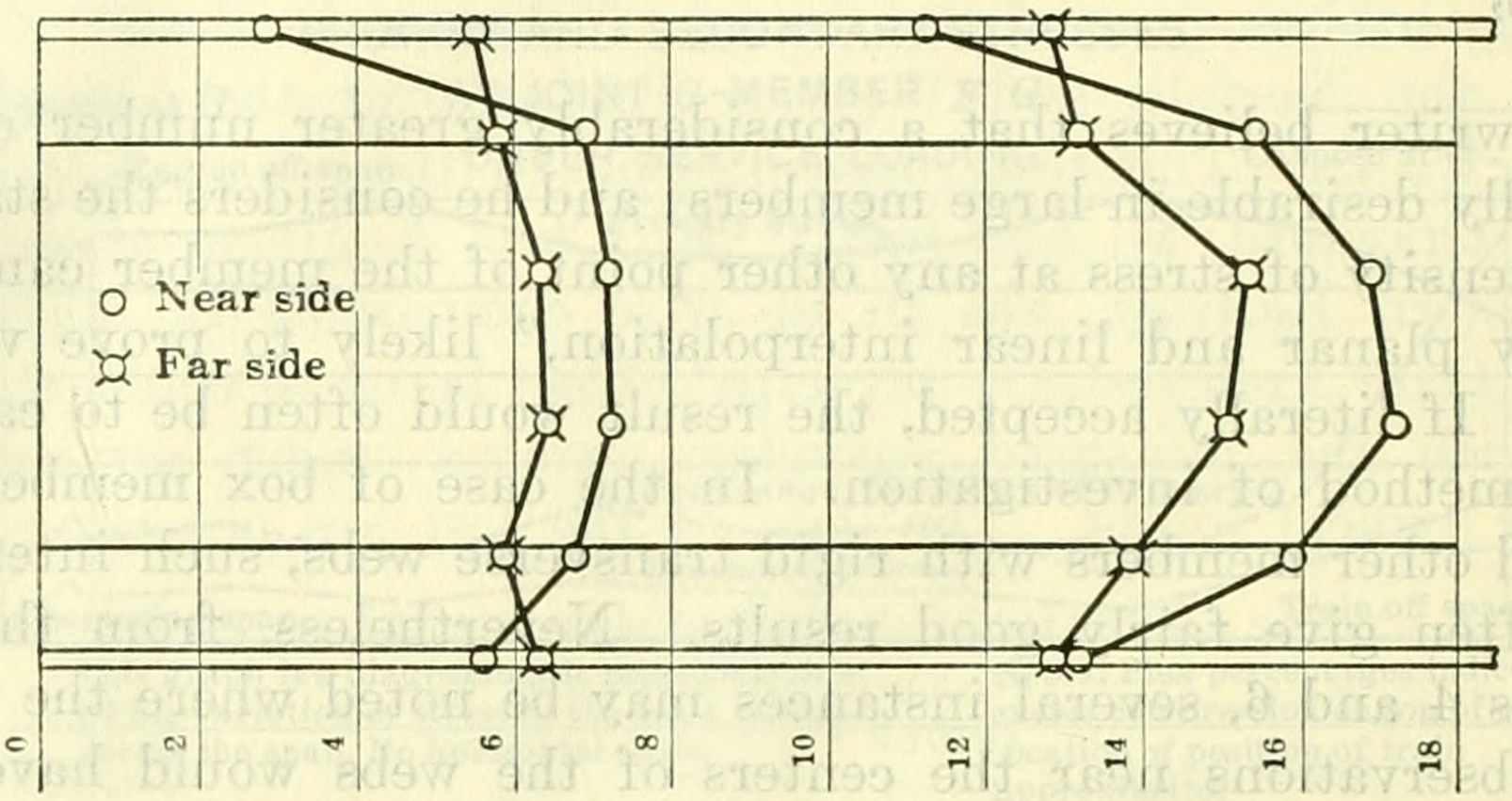
In the case of latticed members and others in which the component parts are less rigidly connected than are those of the lower chord of the Hell Gate Arch, the importance of making a considerable number of observations at each section is greater. Fig. 12 shows the stress distribution, at two stages of loading, in a latticed member tested by the writer, and consisting of four 4 by 4 by $\frac{5}{8}$-in. angles and two 22 by $\frac{5}{8}$-in. webs. The load was applied through 7-in. pins, and the section represented was near the inner end of the pin-plates. The stresses are plotted on an elevation of the member from a base line at the left of the figure, so as to show the distribution at a glance. Taking *E* as 29 000 000 lb. per sq. in., we have:

Average measured stress....	— 6 025	— 14 430
Actual stress.....	— 6 380	— 14 720

Although the pin-plates were of ample length and well designed, the concentration of stress near the center of the web is clearly marked. Many such instances might be given, the condition being quite characteristic of the stress distribution near the ends of pin-connected members. It is clear that in such cases planar interpolation from three or four points would give very misleading results.

Mr. MacKay.

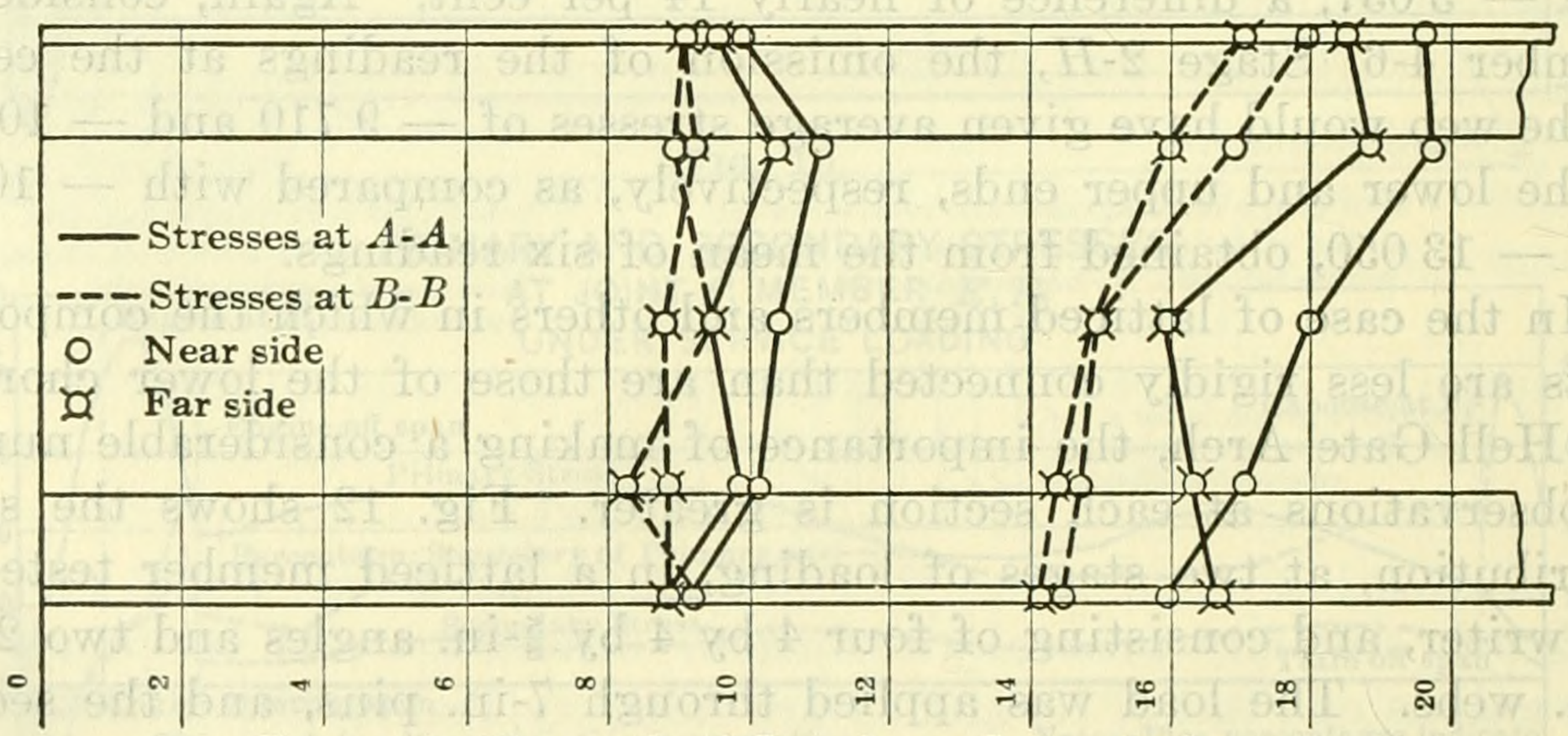
Fig. 13 shows the stress distribution in a similar, but somewhat heavier, member. The full lines indicate the stresses over a 10-in. gauge length, at the center of which a transverse diaphragm is inserted; the dotted lines show the stresses over the same gauge length, the center of which is 10 in. from the diaphragm. Fig. 14 indicates the relative positions of the lattice bars, diaphragm, and points of measure-



Stresses, in Thousands of Pounds per Square Inch.

DISTRIBUTION OF STRESS NEAR PIN-PLATES AT TWO STAGES OF LOADING.

FIG. 12.



Stresses, in Thousands of Pounds per Square Inch.

COMPARATIVE STRESSES AT DIAPHRAGM AND AT ADJACENT SECTION.

FIG. 13.

ment. Under compression, the tendency of the lattice bars is to push the ribs apart, and the restraining action of the diaphragm sets up bending stresses in the ribs, which are clearly indicated, as follows:

Applied load	9 240	lb. per sq. in.
Average measured stress at diaphragm	9 920	" " " "
" " " 10 in. away	9 120	" " " "
Applied load	15 530	" " " "
Average measured stress at diaphragm	17 630	" " " "
" " " 10 in. away	15 770	" " " "

Mr. MacKay.

All measurements were taken on the outside of the ribs. Although the diaphragm was designed so as to be fairly flexible, it increases the stress in the rib nearly 12 per cent. A shorter gauge length would possibly indicate higher bending stress, and a stiffer diaphragm would undoubtedly increase the effect. Linear interpolation between observations taken at the ends of a member would miss such tertiary effects altogether; and those interested in stress measurements cannot be too often reminded that a thorough study of the action of built-up members demands a complete survey of the whole member.

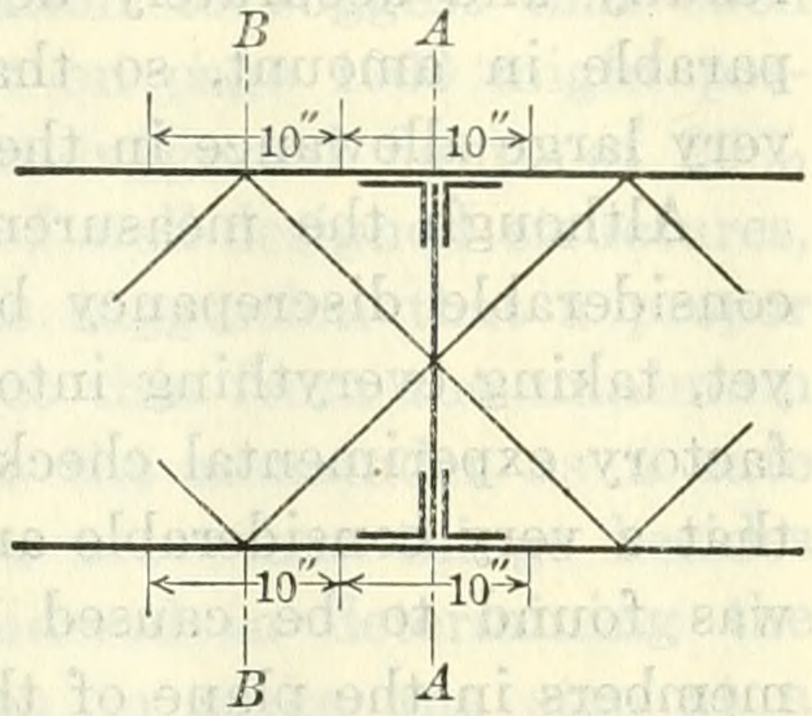


FIG. 14.

The writer cannot share the apprehensions of those who consider the Howard gauge too defective in principle to give good results. It is not easy to protect external holes for a long period on outside work. Any accidental blow in the vicinity of a point of observation may vitiate the results for comparative purposes; also, the writer does not know of any entirely satisfactory material for plugging the holes. Other proposed methods, however, would present similar difficulties. If the gauge is tilted, the contact is between an ellipse and a circle, and, provided that the points are conical, the maximum error is the difference between the semi-major and the semi-minor axes of the ellipse. With a 55° point and a hole 0.03 in. in diameter, a tilt of 5°, which is extravagant, would allow a maximum error of about 0.0001 in.

Mr. Turneaure.

F. E. TURNEAURE,* M. AM. SOC. C. E. (by letter).—The measurements on the Hell Gate Bridge are of great value, not only in connection with the determination of stresses in that structure, but also as an important contribution to our knowledge of the amount of secondary stress actually existing in structures. The writer does not believe that it will ever be necessary or desirable to calculate secondary stresses in the ordinary practice of bridge design; but he does believe that their consideration, both theoretically and experimentally, in such unusual structures as the one under consideration, is of very great importance. The more accurate our information may be concerning the various kinds of stresses which exist in a structure, the more safely and economically can such a structure be designed; and, although the extra margin of safety required to take care of uncalculated and indeterminate stresses may not be a very serious matter in structures of ordinary size, it becomes of extreme importance in those of very large size. Both safety and economy demand that as complete knowledge as possible should be obtained concerning all the

* Madison, Wis.

