An elementary and practical treatise on bridge building

Squire Whipple


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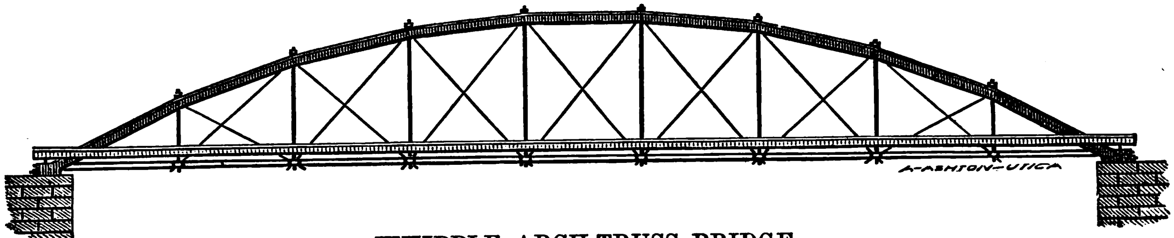
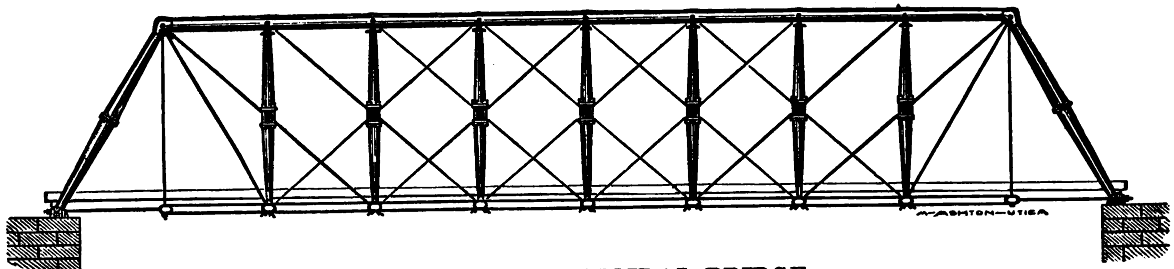


WHIPPLE ARCH-TRUSS BRIDGE.



WHIPPLE TRAPEZOIDAL BRIDGE.

AN

ELEMENTARY

AND

PRACTICAL TREATISE

ON

Bridge Building.

AN

ENLARGED AND IMPROVED EDITION

OF THE AUTHOR'S ORIGINAL WORK,

BY

S. WHIPPLE, C.E.
ALBANY, N. Y.,
INVENTOR OF THE WHIPPLE BRIDGES, &C.

NEW YORK:
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INTRODUCTION.

It is about thirty years since the Author's attention was especially directed to the subject of BRIDGE CONSTRUCTION; and, his Original Essays published in 1847, are believed to have aided considerably toward establishing the foundation upon which a knowledge of the principles involved, and the conditions required in the proper construction of TRUSS BRIDGES, has been built up, and carried to a high state of advancement.

However that may be, the flattering terms in which his former labors in the premises have often been referred to, as well as the frequent applications for copies of his former publication, since the supply became exhausted, have prompted the issue of the present volume.

This work inculcates the same development of GENERAL PRINCIPLES, and treats of essentially the same General Plans, Combinations, and proportions for bridge work, as were discussed and recommended in its humble predecessor; with such additions and improvements as subsequent experience and observation have enabled the Author to introduce.

The design has been to develop from Fundamental Principles, a system easy of comprehension, and such as to enable the attentive reader and student to judge understandingly for himself, as to the relative merits of different plans and combinations, and to adopt for use, such as may be most suitable for the cases he may have to deal with.

It is hoped the work may prove an appropriate Text Book upon the subject treated of, for the Engineering Student, and a useful manual for the Practicing Engineer, and Bridge Builder. But as to this, the decision must be left to those into whose hands it may fall; and to that arbitrement, without further remark or explanation, it is respectfully submitted.

CONTENTS.

BRIDGE BUILDING.

PRELIMINARIES.

I. A bridge is a structure for sustaining the weight of carriages, animals, &c., during their transit over a stream, gulf or valley.

Bridges are constructed of various plans and dimensions, according to the circumstances and objects requiring their erection; and it is the purpose of this work, after a few remarks upon the general nature and principles of bridges, to attempt some analyses and comparisons of the respective qualities and merits of various general plans, with a view of deducing practical results, as to a judicious and economical choice and application of materials in the construction of these useful and important structures.

II. The force of gravity, on which the weight of bodies depends, acts in vertical lines, and consequently, a heavy body can only be prevented from falling to the earth, by a force equal and opposite to that with which gravity impels the body downward. This resisting force must not only act vertically upward, but the line of its action must pass through the centre of gravity of the body it sustains. All the forces in the world, acting parallel with, or perpendicular to, the vertical passing through its centre of gravity, could not prevent a musket ball (concentrated to the point of its centre of gravity) from falling to the centre of the earth, unless it were a horizontal force capable of giving the ball a projection, such that the centrifugal tendency should equal or exceed gravity—a kind of force which could never be made available toward preventing people from falling into the water in crossing rivers; consequently, having no application in bridge building.

In fact, nothing but a continuous series of unyielding material particles, extending from an elevated body downward to the earth, can hold or sustain that body above the earth, by vertical and horizontal action alone, either separately, or in combination.

III. Suppose a body, no matter how great or small, placed above the earth, with a deep void, or an inaccessible space beneath it. Attach as many cords to it as you please, strain them much or little — only horizontally — the body will fall, nevertheless. Thrust any number of rods, with whatever force you may, horizontally against it; still the body will fall. This is obvious from the fact that horizontal forces, acting at right angles with the direction of the force of gravity, have no more tendency to prevent, than to promote the fall of the body.

Moreover, the space beneath being inaccessible, there is no foundation, or foot hold, upon which to rest a post or stud that may directly resist the action of gravity, while the lines of all other vertical forces or resistances, pass by the body without touching it.

In the case here supposed, the body can only be prevented from falling by *oblique* forces; that is, by forces whose lines of action are neither exactly horizontal, nor exactly perpendicular. Attach two cords to the body, draw upon them obliquely upward and outward, in opposite directions, or from opposite sides of the void, with a certain stress, and the body will be sustained in its position. Apply two rods to it obliquely upward, of a proper degree of stiffness, in the same vertical plane, and on opposite sides of the perpendicular, a certain thrust exerted upon those rods, will prevent the descent of the body.

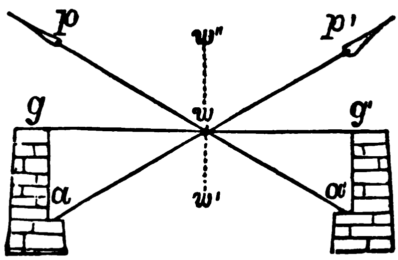
IV. Here, then, we have the elementary idea — the grand fundamental principle in bridge building. Whatever be the form of structure adopted, the elementary object to be accomplished is, to sustain a given weight in a given position, by a system of *oblique forces*, whose resultant shall pass through the centre of gravity of the body in a vertically upward direction, in circumstances where the weight can not be conveniently met by a simple force, in the same line with, and opposite to, that of gravity.

For a more clear illustration of this elementary idea, let us suppose a a', Fig. 1, to represent the banks of a river, or the abutments of a bridge; and gg', the line of transit for carriages, &c.; and, let us further suppose a load of a certain weight, w, to have arrived at a point centrally between a a'. The simplest method of sustaining the weight is, perhaps, either to erect two oblique braces aw. . $a'w$, or suspend two oblique chains or ties pw, $p'w$, from fixed supporting points a a', or p p'.

It is not necessary that the weight be at the angular point w, of the braces or chains, but it may be sustained by simple suspension at w' below, or simple support at w'' above, and such obliquity may be given to the braces or chains as may be most economical; a consideration which will be taken into account hereafter.

V. Thus we see how a weight may be sustained centrally between the banks of a river, or the extremities of a bridge. But the structure must not only provide for the support of weight at this point, but also at every other point between *a a′*, or *g g′*; and it is obvious that the same plan and arrangement will apply as well at any other point as at the centre, with only the variation of making the braces or chains of unequal length.

FIG. 1.



This, however, would require as many pairs of braces or chains as there were points between *g g′*, a thing, of course, impracticable, since the oblique members would interfere with one another, and be confounded into a solid mass. We therefore resort to the transverse strength and stiffness of beams, — phenomena with which all have more or less acquaintance, and without digressing in this place to investigate their principles and causes, it will be assumed as a fact sustained by all experience, that, for sustaining weight between two supporting points upon nearly the same level, a simple beam affords the most convenient and economical means, until those points exceed a certain distance asunder, which distance will vary with circumstances;

but in bridge building, will seldom be less than 10 to 14 feet, where timber beams are employed. Hence, for bridges of a length of 12 to 14 feet, usually, nothing better can be employed than a structure supported by longitudinal beams, with their ends resting upon abutments or supports upon the sides of the stream.

Of course, no reference is here had to stone or brick arches. For, though these are advantageously used for short spans, and in deep valleys, where the expense of constructing high abutments for supporting a lighter superstructure, would exceed or approximate to that of constructing the arch, it is the purpose of this work to speak only of those lighter structures, composed mostly of wood and iron, and supported by abutments and piers of stone, or by piles, or frames of wood.

Having then adopted the use of beams for supporting weight upon short spaces, it is only necessary upon longer stretches, to provide support for a point once in 10 or 14 feet, by braces, &c., from the extremities; and for intermediate points, to depend on beams or joists extending from one to another of the principal points provided for as above.*

VI. For a span of 20 or 30 feet, it would seem that no better plan could be devised, than to support a transverse beam midway between abutments, by two pairs of braces or suspension chains, proceeding from points at or over the abutments, one pair upon each side of the road-way; this transverse beam affording support for longitudinal beams or joists extending

* It is susceptible of easy demonstration that the power of beams to sustain weight by lateral stiffness, forms no exception to the principle that oblique forces alone can sustain heavy bodies over inaccessible spaces. But this matter is deferred for the present.

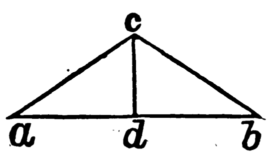
therefrom to the abutments. When suspension chains are used, it may properly be called a *suspension bridge.* If braces be employed, it is usually termed a *truss-bridge.*

HORIZONTAL ACTION OF OBLIQUE MEMBERS.

VII. Before advancing further, it will be proper to refer to an important principle or fact which has not yet been taken into account, though a fact by no means of secondary interest.

The sustaining of weight by oblique forces, gives rise to *horizontal* forces, for which it is necessary to provide counteraction and support, as well as for the weight of the structure and its load. The two equal and equally inclined braces, *ac* and *bc*, Fig. 2, in supporting the weight *w* at *c*, act in the directions of their respective lengths, each with a certain force, which is equivalent to the combined action of a vertical and a horizontal force, [*Elementary Mechanics—Statics,*] which may be called the vertical and horizontal *constituents* of the oblique force. These two constituent forces bear certain determinate relations to one another, and to the oblique force, depending upon the angle at which the oblique is inclined.

FIG. 2.



Now, we know that the vertical constituent alone contributes to the sustaining of the weight, and consequently, must be just equal to the weight sustained, in this case equal to $\frac{1}{2}w$. We know moreover, from the principles of statics, that three forces in equilibrio, must have their lines of action in the same plane, and meeting at one point; and must be respectively proportional to the sides of a triangle formed by lines drawn parallel with the directions of the three forces; and that each of the three forces is equal and opposite to the resultant of the combined action of the other two. We have, then, at *c*, the weight $\frac{1}{2}w$, the oblique force in the line *ac*, and a third force, equal and opposite to the horizontal constituent of the oblique force in the line *ac*. Then, letting fall the vertical *dc*, and drawing the horizontal *ad*, the sides of the triangle *acd*, are respectively parallel with the three forces in equilibrio at the point *c*. Hence, representing the vertical *cd*, by *v*, the horizontal *ad*, by *h*, and the oblique by *o*; and calling the horizontal force *x*, and the oblique force, *y*, we have the following proportions:

(1). $\frac{1}{2}w : x :: v : h$, whence, $x = \frac{1}{2}w\frac{h}{v}$

(2). $\frac{1}{2}w : y :: v : o$, whence, $y = \frac{1}{2}w\frac{o}{v}$

But $\frac{1}{2}w$ equals the weight sustained by the oblique *ac*. Therefore, from the two equations above deduced, we may enunciate the following important rule:

The horizontal thrust of an oblique brace, equals the weight sustained, multiplied by the horizontal and divided by the vertical reach of the brace; and the *direct* thrust (in the direction of its length), equals the weight sustained multiplied by the length, and divided by the vertical reach of the brace.

VIII. Now, it is obvious that the brace exerts the same action, both vertically and horizontally, at the lower, as at the upper end, though in the opposite directions; the brace being simply a medium for transmitting the action of weight from the upper to the lower end of the brace. Hence, the weight sustained by the brace *ac*, exerts the same vertical pressure at the point *a*, as it would do if resting at that point, while the brace requires a horizontal resistance to prevent its sliding to the left, as would be the case if its foot simply rested upon a smooth level surface. This horizontal resistance may be provided by abutments of such form, weight, and anchorage in the earth, as to enable them to resist horizontally as well as vertically, or by a horizontal tie, in the line *ab*, connecting the feet of opposite braces.

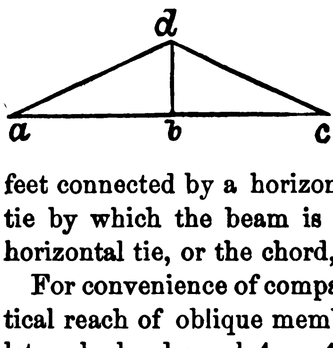
These two methods are both feasible to a certain extent, and in certain cases; and, both involve expense. Under particular circumstances, it may be a question whether the former should not be resorted to, wholly or partially. But for general practice, in the construction of bridges for heavy burthens, such as rail road bridges, and especially iron truss bridges, where expansion and contraction of materials produce considerable changes, it is undoubtedly best to provide means for withstanding the horizontal action of obliques, within the superstructure itself; and this principle will be adhered to in the discussions following.

The preceding remarks and illustrations as to the action of braces, or thrust obliques, obviously apply in like manner to obliques acting by tension, with only the distinction, that in the latter case, the weight is applied at the lower, and its action transmitted to the upper end of the oblique, and the horizontal action (at the remote end), is inward, and toward the vertical through the weight, instead of outward; and consequently, must be counteracted by outward thrust, as by a rigid body between the points *p p'*, Fig. 1, or by heavy towers, and anchorage capable of withstanding the inward tendency. Hence, in applying the rule before given, to *tension obliques*, and their vertical and horizontal constituents, the word *pull* should be substituted for the word *thrust*, wherever the latter occurs in said rule.

TWO PANEL TRUSSES.

IX. There are three forms of truss adaptable to bridges with a single central beam or cross bearer (which may be called two panel trusses), the general characteristics of which, are respectively represented by Figures 3, 4 and 5. Fig. 3 represents a pair of rafter braces, with feet connected by a horizontal tie, and with a vertical tie by which the beam is suspended at or near the horizontal tie, or the chord, as usually designated.

FIG. 3



For convenience of comparison, let $bd = v, = 1, =$ vertical reach of oblique members in each figure. Also, let each chord equal $4v, = 4$, and the half chord $= 2 = h =$ horizontal reach of obliques in Figs. 3 and 4. Then ad, Fig. 3, equals $\sqrt{h^2+v^2} = \sqrt{5}$, and if the truss be loaded with a weight w, at the point b, bd will have a tension equal to w, and abc, [see rule at end of Sec. VII], a tension equal to $\frac{1}{2}w$, (= weight sustained by ad), multiplied by the horizontal, and divided by the vertical reach of ad; that is, equal to $\frac{1}{2} w\frac{h}{v}, = \frac{1}{2}w \frac{2}{1}, = w$; while ad suffers compression from end to end, equal to $\frac{1}{2} w \frac{ad}{v}$. But $ad = \sqrt{5}$, and $v = 1$. Whence $\frac{1}{2}w\frac{ad}{v} = \frac{1}{2}w\sqrt{5}$.

Now, as the cross-section of a piece, or member, exposed to tension (or to thrust, when pieces are similar in figure), should be as the stress, it follows that the *weight* of each such member should respectively, be as the stress sustained, multiplied by the length, + an additional amount taken up in forming connections; which latter, for purposes of comparing the general economy of different plans, may be neglected.