Mr. Turneaure.

elements involved. Secondary stress is one of those elements; and, although ordinarily not as large as the primary stress, and not as readily and accurately determined, it is, in many cases, quite comparable in amount, so that it cannot be ignored without making a very large allowance in the margin of safety.

Although the measurements made by Mr. Steinman show a very considerable discrepancy between the calculated and observed values, yet, taking everything into consideration, they constitute a very satisfactory experimental check of calculated results. Nor is it surprising that a very considerable amount of the secondary stress, as measured, was found to be caused by other factors than the bending of the members in the plane of the truss, which is that part of the secondary stress subject to calculation. In tests made by the writer, the same result was obtained in many cases, but where conditions were perfectly symmetrical, the experimental and calculated results agreed very well.

It does not seem to the writer that it is quite safe to draw conclusions relative to secondary stresses in ordinary trusses from the results on the Hell Gate Arch. It is apparent that the arch as designed is in a very favorable condition relative to secondary stresses when under either full dead or full live load. Under such load the main arch rib or lower chord receives its maximum stress, and, therefore, the significant secondary stresses in this member will be those occurring for this condition of loading; but, when the structure is thus loaded, the upper as well as the lower chord is in compression, and the web members are not very highly stressed. This condition results in comparatively low secondary stresses in the chords, much lower relatively than those which occur in an ordinary truss in which the lower chord is in tension and the upper chord is in compression. In the latter case there will evidently be greater distortion of joints than where the chords are subjected to the same kind of stress. As an illustration of a most favorable condition, consider the legs of an elevated water tank If these are made straight and are centrally loaded at each end, there will be practically no secondary stress except for wind pressure. It would appear, therefore, that the secondary stress to be anticipated in the main chord of the arch would be relatively small, as results of measurement as well as calculations show to be the fact. Calculations also show that, for a live load covering a half span, the secondary stresses reach fairly large values, but these are of no particular significance for the main chord, as the primary stresses for this condition are small. It will be noted on Plate XLVI that the calculated secondary stresses for this condition of loading reach a value as high as 2 000 lb. per sq. in., which is an indication of what may readily occur in other types of structures where the maximum primary stress occurs at the same time. It would seem,

Mr. Turneaure.

therefore, that no general conclusions can be drawn from these calculations and measurements which can be safely applied to other types. It probably was not Mr. Steinman's intention to suggest that such application could be made, but a statement on page 1063 might possibly be interpreted in this direction. His suggested figure of 25% + 4 000 would probably cover most cases of well-designed structures, but the writer would hardly agree with his suggestion that a proper way to treat the matter would be to deduct this from the minimum elastic limit of the material and then use the remainder as a safe working stress for bridge members. There are other things besides secondary stresses which need to be considered in determining the working stresses, and care should be taken to make one step at a time. If it is found that the secondary stresses in any particular structure can be represented safely by a particular percentage of the primary stresses or a percentage plus a constant, then we have arrived at a fair value of the maximum fiber stress, and are much better prepared to discuss the subject of working stresses than if no estimate of the secondary stresses had been made.

It is evident that secondary stresses are much more serious with respect to compression members than tension members, and the precautions taken in the construction of the Hell Gate Bridge to secure fairly concentric loading of segments should be effective in preventing any very extreme secondary stress. Assuming the pressures along the line of contact of the webs (one-third of their width) to vary as much as from zero at one edge to a maximum at the other, the greatest secondary stress which could be produced under such conditions would be approximately 35 per cent. It was to be expected that the actual stresses would be much less than this.

The great care taken in the design and erection of the Hell Gate Arch and the special precautions observed in other large structures of recent design show the increasing appreciation of the importance of scientific methods of design and accurate fabrication of important structures. Where such conditions prevail, the objection to the use of statically indeterminate structures, which has been so general in the past, loses its force, and there would seem to be no reason for excluding such forms of structure where, for other reasons, they are desirable. These are often called indeterminate structures, but such is not the case. They are simply statically indeterminate, and the calculation of the stresses requires a little more work than for structures that are statically determinate. The results of calculation, also, are not quite so accurate, as they are affected somewhat by temperature variations and inaccuracy of fabrication; but they have exactly the same degree of accuracy as the calculations of deflections of statically determinate structures, and it is well known that deflection calculations are very reliable.

Mr. Turneaure.

The same comments hold true, to a large extent, with reference to secondary stress, and such stresses are just as much a real part of the total stress which may prevail in any fiber of a member as the primary stress; and they are just as certain to exist as it is certain that the structure will deflect under load. The accuracy of their determination, however, is not so great as that of the primary stresses, on account of the indeterminate effect of temperature variations and of joint plates, rivets, and other details.

The very excellent and painstaking work represented by this paper should be an example of what can be done in the scientific treatment of important structures, and the value of the results secured should serve as a strong incentive to other engineers to conduct similar investigations.

Mr. Jacoby.

HENRY S. JACOBY,* ASSOC. AM. SOC. C. E. (by letter).—By having stress measurements made on the Hell Gate Arch Bridge during its erection, an important service has been rendered to the Engineering Profession. The design of bridges, as well as other structures, cannot be carried out with the highest regard for both security and economy without the aid of continuous scientific investigation. When observations are made on an actual structure, especially one in which the members are so large and where the resources of construction in equipment and workmanship are taxed to a much higher degree than usual, the results are of far greater value than laboratory experiments could possibly give.

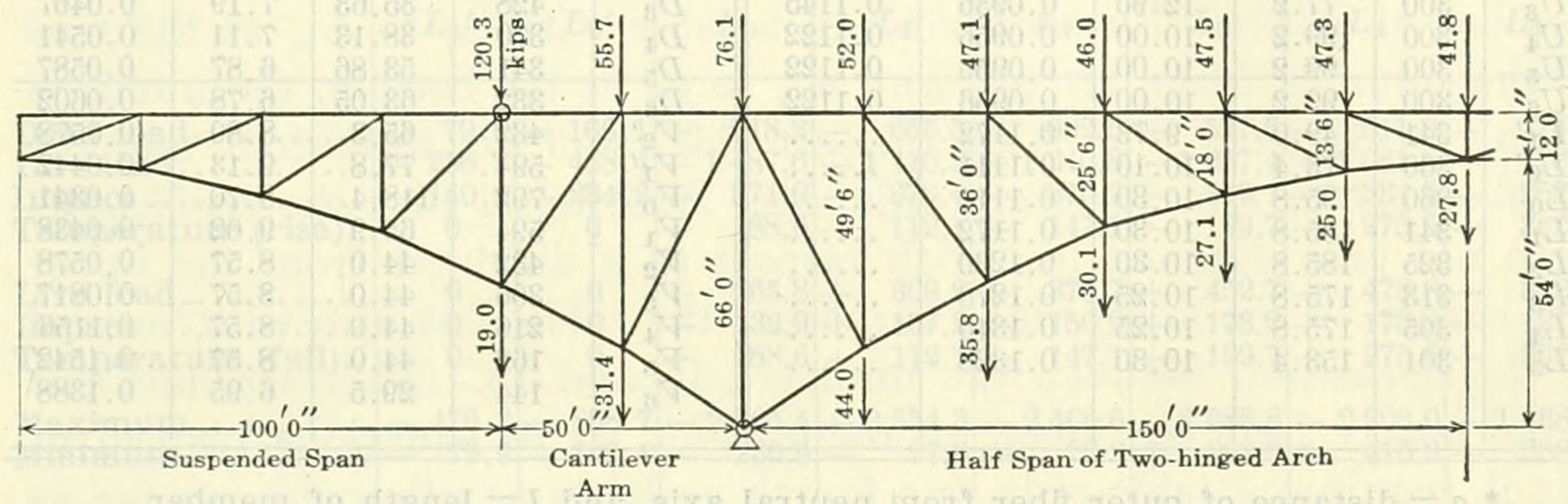
The Chief Engineer, Mr. Lindenthal, therefore merits the appreciation of every engineer who is actively interested in the progress of bridge design and construction, for deciding to take advantage of this unique opportunity. If a larger number of engineers in charge of construction were willing to combine a relatively small amount of scientific research with important works in construction, the rate of engineering progress would be materially increased. The value of the results thus secured, under wise direction, may be far greater than the extra financial outlay involved. The author deserves credit for the systematic form in which the paper is presented. It is gratifying to learn that the actual secondary stresses were found to be lower than the computed values.

As a part of this discussion it may be interesting to present the computed secondary stresses in a two-hinged, spandrel-braced arch with cantilever arms. The design of the arch and of the adjacent suspended spans, a critical discussion of various methods for determining the primary stresses, and the computation of the secondary stresses, were presented to the Faculty of the Graduate School of Cornell Uni-

* Ithaca, N. Y.

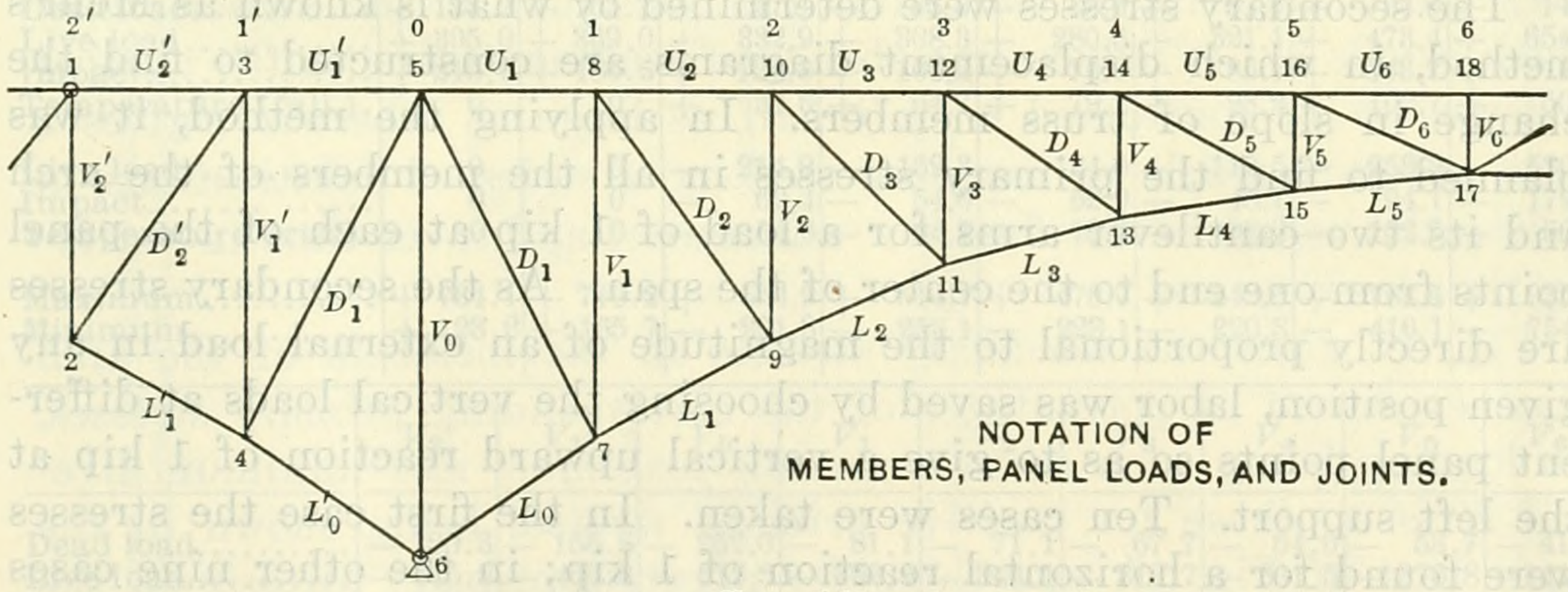
versity as a thesis for the degree of M. C. E. by Mr. Thomson Mao in June, 1917. Mr. Jacoby.

The general dimensions of the trusses, and the dead panel loads, expressed in kips or units of 1 000 lb., are shown in Fig. 15, and the notation used for the truss members, the live panel loads, and the joints, is shown in Fig. 16. The live panel loads on the right half of the arch are 7, 8, 9, 10, 11, and 12; those on the right cantilever are 11′ and 10′. Various properties of the truss members required in the computations are given in Table 7. The revised primary stresses in the members of the half arch and a cantilever arm are given in Table 8. The live panel load per truss is 141.6 kips, the panel load 2′ at the end



GENERAL DIMENSIONS, AND DEAD PANEL LOADS.

FIG. 15.



NOTATION OF MEMBERS, PANEL LOADS, AND JOINTS.

FIG. 16.

of the cantilever being 445.8 kips, of which 375.0 kips equals the reaction of the suspended span. The impact was computed by the formula, $I = \frac{300\,S}{(300 + L)}$, in which I is the impact stress, S is the live-load stress, and L is the loaded length producing the greatest live-load stress. The stresses due to temperature were computed for a range of $\pm$ 75° from the mean. The wind stresses were found to be less than 25% of the other stresses, except in the two upper chord members of the cantilever arm. These two members have a large excess of section, as the section of the upper chord is the same for five panels.

Mr. Jacoby.

TABLE 7.