X. Then, representing by M, the amount of material required to sustain a stress equal to w, with a length equal to bd, = 1, we have only to multiply the stress of a member in terms of w, by the length in terms of bd, or v, and change w to M, to obtain the amount of material required for the member in question, omitting the extra amount in the connections. Hence, the length of the vertical tie bd, being equal to 1, and having a stress equal to w, requires an amount of material equal to 1M.

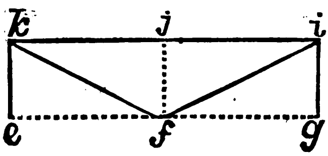
For the horizontal tie, or chord, length = 4, and stress (as seen above), = w, whence material = 4M. This added to 1M, required for the vertical, makes a total of 5M, for material exposed to tension in truss 3.

The two thrust braces, as already seen, sustain compression equal to $\frac{1}{2}w\sqrt{5}$, which multiplied by length, $\sqrt{5}$, and w changed to M, give material = $\frac{5}{2}$ M, for each, or 5 M, for the two.

XI. In the case of truss Fig. 4, the obliques manifestly sustain a weight = $\frac{1}{2}w$, by tension, giving stress = $\frac{1}{2}w\sqrt{5}$, which multiplied by length, = $\sqrt{5}$, gives $\frac{5}{2}$M = material for each, and 5M, for the two. The compression of ki, equals $\frac{1}{2}w \times h = \frac{1}{2}w \times 2 = w$, while the length = 4, whence, material = 4M; and, each end post sustaining $\frac{1}{2}w$, with length = 1, the two require material = M, making the whole amount of thrust material = 5M.

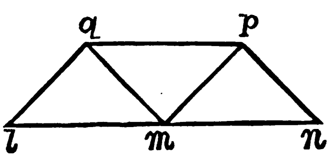
Thus we see that the two plans require each the same precise amount of material for sustaining both tension and thrust, upon the supposition that the material is capable of sustaining the same stress to the square inch of cross-section, in the one plan as in the other. This is true as to tension material; but with regard to thrust material, the power of withstanding compression, varies with the ratio of length to diameter of pieces, as well as with the form of cross-section; and it will hereafter be seen, that in this respect, plan 3 has the advantage in having the compression sustained mostly by shorter pieces, unless *ki* be supported vertically and laterally by a stiff connection with the beam at *f*, which would increase the amount of material.

FIG. 4.



XII. Plan Fig. 5 has three members (*ln*, *mp* and *mq*) exposed to tension, and the remaining three exposed, to compression. With the same length and depth of truss, and the same load = w, at m, and with obliques equally inclined (at 45°), it is manifest that the vertical and horizontal reaches, each for each, is equal to 1, and the length, equal to $\sqrt{2}$; while the weight sustained by each equals $\frac{1}{2}w$. Hence, the action (of tension or compression), equals $\frac{1}{2}w\sqrt{2}$, and the material equals $\frac{1}{2}\sqrt{2}\times\sqrt{2}$.M = 1M;

FIG. 5.



making for the four pieces, 2M for tension, and 2M for compression.

The tie or chord *ln*, suffers tension equal to the horizontal constituent of the thrust of *ql*, manifestly equal to the weight sustained by *ql*, or equal to ½ *w*. Therefore, the length being equal to 4, the material required in its construction, equals 2M. The remaining member *pq* (= 2) sustains compression equal to the combined horizontal constituents of the tension of *mq*, and the compression of *ql*, each of said constituents equal to ½*w*, making compression of *pq*, equal to *w*, and length being 2, material = 2M

We have therefore, for this plan of truss, 4M, for thrust material, and 4M for tension material, which is ⅙ less than in case of Figs. 3 and 4. Consequently, this plan is decidedly more economical than either of the others, unless the compression material acts with better advantage in the latter than the former; that is, unless the thrust members in 3 and 4, have a greater power of resistance to the square inch of cross-section, than those in Fig. 5.

XIII. As to this, both theory and experiment prove, as will be shown in a subsequent part of this work, that the long thrust members in bridge trusses, are liable to be broken by deflection, rather than by a crushing of the material; that in pieces with similar cross-sections, with the same ratio of length to diameter, the power of resistance to the square inch is the same. That, since the cross-section is as the square of the diameter, and the diameters (in similar pieces), as the lengths, the *absolute* powers of resistance (being as the cross sections), are as the squares of the lengths.

Hence, if the compressive forces acting upon two pieces of different lengths, be to one another as the squares of the lengths of pieces respectively, and the diameters be as the lengths, the forces are as the cross-sections, and proportional to the power of resistance in each case, and the material in the two pieces, acts with equal advantage, as far as regards cross-section, so that the products of stress into length of pieces, are the true exponents of amount of material required in the two pieces respectively. It follows, that, if on dividing the forces acting upon the pieces in question respectively, by the squares of the lengths, the quotient be the same in both cases, the two pieces have the same power of resistance to the square inch, and in general, the greater the value of such quotient, the greater the power per inch, and the greater the economy, though not necessarily in the same precise ratio.

XIV. Applying this rule to thrust members in plan Fig. 3, being the braces, the compressive force equals $\frac{1}{2}w\sqrt{5}$, and square of length $=5$. Hence the quotient $\frac{1}{10} w \sqrt{5} = 0.2236w$.

The piece *ki* Fig. 4, has length $=4$ and compression $=$ w, whence, force divided by square of length gives $\frac{1}{16}w$ $= 0.0625w$. This shows the material to be capable of sustaining much more to the square inch in the former, than in the latter case, though it does not give the true ratio. On the other hand, *ek* and *gi*, with length $=1$, and stress $= \frac{1}{2}w$, give quotient $= \frac{1}{2}w = 0.5w$. Hence, with similar cross-sections, these parts have greater power to the inch than either of the former, but not enough to balance the inferiority of *ki*, as compared with *ad* and *dc*, in Fig. 3.

With regard to truss Fig. 5, *ql* and *pn*, suffer each compression equal to $\frac{1}{2}w\sqrt{2}$, with square of length = 2, giving quotient = $\frac{1}{4}w\sqrt{2}$, = $0.371w$, while *pq*, has compression = w, and square of length = 4, and quotient = $\frac{1}{4}w = 0.25w$. Hence it appears that this plan not only possesses a decided advantage in the less *amount of action** upon materials, but also, a considerable advantage as to ability of compression, or thrust members, to withstand the forces to which they are exposed.

XV. Still another modification for a truss to support a single beam, is formed by reversing Fig. 3, thus converting tension members into thrust members, and *vice versa;* the oblique members falling below, instead of rising above the grade, or road-way of the bridge. In this case, the long horizontal thrust member *ac*, is divided and supported in the centre, and its economy of action becomes the same as that of *pq*, in Fig. 5; and the truss gives the same exponents for both thrust and tension material as when in the position of Fig. 3. This arrangement affords no side protection, and is not always admissible, on account of interference with the necessary open space beneath.

DEDUCTIONS.

XVI. We seem to learn from what precedes, that:

(1). Since all heavy bodies not in motion toward, or not approaching the centre of the earth (or receding from it under the influence of previous impulse), exert a pressure equal to their respective weights [VIII],

* By the expression, *amount of action*, is meant, the sum of products of stresses into lengths of parts, or members.

either directly or indirectly upon the earth; and, since, a body crossing a bridge, having (as bridges are always supposed to have), a void space underneath, preventing a *direct* pressure, it follows, that every such body exerts an *in*direct pressure at some point or points at greater or less horizontal distance from the body.

(2). That the pressure of a body at a point or points not directly below it, can only take place through one or more intermediate bodies, or members, capable of exerting (by tension or thrust), one or more oblique forces upon the first named body, and it is the office of a bridge to furnish the medium of such horizontal transfer of pressure [IV].

(3. That a single oblique force can not alone prevent a heavy body from falling toward the earth (since two forces can only be in equilibrio when acting oppositely in the same line), and that each oblique force is equal to the combined action of a vertical and a horizontal constituent, of which the first alone is equal to the weight sustained and transferred by the oblique member, while the horizontal constituent, acting at both extremities of the oblique medium, must be counteracted by means outside of the oblique and the weight sustained by it; which means are usually to be supplied by other members of the structure [VIII].

(4). The direct force exerted by an oblique member (in the direction of its length), is equal to the weight sustained, multiplied by the length, and divided by the vertical reach of the oblique, while the horizontal constituent equals the weight sustained multiplied by the horizontal, and divided by the vertical reach of the oblique [VII].

(5). The amount of material required in a tension member, is as the stress multiplied by the length of the member [IX] (disregarding extras in connections), and the same is true of thrust members of similar formed cross-sections, sustaining stress proportional to the square of the length of pieces respectively.

(6). The respective stresses of two thrust members, divided by the squares of respective lengths, give quotients indicative of, though not proportional to, the relative efficiency of material in the two members, — the greater quotient showing the greater efficiency, or greater power of resistance to the square inch of cross-section [XIII].

With these rules or principles in view, we may proceed advantageously with general analyses and comparisons of different plans, or systems of bridge trussing, adapted to different lengths of span.

THREE PANEL TRUSSES.

XVII. In structures exceeding 25 or 30 feet in length, the length of joists from the centre to the ends, would require cross-sections so great, to give them the requisite stiffness, that their weight and cost would become objectionable. It becomes expedient, then, in such cases, to provide support for more than one principal point, or transverse beam, or bearer. A superstructure from 30 to 40 feet long, may be constructed with two cross beams, supported by two trusses with two pairs of braces each, with the feet connected by a horizontal tie or chord, as seen in Fig. 6.

The cross beams, may be at *b b′*, or suspended at *c* and *d*, at equal horizontal distances from *a a′*, and from one another; which latter position they will be regarded as occupying in this instance. Or, the figure may be inverted, thus reversing the action of the several thrust and tension members.

XVIII. Another, and a more common form of truss for two beams, is shown in Fig. 7. These may be called three panel trusses.

FIG. 6.

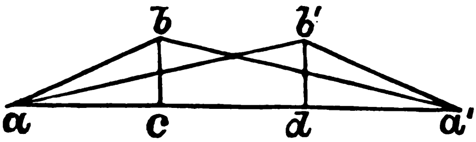
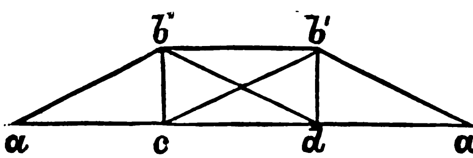


Fig. 7.



To compare these two trusses, suppose the two to have the same length and depth, and to be loaded with uniform weights, w, at the two points c and d in each. Then, since we know from the principle of the lever, that each weight produces upon each abutment, a pressure inversely as its horizontal distance from them respectively, and that the pressure upon the two abutments is equal to the weight producing it, it follows that ab, Fig. 6, sustains $\frac{2}{3}w$, and compression $= \frac{2}{3}\frac{ab}{bc}w$. Hence making $ab = \text{D}, \ldots\ bc = v$, and $ac = h, \ldots\ \frac{2}{3}\frac{ab}{bc}w$, becomes $\frac{2}{3}\frac{\text{D}}{v}w$, and multiplying this stress by length, $= \text{D}$, and changing w to M*, we have for material in ab,... $\frac{2}{3}\frac{D^2}{v}$ M. But $D^2 = h^2 + v^2$, whence $\frac{2}{3}\frac{D^2}{v}M = \frac{2}{3}\left(\frac{h^2}{v} + v\right)M = \left(\frac{2h^2}{3v} + \frac{2v}{3}\right)M$.

Again, ab' sustains $\frac{1}{3}w$, with length $= \sqrt{4h^2 + v^2}$, and by multiplying and changing as in case of ab, we obtain material in ab', $= \left(\frac{4h^2}{3v} + \frac{v}{3}\right)$ M, which added to amount for ab, gives $\left(\frac{6h^2}{3v} + v\right)$ M for the two braces, and $\left(\frac{4h^2}{} + 2v\right)$M for the four.

The horizontal thrust of $ab = \frac{2}{3}w\frac{h}{v}$ while that of ab' $= \frac{1}{3}w\frac{2h}{v} = \frac{2}{3}w\frac{h}{v}$. Hence the horizontal thrust of ab and $ab = \frac{4}{3}w\frac{h}{v} =$ tension of aa', and material for chord aa', equals $3 \times \frac{4}{3}\frac{h^2}{v}M = \frac{4h^2}{v}M$. Tension of bc and $b'd$, each, equals w, and material for the two $= 2\,v$ M, which added to amount in aa', makes the whole tension material equal to $\left(\frac{4h^2}{v} + 2v\right)$ M, being the same co-efficient of M as was obtained for compression.

In truss Fig. 7, ... ab and $a'b'$ $(= D = \sqrt{h^2 + v^2})$, evidently sustain each a weight equal to w, and a stress $= \frac{\sqrt{h^2 + v^2}\,w}{v}$. Whence, material $= \left(\frac{h}{v} + v\right)$ M for each, and $\left(\frac{2h^2}{v} + 2v\right)$ M for both, while bb', equal to h, sustains compression equal to the horizontal thrust of ab, equal to $\frac{h}{v}w$, and requires material equal to $\frac{h^2}{v}$ M, making, with amount in braces ab $a'b'$, $\left(\frac{3h^2}{v} + 2v\right)$ M.

Now we have just seen that the horizontal thrust of ab, equal to the tension of chord aa', equals $\frac{h}{v}w$, and the

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* When v is used in the co-efficient of M, then M represents the product of the stress, in terms of w, by length according to any assumed unit, which may be equal to v or not.

length being $3h$, the material consequently equals $\frac{3h^2}{v}$ M, to which add $2v$M for verticals, and we have $\left(\frac{3h^2}{v}+2\,v\right)$M = whole amount of tension material in truss 7, which is less by $\frac{h^2}{v}$ M, than in the case of truss Fig. 6.

XIX. With regard to thrust material, it is clear that while in truss 6, the weight on either pair of braces, is transferred in due proportion to both abutments, independently of the other braces, whether one or both pair be loaded; on the contrary, truss 7, when c only is loaded, must transfer $\frac{1}{3}w$ to a', which can only be done through $a'b'$, the only oblique member acting at the point a'. Moreover, the weight must be communicated to $a'b'$, at the point b', through the thrust of bd, and the tension of db', assuming bd and cb' to be thrust members. Now, as either c, or d, is liable to be loaded with weight equal to w, while the other is unloaded, it follows that both bd and cb' are liable to sustain weight equal to $\frac{1}{3}w$, and require thrust material equal to $\frac{1}{3}\left(\frac{h^2}{v}+v\right)$ M for each, or $\left(\frac{2h^2}{3v}+\frac{2h}{3}\right)$M for the two. The whole amount of thrust material for truss 7, then, equals $\left(\frac{3h^2}{v}+2\ v\right)$M, (the amount found above) + $\left(\frac{3h^2}{2v}+\frac{2}{3}v\right)$M, equal to $\left(\frac{3\frac{2}{3}h^2}{v}+2\,\frac{2}{3}\,v\right)$M, against $\left(\frac{4h^2}{v}+2v\right)$M for truss 6; the difference being $\left(\frac{\frac{1}{3}h^2}{v}-\frac{2}{3}v\right)$M. If this be a positive quantity, the balance is in favor of truss 7, and if negative, in favor of tress 6, as regards amount of action on thrust material; while, if $\frac{\frac{1}{3}h^2}{v}-\frac{2}{3}\,v$ = zero, the amount of thrust action is the same in both trusses.

Either of these suppositions may be true, according to the relative values of h and v. If $h = v\surd 2$, then $\frac{\frac{1}{3}h^2}{v} - \frac{2}{3}v = o$. If h, be greater than $v\surd 2 - \frac{1}{3}\frac{h^2}{v} - \frac{2}{3}v$ is positive, and if h be less than $v\surd 2$, the value is negative. But the amount is trifling in any probable relation of h and v, and may be disregarded in this general comparison.

Calling then, the *amount of action* upon thrust material in the two plans equal, there is a probable advantage in favor of truss 7, as to efficiency of thrust material, while the latter truss, shows a positive advantage over truss 6 in amount of tension material, equal to $\left(\frac{4h^2}{v} + 2v\right) - \left(\frac{3h^2}{v} + 2v\right) \text{M} = \frac{h^2}{v} \text{M}$. This is equal to 4M, when $h = 2v = 2$; which is in tolerable proportion for the trusses under discussion, and, substituting these values of h and v in the expressions of tension material in the trusses respectively, we have $(16 + 2)\text{M}$ for truss 6, and $(12 + 2)\text{M}$ for truss 7, being about $28\frac{1}{2}$ per cent more for 6 than for 7.