Truss members.	Length, in inches.	Section area, in square inches.	Least radius of gyration, in inches.	Values of $\frac{2c^*}{l}$.	Values of $\frac{2c}{l}$.	Truss members.	Length, in inches.	Section area, in square inches.	Least radius of gyration, in inches.	Values of $\frac{2c}{l}$.
U_2'	300	77.2	12.90	0.0936†	0.1145‡	D_2'	526	51.37	6.95	0.0381
U_1'	300	77.2	12.90	0.0936	0.1145	D_1'	665	51.37	6.95	0.0301
U_1	300	77.2	12.90	0.0936	0.1145	D_1	665	38.13	7.11	0.0301
U_2	300	77.2	12.90	0.0936	0.1145	D_2	526	35.63	7.19	0.0381
U_3	300	77.2	12.90	0.0936	0.1145	D_3	428	35.63	7.19	0.0467
U_4	300	99.2	10.00	0.0956	0.1122	D_4	370	38.13	7.11	0.0541
U_5	300	99.2	10.00	0.0956	0.1122	D_5	341	53.86	6.87	0.0587
U_6	300	99.2	10.00	0.0956	0.1122	D_6	332	63.05	6.78	0.0602
L_1'	341	49.0	9.73	0.1172	.....	V_2'	432	65.3	8.80	0.0578
L_0'	360	73.4	10.10	0.1111	.....	V_1'	594	77.8	9.13	0.0442
L_0	360	195.8	10.30	0.1111	.....	V_0	792	118.4	8.70	0.0341
L_1	341	185.8	10.30	0.1172	.....	V_1	594	65.9	9.02	0.0438
L_2	325	185.8	10.30	0.1230	.....	V_2	432	44.0	8.57	0.0578
L_3	313	175.8	10.25	0.1278	.....	V_3	306	44.0	8.57	0.0817
L_4	305	175.8	10.25	0.1310	.....	V_4	216	44.0	8.57	0.1156
L_5	301	153.4	10.30	0.1328	.....	V_5	162	44.0	8.57	0.1542
						V_6	144	29.5	6.95	0.1388

* c = distance of outer fiber from neutral axis, and l = length of member.

† Upper side of member.

‡ Lower side of member.

The secondary stresses were determined by what is known as Mohr's method, in which displacement diagrams are constructed to find the change in slope of truss members. In applying the method, it was planned to find the primary stresses in all the members of the arch and its two cantilever arms for a load of 1 kip at each of the panel points from one end to the center of the span. As the secondary stresses are directly proportional to the magnitude of an external load in any given position, labor was saved by choosing the vertical loads at different panel points so as to give a vertical upward reaction of 1 kip at the left support. Ten cases were taken. In the first case the stresses were found for a horizontal reaction of 1 kip; in the other nine cases the stresses were found for vertical loads as follows: 1 kip for panel load No. 0; 2 kips for panel load No. 6; 2.4 kips for No. 7; 3 kips for No. 8; 4 kips for No. 9; 6 kips for No. 10; 12 kips at No. 11; 12 kips upward for No. 11′; and 6 kips upward for No. 10′. The stresses for a panel load of 1 kip at each panel point were found from the foregoing by combining the stresses due to corresponding vertical and horizontal reactions.

After the values of various terms required by Mohr's method for secondary stresses were obtained, the equations were formed for every joint of the truss for each case of loading. Accordingly, it was necessary to solve ten sets of thirty-four simultaneous equations. They

TABLE 8.—REVISED PRIMARY STRESSES.

	U_2'	U_1'	U_1	U_2	U_3	U_4	U_5	U_6
Dead load	0	+ 70.4	+ 117.3	+ 97.2	+ 73.8	+ 43.8	+ 13.3	0
Live load	0	+ 225.3	+ 489.1	+ 582.1	+ 652.4	+ 622.6	+ 406.6	+ 217.9
Impact	0	+ 122.7	+ 154.3	+ 183.1	+ 211.1	+ 208.5	+ 143.2	+ 72.6
Temperature (rise)	0	0	+ 24.6	+ 61.7	+ 117.4	+ 197.2	+ 288.0	+ 332.7
Live load	0	0	— 150.3	— 322.3	— 511.6	— 653.4	— 640.4	— 511.2
Impact	0	0	— 54.6	— 115.1	— 176.3	— 219.2	— 203.3	— 170.1
Temperature (fall)	0	0	— 24.6	— 61.7	— 117.4	— 197.2	— 288.0	— 332.7
Maximum	0	+ 418.4	+ 785.3	+ 924.1	+ 1 054.7	+ 1 072.1	— 1 118.4	— 1 014.0
Minimum	0	+ 70.4	— 112.2	— 401.9	— 731.5	— 1 026.0	+ 851.1	+ 623.2

	L_1'	L_0'	L_0	L_1	L_2	L_3	L_4	L_5
Dead load	— 79.9	— 166.1	— 718.2	— 655.5	— 602.5	— 557.6	— 511.3	— 474.7
Live load	— 256.2	— 468.4	— 1 487.6	— 1 410.3	— 1 340.8	— 1 257.4	— 1 081.8	— 778.9
Impact	— 140.2	— 234.2	— 371.0	— 376.4	— 376.2	— 369.1	— 334.3	— 252.3
Temperature (rise)	0	0	— 88.6	— 112.1	— 147.1	— 199.7	— 275.6	— 362.1
Live load	0	0	+ 265.8	+ 309.2	+ 376.7	+ 452.7	+ 478.8	+ 372.2
Impact	0	0	+ 132.9	+ 137.2	+ 150.9	+ 173.2	+ 173.8	+ 127.8
Temperature (fall)	0	0	+ 88.6	+ 112.1	+ 147.1	+ 199.7	+ 275.6	+ 362.1
Maximum	— 476.3	— 868.7	— 2 665.4	— 2 554.3	— 2 466.6	— 2 383.8	— 2 203.0	— 1 868.0
Minimum	— 79.9	— 166.1	— 230.9	— 97.0	+ 72.2	+ 268.0	+ 416.9	+ 388.4

	D_2'	D_1'	D_1	D_2	D_3	D_4	D_5	D_6
Dead load	+ 123.2	+ 165.7	+ 47.1	+ 35.5	+ 33.5	+ 37.1	+ 34.6	+ 14.9
Live load	+ 395.0	+ 369.0	+ 332.9	+ 308.3	+ 280.6	+ 321.1	+ 473.4	+ 654.8
Impact	+ 215.9	+ 184.5	+ 120.3	+ 106.2	+ 76.9	+ 86.1	+ 143.7	+ 215.4
Temperature (fall.)	0	0	+ 54.6	+ 64.7	+ 79.7	+ 98.2	+ 102.7	+ 50.2
Live load	0	0	— 214.9	— 169.3	— 124.0	— 110.5	— 253.9	— 530.1
Impact	0	0	— 69.4	— 54.6	— 52.9	— 48.7	— 94.1	— 179.0
Temperature (rise)	0	0	— 54.6	— 64.7	— 79.7	— 98.2	— 102.7	— 50.2
Maximum	+ 734.1	+ 719.2	+ 554.9	+ 514.7	+ 470.7	+ 542.5	+ 754.4	+ 935.3
Minimum	+ 123.2	+ 165.7	— 291.8	— 253.1	— 223.1	— 220.3	— 416.1	— 754.4

	V_2'	V_1'	V_0	V_1	V_2	V_3	V_4	V_5	V_6
Dead load	— 120.3	— 156.8	— 252.0	— 81.1	— 71.1	— 67.7	— 64.0	— 53.7	— 41.8
Live load	— 445.8	— 465 6	— 613.3	— 392.9	— 346.1	— 312.7	— 331.5	— 376.8	—141.6
Impact	— 243.2	— 232.8	— 204.3	— 133.2	— 94.2	— 75.2	— 95.9	— 115.2	— 73.2
Temperature (fall)	0	0	— 48.8	— 53.2	— 56.9	— 57.4	— 48.8	— 21.6	0
Live load	0	0	+ 166.5	+ 137.7	+ 82.5	+ 47.9	+ 93.8	+ 189.7	0
Impact	0	0	+ 55.4	+ 67.3	+ 40.7	+ 26.1	+ 37.1	+ 69.5	0
Temperature (rise)	0	0	+ 48.8	+ 53.2	+ 56.9	+ 57.4	+ 48.8	+ 21.6	0
Maximum	— 809.3	— 855.2	—1 118.4	— 660.4	— 568.3	— 513.0	— 540.2	— 567.3	—256.6
Minimum	— 120.3	— 156.8	+ 18.7	+ 177.1	+ 109.0	+ 63.7	+ 115.7	+ 227.1	— 41.8

NOTE.—All stresses are expressed in kips, or units of 1 000 lb.

were solved by the method of Gauss, as they have the same form as the normal equations in geodesy. This fact was first noted by Mr. José Páez, a graduate student at Cornell University in 1912-13, who combined a minor in geodetic engineering with his major subject in bridge

Mr. Jacoby.

engineering. Mr. Páez wrote a thesis on secondary stresses in which he made a critical comparison of all known methods for their computation, and applied each of them to the same truss. He also discovered an additional approximate method. By the method of Gauss, the solution of the equations becomes quite mechanical, and may be checked at every step of the process. This method of solution, therefore, has a great advantage over any other.

Mr. Mao made an additional simplification of the solution by numbering the joints in the left half of the truss as shown in Fig. 16, and numbering those in the right half of the truss so that the sum of the numbers for two symmetrical joints is 35, or one more than the total number of joints. By arranging in a table with ϕ_1 to ϕ_{34} at the tops of the columns from left to right, ϕ_{34} to ϕ_1 at the bottom, the numbers of the joints from 1 to 34 on the left side from top to bottom, and the numbers 34 to 1 at the right side, Mr. Mao found that the coefficients were not only symmetrical with respect to a diagonal from the upper left corner to the lower right corner of the table, but also with respect to the horizontal between the numbers 17 and 18. This arrangement saves nearly half the labor by reducing it to 18 equations instead of 34; it requires a table only one-fourth as large as otherwise, so far as coefficients only are concerned; makes a further saving in work in substitutions after some of the values are found, as one substitution gives the coefficients of two unknowns, the sum of the subscripts of which equals 35; it reduces the chance of making errors, as the number of terms to be considered is only half as large as otherwise; gives a more accurate result by avoiding so long a series of substitutions in finding the values of the first few unknowns; and saves time in verifying the check terms.

Fig. 17 gives the secondary stresses in the members of the arch truss, expressed as percentages of the corresponding primary stresses, the loading being such as to cause the maximum and minimum live-load stresses, those for positive primary stresses being shown in the upper part of Fig. 17, and those for negative primary stresses in the lower part. The secondary stresses are for the top and bottom fibers of the upper-chord members, for the top fibers of the diagonals and lower-chord members, and for the outer fibers toward the center of the span in the verticals.

As the members of the cantilever arm have primary stresses due only to one or two live panel loads on the cantilever arm, and the secondary stresses are caused by panel loads over the entire length of the truss, it is necessary to give two sets of percentages for these members. One of these is the percentage of the primary stress for the maximum secondary stress in tension, and the other for that in compression. As U_2' has no primary stress due to vertical loads, the values

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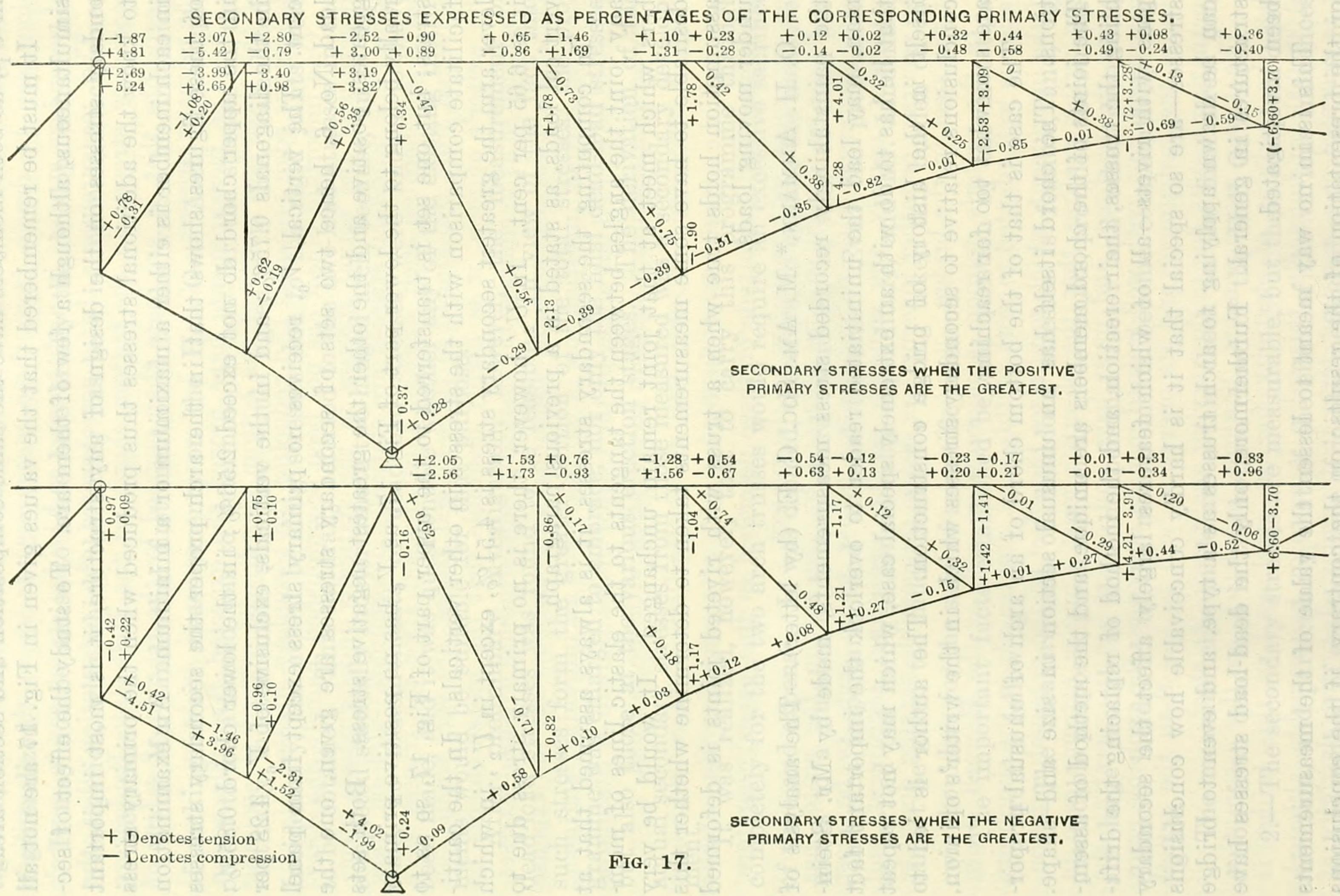

FIG. 17.

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for its secondary stresses are given as percentages of the primary stress in U_1', as both members have the same composition and section area.

It must be remembered that the values given in Fig. 17 are not all simultaneous, although a few of them are. To study the effect of secondary stresses on the design of any structure, it is most important to know the additional stresses thus produced when the primary stress in each member is either a maximum or a minimum. An examination of these figures shows that in the arch proper the secondary stresses in the upper chord do not exceed 2.56%; in the lower chord 0.85%; in the diagonals 0.75%; and in the verticals, exclusive of V_6, 4.28 per cent. The vertical, V_6, receives no primary stress except from panel load No. 6, hence two sets of secondary stresses are given, one the greatest positive and the other the greatest negative stress. Both sets really belong to the lower part of Fig. 17, as V_6 has no positive primary stress; but one set is transferred to the upper part of Fig. 17, so as to facilitate comparison with the stresses in other verticals. In the cantilever arm the greatest secondary stress is 4.51%, except in U_2', in which it is 6.65 per cent. In U_2' however, there is no primary stress due to vertical loads, as stated in a previous paragraph.

In computing the secondary stresses, it is always assumed that at any joint the angles between the tangents to the elastic lines of members which meet at that joint remain unchanged. It would be very desirable to have some measurements taken to determine whether this assumption holds true when a truss with riveted joints is deformed under moving loads.

Mr. Ammann.

O. H. AMMANN,* M. AM. SOC. C. E. (by letter).—The analysis of the painstakingly recorded stress measurements, made by Mr. Steinman, may lead the uninitiated reader to overlook the important fact that he has to do with an extremely special case, which may not repeat itself in the history of bridge construction. The author is led to conclusions relative to secondary stresses which, in the writer's opinion, are somewhat too far reaching.

The case is that of the bottom chord of an arch of unusual proportions. The chord itself has an unusual section in size and shape. The joints of the chord members are unique, and the method of assembling the trusses, their erection, and the method of replacing the drift-pins with rivets—all of which features largely affect the secondary stresses—are so special that it is hardly conceivable how conclusions can be drawn applying to arch trusses as a type, and even to bridge structures in general. Furthermore, only the dead-load stresses have been investigated.

This is in no way meant to lessen the value of the measurements or the presentation of the results; on the contrary, if the conclusions

* South Amboy, N. J.

drawn by the author are taken to apply strictly to the case under investigation, or to very similar cases, they are infinitely more valuable than if generalized. Mr. Ammann.

It may not be amiss to state here briefly the development of this investigation, with which the writer has been in intimate touch since its inception.

When Mr. Lindenthal first mentioned to the writer his intention of making strain measurements on the Hell Gate Arch, the simple object to be attained was to determine in how far the stresses in the statically indeterminate structure would agree with those calculated. Incidentally, the actual bending stresses due to the rigid joints were to be obtained. This naturally referred to the live-load stresses in the completed statically indeterminate two-hinged arch, because, up to that time, no measurements of dead-load stresses were known to have been made, and it was questionable whether any of the existing instruments would be suitable for that purpose.

After preliminary investigation, however, the measurement of dead-load stresses was found to be feasible. In view of the fact that such stresses form the greater portion of the total stresses, the programme was enlarged to embrace, not only the final dead-load stresses, but also those at the various erection stages. The derivation of the actual secondary stresses from those measured was a secondary development.

It was realized, however, that to carry this programme through completely for the two arch trusses would require an excessive sacrifice of time and expense. It was decided, therefore, to confine the measurements to a number of bottom chord members, in view of their preponderant importance and unusual features. Even the programme as carried out involved considerable expenditure, and Mr. Lindenthal cannot be given too much credit for having taken this burden on his own shoulders for the sake of scientific research.

The important facts established beyond question by the investigation are the following:

1.—The measurement of dead-load stresses is feasible, and, for practicable purposes, gives sufficiently close results, provided the stresses are large enough so that the personal factor and other disturbing elements incidental to the measurements become comparatively negligible.

The measurements made are a fair check on the final stress condition from dead load in the bottom chord of the Hell Gate Arch.

2.—The secondary stresses are measurable, but their complete determination, and particularly that of the effect of the various influences, such as method of erection, type of joints, method of replacing drift-pins by rivets, etc., requires far more numerous measurements than have

Mr. Ammann.

been feasible without undue cost in the case of such a large structure as the Hell Gate Bridge. For this reason only hypothetical conclusions can be drawn from these measurements as to the magnitude of secondary stresses in arch trusses and bridge structures in general.

The measurements have corroborated the expectation that the secondary stresses in the bottom chords of the Hell Gate Bridge are negligible, that is, are more than covered by the margin of safety of the primary stresses.

3.—The measurements have proved the expected favorable action of the three-face joints of the bottom chord, with regard to avoiding dangerous edge pressures and reducing the secondary stresses.

One important object has not been accomplished, namely, the determination of the actual stresses in the statically indeterminate structure. The dead-load stresses are statically determinate—at least, very nearly so—because such stresses, superimposed after the trusses were converted from two-hinged to three-hinged arches, are very small. In the writer's opinion, it is highly desirable that the measurements be continued so as to embrace the statically indeterminate live-load stresses. They would furnish valuable additional information. As the expense for such further investigation is too heavy for an individual engineer, it should be carried out either by the United States Bureau of Standards or by an Engineering Society in co-operation with the railroad.

Mr. Randorf.

C. A. RANDORF,* M. AM. SOC. C. E. (by letter).—Few engineers have had the experience or the opportunity of conducting tests of the importance and magnitude of those described in this paper; therefore, few can speak with authority on the subject.

In reviewing the results obtained by the author, the writer has been impressed with regard to two points which seem to be most important, namely, the accuracy of the measuring instrument, and the accuracy or rather inaccuracies due to the possible varying ductility of the steel in the bridge.

In the writer's opinion, a micrometer such as the Howard strain gauge is well adapted to work of this kind, due mainly to the simplicity of the instrument, which minimizes the effect of the personal equation. The writer realizes that the observer must exercise extreme care in taking the readings, and must have had experience in operating the instrument, but, with well-trained men as observers, the probable error due to the instrument and its operation, will be less than the actual fluctuating results due to the varying qualities in the steel.

The stress measurements, however, are based on the assumption that the modulus of elasticity for the steel used in this bridge is 30 000 000; undoubtedly, this value is more or less variable, although it is accepted as a constant term in formulas where the elasticity of

* Buffalo, N. Y.

the material is concerned; but, when stresses are computed from actual strain measurements, the modulus of elasticity of the material should be verified by special tests, if possible, in order to ascertain its extreme variations. It is evident that this cannot be done after the structure to be tested has been erected; but it can be done when the regular tests are being made in the laboratory. Mr. Randorf.

The writer refers to the paper by O. H. Ammann, M. Am. Soc. C. E., giving an account of the design and construction of Hell Gate Bridge,* in which is given a résumé of the chemical analysis of the steel used, indicating a variation in the carbon of from 0.27 to 0.34 per cent. This would have a direct bearing on the ratio of the unit elongation or strain to the unit stress, in other words, the modulus of elasticity.

A certain amount of variation or segregation of the carbon will occur in the best commercial steels. Manufacturers have adopted methods to eliminate as far as possible the unequal distribution of carbon and other impurities, but the art of making steel has not as yet become perfect.

With these facts in mind, however, it would seem reasonable to assume that the author did arrive at quite accurate results, inasmuch as the data are based on the averages of a great many extensometer readings.

The writer believes that the data offered in this paper are of inestimable value to the Engineering Profession and well worth careful study by all engineers interested in bridge construction.

David A. Molitor,† M. Am. Soc. C. E. (by letter).—The results of stress measurements presented in this paper certainly merit the highest appreciation from the Engineering Profession. Special thanks are due to Gustav Lindenthal, M. Am. Soc. C. E., for his keen forethought and insistence on proving the correctness of the theoretic stress computations in this monumental structure. Mr. Molitor.

These stress measurements undoubtedly possess great scientific value, and, from the engineering viewpoint, it would seem that their main purpose should consist in substantiating the correctness of modern methods of stress analysis, regardless of whether these are simple or complex, so long as the results of stress computations can be accepted as sufficiently reliable to warrant their adoption as the basis for safe designs. In the event that stress measurements could be shown to invalidate the results obtained from modern methods of analysis, the dictates of conscience should prompt us to search farther for truths not yet discovered.

The author, on page 1047, states: "The final object of the extensometer measurements was to provide a comparison between the calculated

* *Transactions*, Am. Soc. C. E., Vol. LXXXII, p. 852.

† Detroit, Mich.

Mr. Molitor.

and the actual secondary stresses in the structure." He then mentions the peculiarities in the design and methods of erection and riveting, concluding with the statement: "These features, separately and combined, modify the secondary stresses materially, and render it extremely difficult, if not impossible, to arrive at the true secondary stresses by calculation".

The writer did not find any reference in the paper which would explain how any of these features, especially the three-faced butt-joints, were considered in the secondary stress computations; yet on this point hinges the whole question of how close an agreement could be expected between actual and computed secondary stresses.

These features, as described by the author, and applied to a bridge of this type (two- or three-hinged braced arch), would necessarily result in very small secondary stresses. Therefore, it is most gratifying to know that the good judgment of the designer was verified by the stress measurements which gave secondary stresses averaging about two-thirds of those computed. However, the 50% excess of the computed over the measured secondary stresses may have been due to the manner of computation, if the three-faced butt-joints were not properly considered.

In order to prove or disprove the correctness of the more accurate methods of secondary stress computations, a more suitable structure would have been a simple truss with subdivided panels, wherein the secondary stresses usually attain high values. If the computed values were found to be in close agreement with the measured stresses for such a structure, a fact which is only partly substantiated at present, it would be safe to conclude that the methods are reliable for bridges of all types. To draw such a conclusion from the facts presented in the paper, however, is unwarranted, and is apt to minimize the importance of the question of secondary stresses in ordinary bridges.

The author, on page 1063, in attempting to establish conclusions for guidance in specifying extreme working stresses for bridge members, offers a suggestion which no doubt would meet with great favor among those who are willing to take a chance on the "factor of ignorance" rather than familiarize themselves with modern methods, which are often avoided on the pretext that they are too complicated and laborious. The author states "that a rule of this form can properly apply only to structures in which the secondary stresses have normal or average values", but fails to give any criterion by which to recognize such structures; neither does he convey any good idea of what constitutes a normal value.

The writer's practice is to ascertain the maximum total stress due to dead-live-impact loads, plus secondary stress and temperature effect,

if any, and then design the member for the allowable unit stress. By making proper allowances for live-load stress reversals, he aims to produce a design of uniform strength throughout, for 100% over-load in live plus impact loads, without exceeding the elastic limit. Mr. Molitor.

The ordinary method of analysis, used by the author, involving the solution of a series of simultaneous equations equal in number to the total number of panel points in the span, for each case of loading, is admittedly laborious, and might have been somewhat simplified.

By referring to any treatise on secondary stresses, it is apparent that the theoretic analysis of the problem is quite simple, although the numerical work is laborious, on account of the solution of the final set of simultaneous equations, and, as stated by the author, "the expediting of the entire computation depends largely on the method selected for solving these equations". The method of Gauss is often used to obtain a direct algebraic solution, though many simplifications have been suggested to minimize the labor involved by this method. The method of successive approximations, used by the author, has some merit, though much labor is still required, especially when the process is carried to the third approximate values.

In view of these facts, it would appear that a method of computing secondary stresses which avoids the laborious solution of a set of simultaneous equations would commend itself to bridge specialists and students of this subject.

Such a method has been given, in the chapter on secondary stresses, in a treatise* by the writer, but seems to have entirely escaped the attention of the author as well as others who have dealt with this subject.

It would be transgressing the rules of the Society to quote this analysis here, and as it is treated very exhaustively and illustrated by a complete problem, the reader is referred to the original text. A very much simplified method of solving symmetric equations—known in mathematics as equations of Clapeyron—is also given on page 294 of this treatise, where the solution was effected with a 10-in. slide-rule. This method was also applied by the writer in solving normal equations, up to fourteen in number, and with the same slide-rule, as early as 1907; hence, the discovery by Mr. José Páez, announced by Professor Jacoby in his discussion of this paper, was antedated by at least 5 years, and possibly by more than 40 years, as Mohr's work equations, when applied to many redundants, become identical with normal equations.