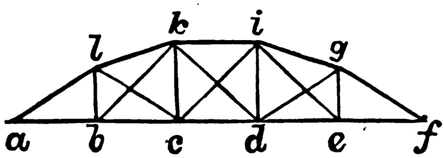
The same difference would appear with Fig. 6 inverted, the thrust and tension action being the same in amount of each, only sustained by different members, thrust members in one case becoming tension members in the other.

FIVE PANEL TRUSSES.

XX. Truss Fig. 7, may be increased in length and number of panels, by introducing additional panels between the end triangular panels, and the rectangular centre one either of an oblique form, as in Fig. 8, which represents an arch-truss, or of a rectangular form as in

Fig. 10. The truss on the plan of Fig. 6, may be lengthened by introducing additional pairs of independent braces, as seen in Fig. 9.

FIG. 8.



For the analysis of these trusses, using the same notation as before, as far as applicable, that is, making v = verticals ck, in 8 and 10, and equal to nn', pm', etc., in Fig. 9; $h = al$, = width of panel in each figure, = $\frac{1}{5}$ whole chord; w = uniform weights at the four bearing points in each, and M = weight of material required to sustain a stress equal to w, with length equal to 1; then, making $lb = \frac{2}{3}v$ in truss 8, it is obvious that the two abutments at a and f together sustain $4w$, with the common centre of gravity of all the weights midway between abutments, whence each abutment sustains $2w$, equal to weight sustained by al. The compression of al therefore equals $\frac{2al}{\frac{2}{3}v}w$. But $al = \sqrt{h^2 + \frac{4}{9}v^2}$, which substituted in last preceding expression, gives $\frac{3}{2}\frac{\sqrt{h^2 + \frac{4}{9}v^2}w}{v}$, = compression of al. Whence, multiplying by length, $\sqrt{h^2+\frac{4}{9}v^2}$, and changing w to M, we have $\left(\frac{3h^2}{v}+1\frac{1}{3}v\right)$ M = material for al.

The horizontal thrust of al, [XVI (4)] equals $2w\frac{h}{\frac{2}{3}v} = \frac{3h}{v}w$, = tension of chord af.

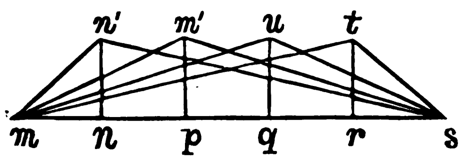
The oblique member lk, sustains weight = w (through the vertical ck), and has a vertical reach = $\frac{1}{3}v$, whence it suffers compression equal to $\frac{lk}{\frac{1}{3}v}w, = \frac{3lk}{v}w, = \frac{3\sqrt{h^2 + \frac{1}{9}v^2}}{v}w$, and requires material equal to $\left(\frac{8h^2}{v} + \frac{8}{9}v\right)$M, while its horizontal thrust equals $\frac{h}{\frac{1}{3}v}w, = \frac{3h}{v}w, =$ compression of *ki*, by which it is contracted. The material required for *ki*, therefore, $= \frac{3h^2}{v}$M. Material for *ig* and *gf*, is the same as above found for *al* and *lk*, and, doubling those quantities, and adding amount just found for *ki*, we obtain $\left(\frac{15h^2}{v} + 3\frac{1}{3}v\right)$ M, = material in the whole arch.

The tension of the chord *af* (Fig. 8), has been seen to be equal to $\frac{3h}{v}w$, whence, multiplying by the length, $5h$, and changing w to M, we have $\frac{15h^2}{v}$M, = material for chord.

The 4 verticals sustain each, weight $= w$, and the aggregate length being $3\frac{1}{3}v$,... material $= 3\frac{1}{3}v$M. This, added to amount in chord, gives $\left(\frac{15h^2}{v} + 3\frac{1}{3}v\right)$M, = tension material required to support a full uniform load, as above assumed. But since any number of the points *b*, *c*, *d*, *e*, are liable to be loaded while the others are unloaded, it is obvious that in such case, the arch will not be in equilibrio, the loaded points tending to be depressed, while the unloaded, tend to be thrust upward. Hence the arch requires the action of the obliques, or diagonals, in the three quadrangular panels, to counteract such tendency; and, as will appear further on, these members will require material equal to about one-third of the amount required in the chord, thus increasing the amount of tension material for the truss to about $\left(\frac{20h^2}{v} + 4\frac{4}{9}v\right)$M.

XXI. In truss Fig. 9, each brace obviously sustains a portion, x, of the weight w, which is to w, as the horizontal reach of its antagonist, as to horizontal action, or its fellow and assistant vertically, is to the whole length of chord; that is, the weight x, bearing at m, through mn', is to w, as ns to ms; or, $x : w :: 4 : 5$. Hence $x = \frac{4}{5}w$. This, multiplied by the horizontal reach, equal to h, and divided by v, gives the horizontal thrust of the brace, equal to $\frac{4}{5}\frac{h}{v}w$.

FIG. 9.



In like manner, mm' sustains a weight x', which is to w, as ps to ms, i. e., $x' : w :: 3 : 5$, whence $x' = \frac{3}{5}w$, and the horizontal thrust $= \frac{3}{5}\frac{2h}{v}w = \frac{6}{5}\frac{h}{v}w$; and in general, the horizontal thrust of a brace in this kind of truss, equals w, multiplied by the product of the number of sections of chord at the right and the left of the point of application (of the weight), and divided by the whole number of chord-sections, and, by the vertical reach (v) of the brace.

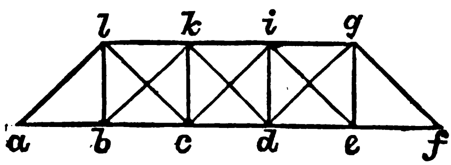
But the horizontal thrust of mn' equals that of $n's$, = that of mt, and the horizontal thrust of mm', equals that of $m's$ = that of mu, whence, horizontal thrust of the 4 braces bearing at m, equals twice that of mn' and mm', together, $= 2\left(\frac{4}{5}\frac{h}{v} + \frac{6}{5}\frac{h}{v}\right)w = \frac{20h}{5v}w = 4\frac{h}{v}w$. This being equal to the tension of the chord, multiplying by length $5h$, and changing w to M, gives material for chord $= \frac{20h^2}{v}$M. Adding to this, $4v$M, for 4 verticals with stress $= w$, and length $= v$, each, makes whole amount of tension material equal to $\left(\frac{20h^2}{v} + 4v\right)$M, being very nearly the same as for truss, Fig. 8.

XXII. As for braces in truss 9, we have already seen that each brace sustains weight equal to w, multitiplied by the number of panels crossed by its fellow, and divided by the number of panels in the whole truss. Hence mn' sustains $\frac{4}{5}w$, with length equal to $\sqrt{h^2+v^2}$. Therefore, stress $= \frac{4}{5}\frac{\sqrt{h^2+v^2}}{v}w$, which multiplied by length, and w changed to M, gives material for $mn' = \frac{4}{5}\left(\frac{h^2}{v} + v\right)$M, $= \left(\frac{4h^2}{5v} + \frac{4v}{5}\right)$M. mm' sustains $\frac{3}{5}w$ with length $= \sqrt{4h^2 + v^2}$, whence material $= \frac{3}{5}\left(\frac{4h^2}{v} + v\right)$M, $= \left(\frac{12h^2}{5v} + \frac{3v}{5}\right)$M. mu sustains $\frac{2}{5}w$ with length $= \sqrt{9h^2 + v^2}$, and material equals $\left(\frac{18h^2}{5v} + \frac{2v}{5}\right)$M, while mt, sustains $\frac{1}{5}w$, with length $= \sqrt{16h^2+v^2}$ requiring material $= \left(\frac{16h^2}{5v} + \frac{v}{5}\right)$M. Then, adding and doubling these amounts, we obtain $\left(\frac{20h^2}{v} + 4v\right)$M, against $\left(\frac{15h^2}{v} + 3\frac{1}{3}v\right)$M, for truss 8; a difference of about 30.6 per cent when $h = v$, and about 32.6 per cent when $h = 2v$.

Thus, truss Fig. 8 has over 30 per cent advantage over truss Fig. 9, in the economy of *amount* of *action* upon thrust material, with the advantage as to *efficiency* of action of this material, undoubtedly, also on the side of truss 8. Tension material is nearly the same in both.

XXIII. If truss Fig. 9 be inverted, dropping the oblique members below the road-way of the bridge, thus reversing the action of thrust and tension members, the thrust material would act with nearly equal advantage in both plans, and with about the same amount of action. But the 30 per cent advantage as to amount of action upon *tension* material, would still be in favor truss 8. Besides, it is only in exceptional cases that this arrangement can be adopted, on account of interference with the necessary space below the bridge.

FIG. 10.



XXIV. In truss Fig. 10, suppose the points *b*, *c*, *d*, *e*, to be loaded successively from left to right, with uniform weights equal to *w* each, and suppose the truss to be without weight, as we have hitherto done. When *b* alone is loaded, $\frac{1}{5}w$ must bear at *f*, [XVIII] which may be effected, either by tension of *bl*, thrust of *lc*, tension of *ck*, thrust of *kd*, &c., by tension vertical and thrust diagonal alternately, till it reaches *f*; acting in its course upon 4 verticals, and 4 obliques, with a weight upon each, equal to $\frac{1}{5}w$. Or, the weight may be transferred by tension of *bk*, *ci* and *dg*, and thrust of *kc*, *id* and *gf*. These alternatives are subject to the control of the builder, and he will form and connect the parts accordingly. Let it be assumed that the truss has tension diagonals, and thrust uprights at *c* and *d*, while *lb* and *eg* are necessarily tension members in all cases, in practice.

Again a weight, w, at c, must cause pressure equal to $\frac{2}{5}w$ at f, through tension of *ci* and *dg*, and thrust of *id* and *gf*. This, with the $\frac{1}{5}w$, from the weight at b, makes $\frac{3}{5}w$, acting on *ci*. But the weight at c, causes pressure equal to $\frac{3}{5}w$ at a, necessarily through tension of *cl*; and since *cl* and *bk* are antagonistic, the action upon one tending to produce relaxation of the other, it follows that only one can act at the same time, unless unduly strained in the adjustment of the truss. Hence, the $\frac{1}{5}w$, which acts upon *bk*, when b alone is loaded, is overbalanced by the $\frac{3}{5}w$, tending to act upon *cl*, on account of the load at c; and the result is, that *bk* is relaxed, the whole weight at b, is necessarily sustained by *bl* and *la*, and the $\frac{3}{5}w$, which must by a statical necessity, bear at f, in consequence of the loads at b and c, is all made up from the weight at c, leaving only $\frac{2}{5}w$ of this weight to bear at a, through *cl*. Now, since it is obvious that all load at c, d, or e, must contribute to the pressure at a, which can only occur through action upon *cl*, it follows that *bk* can only sustain the whole weight of $\frac{1}{5}w$, when the point b alone is loaded; and consequently, that $\frac{1}{5}w$ is the greatest weight that *bk* can ever be subjected to.

Then, applying another weight, w, at d, it must add $\frac{3}{5}w$ to the pressure at f, through tension of *dg* and thrust of *gf*; which last amount, added to $\frac{3}{5}w$, communicated to *dg* through *ci* and *id*, makes $\frac{6}{5}w$, as the weight sustained by *dg*. But the weight at d, also causes pressure at a, equal to $\frac{2}{5}w$, which can only be done through action, or tendency to action upon *dk*, and since *dk* and *ci* are antagonists, only one can act at once, and that, only with a force equal to the excess of tendency to action of the one, over that of the other. Now we have seen that weights at *b* and *c*, tend to throw $\frac{3}{5}w$ upon *ci*, while the weight at *d*, tends to throw $\frac{2}{5}w$ upon *dk*. Hence, in these circumstances, *ci* only sustains $\frac{1}{5}w$, which is transferred to *dg* through thrust of *id*, while *dk* is relaxed, and the whole weight at *d*, is sustained by *dg*; making, with the $\frac{1}{5}w$ from *ci*, just above mentioned, $\frac{6}{5}w$, equal to the pressure due upon the abutment at *f*, on account of weights at *b*, *c* and *d*.

Lastly, a weight, *w*, at *e*, tends to give pressure equal to $\frac{4}{5}w$ at *f*, through *eg* and *gf*, and a pressure equal to $\frac{1}{5}$ *w* at *a*, through *ei*, *dk*, etc. This latter tendency has the effect to diminish by $\frac{1}{5}w$, the tendency of previously imposed weights, to throw $\frac{6}{5}w$ upon *dg*, reducing it to $\frac{5}{5}w$, and to neutralize the balance of $\frac{1}{5}w$ acting upon *ci*, after the imposition of the weight at *d*, leaving *ci* and *dk* both inactive, while *eg* sustains the whole weight applied at *e*, equal to *w*.

Now, as we have seen, any weight at *d* or *e*, tends to throw action upon *dk*, thereby diminishing action upon *ci*, and since weight at *b* and *c*, both contribute to the stress of *ci*, it follows that the maximum action upon *ci*, occurs when *b* and *c* are loaded, and *d* and *e*, unloaded.

For similar reasons, the maximum action upon *dg*, occurs when *e* alone is unloaded.

The maximum weight sustained by *lb*, and *eg*, is the weight applied directly at each of the points *b* and *e*, equal to *w*, and the maximum weights sustained by *ei*, *dk*, and *cl*, are the same as those sustained by *bk*, *ci* and *dg*, each respectively, as just above determined; while *al* and *gf*, both receiving action from weight on any part of the truss, obviously sustain their maximum weight, equal to 2*w*, under the full load of the truss.

The section ab, of the lower chord, suffers a stress equal to the horizontal thrust of al, which of course, is greatest when al sustains the greatest weight. This has just been seen to be equal to $2w$, and occurs under a full load of the truss. Hence the greatest stress upon ab equals $2w\frac{h}{v}$, and is communicated without change to bc, bk being inactive when the truss is fully loaded. The section cd, suffers stress equal to the combined horizontal action of al and lc, which must be greatest when this combined action is greatest. That is also under the full load of the truss. For, though lc sustains $\frac{1}{5}w$ more weight when b is unloaded, the same cause relieves al of the amount of $\frac{4}{5}w$. Consequently, the weight borne by the two, is $\frac{3}{5}w$ less in this case, than when the truss is fully loaded. The greatest combined weights, then sustained by al and lc, being equal to $3w$, the greatest stress of cd equals $3w\frac{h}{v}$. This is also the greatest compression suffered by the upper chord lg, since the latter is also equal to the combined horizontal thrust and pull of al and lc. The stress of this chord is the same throughout, because the obliques meeting at k and i, are inactive when the truss is loaded throughout.

The maximum compression upon ck and id, equals the greatest weight sustained by ci and dk, already found to be equal to $\frac{3}{5}w$.

XXV. Having thus ascertained the greatest weights sustained by the several oblique members, and the greatest stresses of the horizontals and verticals, we may deduce the required amount of material, or, perhaps more properly, the *amount of action* upon the material required for the truss, as compared with like amount of action in trusses 8 and 9, thus: Max. weight on end braces, $2w \times \text{length}\sqrt{h^2 + v^2} = \text{stress} = \dfrac{2w\sqrt{h^2 + v^2}}{v}$.

Hence, action upon material for the two = ...	$\left(\frac{4h^2}{v} + 4v\right)$M.
Max. weight on 2 verticals = $\frac{3}{5}w$ × length of the two (= $2v$), gives......	$1\frac{1}{5}v$ M.
Max. stress of upper chord = $3w\frac{h}{v}$ × length (= $3h$), gives amount of action = ...	$9\frac{h^2}{v}$ M.
Making total amount of action on thrust material = ...	$\left(\frac{13h^2}{v} + 5\frac{1}{5}v\right)$ M
Aggregate max. weight on 6 tension diagonals = $\frac{20}{5}w = 4w$. This by the length (= $\sqrt{h^2 + v^2}$), gives stress = $4\,w\frac{\sqrt{h^2 + v^2}}{v}$; whence amount of action on material, equals	$\left(\frac{4h^2}{v} + 4v\right)$M.
2 tension verticals sustain each, $1w$, with length = v, giving amount of action for the two = ...	$2v$ M.
Stress of middle section, lower chord = $3w\frac{h}{v}$, × length (= h), gives action	$3\frac{h^2}{v}$M.
4 remaining sections, with stress = $2w\frac{h}{v}$ × length (= $4h$), give	$8\frac{h^2}{v}$M.
Making whole amount of action on tension material = ...	$\left(15\frac{h^2}{v} + 6v\right)$M.