The agreement between the calculated and measured values of the secondary stresses in the Hell Gate Arch does not appear to be very satisfactory, from a scientific standpoint, though the disparity is not serious, because these stresses are never excessive in a bridge of this

* "Kinetic Theory of Engineering Structures", Chapter XIII, 1911.

Mr. Molitor.

type, especially when extraordinary conditions are introduced for the express purpose of reducing them still further. The author, on page 1065, evaluates the relation between the measured and calculated secondary stresses in the form of an approximate empiric statement according to which the percentage of measured stress equals half the percentage of the calculated stress, plus 15, and adds that the factor, $\frac{1}{2}$, would probably have a higher value in other structures. Accordingly, one or the other value, attributed to the secondary stress in any member, must be radically wrong. That is to say, either the computed secondary stresses are much too high, owing to some error in theory or its application to the three-faced butt-joints, or else the measured stresses are too low, for some reasons not discernible from the data presented.

On Plate XLVII, the extreme fiber distance for the lower chord is given as 63.6 in. for Member 0-2, with diminishing values to 41.9 in. at Point 22. These are in accordance with Fig. 4, without any allowance for the three-faced joints, and, if these distances were used in computing the fiber stresses from the bending moments, Navier's law has been violated, and the resulting stresses may easily be 50% too high for practically all cases of loading, except the maximum dead-live-impact load, for which no stress measurements were made.

On the other hand, the location of the extensometer points adjacent to the ends of a member may, or may not, give maximum fiber stresses; and as there is only one point at each end of each member for which the fiber stress can be a maximum, there is great likelihood of missing this point in locating the points for measurements, and hence the chances for obtaining consistently low measured fiber stresses are very great. For the direct stresses, this danger is not encountered.

The fundamental equations for computing secondary stresses are expressions representing the relations between the unit direct stresses and the angle distortions resulting from these stresses, for every triangle of the frame. The angle distortions are thus derived on the assumption of frictionless pin connections between all members. These angle distortions, thus found, are then assumed to represent the bending effects in the members which are prevented from taking place by virtue of rigid riveted connections at the ends or joints. Hence, as the material, even at the joints, is elastic, and not rigid, some elastic distortion will take place in the apex angles of every triangle, the extent of which will depend on the quantity of metal and riveting used in the connecting plates, and, whatever this may be, its amount will be lost in the production of bending stresses in the members themselves. For very stiff joints, or heavy details, this will be a small quantity, representing an error on the side of safety in computing the secondary stresses. Therefore, the computed secondary stresses must always be

slightly in excess of the actual values, but the error is relatively small and not in a class with the findings recorded in the paper. Mr. Molitor.

The actual maximum bending stress could hardly occur at the extreme end of a member, but would be developed at some distance back, near the edge of the connecting plate, and that point would have to be located very accurately in order that the measured fiber stresses may be accepted as a true measure of the maximum bending stress. This, again, does not apply in the same way to the direct stress.

Hence, it is quite evident that some disparity will always exist between computed and measured secondary stresses, without casting any serious reflection on the accuracy of either value, unless the doubt can be located definitely.

In conclusion, the writer expresses his sincere appreciation of the author's contribution to this subject, and trusts that the remarks presented herewith may be accepted in the sense of constructive criticism.

D. B. Steinman,* M. Am. Soc. C. E. (by letter).—The writer wishes to take this opportunity of acknowledging his cordial appreciation of the courtesy and interest of all those who have discussed his paper. The number, range, and significance of the discussions have been greater than were anticipated for a subject in so new a field, and the interest evinced by so many high authorities in the Profession has been gratifying. Mr. Steinman.

In eliciting this series of valued contributions of data and thought to the science of structural stresses, the writer feels that his labors in conducting the investigations under consideration, and in analyzing and reporting them, have found their full justification and reward.

The various discussions will be taken up in the order in which they were received by the writer.

Mr. Powell, in commenting (page 1077) on the precision of the extensometer measurements, reports that he has found it difficult in his experience with gauges to take readings correct to a tenth of a thousandth of an inch.

The writer does not claim that this degree of precision was secured, nor does he wish to minimize the difficulty of even approximating it. Throughout the work, a number of readings were taken for each gauge measurement until the results appeared to be consistent within two or three ten-thousandths of an inch, and then the average of the last three or four readings was recorded. The proof of the accuracy secured by this procedure, using the instrument and method of manipulation described in the paper, does not lie in abstract considerations, but in the actual comparison of measurements at the two ends of

* New York City.

Mr. Steinman.

each member, or of observations made on the same gauge points on different days.

Mr. Powell regards as objectionable features the conical measuring points of the extensometer and the conical holes for the insertion of these points. It is true that these would give rise to error if the instrument were not held square to the holes, but this is a precaution which is easily observed; moreover, as pointed out by Professor MacKay (page 1097), a slight deviation from the perpendicular produces an entirely negligible error.

The question of getting a satisfactory micrometer "feel" with conical points bearing against the inclined sides of the holes, and of the difficulty of securing a thorough cleaning out of the holes, are also raised by Mr. Powell; but these and other difficulties were fully recognized by the writer and his assistants, and necessitated extreme care in every phase of the work. It may be stated that the cleaning of the holes before each measurement usually consumed more time than the actual taking of the readings; but, with conscientious and practised observers, using every possible precaution, the effects of the above-mentioned difficulties were minimized, and reliable measurements were secured.

If any more accurate instrument were known, or any better method of preparing and taking care of the gauge points, the writer would have been glad to adopt it for the Hell Gate Arch observations. The suggestion made by Mr. Powell to use slightly projecting ball-ended cylindrical studs, instead of the conical holes, with an ordinary micrometer replacing the Howard instrument, is certainly worth trying; but it appears doubtful whether it would yield sufficient increase in accuracy to justify the greater labor required in preparing the gauge points. Moreover, as Mr. Howard suggests (page 1086), the use of drilled holes better enables the gauge lengths to be preserved and protected.

The writer is grateful to Dr. Waddell for his kind remarks on the value and importance of the measurements; and heartily endorses his recommendation that the experiments on the Hell Gate Arch should be completed, so as to include the effects of live loads.

Mr. Frankland suggests an interesting programme of stress investigations, to embrace measurements on the Quebec and the Sciotoville Bridges, and the completion of the tests on the Hell Gate Arch. The undertaking of this work by the American Society of Civil Engineers, under the supervision of a special committee, would certainly extend our knowledge of stress conditions, and develop improvements in both design and construction.

Professor Waterbury proposes an interesting theory to account for the anomalous results found by the writer in some of the members of

the final stage, where the upper end uniformly presented a larger average stress than the lower end. Professor Waterbury's explanation of the possible influence of the order in which the drift-pins may have been replaced by rivets is very lucid, and is one of the theories which the writer had in mind when he wrote: Mr. Steinman.

"This anomalous result appears to arise from some unexplained disturbance of stress distribution, and does not represent any error of observation."

If Professor Waterbury's theory is correct, there still remains to be explained why the effect described was confined to the members near the end of the span, and why the larger resulting stress was invariably found at the upper end in each of these members.

The point brought out by Professor Waterbury, that the correct average stress at a cross-section can be obtained only by weighting each stress reading in proportion to the area of metal it represents, was recognized by the writer in his early studies. At one time, as a result of a comparison of the respective areas represented by the middle and the corner gauge points, the rule was tried of giving the middle readings twice the weight; but this did not yield as consistent results as the method which was finally adopted of simply averaging the six readings at each cross-section.

Mr. Hughes submits a comparison of the two extremes which he has observed in bridge design practice, *viz.*, excessive and sometimes meaningless precision in computation of stresses and in proportioning of members, on the one hand, as contrasted with slipshod and skimpy detailing of members, in violation of the fundamental principles of safety, on the other. No one will take exception to the recommendation that the most acceptable practice lies between these two extremes.

When Mr. Hughes advises the measurement of stresses on all structures in which there may be uncertainty of condition as a result of erection operations or eccentric loading, he is recommending something highly desirable but probably impracticable. The work of taking stress readings is expensive, and requires carefully selected and well-trained observers.

The writer is indebted to Dr. Fuller for his able and comprehensive discussion, and regards the latter's close study of the paper and his careful scrutiny of the results and conclusions as a high compliment. This contribution eminently succeeds in the object suggested in its opening paragraphs as a guiding purpose for such discussions, namely to "throw even greater light on the subject, and to remove the last shadow of doubt in regard to the interpretation that may be drawn."

Dr. Fuller refers to the writer's statement that measured secondary stresses will usually be lower than computed ones. The only exceptions

Mr. Steinman.

to this rule arise when there are special circumstances in the fabrication or erection of the structure which tend to increase the secondary stresses in a manner not provided for in the computations. An example of such augmentative factor, in the case of the Hell Gate Bridge, was the cantilever method of erection; this unavoidably gave rise to large secondary stresses, which persisted after the structure passed from the cantilever to the three-hinged condition. There was no way of computing the effect of this transition in advance of actual test, consequently the secondary stresses in Stage 3-*H* exceeded the calculated values. This fact was frankly displayed in Table 5 and in the graph on Fig. 5, and the explanation was suggested in the text.

Consequently, when Dr. Fuller points out that wherever the primary stresses exceeded 3 000 lb. per sq. in. the measured secondaries were greater than the computed values, he is not upsetting the writer's conclusions in this regard; he is focussing attention on what is merely an example of the qualifying circumstances outlined in the foregoing paragraph; for it happens that all stresses exceeding 3 000 lb. per sq. in. occurred in Stages 3-*H* and 2-*H*; and in these stages the special conditions just described came into play.

In the more common methods of truss erection, there is no abrupt transition from one structural form to another, like that involved in the cantilever erection of the arch; consequently, in such cases, there will be nothing to invalidate the correctness of the calculation of secondary stresses. This may be regarded as the normal or usual condition obtaining; the final stages of the Hell Gate Arch erection represent an unusual or exceptional condition. With these considerations, the writer does not see any reason for renouncing his belief that the measured secondaries will normally be smaller than the calculated values.

The fact that this relation was fulfilled by the measurements taken in all the eleven erection stages preceding the closure of the arch, and that the exceptions are found only in Stages 3-*H* and 2-*H*, would seem to be a convincing corroboration of the writer's thesis.

If there were a satisfactory method of providing in the computations for the complicating effect of the transformation from cantilever to arch, and for any similar cause of augmented secondary stresses, then there would be no need of appending any qualification to the writer's conclusion as to the relative values of measured and calculated secondaries.

As a consequence of the above-described effect of the cantilever method of erection, the final stresses in the extreme fibers of the Hell Gate Arch chord members would have been seriously larger than the computed values, were it not for the provision of the three-faced joints; and this measure would have been of little value without the additional

precaution which was adopted, of postponing the riveting until after the arch was closed. Mr. Steinman.

If the joints had been riveted as the erection proceeded, instead of waiting for the closure of the span, it would have been possible to calculate the secondary stresses more definitely; the complicating effect of the transformation from cantilever to arch would have been obviated, and the final measured secondaries, in all probability, would not have exceeded the computed values. The only question is whether these advantages would have outweighed those which were secured by the use of the three-faced joints in conjunction with deferred riveting.

In connection with the points just discussed, Dr. Fuller asks for the results of the measurements of secondary stresses in Stage 3-*H*, and the writer is glad to supply these values in Table 9.

TABLE 9.—STAGE 3-*H*.

(All values are in pounds per square inch.)

Member.	Calculated S. S.	Measured S. S.	Member.	Calculated S. S.	Measured S. S.	Member.	Calculated S. S.	Measured S. S.
0–2	104	238	8–10	264	1 125	16–18	710	994
2–0	445	813	10–8	395	925	18–16	897	844
2–4	319	388	10–12	425	688	18–20	1 079	1 050
4–2	104	738	12–10	565	738	20–18	959	1 182
4–6	8	788	12–14	613	513	20–22	1 124	1 482
6–4	324	1 013	14–12	728	1 013	22–20	935	750
6–8	328	463	14–16	752	1 200	22–22 A	1 187	900
8–6	200	713	16–14	660	850	22A–22	258	563
Average							557	832

The averages shown at the foot of Table 9 are the values which were included in Table 5. It should be noted that the measured secondary stresses in Stage 3-*H* averaged 50% higher than the calculated values, as a consequence of the conditions discussed in the preceding paragraphs.

The erratic character of the individual results in this stage is to be attributed to the fact that the calculated and measured values do not refer to identical conditions of the structure. The calculated secondaries were obtained for the completed stage, with the entire floor erected, whereas the measured values were secured during early and varying stages of the floor erection. In consequence of the rapid progress made in this erection stage, as compared with the slow operation of taking the observations, the different measurements were obtained

Mr. Steinman.

with changing positions of the erecting travelers and with varying quantities of floor system completed. Without the excessive labor of computing secondary stresses for so many different conditions of dead load on the structure, a proper comparison of calculated and measured values could not be submitted; for this reason, these percentages were omitted from the tabulation on Plate XLIV.

Dr. Fuller calls attention to the low percentages of computed secondary stress tabulated on Plate XLVII as contrasted with the higher values of the measured secondaries in the same stage (2-*H*) given on Plate XLIV. There really was no need of going to two different plates to find this relation, as it is frankly exhibited in direct comparisons in Plate XLIV. It is also indicated by the upper branch of the graph of measured secondaries on Fig. 5. It should be remembered, however, that these values of the measured stresses correspond to the usual theoretical definition of secondaries, as one-half the difference between the top and bottom fiber stresses; whereas, in this instance, on account of the effect of the three-faced joints, a more suitable definition of the secondaries is the excess of extreme fiber stress over the average stress in the section.

Ordinarily, these two definitions of secondary stress are equivalent; in the Hell Gate Arch chord members, however, the distinction is important, and it is evident that the second definition is the one of greater practical significance. In Fig. 5, the two methods of defining the measured secondaries are represented by the two branches of the graph for Stage 2-*H*; the divergence of these two branches shows the benefit secured by the use of three-faced joints, namely, to reduce the measured secondaries from an average of 220% down to an average of 58% of the calculated values.

The question is raised by Dr. Fuller as to the possibility of variation of stresses within the 20-in. gauge length that was used, and he suggests that smaller lengths are needed to catch the maximum values. This may be the case in short-span reinforced concrete beams and slabs, where, on account of the rapid drop in maximum stresses from the points of connection of members, it is found desirable even to reduce the customary 8-in. gauge length to 2 in. In the case of the 45-ft. members of the Hell Gate Arch, however, the variation of stress in a 20-in. length, at the points where the measurements were taken, was practically negligible, and the use of a shorter instrument would offer no advantage to compensate for the reduced accuracy of the results.

Investigators whose experience in strain measurements has been on concrete structures should remember that, on account of the difference in elastic coefficients, extensometer observations on steel require about fifteen times as great accuracy in order to secure the same precision in the values of the stresses; consequently, a longer gauge line is necessary in work on steel structures. Furthermore, in reinforced concrete spans,

the flexural stress declines rapidly, along a parabolic curve, from the end connection to the point of contraflexure; whereas, in steel chord members, the secondary stress has a linear variation which is very gradual in comparison. Mr. Steinman.

For the Hell Gate Arch investigations, both 10-in. and 20-in. extensometers had been provided; but the considerations just outlined prompted the writer to adopt the 20-in. instrument for the measurements on the main members, and to reserve the 10-in. instruments for observations on gusset-plates and on other connection details. It is regretted, however, that lack of time made it impossible to go very far with the latter measurements, or to take any observations on the distribution of stress between plates and angles composing a section.

The suggestion is made by Dr. Fuller that a Berry instrument might have given more satisfactory results than the Howard strain gauge, but he confesses that he has never used the latter. In the Department of Bridges, of New York City, the Howard instrument was adopted for all stress investigations, after consideration and trial of other types. With the Howard strain gauge, accurate results are secured by repeating the readings until they prove consistent and then recording the average. Although the Berry extensometer may permit quicker readings, it is doubtful whether the same accuracy is afforded, as a larger number of moving parts are involved. The Berry gauge gives all desired accuracy for getting stresses in concrete, on account of the lower value of the modulus of elasticity; and it has its advantage of speed for observations under changing load; but, for the work which was done on the Hell Gate Arch, it is believed that the Howard gauge was the more suitable.

Mr. Howard mentions some of the interesting possibilities of measurements with his extensometer, and he certainly merits the thanks of the Profession for his invention of the instrument and for developing its application. It is to be regretted that he has never found time to prepare, for the Society, an account of his measurements along the different lines to which they have been applied.

The writer communicated with Mr. Howard in reference to several points raised in these discussions and received the following letter in reply:

"INTERSTATE COMMERCE COMMISSION
"DIVISION OF SAFETY
"WASHINGTON

"DR. D. B. STEINMAN, "MAY 11TH, 1918.
"35 Nassau Street,
"New York.

"DEAR DOCTOR: Please excuse this tardy response to your letter of the 7th inst. I have been out of town part of the time.

"In regard to Mr. Powell's misgivings that the use of the strain gauge does not permit the necessary accuracy, I might mention that

Mr. Steinman.

so far as check readings go they very often come out better than might be expected. Some readings taken on the lower chord of the Missouri River bridge at Kansas City in the forenoon and repeated in the afternoon of the same day chanced to accord to the same ten-thousandth throughout the series, of some twenty or more readings, corrections for temperature made in each case. The drilled and reamed holes in the bridge member are however rarely so well made that readings can be repeated as above. Usually the readings on the reference bar check, without variation greater than one-thousandth. The degree of accuracy depends upon the care exercised in drilling and reaming the holes. The holes should be smoothed with a conical set after drilling and reaming. A series of readings where successive changes in the loads occur very generally yield results which indicate the readings are reliable to one or two ten-thousandths. I do not make use of readings down to a ten-thousandth but place reliance upon the stresses to less than 500 lbs. per sq. in. If there was any need of greater refinement in the work I think it could be had by exercising more care. Differences in temperature limit the degree of accuracy in many cases. My impression is that the gauge readings are better than I claim for them. If Mr. Powell's experience is with plate gauges or with plug or ring gauges and he finds it difficult to attain an accuracy of a ten-thousandth of an inch then his aptness for fine measurements is not to be commended. No good machinist would think he had a satisfactory fit if it was not within a ten-thousandth of an inch on fine work. Sensitiveness of touch, in some places, is such that measuring to a ten-thousandth can be considered as more than ordinary work.

"In respect to holding the gauge in position normal to the work. Provided the drilled and reamed holes are strictly normal to the plane of the work, I do not think a slight inclination of the gauge would be found to give a change in the reading. The conical points of the gauge would make contact very nearly as a cylinder and center themselves on an elliptical contact zone. Reference lengths in the field are not paralleled as a rule and normal to the surface of the work. In order to avoid error in placing the gauge, it is my practice to use the same end of the gauge over the same hole in each observation, and judge of the gauge being normal to the work by noting the annular space around the conical point at the same end of the instrument on each occasion. One person should hold the instrument in place against the work, another advance the micrometer and make contact, and read the gauge. With suitable conditions, the holes being in good order, there is hardly any trouble in noting contact by sensitiveness of touch, with an accuracy of one ten-thousandth. Mr. Powell's suggestion of using a projecting pin with spherical end is not a practical one. Without going into the reasons why, if Mr. Powell will try such a method he will at once see the many difficulties which will defeat the accuracy easily attained with conical points and drilled and reamed holes. I doubt whether he would offer such a suggestion if he ever had experience with his suggested method. Of course the holes have to be cleaned at each reading. Dust has measurable thickness or diameter and must not be included with the steel member.

Mr. Steinman.

"In regard to Dr. Fuller's remarks about the use of a Berry gauge, which has a bell crank lever in its construction, strictly considered this is not a satisfactory detail of design. There is a difference between tilting a gauge of my type in which both points remain in the same plane and the action of a bell crank contact at one end. The rectilinear movement of the inner member in my gauge is mathematically correct. With this rectilinear movement there is no reasonable limit to the range of the instrument. I use a micrometer screw of one-half inch travel. It could be used with an inch travel or more if desired. The action of the gauge would not be impaired if the gauge length on the work was an inch longer or shorter than the reference bar. You will readily see how limited the use of a bell crank lever is in such an instrument as a strain gauge. The readings might be repeated and check in each case and still the measured length be in error. Parallel and rectilinear movements are desirable in instruments of precision.

"I note your kind allusion to the desirability of hearing from me in some of the strain gauge work not yet published, and much regret that I have not done so. Perhaps at an early date I may have opportunity to prepare some notes on the Missouri River Bridge and some structural work done in New York.

"Yours very truly,

"(Signed) JAMES E. HOWARD."

Dr. Lindenthal explains the considerations which prompted him to undertake the measurement of the stresses in the Hell Gate Arch. In a review of the erection of the Eads' Bridge at St. Louis he presents a striking illustration of the possible serious effect of stresses created in a bridge during construction, and he thus emphasizes the importance of ascertaining the actual stresses left in a bridge after erection. He succinctly expresses the guiding consideration in his concluding paragraph:

"The object of the strain measurements, as related by the author, was fully attained. There are no unknown stresses in the Hell Gate Arch structure."

Dr. Lindenthal certainly deserves all possible credit for his initiative and public spirit in personally defraying the expenses of this investigation as a contribution to engineering science.

Mr. Delson presents an account of a very interesting series of extensometer observations conducted by the New York Department of Bridges in connection with one of the operations of strengthening the side spans of the Williamsburg Bridge. This experience of the Bridge Department afforded a striking instance of the value of the extensometer in such work, as the measurements supplied the information necessary for the solution of a perplexing and critical erection problem.

The stress readings presented by Mr. Delson in connection with this

Mr. Steinman.

incident also indicate the high precision attainable with the Howard extensometer.

Professor Parcel, in a very able discussion, raises some doubts as to the validity of two conclusions which he credits to the writer. The first of these conclusions is stated as follows:

"1.—Actual secondary stresses, in general, will be less than the computed stresses, due largely to a constant automatic re-adjustment of the internal strains tending toward the relief of secondary stresses."

This is a correct and clearly expressed paraphrase of statements made in the paper, but the writer wishes to make it clear that he did not offer this principle as a direct conclusion from the results of the investigation under discussion. It was merely an expression of a relation which he derived from outside considerations, and in the validity of which he had, and still has, firm conviction. It was interjected as a partial explanation of certain findings, and not as a direct deduction from the data of this experiment.

Professor Parcel admits that the principle is perhaps entirely sound, but he questions its consistency with the results of the measurements. On this point the writer has already presented his case in replying to the discussion of Dr. Fuller. For the sake of clearness, even at the risk of reiteration, it may be stated here that all apparent violations of the above-formulated relation in the comparison of the final results were associated with the unique and exceptional features in the Hell Gate Arch erection—features that do not occur in the usual cases of truss building. The most significant of these special features was the cantilever erection, resulting in the fixation of large secondary stresses at the joints during the cantilever stages, which stresses were not released in the transformation to the arch condition.

The special erection feature just described would naturally affect the results in Stages 3-*H* and 2-*H*, tending to increase the secondary stresses in those stages above the calculated values. This is exactly the phenomenon observed, for it is only in those stages that the measured secondaries exceed the calculated; and this fact, to which both Professors Fuller and Parcel call attention, instead of contradicting the writer's conclusions, serves as a clear confirmation of the correctness of his reasoning.

Apparently ignoring the above considerations, Professor Parcel thinks that the following additional conclusion might fairly be drawn: "For the higher ranges of primary stress, the actual secondary stresses will probably exceed the calculated values by a considerable margin." To this conclusion the writer takes distinct exception. It is based on a comparison of the results in the arch stages with those in the cantilever stages—a comparison which yields misleading conclusions, or

Mr. Steinman.

conclusions the generality of which is vitiated, on account of the special considerations already submitted. The only comparisons from which a conclusion of this character could legitimately be drawn would be between measurements of stresses of different magnitudes in the same stage or condition of the structure; and such comparisons, in the present investigation, distinctly contradict Professor Parcel's suggested conclusion, and confirm that of the writer instead.

The writer maintains that the apparent exception in Stages 3-*H* and 2-*H* to his rule, that actual secondary stresses will generally be less than computed stresses, is due merely to the fact that in these stages there was a gross but unavoidable incorrectness in the calculated secondaries. He regrets the inadequacy of the available methods of computing secondary stresses to take into account the effect of such complicating factors as the transformation of the structure from a cantilever into an arch. If a satisfactory solution of this mathematical problem were obtainable, there is little doubt that the anomalous results under discussion would be entirely dispelled.

It may be suggested here that the problem just described would be simple of solution if the joints were securely riveted before the transformation. It is the partial and unknown slipping or yielding of the joints before or during riveting that complicates the theoretical problem.

The second conclusion presented by Professor Parcel for discussion is:

"2.--For most bridges it will probably be satisfactory to provide for secondary stresses by an allowance in the specified unit stresses of about 20% of the total primary stress, plus 3 000 lb."

A comparison of this version with that of the writer in Conclusion 4, on page 1071, will indicate certain differences. The writer did not suggest the deduction of 25% plus 4 000 lb. from the elastic limit as a "satisfactory" allowance but as a minimum "necessary" allowance. In doing this he was merely calling attention to the necessity, in fixing extreme values of working stresses, of leaving a sufficient margin below the elastic limit to provide for secondaries and other anticipated additions to calculated primary stresses; and he wished to suggest the possible usefulness of investigations of this character in supplying information to help fix the proper magnitude of this margin or allowance. The value suggested by the writer was simply the best one that he could deduce with the aid of the data of this experiment; and it was offered for whatever it was worth, merely as an initial contribution to the fund of knowledge requisite to form a proper judgment on this point. No one realizes better than the writer the importance of securing similar data from measurements on a great number and variety of structures before any final decision on such a question can be reached.

Mr. Steinman.

The writer's reasons for including this matter in the paper were twofold: First, to indicate an additional objective for extensometer and secondary stress investigations; and, second, to suggest a form for an empirical rule for a margin above extreme working stresses.

In the original presentation of this tentative rule, on page 1063, it will be observed that judgment based on outside considerations generously supplemented logical deduction in the formulation of the conclusion. It was more in the nature of a suggested line of thought and future investigation than a final and direct conclusion from the data of the investigation.

That the limitations and present indefiniteness of such an empirical rule were clearly realized by the writer is shown by the following quotation from the paper:

"It is recognized, of course, that a rule of this form can properly apply only to structures in which the secondary stresses have normal or average values. Where unusually large secondaries may be anticipated, these should receive special investigation and attention."