SYNOPTICAL STATEMENT IN REGARD TO TRUSSES (Figs. 8, 9 and 10.

No. of Plan.	References.	Amount of Action upon Materials.		
		Tension.	*Compression.*	*Total.*
8	XX	$\left(\frac{20h^2}{v}+4\frac{1}{3}v\right)$ M	$\left(\frac{15h^2}{v}+3\frac{1}{3}v\right)$ M	$\left(\frac{35h^2}{v}+7\frac{2}{3}v\right)$ M
9	[XXI XXII]	$\left(\frac{20h^2}{v}+4v\right)$ M	$\left(\frac{20h^2}{v}+4v\right)$ M	$\left(\frac{40h^2}{v}+8v\right)$ M
10		$\left(\frac{15h^2}{v}+6v\right)$ M	$\left(\frac{13h^2}{v}+5\frac{1}{5}v\right)$ M	$\left(\frac{28h^2}{v}+11\frac{1}{5}v\right)$ M

Making $h = v = 1$, the above table will be as follows:

8	$24\frac{1}{3}$M	$18\frac{1}{3}$M	$42\frac{2}{3}$M
9	24M	24M	48M
10	21M	$18\frac{1}{5}$M	$39\frac{1}{5}$M

XXVI. This shows very nearly the relative amount of tension material required in the several plans; while, as previously stated, the amount of compression material is not so nearly indicated by the figures and expressions giving the *amount of action* (sum of stresses into lengths of pieces), as in case of tension members. The compression material in No. 8 (the arch truss), is undoubtedly more efficient in action than in either of the others, while that in No. 9, is unquestionably the least so. In fact, this truss will be hardly considered as possessing advantages of any kind, sufficient to induce its adoption; and it will not be considered in the discussions and comparisons in regard to trusses of greater span, to which we may now proceed.

TRUSSES WITH SEVEN PANELS.

THE ARCH TRUSS.

XXVII. In Fig. 11, let $md = v$, and $h = ab = \frac{ai}{7}$. If each of the points b, c, d, etc., be loaded with a weight equal to w, then, in order that the arch may be in equilibrio under the effects of these weights, without any action of the diagonals, it is necessary that each section of the arch have the same horizontal thrust, since, if one section have a greater horizontal thrust than the one opposed to it at either end, the diagonals alone can sustain the surplus. And, that the sections may have the same horizontal thrust each must have a vertical reach (the horizontal reach being the same for all), proportional to the weight (W) sustained by each. For illustration, horizontal thrust being equal to $W\frac{h}{v}$, in order that this expression may represent a constant quantity, h remaining constant, v must be as W.

Now, ml being horizontal, can sustain none of the weight acting at the point m, through the vertical md; hence mn must sustain a weight equal to w. This is transferred to no [VIII], and in addition to the weight at c, makes $2w$ sustained by no. The latter weight is in turn transferred to oa, and, in addition to the weight at b, makes $3w$, to be sustained by ao. The vertical reaches, therefore, beginning with ao, should be as 3, 2 and 1; whence, ob should equal $\frac{1}{2}v$, and nc should equal $\frac{5}{6}v$.

The thrust of ao, then, equals $3w\frac{ao}{\frac{1}{2}v}$, $= 6w\frac{ao}{v}$, $=$ $6w\frac{\sqrt{h^2+\frac{1}{4}v^2}}{v}$; whence, amount of action on material* $=$ ....$\left(\frac{6h^2}{v} + 1\frac{1}{2}v\right)\times$M.

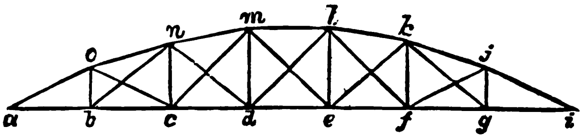
Thrust of on, $= 2w\frac{on}{\frac{1}{3}v}$, $= 6w\frac{on}{v}$, $=$ $6w\frac{\sqrt{h^2+\frac{1}{9}v^2}}{v}$, and material $=$ ....$\left(\frac{6h^2}{v} + \frac{2}{3}v\right)\times$M.

Thrust of $nm = w\frac{nm}{\frac{1}{6}v} = 6w\frac{nm}{v} =$ $6w\frac{\sqrt{h^2+\frac{1}{36}v^2}}{v}$, and material $=$...............$\left(\frac{6h^2}{v} + \frac{1}{6}v\right)\times$M.

Thrust of ml ($=$ horizontal thrust of nm), $= w\frac{h}{\frac{1}{6}v} = 6w\frac{h}{v}$, and material $=$ ....$6\frac{h^2}{v}\times$M.

Adding these amounts, and repeating the first three, we have $\left(\frac{42h^2}{v} + 4\frac{2}{3}v\right)\times$M, equal to amount of action upon the arch when fully loaded.

FIG. 11.



The stress of the chord obviously equals the horizontal thrust of ao, equal to $3w\frac{h}{\frac{1}{2}v}$, $= 6w\frac{h}{v}$; and is the same throughout, when the truss is fully loaded throughout. Hence, for the whole chord, we have, stress $= 6w\frac{h}{v}$ multiplied by length ($= 7h$), and w changed to M, $= 42\frac{h^2}{v}\times$M, representing the material required for the chord.

The above are assumed, for the present, to be the greatest stresses that any part of the chord or arch can

* By amount of action upon material, is meant the stress of a member multiplied by its length.

be subjected to, in any condition of the load ; w, being the maximum weight for any one of the sustaining points, b, c, d, &c. This is a point we shall be better enabled to verify after considering the

STRESSES OF VERTICALS AND DIAGONALS.

XXVIII. As the diagonals do not act under a full load of the truss, the verticals must each sustain a tension equal to w, when the weights are applied at the chord; and, the diagonals acting by tension, serve, when in action, to diminish the tension of verticals, or to subject them to compression, but can never *increase* their action of tension. Hence, the maximum tension stress of each vertical equals w.

In order to bring the diagonals into action, the truss must obviously be unequally loaded ; and, to determine the *maximum* stresses of the several diagonals respectively, we may begin by removing the weight at g ; (see Fig. 11A), and, to facilitate the process, let w'' represent w divided by the number of panels in the truss (= 7 in this case), i. e., $w'' = \frac{1}{7}w$. Then, the full load of the truss bearing with a weight of $3w$, $=21w''$, at i, ... with load removed from g, the bearing at i, equals $21w'' - 6w''$, $= 15w''$; and produces a thrust upon ij equal to $15w''\frac{\sqrt{h^2+\frac{1}{4}v^2}}{\frac{1}{2}v}$. Then, taking jq by any convenient scale, on ij produced, to represent the thrust of ij (reduced to w'' with a numerical coefficient, according to the proportions of the truss), and drawing qr parallel with fj, and meeting jk produced in r, it is obvious that the three forces acting at j, namely, the thrust of ij and jk, and the tension of fj, will be represented respectively by the sides of the triangle jqr, parallel respectively with the directions of those forces ; and may be measured by scale and dividers, or calculated trigonometrically.

Now, it will be seen that the greater the pressure at *i*, the greater the thrust of *ij*, represented by *jq*, and consequently, the greater the line *qr*, representing the tension of *fj*. But the pressure at *i*, is manifestly the greatest possible in the case here supposed, except when the weight at *g* is wholly or partially restored, in which case the tension of *fj* would be wholly or partially relieved. Hence, it follows that the maximum stress of *fj*, occurs when all the supporting points except *g*, have their full load, and the point *g* is without load.

Taking, then, *fs* = *qr*, and drawing *st* parallel with *gf*, *st* will represent the horizontal, and *ft*, the vertical effect (equal to $3w''$) of the action of *fj*; the former

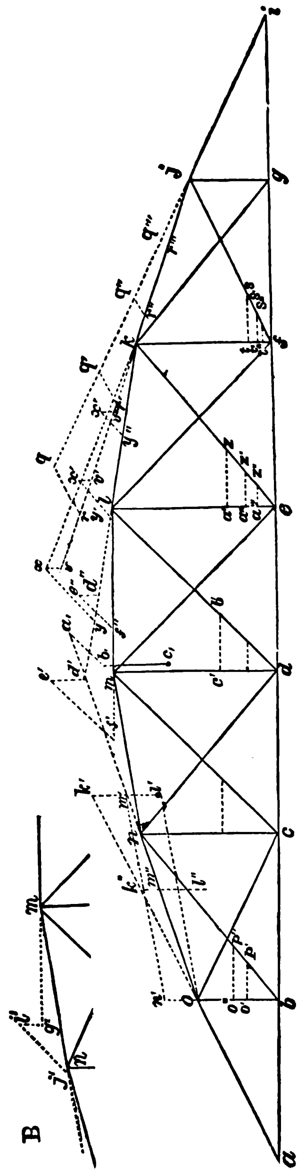


FIG. 11A.

effect being, in addition to the tension of *f i*, resisted by the tension of *ef*, while the latter is counteracted by the weight at *f*, and the tension of *fk* is thereby diminished, but not exhausted*. But if the weights were applied at the arch instead of the chord, then *fk*, in this condition, would suffer compression represented by *ft*.

XXIX. If the points *g* and *f* be unloaded, the pressure at *i* is reduced to $10w''$, and the thrust of $ij = 10w'' \frac{\sqrt{h^2+\frac{1}{4}v^2}}{\frac{1}{2}v}$. Then, taking jq' by the scale, to represent this quantity, and drawing $q'r'$ parallel with *fj*, $q'r'$, compared with the same scale, will give the stress of *fj*; from which, as in the preceding case, we obtain ft' ($= 2w''$†), to represent the vertical effect of the action of *fj*; and, there being no weight at *f*, this force

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* Since the vertical reach of *jk* is $\frac{2}{3}$ that of *ij*, and their vertical actions (horizontal thrust, and horizontal reach being the same), as their respective vertical reaches, *jk* must react downward at *j*, with $\frac{2}{3}$ of the lifting force exerted by *ij*. Then, if a weight be suspended at *g*, by the vertical *jg*, equal to $\frac{1}{3}$ of the weight bearing at *i*, the forces acting at *j*, through *ij*, *jk*, and *jg*, are in equilibrio. But if the weight at *g* be less than $\frac{1}{3}$ of the pressure at *i*, the tendency of the point *j* is upward, and exerts a lifting force upon *fj*,. But the action of *fj*, brings into play horizontal reaction in *jk*, equal to that of *fj*, which gives *jk* a depressing action at *j*, equal to $\frac{2}{3}$ of the lift of *fj*. This depressing power ot *jk*, depends on forces acting directly or indirectly at *k*, and which go to make up part of the pressure at *i*. Hence, *jk* supports at the upper end, at *k*, first, $\frac{2}{3}$ of the weight bearing at *i*, in virtue of the horizontal thrust received through *ij*, and second, $\frac{2}{5}$ of the other $\frac{1}{3}$ (when there is no weight at *g*), in virtue of the horizontal thrust communicated through *fj*. Now, *g* being alone unloaded, the bearing at *i* is $15w''$, of which, $10w''$ is received through *jk*, in virtue of horizontal thrust counteracted by *ij*. and $\frac{2}{5}$ of the other $5w''$, in virtue of horizontal thrust counteracted by *fj*, in consequence of the latter sustaining $\frac{3}{5}$of said $5w''$. Hence, *fj* sustains $\frac{3}{15}$ or $\frac{1}{5}$, while *jk* sustains $\frac{12}{15}$, or $\frac{4}{5}$ of all the weight bearing at *i*, when *g* alone is unloaded; and $3w''$, therefore, is the maximum weight sustained by *fj*.

† Since *jq* represents the action of *ij*, due to a pressure of $15w''$ at *i*, and jq', the action of *ij*, due to a pressure of $10w''$, it follows that $jq' = \frac{2}{3}jq'$; whence $q'r'$ obviously equals $\frac{2}{3}$ *qr*, and $fs' = \frac{2}{3}fs$. Consequently, $ft' = \frac{2}{3}ft$. But *ft* represents the lift of *fj*, $= 3w''$, whence ft' represents $2w''$.

is expended in causing compression upon *fk*; and is the measure of the greatest compression that member can receive through *fj*. But *fk* is also liable to compression, or a tendency thereto, from the tension of *fl* when *f* and *g*, are loaded, or *g* alone, and the other parts *un*loaded. This, however, in the former case, will never equal the weight at *f*, and in the latter, the compression will not exceed that just found, resulting from action of *fj*; as will be better understood hereafter.

Now, as *jr'* represents the thrust of *jk*, if we take *kv*, on *jk* produced, equal to *jr'*,—raise the vertical $vx = ft'$, and join *kx*, the line *kx* represents the resultant of the forces *kv* and *ft* (representing thrust of *jk* and *fk*); and *xy*, drawn parallel with *ke*, represents the tension of *ke*;* This is the maximum stress of the diagonal *ke*.

XXX. For, when the left half of the truss has more load than the left hand abutment is required by the statical law to sustain, it is clear that a part of the weight on the left, is transferred from left to right past the centre through *dl*; that being the only member capable of effecting the transfer. It is also clear, that such transferred weight, together with the weight at *e*, if any, is sustained by *lk* and *ek*, and causes pressure at *i*, equal to the weight sustained by *lk* and *ek*. Also, that this pressure at *i*, causes a horizontal thrust in *ij*, which is all transferred to *lk* (except when *kg* is in action), and gives a lifting power to *lk*, equal to ⅓ of that exerted by *ij* (the vertical reach of the former being ⅓ that of the latter), that is, equal to ⅓ of the weight

* The sides of the triangle *kxy*, being parallel with the directions of the thrust of *lk*, the tension of *ek*, and the resultant *kx*, of the thrust of *jk* and *fk*, which 4 forces are in equilibrio at the point *k*.

transferred by *lk* and *ek* together. But the lifting power of *lk* is further increased by $\frac{1}{3}$ of the weight sustained by *fj*, which increases the horizontal thrust of *lk*, the same as a like amount sustained by *ij*; also, by $\frac{1}{5}$ the weight sustained by *ek;* this member having 5 times the vertical reach of *lk*. Now, as each one of these items results from the weight transferred through *lk* and *ek*, and is greater or less in proportion as the last named weight is greater or less (*f* and *g* being unloaded), since all the conditions are the same, except as to amount of weight, it must follow that the greatest stress of *ek*, is when *f* and *g* are unloaded, and all the other points *b*, *c*, &c., are fully loaded — unless it be when *f* and *g*, or one of them, be wholly or partially loaded. But any weight at *f*, increases the thrust and lifting power of *lk*, through increased action of *ij* and *fj* both, while it diminishes the amount sustained by *ek* and *lk*, whence the action of *ek*, is diminished, inasmuch as it transfers to *k*, a less proportion of a less weight.

Again, weight applied at *g*, while *f* is unloaded, relieves the tension of *fj*, and diminishes its lifting power represented by *ft'* and *vx*, and if of sufficient amount, relaxes *fj*, and brings tension upon *gk*; so that, when the weight at *g* equals *w*, or $7w''$, *lk* has a lifting power $= \frac{1}{3}$ pressure at *i*, *less* what is due to the horizontal pull of *gk*, *plus*, amount due to horizontal pull of *ek*; while the weight bearing at *k*, equals $9w''$ (being weight at *e* ($= 7w''$) $+ 2w''$ through *dl*). Now if *ek* lifts as much as *kg*, *lk* must have as great a horizontal thrust as *ij*, and be capable of lifting $\frac{1}{3}$ $16w''$ ($=$ weight bearing at *i*), $=5\frac{1}{3}w''$; which taken from $9w''$ bearing at *k*, leaves $3\frac{2}{3}w''$ sustained by *ek*. Then it remains to be seen whether gk sustains more than $3\frac{2}{3}w''$, so as to reduce the horizontal thrust of lk below that of ij.

With the truss fully loaded except at the point f, ij sustains vertically, $16w''$, whence jk, having the same horizontal thrust exerts a depressive force$=\frac{2}{3}16w''$,$=10\frac{2}{3}w''$, at j, leaving a balance of $5\frac{1}{3}w''$, exerted by ij toward lifting the $7w''$ at g. Hence, only $1\frac{2}{3}w''$ remains as the weight sustained by gk. Therefore, the horizontal pull of ek, is not less than that of gk, the horizontal thrust of lk, is not less than that of ij, and its lifting power, not less than $5\frac{1}{3}w''$, and ek does not lift more than $3\frac{2}{3}w''$, nor as much as when f and g are without load, as determined by the process above explained.

XXXI. To determine the greatest stress to which dl is liable, let the weights at e, f and g be removed. Then the pressure at i, due to the weights at b, c and d, equals $6w''$, that is, $1w''$ for weight at b, $2w''$ for that at c, and $3w''$ for that at d. We therefore take jq'' on ij produced, to represent the thrust of ij, produced by $6w''$—draw $q''r''$ parallel with fj, and from $q''r''$ find ft'' (of course less than ft'), and having taken kv' on jk produced, equal to jr'', raise the perpendicular $v'x' = ft''$, and draw $x'y'$ parallel with ek. Then, $x'y'$ represents the tension of ek, from which we find ea'', representing the vertical thrust of el at its maximum. Also ky' represents the thrust of kl; and, having taken ld' on kl produced, equal to ky', raise the vertical $d'e'$, equal to ea'', from e', draw $e'f'$, parallel with dl, and meeting lm (produced, if necessary), in f', and $e'f'$ represents the tension of dl.

We have a short way of verifying the correctness or otherwise of the last result, since we know that, in the state of the load here assumed, $6w''$, is transferred from the left to the right of the centre, necessarily through the tension of *de*, the only member capable of performing that office. Hence the tension of *dl* in this case, can be neither more nor less than what is due to a lifting power equal to $6w''$. Then, taking *dc'* on *dm*, to represent $6w''$, and drawing the horizontal *c'b'*, we have *db'* to represent the tension of *dl*, and, if *e'f'*=*db'*, the result is probably correct. We know, moreover, that $6w''$ is the greatest weight ever transferred past the centre of the truss, the left hand side having the greatest possible load, and the right hand side, the least possible. Therefore, *db'* represents the maximum stress for *dl*, which is equal to $6w''\frac{\sqrt{h^2+v^2}}{v}$

XXXII. If the points *b* and *c* alone be loaded, we know that $3w''$ is transferred through *dl*, and there being no weight at *d*, this lifting force of *dl*, must be sustained by the thrust of *dm*. Having then, found *lf''* representing thrust of *ml*, by a process similar to that by which we obtained *lf'* in the preceding case, that is, commencing with *jq'''*, representing the thrust of *ij* under a weight equal to $3w''$, we take *mg'* = *lf''*,* on *lm* produced, raise the vertical *g'i'* equal to a line representing $3w''$, and draw *i'i'* parallel with *cm*, when we have *i'j'* to represent the tension of *cm*.

XXXIII. Or, we may take *ok'* on *ao* produced, to represent the thrust of *ao*, due to the vertical pressure ($=11w''$) at *a*, resulting from the weights at *b* and *c*,—draw the vertical *k'l'*, representing $7w''$, = weight at *b*, and cutting *on* produced in *m'*, and *l'm'* represents the lifting force exerted by *bn*; as is made obvious by forming the parallelogram *l'n'*, upon the diagonal *om'*.

* *g' i'* and *j'* shown in Diagram B, to avoid complication

Take $bo' = l'm'$, and draw the horizontal $o'p'$, then bp' represents the tension of bn, and om', the thrust of on. Take $na_{,,} = om'$, on on produced, draw $a_{,}b_{,} = bp'$, parallel with bn, and, from $b_{,,}$ let fall $b_{,}c_{,}$ representing the weight ($=7\,w''$) at c, and the part below nm, represents the lift of cm, whence we derive the tension of cm. The result should be the same as that obtained by the former operation.

XXXIV. If the point b only, be loaded, we may take ok'' to represents the thrust of ao resulting from a pressure of $6w''$ at a, let fall $k''l''$ cutting on in m'', to represent the $7w''$ at b, and $m''l''$ represents the vertical lift of bn. Make $bo'' = m''l''$, and draw the horizontal $o''p''$, and we have bp'' representing the tension of bn. This is the maximum stress of bn, since bn, can only sustain the weight at b, *less* the excess of lifting power of ao over the depressing power of on, both having the same horizontal thrust; which excess is represented by $k'm'$ and $k''m''$, and is least when the weight bearing at a is least. But the bearing at a (and the lift of ao), can never be less than $\frac{6}{7}$ of the weight at b, and $k''m''$ etc., can never represent less than $\frac{2}{7}$ weight at b, or $\frac{1}{3}$ of the lift of ao; whence $m''l''$ etc., can never represent more than $\frac{5}{7}$ weight at b; consequently bn can never sustain a weight greater than $5w''$ which is the amount represented by $m''l''$ when b is fully loaded, and the remainder of the truss without load.

XXXV. With regard to cm, no simple and conclusive reason presents itself, why the result above obtained for the stress of that member when b and c alone are loaded, is the actual maximum. But, as the assumed condition is precisely analogous, as far as the case permits, to the conditions under which all the other diagonals have been shown to suffer their maximum stresses, it is reasonable to conclude from analogy, and the general nature of the case, that we have obtained the true maximum stress for *cm*. Should there be doubt whether some supposable distribution of load upon the truss, would not produce greater stress than that above shown for the member in question, it is presumed that such doubt would readily be set aside by an analysis similar to what has already been gone through with.

UPRIGHTS, OR VERTICAL MEMBERS.

XXXVI. It has already been seen [XXVIII], that the maximum *tension* of verticals equal *w*, throughout. We have also seen [XXXIV], that the lift of *bn*, is always less than the weight at *b*; consequently *ob* is never exposed to compression, unless the load be applied at the arch, which will seldom be found advisable.

When *cm* exerts its maximum lift, the point *c* is loaded, and the lifting force of *cm* is all expended upon the weight at *c*. But when the point *b* alone is loaded, *cm* exerts a lift, which is the measure of the maximum compression of *cn*, resulting from tension of *cm*, since any weight at the right hand of *c*, would bring more or less downward action at the point *m*, thereby relieving some of the tension of *cm*, and consequently, diminishing its compressive action upon *bn*. The compression exerted upon *cn* by the tension of *cm*, is found to be about equal to $2w''$, and very nearly the same as that exerted by *fj*, and represented on the diagram by *ft'* [XXIX], $2w''$, then, may be regarded as the maximum compression of *cn*.

The vertical *dm*, can never suffer compression to exceed 3 w'', = greatest weight sustained by *dl* when *d* is without load; and if *d* be loaded, so as to add to the lift of *dl*, any weight at *d*, relieves *dm* of 4 pounds of compression, to every three pounds added to the lift of *dl*, so that $7w''$ at *d*, while it increases the lift of *dl* from $3w''$ to $6w''$, changes the action of *dm*, from compression of $3w''$, to tension of $1w''$.

XXXVII. Having determined the maximum weights sustained by the several diagonals and verticals, we proceed to ascertain their respective stresses, and required amounts of material; as depending upon the length of each member, multiplied by its maximum stress.

The greatest weight sustained by *fj*, as measured on the diagram, and verified by calculation, is equal to $3w''$, or $\frac{3}{7}w$; and the length being equal to $\sqrt{h^2+\frac{1}{4}v^2}$, the stress equals $\frac{3}{7}\frac{\sqrt{h^2+\frac{1}{4}v^2}}{v}w$, and the required material equals $\left(\frac{3}{7}\frac{h^2}{v}\times\frac{3}{28}v\right)$M. The 4 diagonals *gk*, *ek*, *bn* and *nd*, sustain, each, a maximum weight of $5w''$, [XXXIV], with length $=\sqrt{h^2+\frac{25}{36}v^2}$. Hence, stress equals $\frac{5}{7}\frac{\sqrt{h^2+\frac{25}{36}v^2}}{v}w$, and, material for each, $=\left(\frac{5}{7}\frac{h^2}{v}+\frac{125}{252}v\right)$M.

The four remaining diagonals sustain each a maximum weight of $6w''$, $=\frac{6}{7}w$, with length $=\sqrt{h^2+v^2}$, giving stress equal to $\frac{6}{7}\frac{\sqrt{h^2+v^2}}{v}w$, whence material equals $\left(\frac{6}{7}\frac{h^2}{v}+\frac{6}{7}v\right)$ M. Then, multiplying the last two coefficients of M by four, and the preceding one by two for the number of pieces in each class; adding the products, and annexing the common factor M, we obtain for the ten diagonals, an amount of material equal to $(7.14\frac{h^2}{v}+5.626v)$M.

The aggregate length of verticals, equals $4\frac{2}{3}v$, and their greatest tension stress equals w. Hence, $4.666v$,M represents the amount of tension material they require.

The two longest verticals sustain a maximum compression of $3w''$ [XXXVI], or $\frac{3}{7}w$, with length $= v$. Hence material $= 0.857v$M. The two next in length sustain compression $= \frac{2}{7}w$, with aggregate length $= 1\frac{2}{3}v$, and require compression material $= 0.476v$M, making the whole amount of required material, as represented by the amount of action by compression on verticals $=$ 1.333M.

We have, then, material for the whole truss, as represented by the *amount of action*, as follows:

Under Compression.

Arch, [XXVII]	$(42\frac{h^2}{v} + 4\frac{2}{3}v)$M
Verticals,	$1\frac{1}{3}v)$M
Total,	$(42\frac{h^2}{v} + 6v)$M

Under Tension.

Chord, [XXVII]	$(42\frac{h^2}{v}$ ....M)
Diagonals,	$(7.14\frac{h^2}{v} + 5.626v)$M
Verticals,	$4.666v$M
Total,	$49.14\frac{h^2}{v} + 10.292v)$M

Making $h = v = 1$, these amounts become:
Under Compression, 48M. Under Tension, 59.432M.*

* The difference between this result, and that given in the synopsis on page 20 of my original work, arises from the fact that one was based on a circular arch, and the stresses taken from the diagram, and

XXXIII. The preceding results are based on the assumption [XXVII] that the maximum tension of the chord, and the maximum horizontal thrust of the arch in all parts, are equal to the horizontal thrust of the end sections of the arch when the truss is fully and uniformly loaded. Although this may seem self-evident, it may not be amiss to make particular mention of some of the conditions affecting the case, which may lead to a better understanding of the subject, if it fails to amount to an absolute demonstration.

The arch and chord obviously act and react upon one another horizontally from end to end, and, as weight removed from any part of the length, diminishes the amount of bearing at both ends, which bearing governs the stress at the ends, it follows that the ends, at least, of both arch and chord, have their greatest stresses under a full load of the truss.

It is obvious, also, that no part of arch or chord can have greater stress than the ends, unless it be communicated by the tension of diagonals. When the acting diagonals all incline one way, their united horizontal action only equals the difference between the horizontal thrust of the two end sections of the arch, and the action of chord and arch can no where be greater than at the end *from* which the acting diagonals incline.

When acting diagonals incline inward from the ends, the intermediate portions of chord and arch are under less stress than the end portions, and consequently, less than they sustain under a full load of the truss. But when acting diagonals incline outward, toward the

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the other, on a parabolic arch, and stresses mostly calculated numerically; and, from the further fact that in one case, the weight was assumed to be applied at the arch, and in the other, at the chord; the former producing more compression, and the latter, more tension upon the uprights.

ends of the truss, the intermediate portions of arch and chord are under horizontal stress greater than that of the end portions, by an amount equal to the aggregate horizontal action of all the acting diagonals inclining toward the respective ends.

Now, as no more than two diagonals inclining toward the end bearing the greatest weight, can be in action at the same time, in a seven panel truss, the question resolves itself into — whether two diagonals acting in one direction, can ever exert force enough to over balance the loss of action of the end section, resulting from diminished bearing at the abutment, consequent upon the removal of load, on which removal, the action of diagonals depends?

As to that question, the removal of weight from the central portion of the truss, must bring into action inwardly inclined diagonals, while removing weight from one end only, can bring into action no diagonals inclined toward the full loaded end, whence the weight bearing at that end indicates the greatest stress of any part of chord and arch which, of course, is less than under the full load of the truss.

There remains then, only the case of removal of load from both ends of the truss, which can produce any considerable action upon diagonals inclined outward, so as to give greater stress to the middle, than the end portions of the arch and chord. If the weights at *b* and *g* be removed, the pressure at each abutment is diminished by $\frac{1}{3}$ of the maximum, or, by $7w''$; and *fj*, sustaining only $3w''$ at the maximum, and having the same inclination as *ij*, its horizontal action could only balance the effect of $3w''$ removed from *ij*; while *ek*, even if it sustained its greatest weight of $5w''$, as it evidently does not, in this case, would only exert the same horizontal action as ij would do under $3w''$. Hence these two diagonals, under their maximum stresses, which neither suffers, in the present case, would only compensate $\frac{6}{7}$ of the loss on stress of arch and chord, due to removal of weight from the truss.

It may, therefore, be regarded as a matter of extreme probability, if not a rigidly demonstrated fact, that the arch and chord of an arch truss, undergo their maximum stress in all parts, under the full uniform load of the truss.

It is hoped and believed that the foregoing illustrations of the manner of determining the strains of the several parts and members of an arch truss of seven panels, will be sufficient to enable the same to be done in the case of trusses of any desired number of panels.

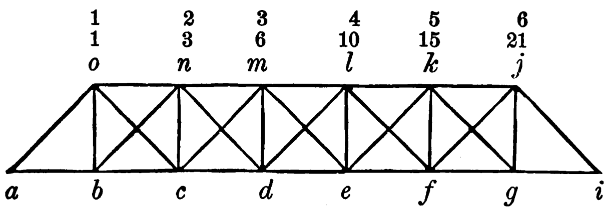
THE TRAPEZOIDAL TRUSS.

So designated from the figure of its outline.

XXXIX. This truss may be constructed with diagonals and verticals, as in Fig. 12, or without verticals, except at b and g, as seen in Fig. 13. To explain the operation of these trusses, and determine the maximum, stresses of their various parts, we may use the same notation, generally, as heretofore; that is, let h represent the horizontal, and v, the vertical reach of the diagonal or oblique members, and D, the length of diagonals. Also, let w represent the greatest movable load for a panel length, supposed to be concentrated at the nodes b, c, d etc., of the lower chord; and, let w'' be equal to w, divided by the number of panels in the truss (7, in this case), *i.e.*, let $w = 7w''$.

Then, supposing the diagonals (Fig. 12), not including the king braces, ao and ij at the ends; the verticals *ob* and *jg*, and the lower chord, to act by tension; and the upper chord, or boom, the king braces, and the four intermediate verticals, to act by thrust, or compression — if a weight (w) be applied at b, it will obviously cause a downward action equal to w'' at i, and one equal to $6w''$ at a.

FIG. 12.



Now, from what has already been seen, in the discussion in relation to Fig. 10, the weight acting at i, can only do so by acting successively, or simultaneously, upon *bn*, and each diagonal parallel with *bn* on the right, by tension, and upon each compression upright and the king brace *ij*, by thrust; causing upon each of these 10 members, a stress equal to w'' upon verticals, and equal to $w''\frac{D}{v}$ upon obliques; D representing the length of obliques, or diagonals.

A weight (w) at c, in like manner, causes a pressure of $2w''$ at i, through *cm*, and other diagonals inclining to the right, on the right hand of c. Also, a pressure of $5w''$ at a. But *co* being the only member that can transfer weight from c to the left, and, *co* and *bn* being antagonistic — stress upon the one tending to relax the other, the result must be, that both can not act at the same time, from the effects of weight at b and c, and only that one can act, to which the greater weight is applied; and that, only with the excess of weight acting upon it, over what is acting, or tending to act upon the other. Now, as the load at c, tends to throw a weight of $5w''$ upon *co*, while the load at *b* tends to throw $1w''$ on *bn*, the former tendency must preponderate — *co* must sustain $4w''$, while *bn* is relaxed, and the whole weight at *b*, is sustained by the tension of *ob*. In reality then, *cm* sustains $\frac{3}{7}$ of the weight at *c*, and none of that at *b*.

Still, the result is the same, as to pressure at *a* and *i*, the former point supporting $7w''$, = the whole of the load at *b*, plus $4w''$ of that at *c*, making $11w''$, = pressure due at *a*, from the weights at *b* and *c*, while the point *i* supports $3w''$, all out of the weight at *c*. Thus, *cm*, *dl* etc., sustain the same proportion of the aggregate weights at *b* and *c*, as if each weight acted separately, and independently of the other.