Professor Parcel raises the question "whether any empirical relation based on averages is an adequate means of providing for secondary stresses, even in ordinary structures." The writer endeavored to make his proposed allowance sufficiently generous to take care of the maximum, and not the average, secondary stresses ordinarily occurring. That the suggested allowance is adequate is corroborated by the results of Professor Parcel's own measurements of secondary stresses as plotted in Figs. 9, 10, and 11. These graphs show that the secondary stresses in a railway bridge under service loading ranged between 10 and 30%, and never exceeded the latter amount except when the primary stresses fell very low; and, as Professor Parcel himself remarks, "it is only the secondary stresses that co-exist with the maximum primary stresses that affect the design of the structure."

The writer does not take issue with Professor Parcel's suggestion of the desirability of making some analysis of the secondary stresses in every design of a bridge structure. That, indeed, would be a most desirable consummation; but it is going to be very difficult to convert the Profession to such procedure in the case of ordinary structures, or to get clients to pay for the additional labor involved.

In proposing the use of empirical allowances for secondary stresses, the writer was not actuated by any distrust of the reliability of secondary stress computations, nor by any doubt as to their desirability, but rather by a conviction of the difficulty of changing present practice in this regard. So long as engineers have to deal with actual conditions where secondary stress computations are generally omitted, it is certainly better to have some guide for estimating a proper allowance than to get along without any. Therefore the writer offered, as a

suggestion, a rule for this purpose, derived from his best judgment, guided by the results of this investigation. No data tending to contradict or modify this rule have yet been advanced. Mr. Steinman.

Professor Parcel writes:

"If we assume that theoretical computations for secondaries are wholly untrustworthy, and thus justify reliance on an empirical formula for ordinary bridges, how are we to rely on our analysis for the cases where 'exceptionally large secondaries may be anticipated', and what basis is there for the elaborate and very expensive correction methods used in the erection of the Quebec and Sciotoville spans, methods which have been referred to in current technical literature as the most remarkable single feature of these great engineering projects?"

In reply the writer wishes to make it clear that he does not regard theoretical computations for secondaries as untrustworthy; on the contrary, he is convinced that, with the exception of cases involving complicated erection features, which cannot be accurately considered in the computations, the actual secondaries will be either equal to or slightly less than the calculated values. The use of an empirical formula is recommended, not as being more reliable than theoretical computations of secondary stresses, but as constituting a more or less satisfactory substitute when time and labor have to be conserved.

Referring to cases where "exceptionally large secondaries may be anticipated", the writer submits that such cases can readily be recognized by any one who has had proper experience in the study of secondary stresses. Exceptional stresses will be found at points where there is any interference with the production of a smooth curve of deformation of a span or of a single member. Such points are: at the feet of main hangers in simple Pratt trusses; at the feet of sub-hangers in trusses with subdivided panels; at the connections of collision struts; at the points of attachment of back-stays in cantilever erection of arches; at intermediate supports of continuous and cantilever bridges; at the connections of intersecting diagonals, etc. Fortunately, in nearly all such cases, it is possible to ascertain the approximate value of the critical secondaries by some simplified method based on local deflections without undertaking a complete theoretical analysis of the secondary stresses in the entire structure. It would seem that such procedure, in combination with the use of an empirical rule for the secondaries of ordinary magnitude in the structure, would provide a safe and economical solution of the problem of providing for secondary stresses.

Professor Parcel refers to the elaborate and expensive erection methods which were adopted for the elimination of the secondary stresses in recently completed long-span bridges. In the case of the Sciotoville Bridge, the writer can speak from experience, as he was

Mr. Steinman.

identified with the design and computations for that structure. The Sciotoville Bridge presents a striking instance of a case where "exceptionally large secondaries may be anticipated." Over the middle support of this continuous structure, there is necessarily a large upward cusp in the curve of deflections, and the secondary stresses from this cause, for live load on both spans, would have amounted to nearly 20 000 lb. per sq. in. Although elaborate secondary stress computations were prepared for the entire structure, the critical stress just mentioned was easily foreseen; and it was computed in a few minutes from the deflection diagram of the trusses, quite independently of the more complete analysis of secondaries. Similarly, the secondary stresses in the bottom chords at the connections of the various subhangers could have been foreseen and computed independently by approximate methods. Such approximate investigations would have been sufficient to disclose the serious character of the secondary stresses in this structure, and to emphasize the necessity of some such method of erection as was adopted to counteract these dangerous stresses.

Professor Parcel adds:

"We need a great many more experimental data to settle the question of actual *versus* calculated secondary stresses, no doubt; but the bulk of the evidence, up to the present, would seem to justify considerable confidence in our theory."

It is difficult to understand what is referred to in the phrase "the bulk of the evidence, up to the present"; since, as the writer endeavored to point out in the introductory part of the paper, there had been practically nothing published in the form of a comparison of calculated and measured secondary stresses before the presentation of this investigation. Nevertheless, the writer joins Professor Parcel in placing considerable confidence in the theory of secondary stresses; in fact, it does not appear that any one has questioned it.

The results of stress measurements on the Kenova Bridge, presented by Professor Parcel, are certainly interesting. It is unfortunate that the data have not yet been worked up to show a comparison between measured and computed secondaries, and it is to be hoped that this analysis will soon be completed and presented to the Profession.

The writer is grateful to Professor MacKay for directing attention to a point of possible misunderstanding, namely, the statement in the paper about the number of measurements required to determine the distribution of stress in any member. When he wrote "It is necessary to take measurements at not less than three, and preferably four, points of the cross-section near each end of the member", the writer intended this merely as a statement of minimum requirement, to apply only to members of ordinary size and section; and he is in complete accord with Professor MacKay in considering that a greater

number of points are usually desirable in large members. The writer recognized this fact in the case of the Hell Gate chord members, of which he wrote (page 1052), that "measurements at the four extreme corners of the section would not be fairly representative of the conditions throughout the entire area." Mr. Steinman.

The figures cited by Professor MacKay from the Hell Gate measurements to show that the omission of the readings near the center of the webs would have altered the results, serve as a confirmation of the writer's judgment in deciding to use six observations at each cross-section. The two readings at mid-height were necessary, not only on account of the mass of metal concentrated at and near the middle diaphragm, but also to determine and provide for the effect of the concentration of bearing in the middle thirds of the joints.

Whether a larger number than six observations at each section should have been taken, it is difficult to decide. It is doubtful whether there would have been any appreciable increase in the reliability of the results to compensate for the curtailment of programme which the extra labor would have necessitated.

The data of tests on latticed members submitted by Professor MacKay are extremely interesting. His plats of stress distribution near the end of a pin-connected member show a concentration of stress toward the center of the web, as might be anticipated. He also shows the disturbance of uniform stress condition at the points of connection of diaphragms and lattice bars, on account of local bending in the ribs composing the member. It was in anticipation of such effects that the writer avoided the proximity of diaphragms, splice-plates, and similar factors of stress disturbance, in fixing the locations of the gauge points for his stress measurements on the Hell Gate Arch.

In correspondence with the writer, Professor MacKay described some of his measurements on bridges of the Canadian Government Railways, and it is to be regretted that these came too late to be included in his published discussion. With Professor MacKay's permission, the following is quoted from one of his letters:

"My work on the C. G. R. bridges was limited to a single type of riveted pony truss span, and my object was to check primary, secondary, and impact stresses under live load. The results would dispose me to endorse all of your conclusions applicable to such widely divergent cases, with the possible exception of the point referred to in my discussion."

The following are some of the results as regards precision in the span most thoroughly examined. The range of stress measured was from 5 000 to 8 000 lb. per sq. in.

Difference between calculated and observed primary stresses:

Average	390 lb. per sq. in.	(525)
Maximum	780 " " "	(1 290)

Mr. Steinman.

This omits some lower chord members which were affected by stringer and floor-beam action, and by the failure of roller bearings to work.

Difference between stresses in two ends of same member:

Average	270	lb.	per	sq.	in. (140)
Average, omitting one section	160	"	"	"	"
Maximum	960	"	"	"	" (500)
Maximum, omitting one section	320	"	"	"	"

Most of the work was done under favorable temperature conditions.

As regards secondary stresses, the most important effects agreed fairly well with calculated stresses.

The figures in parentheses, representing corresponding values from the Hell Gate Arch investigation, have been inserted by the writer to permit a comparison with the results secured by Professor MacKay. When the difference in the characters of the structures and loadings is considered, the two sets of results may be regarded as mutually confirmatory.

In the discussion by Professor Turneaure, we have a clear presentation of the significance of secondary stresses by one who is an acknowledged authority on the subject. He writes that he "does not believe that it will ever be necessary or desirable to calculate secondary stresses in the ordinary practice of bridge design; but he does believe that their consideration, both theoretically and experimentally, in such unusual structures as the one under consideration, is of very great importance."

It was in connection with the ordinary cases of bridge design, for which no calculation of secondary stresses is undertaken, that the writer suggested the use of an empirical correction to be applied to the working stress. Referring to the suggested figure of $25\% + 4\,000$, Professor Turneaure agrees that this "would probably cover most cases of well-designed structures." In arriving at this estimate of a proper deduction from the minimum elastic limit, the writer took into consideration, not only the secondary stresses usually to be expected, but also other possible variations from computed stress.

The writer, as previously stated, did not intend to present the foregoing recommendation as a direct and rigorous conclusion from the results of the calculations and measurements on the Hell Gate Arch. Other considerations entered in arriving at this figure, and it was offered merely as a guide or basis for future investigation and discussion. Consequently, the following statement by Professor Turneaure on the possible value of the suggestion is appreciated:

"If it is found that the secondary stresses in any particular structure can be represented safely by a particular percentage of the primary stresses or a percentage plus a constant, then we have arrived at a

fair value of the maximum fiber stress, and are much better prepared to discuss the subject of working stresses than if no estimate of the secondary stresses had been made." Mr. Steinman

In reference to the reliability of secondary stress calculations, Professor Turneaure writes:

"Such stresses are just as much a real part of the total stress which may prevail in any fiber of a member as the primary stress; and they are just as certain to exist as it is certain that the structure will deflect under load. The accuracy of their determination, however, is not so great as that of the primary stresses, on account of the indeterminate effect of the temperature variations and of joint plates, rivets, and other details."

It is on account of this uncertainty which characterizes secondary stress calculations that the writer recommends the application of the extensometer to the actual determination of the fiber stresses in all structures of unusual magnitude or construction.

Referring to the great care taken in the design and erection of the Hell Gate Arch and of other large structures as evidence of the increasing appreciation of scientific methods of design and accurate fabrication, Professor Turneaure writes:

"Where such conditions prevail, the objection to the use of statically indeterminate structures, which has been so general in the past, loses its force, and there would seem to be no reason for excluding such forms of structure where, for other reasons, they are desirable."

This statement cannot be too heartily endorsed; it expresses what is probably the most important lesson to be learned by the Profession from the work on such structures as the Hell Gate Arch and the Sciotoville Bridge.

Professor Jacoby emphasizes the value of scientific investigation as conducive to security and economy in structural design. He adds:

"When observations are made on an actual structure, especially one in which the members are so large and where the resources of construction in equipment and workmanship are taxed to a much higher degree than usual, the results are of far greater value than laboratory experiments could possibly give."

The realization of this opportunity in connection with the erection of the Hell Gate Arch was the incentive for undertaking the stress measurements, and the justification of the financial outlay on the part of Dr. Lindenthal.

The summary of the investigation of secondary stresses in a two-hinged, spandrel-braced arch with cantilever arms, computed by Mr. Thomson Mao, of the Cornell University Graduate School, and presented by Professor Jacoby, constitutes a valuable contribution to this discussion. It exemplifies the diversity of methods applicable

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to the calculation of secondary stresses. In Mr. Mao's thesis, Mohr's method was used, involving the use of displacement diagrams to find the change in slope of the truss members. The method of Winkler, in which angle changes are found analytically, is preferred by the writer, because of its greater precision and ease of checking.

Another distinctive feature in the method of computation followed by Mr. Mao was the calculation of the secondary stresses in all the members of the structure for a unit load at each of the panel points. Except for spans having a very small number of panels this procedure would involve too great an expenditure of time and effort for application in actual practice. All essential purposes of secondary stress computations will be served if the analysis is made for a much smaller number of cases of loading. As a rule, the simplest considerations of the natural deflection characteristics of a structure will indicate the proper loading assumptions for maximum secondary stresses; and an inspection of the load lengths for maximum primary stresses will indicate the one or two loading assumptions which will give, with sufficient accuracy, the critical secondary stresses, namely, those occurring simultaneously with the maximum primary stresses.

Relative to this matter, the writer would offer, as a recommendation for future secondary stress computations, the suggestion to compute the stresses for just two load conditions, *viz.*, the first and the second quarters of the span, respectively, loaded. By simple reversal of the diagram and algebraic addition of the sets of stresses, there will be obtained the secondary stresses for load covering one-quarter, one-half, three-quarters, or full span, or any other combination of quarters of the span. The resulting values will suffice for all practical purposes, and by this procedure the analysis will involve only two solutions of secondary stresses, instead of the ten or more required in Mr. Mao's work.

Professor Jacoby recommends the method of Gauss for the solution of the sets of simultaneous equations involved in the computations. He writes:

"By the method of Gauss, the solution of the equations becomes quite mechanical, and may be checked at every step of the process. This method of solution, therefore, has a great advantage over any other."