Again, applying a weight (w), at *d*, $3w''$ tends to bear at *i*, and $4w''$ at *a*, through *dn* and *co*. But as we have $3w''$ tending to act on *cm*, as already seen, this is neutralized by the tendency to action upon *dn*, and only a surplus of $1w''$, really acts upon *dn* in this case, while the $6w''$, = pressure due at *i*, from the weights at *b*, *c* and *d*, is all made up out of the single weight at *d*, and the whole of the weights at *b* and *c*, together with $1w''$ from that at *d*, comes to bear at *a*, giving a pressure of $15w''$, at that point; still the same as if each weight acted independently of the others. And, in general, each diagonal, at all times, sustains the preponderance of weight *tending* to act upon it, over that which tends to act at the same time upon its antagonist. Hence, the greatest weight sustained by any diagonal, is when all the weight tending to act upon it, is upon the truss, and none of the weight tending to produce action upon its antagonist, or counter.

Thus, when *b* alone is loaded, $1w''$ is sustained by *bn*, but when any point on the right of *b* is loaded, there is tendency to action by *co*, and the action of *bn* is destroyed or diminished. Therefore $1w''$ is the maximum weight sustained by *bn*. When *b* and *c* alone are loaded with the weight (w) at each, *cm* sustains $3w''$, as already seen, with no tendency to action in *dn*. But if *d*, or any point on the right of *c*, be loaded, there is tendency to action in *dn*, which must diminish or destroy the action of *cm*. Hence, *cm* sustains its maximum weight ($= 3w''$), when the points *b* and *c* alone are under their full load. And, it must be obvious that the maximum weight is sustained by each diagonal inclining to the right, when the point at its lower end, and all the nodes at the left are fully loaded, and all those at the right are without load. Hence we establish the following easy and expeditious practical method of determining the maximum weights and stresses upon this class of members, in trusses with any number of panels.

XL. Having made a rough diagram of the truss, as Fig. 12, for instance, place over the nodes *o*, *n*, *m*, &c., the numbers 1, 2, 3, &c., high enough to admit of a second series under the first, formed by repeating the 1 under itself, adding the 1 and 2 together and placing the sum (3), under the 2 in the upper series.. Then add 1, 2 and 3, and place the sum (6) under 3, and so on, placing under each figure of the upper series, the sum of that figure, and all those at the left, in said upper series.

Then, it will be seen that each figure in the upper line, prefixed to w'', shows the pressure caused at the right hand abutment, by the weight (w) directly under the figure, *e. g.*, the upper figure 3 over *d*, indicates that $3w''$ is the bearing at *i*, produced by the weight (w) at *d*, and so of the other figures in the upper line.

In the mean time, the figures in the lower line, show the accumulation of the effects of the different weights upon successive diagonals from left to right. Thus, the figure 6 over the point *d*, shows that *dl* sustains $6w''$, = pressure due at *i*, from weights (w) at *b*, *e* and *d*, when those points only are loaded; in which case, *dl* sustains its maximum weight, as before seen.

In like manner, the figures 10 & 15 over *e* and *f*, indicate that $10w''$ and $15w''$, are respectively the maximum weights sustained by *ek* and *fg*, while $21w''$ ($= 3w$), equals the maximum weight sustained by *ij*, (by compression, of course), when the whole truss is loaded.

XLI. Having thus ascertained the greatest *weights* the several oblique members are liable to sustain (those inclining to the left being obviously exposed to the same stresses as those inclining to the right), we find their maximum stresses by rule 4, [XVI]; i. e., multiply the weight by the length, and divide by the vertical reach of the member. Thus, the maximum compression of *ij*, equals $3w\frac{\text{D}}{v}, = 3w\frac{\sqrt{h^2+v^2}}{v}$, and the representative of required material, is $\left(\frac{3h^2}{v} + 3v\right)$ M.

The maximum stress of *ek* equals $10w''\frac{\text{D}}{v} = 1\frac{3}{7}w\ \frac{\sqrt{h^2+v^2}}{v}$ and its representative for material is $\left(1\frac{3}{7}\frac{h^2}{v} + 1\frac{3}{7}v\right)$M. Or, the lengths and inclinations being the same, we may take the aggregate maximum weights sustained by tension diagonals, reduced to terms of w, multiply by the square of the common length, divide by v, and change w to M. The ten tension diagonals sustain maximum weights equal to w'' multiplied by twice the sum of all the figures in the lower line over the diagram, except the last, making $70w''$,$=10w$, and require material $= \left(10\frac{h^2}{v} + 10v\right)$ M.

The tension verticals *ob* and *jg*, sustain the weights (w) acting at b and g when the truss is fully loaded, which is their maximum stress, and they require material for the two, $= 2v$M.

The thrust uprights *el* and *fk*, receive and sustain the maximum sustained by *dl* and *ek* respectively, which are the measures of their respective stresses of compression, being $6w''$ for *el*, and $10w''$ for *fk*, and the same for *dm* and *cn*, making an aggregate weight of $4\frac{4}{7}w$, whence, their representative for material is $4\frac{4}{7}v$M.

XLII. With regard to stress of the lower chord, the tension of *ac* equals the horizontal thrust of *ao*, and of course is greatest when *ao* sustains the greatest weight; which is manifestly under the full load of the truss. The tension of *cd* equals the horizontal thrust of *ao* (through *ac*), and the horizontal pull of *oc*; and must be greatest when the combined action of *ao* and *oc* is greatest. Now, although the weight borne by *oc* is greater by w'' when the point b is unloaded, than under the full load, on the other hand, the weight on *ao* is less by $6w''$, so that the combined action of the two members, must be greatest when the truss is fully loaded; since no other change can increase the action of either.

Again, *de* sustains the horizontal action of *ao*, *oc* and *nd*, when the truss is under a full load; *dl* and *em* being inactive in that case, since the tendency to action is the same in each, whence neither can act. Therefore *nd* sustains simply the weight (w) at d; *oc* sustains the two weights at c and d, $= 2w$, while *ao* sustains the same, with the addition of the weight at b, making $6w$

sustained by the three members contributing to the tension of *de*. Now while the *maximum* weights sustained by *oc* and *nd*, exceed by only $4w,''$ what they sustain under a full load, *neither* can be brought under its maximum stress, without removal of load from *b* in one case, and from both *b* and *c* in the other, thereby diminishing the weight on *ao*, by $6w''$ in one case, and by $11w''$ in the other. Hence the stress of *de* is greatest under a full load of the truss, and as already seen, equals the horizontal action of weight equal to $6w$ upon oblique members of vertical reach equal to v, and horizontal reach equal to h. The maximum stress of *de*, then, equals $6w\frac{h}{v}$, and that of *cd* equals the same, less the horizontal pull of *dn*, due to the weight (w) at *d*, and is therefore equal to $5w\frac{h}{v}$.

We have for the lower chord, then, one section, *de* (with length equal to h), exposed to stress...... $= 6w\frac{h}{v}$,
Two sections, *cd* and *ef*, with stress.............. $= 5w\frac{h}{v}$,
Four do., *ac* and *fi*,.............................. $= 3w\frac{h}{v}$;
whence, adding, multiplying by h, and changing w to M, we have to represent required material, $28\frac{h^2}{v}$M.

XLIII. It is scarcely necessary to state that the oblique members *ao*, and *oc*, exert at *o* in the direction from *o* to *n*, the same force that they exert in the opposite direction upon *cd*. In fact, the thrust of *on*, and the tension of *cd*, simply act and react upon one another through the media of *ao*, *oc* and *ac*, whence the compression of *on*, must be just equal to the tension of *cd*; and furthermore, the thrust of *nm* is the indirect counteraction of the tension of *de*; and, as the two forces are in opposite and parallel directions, they must be equal, being in equilibrio. Also, *mlk* must sustain the same compression as *nm*, throughout, since the diagonals meeting the chord at *m* and *l*, are inactive under the full load of the truss. Of course, *kj* is liable to the same maximum action as *on*.

From what precedes, it cannot fail to be obvious that the maximum action of all parts of the upper chord occurs at the same time with that of the lower chord, namely, under the full load of the truss. We have then, two sections liable to a compression of $5w\frac{h}{v}$, and three, liable to $6w\frac{h}{v}$; whence the representative of required material, is $28\frac{h^2}{v}$M. We may now sum up the material for the truss, required to support the assumed movable load through all the changes liable to take place, as follows:

Material under Compression.

Chord,........	$(28\frac{h^2}{v} \times$ M
King Braces, [XLI]	$(6\frac{h^2}{v} + 6v)$M
Posts, [XLI]..................................	$4\frac{4}{7}v)$M
Total,..	$(34\frac{h^2}{v} + 10\frac{4}{7}v)$M

Under Tension.

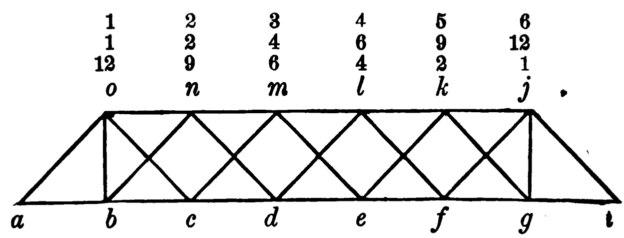
Chord, [XLII]................................	$(28\frac{h}{v} \times$ M
Diagonals, [XLI]............................	$(10\frac{h^2}{v} + 10v)$M
Verticals, [XLI]..............................	$2v$M
Total,..	$(38\frac{h^2}{v} + 12v)$M

Making $h = v = 1$, we have:

Compression material, ...	$= 44\frac{4}{7}$M
Tension material, ...	$= 50$M
For Arch Truss, Comp. Mat., [XXVII]..........	$= 48$M
" " Tension do., ...	$= 59$M

XLIV. A trapezoidal truss without verticals (except at one panel width from the ends), is represented in Fig. 13. The members *ob* and *oc*, as also *jg* and *fj*, are supposed to be so formed and connected as to act by tension only, and the other diagonals, so as to be capable of acting either by thrust or tension.

FIG. 13.



If a weight (w), be applied at b, it will cause a bearing of $1w''$ at i, and $6w''$ at a, the same as in the case of Fig. 12. Now, this weight at b, might (if *bn* were removed, and *oc* were capable of withstanding compression), be suspended entirely by *ob*, and supported by *ao* and *oc*, in proportion to the bearing produced by it at a and i respectively. But as *bn* is able to act by tension, and *oc* unable to act by thrust, the $6w''$ bearing at a, acts through *bo* and *oa*, while the $1w''$ bearing at i, must first act by tension upon *bn*; secondly, by thrust upon *nd*, since that is the only member meeting *bn* at n, capable of sustaining weight. Hence, the action of the weight is transferred to d, and through *dl* to l, thence to f, and so on through *fj*, and *ji*, to the abutment at i; acting alternately by tension and thrust, upon six oblique members, producing the same amount of action ($= w''\frac{\text{D}}{v}$), upon each.

Another weight (w) at c, must cause pressure at i, equal to $2w''$, and at a, equal to $5w''$. The action therefore, must be divided between cm and oc, in the proportion of 2 to 5, producing alternately tension and thrust, through the points m, e, k, y, j to i; on the right, and through co and oa, to a, on the left.

Thus far, the weights have acted upon independent systems of oblique members (except as to king braces), neither weight acting upon any member acted on by the other. But when a weight (w) is imposed at d, it must act upon the same members acted on by the weight at b. The $3w''$ to be transferred to i, must act by tension upon dl, in concert with the $1w''$ of the weight at b. But the pressure at a, must be increased by $4w''$, which can be transferred from d, only through tension of dn, and dn being previously occupied in carrying $1w''$ from b, by compression, it follows, since the same member can not be under compression and extension at the same time, that the greater force must preponderate, dn being brought under tension due to the difference of $4w''$ tending to act by tension, and $1w''$, tending to act by thrust, the action of dn, being changed from thrust under $1w''$, to tension under $3w''$. All the weight at b, then, is sustained by bo, together with $3w''$ from weight at d, making $10w''$, which, with $5w''$ from weight at c, makes $15w''$ = pressure due at a, from weights at b, c and d.

In the mean time, the weight at d, having obstructed the passage of $1w''$ from b to the right hand abutment, has been obliged to make compensation, by sending $4w''$ of its own gravitating force, nstead of $3w''$ owed by it to the bearing at i.

Similar changes of action take place when another weight (w) is applied at e; which tends to throw $3w''$

by tension upon *em*, while the weight at *c* tends to throw $2w''$ upon it by thrust, as seen above. The result is, that *em* sustains $1w''$ by tension, and *cm*, $1w''$ by thrust, while *co* sustains the whole weight at *c*, in addition to $1w''$ from *e*.

Again, a weight (w) at *f*, tends to throw $2w''$ upon *fl*, to act by tension; but as *fl* is already occupied by $4w''$ acting by thrust, *f* is obliged to depend entirely upon *fj*; at the same time turning back $2w''$, and reducing the weight previously on *lf*, by that amount, or, to $2w''$.

In like manner, a weight applied at *g*, finds *gk* sustaining $6w''$ by thrust, whereby it is prevented from sending $1w''$ (due from it at *a*), to the left, through tension of *gk*. Hence, the whole weight at *g* is sustained by *jg*, and the weight acting on *kg* is reduced to $5w''$.

XLV. Thus we see that each diagonal (except *oc* and *fj*, excluded by hypothesis), is liable to compression from weights at certain points, and tension from weights at other points; and, it is manifest that the greater stress of either kind, on each diagonal, is when all the weights are on the truss, which tend to produce upon it one kind of stress, and none of those which tend to produce the opposite stress.

Hence, if we place the numbers 1, 2, 3, &c., over the diagram, as in case of Fig. 12. [XXXIX], it is clear that only alternate weights act upon the same system of diagonals; that only weights under the odd numbers 1, 3 and 5 act upon diagonals meeting the lower chord at points under those numbers; and so of the weights under the even numbers 2, 4 and 6. We therefore form a second series of figures under the first, by placing under each odd number, the sum of that number and all the preceding odd numbers, and under each even number, the sum of that and all preceding even numbers. Then, the number in the second line, is the coefficient of w'', to express the maximum weight acting by tension upon the diagonal inclining to the right, from the point under that number, and by thrust, upon the diagonal meeting the former at the upper chord.

For instance, the figure 4 in the second line, over *d*, shows that *dl* and *lf*, sustains $4w''$, the former by tension, and the latter by thrust. This is the weight which must bear at *i*, in consequence of the weights at *b* and *d*, the only weights that can produce those specific actions upon those members. On the contrary, this action upon *dl* and *lf*, is only liable to diminution from weight at *f*, which tends to throw $2w''$ upon the left abutment through tension of *fl* and thrust of *dl*, and consequently diminishes the action upon those members, due to weights at *b* and *d*. Therefore, $4w''$ is the greatest weight sustained by *dl* and *lf*, and $2w''$, the weight sustained by them when the points *b*, *d*, and *f* are loaded, whether the other nodes are loaded or not. There is, however, an alternative in this case, which will be noticed hereafter.

The figure 1 over *b*, indicates that *bn* sustains $1w''$ by tension, and *nd* the same by thrust, which action is reversed by weights at *d* and *f*, which tend to throw $6w''$ upon these members, namely, $4w''$, from *d*, and $2w''$ from *f*. Hence, *bn* is liable to $1w''$ by tension, and $6w''$ by thrust, the latter, when *d* and *f* are loaded, and *b* unloaded; and to $5w''$ (acting by thrust), when all the three points are loaded.

Then, if we form a third series of numbers under the second, by reversing the order of the second, the one series shows the tension, and the other the thrust to which a member is liable. But as thrust action is not received by any diagonal directly from the weight producing it, but from a tension diagonal meeting it at the upper chord, we do not learn the thrust of a diagonal from the figure over it, at either end, but from the figure over the foot of the diagonal by which the compression is communicated.

Having arranged the diagram as above explained, we form from it a table of greatest weights sustained by the several diagonals, and stresses produced thereby, both of tension and thrust, remembering that tension weights are shown by one series of figures and thrust weights, by the reversed series.

Diagonals.	*Compression.*		*Tension.*		*Under full load.*	
	Weights.	*Stresses.*	*Weights.*	*Stresses.*	*Weights.*	
					THR.	TEN.
bn and *kg*,	$6w''$	$6w''\frac{D}{v}$	$1w''$	$1w''\frac{D}{v}$	$5w''$	
cm and *lf*,	4 "	$4w''\frac{D}{v}$	2 "	$2w''\frac{D}{v}$	$2w''$	
dl and *me*,	2 "	$2w''\frac{D}{v}$	4 "	$4w''\frac{D}{v}$		$2w''$
ek and *dn*,	1 "	$1w''\frac{D}{v}$	6 "	$6w''\frac{D}{v}$		5 "
fj and *co*,	0		9 "	$9w''\frac{D}{v}$		9 "
	13		22			

Adding and doubling the several weights, we deduce the representatives of material.

Under Compression,.......................... $(3\frac{5}{7}\frac{h^2}{v} + 3\frac{5}{7}v)$M

Under Tension,................................ $(6\frac{2}{7}\frac{h^2}{v} + 6\frac{2}{7}v)$M

The 2 verticals sustain each $12w''$, giving $3\frac{3}{7}v$M

The king braces *ao* and *ij*, sustain $3w$ each,

requiring material for the two = $(6\frac{h^2}{v} + 6v)$M

XLVI. The stress of chords is, as in case of Fig. 12, due to action of obliques, and may be fairly assumed to be greatest under a full uniform load of the truss. The brace *ao* has a horizontal thrust $= 21w''\frac{h}{v}$, = tension of *ab*. The thrust of *bn* (under the full load), adds $5w''\frac{h}{v}$ at *b*, making $26w''\frac{h}{v}$ = tension of *bc*. This is increased by $9w''\frac{h}{v}$for tension of *oc*, and by $2w''\frac{h}{v}$ for thrust of *cm*, making $37w''\frac{h}{v}$ for tension of *cd*; and, adding $5w''\frac{h}{v}$ for tension of *dn*, and subtracting $2w''\frac{h}{v}$ for tension of *dl* in the opposite direction, we have $40w''\frac{h}{v}$ = tension of *de*.

We have then,

1 section	sustaining	$40w''\frac{h}{v}$	=	$40w''\frac{h}{v}$
2 "	"	37 "		74 "
2 "	"	26 "		52 "
2 "	"	21 "		42 "

Making a total stress$=208w''\frac{n}{v}=29\frac{5}{7}w\frac{h}{v}$acting upon sections of a common length equal to *h*, and therefore, requiring material represented by $29\frac{5}{7}\frac{h^2}{v}$M.

Upon the upper chord, we have the horizontal action *ij* and *fj*, producing compression equal to $30w''\frac{h}{v}$ upon *jk*. Add for horizontal action of *ek* and *kg*, $10w''\frac{h}{v}$ making $40w''\frac{h}{v}$ =stress of *kl*. Again, add $4w''$ for horizontal action of *lf* and *ld*, and we have $44w''\frac{h}{v}$ = thrust of *lm*.

Thus, we have for the whole upper chord, $184w''\frac{h}{v}$ = aggregate stress upon sections of the common length equal to *h*. Hence, representative for material = $26\frac{2}{7}\frac{h^2}{v}$M.

Aggregate for whole Truss.

Material under Compression.		*Under Tension.*	
Chord, ...	$(26\frac{2}{7}\frac{h^2}{v}\times$ M	Chord,.........	$(29\frac{5}{7}\frac{h^2}{v}\times$ M
Diagonals, ...	$(3\frac{5}{7}\frac{h^2}{v}+3\frac{5}{7}v)$M	Diagonals,...	$(6\frac{2}{7}\frac{h^2}{v}+6\frac{2}{7}v)$M
End braces,...	$(6\frac{h^2}{v}+6v)$M	Verticals,.....	$3\frac{3}{7}v$M
Total, ...	$(36\frac{h^2}{v}+9\frac{5}{7}v)$M	Total,.........	$(36\frac{h^2}{v}+9\frac{5}{7}Mv)$

Making $h = v = 1$, Comp. 45.714M. Ten. = 45.714M.
Grand total, ... 91.428M.

We have here a little over 3 per cent less action upon material, than in case of truss Fig. 12, with verticals. The difference is a little less than was shown in my original analysis, that being based on trusses loaded at the upper, and this, at the lower chord; the former giving a trifle more action for the truss with verticals, and a trifle less for the other.

Moreover, the difference was made to show greater still, by assuming that deductions might be made on account of certain diagonals being liable to two kinds of action. For instance, it was supposed that a member formed to sustain a considerable tension stress, might also sustain a small compressive force without additional material (not at the same time, of course), which is undoubtedly the case, on certain occasions; especially in the use of wooden trusses. This would give still greater apparent advantage to truss 13, with regard to economy of material.

XLVII. There is, however, another view as to the action of load upon truss Fig. 13, which may modify the results above shown to a small extent.

If we strike out the diagonals *cm* and *me*, and also *dl* and *lf*, all the determinate forces necessary to sustain uniform weights at the nodes of the lower chord, would be exerted by remaining members, although we have assigned to those members, each, the sustaining of weight equal to $2w''$ under the full load, and twice that weight under certain conditions of partial load; and it is quite certain that they are necessary to the stability of the truss when partially loaded. But with both halves loaded uniformly, the weight upon each half could be transferred to the nearest abutment, producing equal thrust in both directions upon the central portion of the upper, and equal tension in opposite directions upon the lower chord; whereas, with one-half loaded, there is no means by which the pressure due at the farther abutment could be transferred past the centre, without oblique members in the centre panel. Still, which mode of action takes place under the uniform load, when the diagonals are in place, is a matter involved in a degree of uncertainty. If the centre diagonals do not act, under the uniform load, then *ek* and *fj* must sustain each $7w''$, instead of $6w''$ for the former, and $9w''$ for the latter, as above estimated. Also, *kg* would sustain $7w''$ by thrust, and different results would be produced as to stresses of various parts of upper and lower chords.

The maximum stress for *ek* and *dn*, and for *nb kg*, would be $7w''\frac{D}{v}$ instead of $6w''\frac{D}{v}$, as found above, and would occur under the full, instead of the partial load. The tension of *gj* and *ob*, also, would be increased to $14w''$. The weight sustained by *fj*, would be only $7w''$ under the full load, though liable to the same maximum weight of $9w''$, under a partial load.

For the lower chord, we should have the same co-efficient, (21) of $w''\frac{h}{v}$ to express the tension of *ab* and *ig*, 28 for *bc*, 35 (a decrease), for *cd*, and 42 for *de*.

For upper chord, the co-efficients of $w''\frac{h}{v}$ would be 28 for *on* and *kj*, and 42 for the three middle sections; no action being imparted by diagonals at *m* and *l*.

XLVIII. This uncertainty of action has no place in trusses of an even number of panels, as in such cases, no transfer of the action of weight can be supposed to take place past the centre, under a uniform load, without involving the absurdity of supposing the same member to carry weight by tension and compression at the same time; except, however, that in case of diagonals crossing two panels, or having a horizontal reach equal to twice the space between nodes of the chords, there will be diagonals filling the same condition of crossing in the centre of the truss, both vertically and longitudinally, as in Fig. 13.

We may obviate mostly, any mischief liable to result in cases of the kind under consideration, by estimating the stresses upon the several parts under both hypotheses, and taking for each member the highest estimate, which will *mostly* meet all contingencies. Estimating action upon truss 13 in this manner, we obtain the following representative expressions for material:

Compression.		*Tension.*	
Chord,.........	$26\frac{6}{7}\frac{h^2}{v} \times$ M	Chord,.........	$30\frac{4}{7}\frac{h^2}{v} \times$ M
Diagonals,.....	$(4\frac{h^2}{v} + 4v$M	Diagonals,...	$(6\frac{4}{7}\frac{h^2}{v} + 6\frac{4}{7}v)$M
End braces,....	$6\frac{h^2}{v} + 6v)$M	Verticals,......	$4v$M
Total,	$(36\frac{6}{7}\frac{h^2}{v} + 10v)$M	Total,	$(37\frac{1}{7}\frac{h^2}{v}\ 10\frac{4}{7}v)$M

Making $h = v = 1$. These expressions give, Compression material $= 46.855$M $+$ Tension do, 47.714M $=$ total 94,571M.

This shows an aggregate amount of compression and tension action, identical with that of truss Fig. 12, [XLIII.]

DECUSSATION AND NON-DECUSSATION.

XLIX. The elasticity of materials affords a means of answering the question as to decussation of forces through diagonals crossing in the centre of the truss, vertically and longitudinally (as in Fig. 13), in specific cases. But the results will vary in trusses of different numbers of panels, and different inclinations of diagonals.

Suppose the truss Fig. 13 to be so proportioned that the maximum stresses of the several parts and members, will produce change of length equal to E, multiplied by the lengths of parts respectively; the vertical *ob*, $= v$, being the unit of length. Then, the truss being uniformly and fully loaded, and the chords being under their maximum stress, the upper chord is contracted, and the lower one extended at a uniform degree; and, if the diagonals be unchanged in length, their vertical and horizontal reaches have not been changed by the change in length of chords. Hence, the *distance* between chords is not altered by change in their length. But the diagonals being under stress, by which some are extended and others contracted, according to the stress they are under, as compared with their maximum stresses respectively, the nodes of the chords are allowed to settle to positions below what they are brought to by the mere change in lengths of chords.

Hence, the panels are (generally) thrown into more or less obliquity of form, in consequence of inequality in length of diagonals in the same panel. But the centre panel can not assume obliquity, because any tendency of forces to change the length of one diagonal, is attended by a like tendency of equal forces to produce exactly the same change in the other; so that the vertical reaches of both must suffer the same change, if any, and both must be under tension or compression, according as the acting forces tend to bring the chords at the centre, nigher together or farther apart.

Now, the forces produced by the load being all concentrated at the points *o* and *j* (Fig. 13), the point *d* is depressed with respect to *o*, by the extension of *ob* and *nd*, and by the compression of *bn*. Hence, assuming decussation to have place, giving tension to the diagonals *dl* and *me*, equal to what is due to a weight of $2w''$, *ob* is under maximum tension and gives depression equal to E, to the point *b*, — *bn* and *nd* are under $\frac{5}{8}$ maximum stress, and give depression, each equal to $\frac{5}{8}$ E $\times$ D^2* = $(1.666h^2 + 1.666)$ E, for the two (D representing length of diagonals, $=\sqrt{h^2+1}$). Then, adding 1E for effect of *ob*, we obtain $(1.666h^2 + 2.666)$ E, = depression of point *d*.

The point *m* is depressed by extension of *oc* under a maximum stress, giving an amount equal to D^{2}E = $(h^2 + 1)$ E. Also, by compression of *cm* under one-half maximum stress, to the extent of $(\frac{1}{2}h^2 + \frac{1}{2})$ E. Hence, depression of point $m = (1\frac{1}{2}h^2 + 1\frac{1}{2})$ E.

This shows the point *d* to be depressed more than *m*, by $(1.666h^2 + 2.666)$ E — $(1.5h^2 + 1.5)$E = $(0.166h^2 +$

---

* Let the diagonals bd and Bd, of two rectangular panels ac and Ac, Fig. 14 (c and d, being fixed points), be exposed to tension in proportion to their respective cross-sections, receiving each thereby, extension

1.166) E, and the spaces *md* and *le* to be increased to that extent; of course producing tension upon *dl* and *me*.

Now, by hypothesis, these diagonals are under the weight of $2w''$, giving half maximum stress, and requiring an increase of vertical reach, equal to $(\frac{1}{2}h^2 + \frac{1}{2})$ E. If then, we give such a value to h, as will make the last co-efficient of E equal to the one above, it will show that the chords have receded just enough to give the assumed tension to *dl* and *me*, and the decussation is a demonstrated fact. To find the value of h, producing this result, make, $.5h^2 + .5 = .166h^2 + 1{,}166$, and we deduce $.333h^2 = .666$; whence $h = \sqrt{2}$.

But this requires too great an inclination of diagonals, and a less value of h, gives a space from d to m too great for the supposed tension of *dl* and *me*. Making $h = 1 = v$, we have increase of distance from d to m = 1.333E, requiring a weight of $2.666w''$, to stretch *dl* down to the point d. But as no weight or stress can be added to the $2w''$ assigned to *dl* and *me*, without af-

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equal to b′e and B′E, respectively. This will cause the points b and B to drop to b′ and B′, in ab and AB produced. Join b e and BE.

Then, the infinitesimal triangles bb′e and BB′E, right-angled at e and E, are essentially similar, respectively to the triangles db′a and dBA. Hence, the following relations:

(1). bb′ : b′e : : bd : ab, and

(2). BB′ : B′E : : Bd : ab.

whence, bb′ × ab = b′e × bd, and

BB′ × ab = B′E × Bd,

From these two equations we derive —.

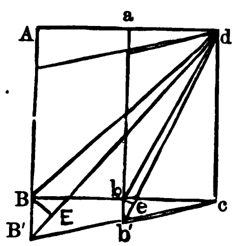
(3). bb′ × ab : BB′ × ab : : b′e × bd : B′E × Bd.

But, by the law of elasticity

(4). b′e : B′E : : bd : Bd; whence,

(5). b′e × bd : B′E × Bd : : bd^2 : Bd^2.

FIG. 14.



Hence dividing the first ratio of proportion (3) by ab, and substituting for the last ratio of (3), its equivalent found in (5), we have,

(6). bb′ : BB′ : : bd^2 : Bd^2.

Hence the depression due to the extension of a diagonal retaining the same vertical reach, is as the stress (per square inch), sustained, multiplied by the square of the length of diagonal.

fecting all the 5 members contributing to the depression of the points d and m, and in all cases, so as to diminish the elongation of distance between d and m, it is reasonable to conclude that by assigning some $\frac{1}{6}$, or thereabouts, of the extra weight of $.666w''$ required on dl alone, it would, by affecting the whole 5 members, be sufficient to correct the error. Let us, then, assume that dl and me sustain $2.15w''$, instead of $2w''$ as by previous supposition. This change requires reduction of weight upon dn, nb and bo, from $5w''$ to $4.85w''$ for the two former, and from $12w''$ to $11.85w''$ for bo. Also an increase of weight on mc and co, to $2.15w''$ on mc, and to $9.15w''$ on co. Then bo sustains $\frac{11.85}{12}$ of the maximum, and gives depression $= \frac{11.85}{12} = .9875E$ bn and nd, sustaining $\frac{4.85}{6}$ of the maximum, give depression for the two, equal to $3.233E$, making $4.2205E$ = whole depression of point d.

With regard to the point m, we have the maximum stress on oc, giving depression equal to $2E$, and $\frac{2.15}{4}$ of the maximum on mc, giving depression $= 1.075E$, making a total of $3.075E$ for the point m. Hence, the elongation of the space dm, equals $(4.2205 - 3.075)\,E$, $= 1.145E$, whereas $2.15w''$ upon dl, increases its vertical reach by only $1.075E$. This shows that dl sustains still a little more than $2.15w''$. On the other hand if we assume a weight of $2.2w''$ on dl, we obtain the opposite result, showing that dl sustains less than $2.2w''$, and hence the actual amount must be between $2.15w''$ and $2.2w''$.

We conclude then, that dl and me are *not* inactive under the full load of the truss, but on the contrary, they sustain, in this case, even more than the decussation theory assigns to them. We learn, moreover, that the question is affected by the horizontal reach of the diagonal, or the value of *h*. And, since in this case, the point *d* being depressed by action upon 3 members, and the point *m* by action upon only 2, we have an elongation of space between *d* and *m* requiring more than the theoretical stress upon centre diagonals, it is natural to conclude, that, in case of a greater number of panels, nine, for instance, where 4 members contribute to the depression of the upper, and only three to that of the lower chord at the centre, the increase of distance between chords, would be *less* than that required to give the theoretical stress upon diagonals in the centre panel; and, such is found to be the case. In a nine panel truss on the plan of Fig. 13, the increase of distance between chords, due to the stresses assigned by the decussation theory, is only about one-third of what would be required to give the centre diagonals the stress assigned them. Hence in this case there is less decussation than the theory requires; and, one or two trials, by assigning different weights as sustained by centre diagonals, in the manner pointed out above, would enable a near approxination to the actual amount of decussation to be arrived at, in the case of the nine panel truss, or any other.

Let us take one more view of this matter, by assuming no action by centre diagonals, under the full load. Then *cm* (Fig. 13), is also out of action, and *oc* alone, under $\frac{7}{9}$ maximum stress, contributes to the depression of point *m*, giving depression equal to $2\times\frac{7}{9}$E, = 1.55E, (assuming $h = v$).

The point *d* is depressed by the maximum change of two obliques, and one vertical, giving depression = 5E. Therefore the distance *md*, is increased by (5 — 1.55) E, = 3.45E. Hence *dl* and *me* must be elongated by 1.725

times the amount due to the maximum stress, in order to escape action. Suppose the member to be of wrought iron, proportioned to a maximum stress of 10,000lb to the square inch. Then, the extension due to 17250lb to the inch, is about $\frac{17250}{25000000}$ × length = .00069 × length, and if length = 15 feet, the extension is equal to .00069×15, = .01035 ft. or, say $\frac{1}{8}$ inch.