The writer doubts whether this method can compare favorably in speed with the procedure of successive substitution, as described on page 1073, which was used for the Hell Gate computations, and yielded a remarkable showing over all other methods in its greater speed and self-checking advantages.

On this point, the writer has had some correspondence with Professor Turneaure, who advocated an improved method of successive elimination as being the most expeditious. To this the writer replied:

"I believe that if you include the time required to check the work, to locate errors and to rectify them, the method of elimination will be seriously handicapped in comparison with the methods of successive approximation by substitution. The figures which I gave you in my last letter for the time required by the different methods included the work of retracing the steps whenever an error was suspected, and of correcting any such error. If we consider the primary desideratum to be correctness of the results, rather than merely securing of maximum speed, it seems to me that the method of substitution is the more desirable, as it supplies an automatic check at every step in the work. It is a self-checking method in that it is impossible to proceed very far without detecting any error that may have been made." Mr. Steinman.

The writer has worked out a systematic arrangement of the computations in the method of successive substitutions so as to minimize the consumption of time and the possibility of mistakes. In actual test he has not yet found any other method to equal it in the desiderata of speed and self-checking.

The writer is grateful to Mr. Ammann for calling attention to the possible danger of applying the conclusions of this paper without due regard to special conditions which might modify the results.

In discussing the findings of his investigation, the writer has been careful to keep in mind at all times the special features in the design and erection of the Hell Gate Arch which would affect the results in any way. He submits that a careful perusal of the ten conclusions presented at the end of the paper (pages 1070 to 1072), will show no violation of this principle.

There is but one paragraph occurring in those pages, which is not a rigorous deduction from the results of the investigation; but its very form indicates that it was not presented as a deduction or conclusion, but rather as a parenthetical adduction of a conviction entertained by the writer and tending to throw light on certain results obtained. It refers to the observed fact that most of the measured secondary stresses during the erection of the arch were less than the calculated values, and reads as follows:

"It is believed that similar results, though not as marked, would be found in other structures. The actual secondary stresses will generally be lower than the calculated values. There is an automatic re-adjustment of strains within a structure in such direction as to relieve the secondary stresses."

The writer trusts that no one will take exception to the foregoing statement, as it is a simple matter to demonstrate, both mathematically and practically, the validity of the proposition therein enunciated. It is not a conclusion from the present investigation, although it is corroborated by the results obtained.

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Apart from this passage, which was not intended as a conclusion, and from Conclusion No. 4, which has already been the subject of extensive discussion, the writer would like to ask Mr. Ammann to point out anything in the summary of conclusions which can possibly be regarded as too far-reaching a deduction from the results of the investigation.

Mr. Ammann refers to the objects for which the measurements were undertaken, but this matter has already been presented in the paper and in Dr. Lindenthal's discussion.

Mr. Ammann points out that one of the objects in the programme of investigation has not been accomplished, namely the measurement of the live-load stresses in the finished structure. It is to be regretted that Dr. Lindenthal interrupted the investigation at this critical point, and the writer joins Mr. Ammann and the others who have expressed the hope that some Federal bureau or technical society may take up the measurements, so as to include the final live-load stresses.

Mr. Randorf contributes an important suggestion as to the possible influence of the varying elasticity of the steel on the results of the measurements. Although a uniform value of 30 000 000 for the modulus of elasticity was assumed in reducing the observations, the effect of possible variation from this value was discussed on page 1061 of the paper.

Professor MacKay, in his investigation, as mentioned on page 1095, adopted a value of 29 000 000 for the elastic modulus. The difference in the quality of the steel in the two structures tested would justify this difference in the assumed values of E.

Mr. Randorf's suggestion that the modulus of elasticity of the material should be verified by special tests is certainly a good one. In the present case, the satisfactory agreement between average calculated and average measured stresses would appear to be a confirmation of the correctness of the assumed value of E; nevertheless, it is readily conceivable that some of the variations in the results might be traced to fluctuations in the elasticity of the material.

Most of the questions raised in Mr. Molitor's valued contribution have already been answered in connection with the preceding discussions. There are a number of points, however, which call for individual consideration.

Referring to the features which gave rise to the observed discrepancies between calculated and measured secondary stresses, Mr. Molitor wants to know why these features were not provided for in the computations, so that a closer comparison might have been secured. Perhaps the best way of answering this question is to present for

individual consideration the following list of factors tending to produce deviations from computed secondary stresses: Mr. Steinman.

1.—The common theory of secondary stresses assumes absolute angular rigidity of the connections, whereas, actually, there must be some elastic yielding or mechanical slip between the members. This condition, occurring to some extent in all structures, tends to relieve the secondary stresses. The amount, however, is unknown, and is not susceptible of analytical determination. The effect is augmented materially by the slip and re-adjustment which take place when the drift-pins and bolts used temporarily during erection are replaced by rivets. No attempt is made to correct for this condition in the computations, as the part due to elastic strain is too small to justify the effort, and that due to mechanical slip is too erratic and arbitrary to permit analysis. It is, of course, on the safe side to ignore this effect.

2.—The common theory of secondary stresses neglects the bending moments produced in the members by the eccentricities resulting from their flexural deformations. This condition tends to augment the secondary stresses in compression members and to reduce them in tension members. Provision for this effect greatly complicates the analysis and computations, and ordinarily it may be disregarded. For long and slender members, however, the effect would be appreciable.

3.—In the generally accepted method of analyzing trusses, the primary stresses are determined on the basis of frictionless joints. There is a small error involved in this assumption, as the joints are not frictionless, but possess rigidity. The elastic curvature assumed by the members under the secondary strains gives them a girder action whereby a certain amount of transverse shear is transmitted from panel point to panel point, and the primary stresses are relieved to that extent. This, in turn, results in a slight modification of the secondary stresses, generally a reduction; but the effect is small, and as the corresponding calculations are laborious, they are neglected.

4.—Shop inaccuracies in laying out the rivet holes for end connections, either as to distance or angle, will produce secondary stresses not covered by the computations.

5.—Temperature differences between parts of the structure or of the same members will produce secondary stresses not included in the calculations.

6.—Inaccurate facing of the ends of compression members may produce eccentric bearing at the butt-joints, and the effect will appear in the measured secondary stresses.

7.—An important factor is the use of drift-pins during erection. When these are replaced by rivets, a certain amount of mechanical slip and re-adjustment of strain occurs at the joints, permitting a

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partial release of the secondary strains. If the secondary stresses then obtaining are contrary in sign to the final secondaries at the same points, the ultimate effect of the mechanical slip will be an augmentation of secondary stress. On the whole, this feature is too erratic and arbitrary to permit analysis; consequently, it is not provided for in the computations.

8.—For large, built-up sections, there is some uncertainty as to the distribution of stress over the cross-section. Navier's law cannot be assumed to apply with positiveness. The division of stress between the plates and angles composing the section may be imperfect, depending on the riveting; and local stress concentrations are produced by splices, diaphragms, and details. All these conditions affect the measured secondary stresses, but cannot be provided for in the computed secondaries.

The foregoing list of disturbing factors should serve to make clear why the exact values of the secondary stresses cannot be anticipated by computation, and why the results of theory should be supplemented by experimental investigations aiming to throw light on the individual and collective amounts of these deviations from the calculated stresses.

In addition to the conditions previously outlined, which are of more or less general occurrence, there were, in the case of the Hell Gate Arch, two other features affecting the secondary stresses, as follows:

1.—The three-faced joints: With regard to these, it is important for clearness to distinguish between their action in relieving the primary stresses in the outer fibers, and their influence in permitting partial hinge action at the connections during erection. The former effect does not change the secondary stresses (defined as one-half the difference between top and bottom fiber stresses); consequently, no correction for it is necessary before comparing measured and calculated secondaries. The hinge effect, on the other hand, does modify the secondary stresses, but it is more or less indeterminate qualitatively as well as quantitatively. It is resisted to an unknown extent by the presence of the splice-plates and drift-pins; and it ceases to occur when the primary stresses attain an undefined value. The ultimate effect on the secondaries will depend on the reversals of secondary flexure taking place during erection. For these reasons it would be practically futile to endeavor to provide for this hinge action in the computations.

2.—The cantilever erection: This produced large secondary stresses during the erection stage, and a part of these persisted in the structure after its transformation into an arch.

The effects of the ten features just enumerated as influencing the secondary stresses are complicated by their action in combination. The action of the three-faced joints is modified by the interference

of splice-plates and drift-pins; the effect of the cantilever erection is controlled by the unknown amount of hinge action taking place as a result of the three-faced joints; the replacement of drift-pins by rivets is accompanied by an uncertain amount of slip and strain redistribution in the connections; and the transformation from cantilever to arch involves unknown re-adjustments of strain at all the joints. Mr. Steinman.

In his computation of the secondary stresses, the writer followed the established procedure and did not provide for the previously discussed modifying factors. To take all these features into account analytically would be impossible; for most of them even a qualitative prediction would be difficult. The best that can be done is to determine their combined effect experimentally for individual cases, until sufficient data are thus accumulated to form a basis for empirical rules to be applied to other structures.

Mr. Molitor suggests that the features just discussed would necessarily result in very small secondary stresses. If he had predicted, in advance of this investigation, that the combined effect of these features would be to reduce the secondary stresses, he would have proved himself a poor prophet; for, as it happens, the secondary stresses in the final stage average about 120% greater than the corresponding computed values. (See Table 5.) He would have been underestimating the intensifying effect of the cantilever method of erection, which, in this case, outweighed the combined influence of all the other features tending to modify the secondary stresses.

What Mr. Molitor apparently has in mind is the easily anticipated relief in the stresses in the outer fibers as a result of the greater concentration of pressure in the middle thirds of the three-faced joints. This, however, is not a secondary stress in the accepted sense of the term; it is merely a redistribution of the primary or axial stress over the cross-section. In the case of the Hell Gate chord members, this action of the three-faced joints proved to be a saving feature; it just about neutralized the resultant augmentative effect of all the other disturbing conditions, so that the final extreme fiber stresses came down to the calculated values (see page 1068). For the sake of clearness, however, it should be understood that this effect of the three-faced joints, although representing a reduction of the outer fiber stresses, is not, strictly speaking, a reduction of the secondary (or flexural) stresses. It does not affect the comparison between measured and calculated stresses so long as these are defined as one-half the difference between top and bottom fiber stresses.

No exception can be taken to the point made by Mr. Molitor that the Hell Gate Arch Bridge is not the most suitable structure for proving or disproving the correctness of secondary stress computations; but that was not the object of these investigations. The

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measurements were undertaken primarily for the purpose of ascertaining the actual stresses in the Hell Gate Arch, and that object was attained. Incidentally, the results secured have been studied and the conclusions reported in order to throw whatever light they can on the effects of the various features tending to modify the stresses.

In this connection, the writer wishes to point out that, although the Hell Gate Arch results represent the composite effect of many stress-modifying elements, nevertheless, the measurements may be studied so as to single out the effects of some of the special features. For example, by comparing the results in the arch stages (3-*H* and 2-*H*) with those in the preceding cantilever stages, information is secured on the effect of the transformation from cantilever to arch; and it is thus found that this single feature of the erection outweighs the combined effect of all the other factors modifying the secondary stresses. Similarly, a comparison of the final secondary stresses according to the respective definitions represented by the two branches of the graph in Fig. 5, yields directly the effect of the three-faced joints in relieving the outer fiber stresses. With these two special features (cantilever erection and three-faced joints) separated out, there is little left in the results and conclusions that may not be of general application to other structures.

Referring to the method used for computing the secondary stresses in the Hell Gate Arch, Mr. Molitor remarks that it is "admittedly laborious, and might have been somewhat simplified". If Mr. Molitor knows of any solution that is less laborious, or of any further simplification that might be introduced, the Profession would certainly welcome his contribution. If, however, he is referring to the method described in the chapter on Secondary Stresses in his book, he is claiming an advantage which is not substantiated by actual comparisons. That solution was invented and published by Müller-Breslau more than 30 years ago, but it has failed to stand the test of time in competition with other methods.

The writer uses Winkler's method which involves only one unknown for each panel point, whereas Müller-Breslau's treatment requires the solution of two unknowns for each member. Thus, for the Pratt truss worked out by Mr. Molitor in his book, he has to determine 22 unknowns; for the same problem, Winkler's method would involve only 6 unknowns. The simultaneous equations in Winkler's solution are one for each panel point, while Müller-Breslau's method requires in addition three equations for each truss triangle; and the method of elimination used for solving these equations is no more expeditious than the procedure of successive approximations used by the writer.

In the discussions there has been practically no mention of two of the most interesting facts established in the course of the investigation.

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One was the hinge action of the three-faced joints, permitting, in the early stages, a certain amount of rotation which was easily measured. The other was the re-adjustment and re-distribution of stress at the joints when the drift-pins were replaced by rivets. Before the measurements were made, doubt was expressed in certain quarters as to the actuality of both of these anticipated effects, but the investigation has set all such doubts at rest.

In conclusion, the writer wishes to repeat his acknowledgment of indebtedness to all those who have discussed the paper, as he realizes that their contributions have greatly enhanced its usefulness to the Profession. On account of the necessary brevity of the paper, some of the thoughts and conclusions were perforce inadequately presented; and the discussions so generously contributed have helped to focus attention on the points of greatest interest and to remove any doubts as to the interpretations to be drawn.

This paper would not be complete without a closing tribute to Dr. Lindenthal for his foresight in conceiving the investigation, and for his public spirit in undertaking it at his own expense.