Hence, in a 7 panel truss, as represented in Fig. 13, with $h = v$, if the diagonals in the middle panel be slack, by $\frac{1}{8}$ inch in 15 feet of length, no decussation will take place, and the centre diagonals will be inactive, under the full load of the truss. If those members have less than that degree of slackness, they will be in action in such circumstances.

It would be a very badly adjusted piece of work in which such a degree of slackness should occur, and we may fairly conclude, that the centre diagonals, in this class of trusses, are never entirely inactive.

But the quantity E, is so very small, with any kind of material, and with any co-efficient that may affect it, in practice, that a slight inaccuracy of adjustment, may so change the practical form the theoretical results deduced by calculation, as to decussation, as to render the latter of no great practical reliability. Hence, after all, perhaps the most unexceptionable course, in this regard is, to follow the rule given before [XLVIII], of estimating stresses on both hypotheses, and taking the highest estimate for each part.

Now, perhaps, this subject has been discussed at greater length than its practical importance demanded, considering the small percentage of error liable to occur in any case; but with regard to this, as well as to other matters, it is well to know, what *may* be known without inconvenient or unreasonable effort at investigation.

THE WARREN GIRDER.

L. There is another form of truss operating upon the same principle as truss Fig. 13, in which one set of oblique members is left out, so that only one diagonal remains to each panel. The diagonals meet and connect with one another and with the chords, forming alternate nodes at the upper and lower chords. This truss, represented in Fig. 15, requires an even number of panels that the two half-trusses may be symmetrical.

This is an extension of truss Fig. 5, with tension verticals for suspending floor beams from the upper nodes, when the travel-way is along the lower chord, and thrust verticals ascending from the lower nodes, in the case of what are technically called deck-bridges.*

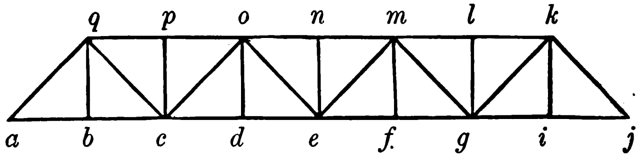
We compute the stresses of the members of this truss, by placing the figures 1, 2, 3, etc., over the diagram as in preceding cases, and from a second line or series of figures, by adding all those in the first series, as in case of truss Fig. 12, because each weight tends to act upon every diagonal. Each figure in the second line, is the co-efficient of w'' in the expression of the greatest weight transferred to the right hand abutment, through the diagonal crossing the panel next on the right hand of the figure; and the action is tension or thrust, according as the diagonal ascends or descends toward the right. Thus, the Fig. 6 over o, indicates that $6w''$ is the greatest weight acting by thrust upon oe, while 10 over the point e, indicates $10w''$ as the maximum weight acting by tension upon em. These

* The author built several small bridges upon this plan, to carry a rail road track over common highways, in 1849 or 1850, believed to have been the first application of this form of truss.

figures only indicate weight transferred from left to right, and it is evident that the same weights in a reversed order, are transferred from right to left, through the same diagonals. Hence, a third series of figures under the second, composed of the same figures in a reversed order, shows the weights carried by the several diagonals from right to left. The figures in the third line, show the weights acting on diagonals next on the *left* of respective figures. It will be seen also, that the figures under odd numbers of the upper line,

FIG. 15.

1	2	3	4	5	6	7
1	3	6	10	15	21	28
28	21	15	10	6	3	1



indicate weights acting by thrust, and those under even numbers, by tension. The figures 6 and 15 over *o*, indicate $6w''$ acting on *oe*, and $15w''$, on *oc*, both by thrust. Again 3 and 21 under 2, indicate $3w''$ acting on *co*, and $21w''$ upon *cq*, both by tension. The figure 28 at the right and left, under 1 and 7, indicate $28w''$ acting by thrust upon *aq* and *jk*.

Now if we add all the figures in the second and third lines standing under odd numbers of the upper line, we obtain the co-efficient of w'' for the aggregate maximum weights acting by thrust upon oblique members, while the sum of all the figures in like manner, under even numbers, forms the co-efficient of w'' for the aggregate maximum weights acting by *tension* upon obliques. The former gives $100w''$,$=12.5w$ for compression, and the latter, $68w''$,$=8.5w$, for tension. Hence, making $h=v=1$, we have as expressions for amount of thrust and tension action upon material in oblique members, 25M for thrust and 17M for tension.

One half of the lower chord obviously sustains a stress of $28w''$, equal to horizontal thrust of the end braces, and the other half, $60w''$,= horizontal action of *aq*, *qc* and *co* (under full uniform load), at one end, and of corresponding diagonals at the other end, giving required material for chord equal to 44M.

The compression of the upper chord, equals the horizontal thrust and pull of *aq* and *qc*,= $48w''$, for $\frac{2}{3}$ of its length, with the addition of $12w''$ for horizontal thrust of *co*, and $4w''$ for pull of *oe*, making $64w''$ for the two middle panels. Hence expression for material is 40M. The verticals obviously require tension material equal to 4M, and the aggregate for the truss, is,

For Compression.		*For Tension.*	
Chord,................	40M	Chord,..................	44M
Obliques,.............	25M	Diagonals,.............	17M
		Verticals, ...	4M
Total,..................	65M	Total,.....................	65M

A corresponding truss with 2 diagonals in each panel, on the plan of Fig. 13, shows the same expressions for materials, or amount of action of both kinds, item for item, and any advantage possessed by either plan, must depend essentially upon the more advantageous action of compression material.

Truss Fig. 15, has fewer intermediate thrust diagonals, and greater concentration of weight upon them; which is favorable; while in the other, the diagonals crossing one another, are enabled to afford mutual support laterally, in certain modes of construction.

The upper chord in Fig. 15, acts at a decided disadvantage, in having no vertical support for a length of 2 panel widths, unless it be especially provided at additional expense. As a deck bridge, with struts, or posts at *p*, *n*, *l*, and lateral tying and bracing, the truss may answer an excellent purpose. But even in that case, it can scarcely be considered as preferable to the truss with a double system of diagonals.

The Ohio river bridge at Louisville, Ky., has its long spans (about 400 ft.), constructed upon the plan of Fig. 15, and no plan which we have considered, shows a less *amount of action* upon material. These are believed to be the longest spans of *Truss Bridging* in the country.

An eight-panel truss upon the plan of Fig. 12, gives the following expressions for amount of material.

Compression.		*Tension.*	
Chord,	43M	Chord,	41M
Ends,	14M	Diagonals,	28M
Verticals,	7M	Verticals,	2M
	64M		71M

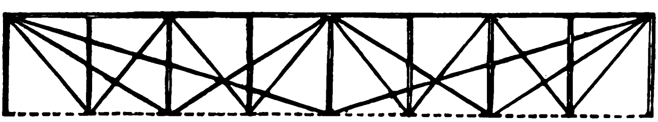
This indicates a difference of nearly 4 per cent, as to amount of action upon material, in favor of the truss without vertical members, generally speaking; *i. e.* in which there is no regular transfer of action from one to another, between diagonal and vertical members, as in truss Fig. 12.

This advantage is made still larger in certain modes of construction, by the circumstance that the same members, in trusses 13 and 15, may sometimes act by tension and thrust, on different occasions, without any more material than would be required to act in one direction only.

LI. It may be proper in this place, to refer to still another form of trussing, which has enjoyed a degree of popular favor, and which differs somewhat from any we have hitherto considered. The plan is seen in outline, in Fig. 16. Each weight is sustained primarily by a pair of equally inclined tension members, and thereby transferred either to the king posts standing upon the abutments, or, to posts sustained by other pairs of equally inclined suspension rods of greater horizontal reach; which in turn, transfer a part to king posts, and another part to a post sustained by obliques of still greater reach, until finally, the whole remaining weight is brought to bear upon the abutments by a single pair of obliques, reaching from the centre to each abutment.

FIG. 16.

THE FINCK TRUSS.



In Fig. 16, are represented four different lengths of obliques, in number, inversely as the respective horizontal reaches. The first set contains 8 pieces reaching horizontally across one panel, and sustaining each $\frac{1}{2}w$. The next longer set, of four pieces, reach across two panels, and sustain each $1w$; one-half applied directly, and the other, through posts and short diagonals. The third and longest set, contains but two pieces, reach across four panels, and sustain together $4w$; of which $1w$ is applied directly, $1w$ through two short diagonals, and $2w$ through two intermediates.

Now, as each set sustains the same aggregate weight, namely $4w$, the material in each set, will be represented by this weight multiplied by the square of the lengths respectively, and divided by v: and, making $k = v = 1$, the squares of respective lengths are 2, 5 and 17, which added together and multiplied by $4w$, and w changed to M, gives 96M=amount of material in tension obliques, the only tension members in the truss.

The upper chord sustains compression equal to the horizontal pull of one oblique member of each class, obviously equal to $10\frac{1}{2}w$, with length = 8. Hence, required material equals 84M. End posts sustain together, $7w$, centre post $3w$, and the two at the quarters, one w each, in all $12w$, and the representative for material is 12M; whence the total for thrust material is 96M, making a grand total of thrust and tension material= 192M.

The 8 panels trapezoid with verticals, requires,... 135M
Do " " without verticals,......... 130M

This comparison exhibits an amount of action in case of the first (Fig. 16), which, considering that it possesses no apparent advantage as to the efficient working of compression material, would seem to exclude it, practically, from the list of available plans of construction.

DISTINCTIVE CHARACTERISTICS OF THE ARCH.

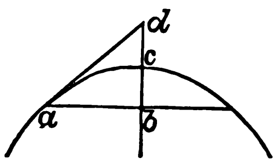
LII. We have seen that all heavy bodies near the earth's surface (except when falling by gravity or ascending by previous impulse), exert a pressure upon the earth equal to their respective weights. We have also seen that the object of a bridge, in general, is, to sustain bodies over void spaces, by transferring the pressure exerted by them upon the earth, from the points immediately beneath them, to points at greater or less horizontal distances therefrom.

We have, moreover, seen that this horizontal transfer of pressure can only be effected by oblique forces (neither exactly horizontal nor exactly vertical), and have discussed and compared, in a general way, various combinations of members, capable of effecting this horizontal transfer of pressure.

But, without going into unnecessary recapitulation, we find two or three styles of trussing, possessing more or less distinctive features, which promise decidedly more economical and satisfactory results than any others; and, to make the properties and principles of action of the best and most promising plans as thoroughly understood as may be within the proposed limits of this work, will form a prominent object in the discussions of succeeding pages.

The distinctive feature of the arch, as a sustaining structure, consists in the fact that all the oblique action required to sustain a uniformly distributed load, is exerted by a single member of constantly varying obliquity from centre to ends; each section sustaining all the weight between itself and the centre, or crown of the arch, and none of the weight from the section to the end; so that the weight sustained at any point, is as the horizontal distance of that point from the centre. Consequently (the arch being supposed in equilibrio under a uniform horizontal load), the horizontal thrust at all points must be the same, and the inclination of the tangent at any point should be such that the square of the sine, divided by the cosine of inclination (from the vertical), may give a constant quotient. For, regarding each indefinitely short section of the arch as a brace coinciding with the tangent at the point of contact, its horizontal thrust equals the weight sustained, multiplied by the horizontal, and divided by the vertical reach of the brace. But the horizontal and vertical reaches are respectively as the sine and cosine of the angle made by the tangent with the vertical; that is, as *ab* and *bd*, Fig. 17, while the weight is also as the sine *ab*, of the angle *adb*. Hence, the weight by the horizontal reach, is as ab^2, or as the square of the sine of *adb*; and the constant horizontal thrust of the arch at all points, is as $\frac{ab^2}{bd}$; or, as $\frac{ab^2}{\frac{1}{2}bd}$.

FIG. 17.



Now this condition is answered by the parabola, in which $bc = cd = \frac{1}{2}bd$, and $\frac{ab^2}{bd} = \frac{1}{2}\frac{ab^2}{cb} =$ constant C, whence $ab^2 = cb \times$ constant $2C$, which is the equation of the parabola.

This quality of the arch truss, allowing nearly all of the compressive action to be concentrated upon almost the least possible length, and consequently, enabling the thrust material to work at better advantage than in plans where this action is more distributed, and acts upon a greater number and length of thrust members, enables it to maintain a more successful competition with other plans than we might be led to expect, in view of the greater amount of action upon materials in the arch truss, than what is shown in trusses with parallel chords. Hence, we should not too hastily come to a conclusion unfavorable to the arch truss, on account of the apparent disadvantage it labors under, as to amount of action upon material. These apparent disadvantages are frequently overbalanced by advantages of a practical character, which can not readily be reduced to measurement and calculation.

The preceding general comparisons are to be regarded only as approximations, and should not be taken as conclusive evidence of the superiority or otherwise, of any plan, except in case of very considerable difference in amount of action, with little or no probable advantage in regard to efficient action of material.

EFFECTS OF WEIGHT OF STRUCTURE.

LIII. In preceding analyses, and estimates of stresses upon the various members in bridge trusses, regard has only been had to the effects of movable load, which may be placed upon, or removed from the structure, producing more or less varying strains upon its several parts.

But the materials composing the structure, evidently act in a similar manner with the movable load, in producing stress upon its members; the only difference being, that the weight of structure is constant, always exerting or tending to exert the same influence upon the members, instead of a varying action, such as that produced by the movable load. In order, therefore, to know the absolute stress to which any member is liable, and thereby to be able to give it the required strength and proportions, we have to add the stresses due to constant and occasional loads together.

The weight of structure evidently acts upon the truss in the same manner as if it were concentrated at the nodes along the upper and lower chords, and of the arch, in case of the arch truss. And, since much the larger proportion of it acts at the points where the movable load is applied, if we regard the whole as acting at those points, the results obtained as to stresses produced by it, will be sufficiently accurate for ordinary practice. Still, more closely approximating results may be obtained by assigning to both upper and lower nodes, their appropriate shares of weight sustained, as may easily be done when deemed expedient.

If we divide the whole weight of superstructure supported by a single truss, by the number of panels, the quotient, which we may represent by w', will show the weight to be assumed for each supporting point, on account of structure; and the stresses produced by such weights, added to the maximum stresses of the several members, due to the movable load, will represent the true absolute stresses the respective members are liable to bear.

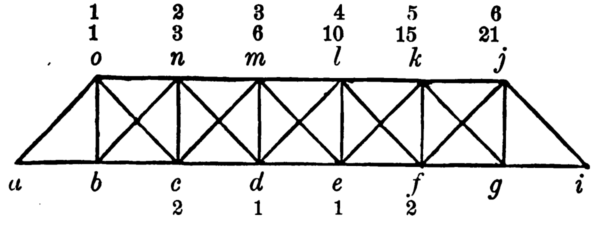
Now, as far as relates to parts suffering their maximum stresses under the full load, such as chords, arches, king braces, and verticals in the arch truss, as to their *tension* strain, we have only to substitute W, $(=w+w')$, in place of w, in expressions obtained for stresses due to movable load. In other cases, w and w' will have each its peculiar and appropriate co-efficient.

The diagonals of the arch truss, are obviously not affected by weight of structure, as they are not so under full and uniform movable load. Moreover, the weight of structure acts in constant opposition to the compressive action of movable load upon verticals. Hence, in truss Fig. 11, where we find the varying movable load gives a maximum compression upon the longest, equal to $3w''$, and upon the next shorter, equal to $2w''$, the weight of structure diminishes those quantities to $3w''-w'$, and $2w''-w'$ respectively. Or, if we would be more exact, we may add in both cases, the weight of a segment of the arch, which has no tendency to produce tension upon the verticals; or we may subtract only $\frac{2}{3}$ or $\frac{3}{4}$ of w'; thus, $3w''$—$\frac{2}{3}w'$, and $2w''$—$\frac{2}{3}w'$, may be taken to represent the compressive action upon the verticals in Fig. 11.

LIV. In the case of truss Fig. 12, the only diagonals acting under uniform load, are *oc*, *fj*, *nd* and *ek*; the two latter sustaining, of weight of structure, $1w'$, and the two former, $2w'$. And, the maximum movable weight borne by those members, being [XL] $10w''$ and $15w''$, the absolute maximum will be $10w''+w'$ for *nd* and *ek*, and $15w''+2w'$ for *oc* and *fj*.

Now, if we place the figure 1 under *d* and *e*, (Fig. 12 A), and the figure 2 under *c* and *f*, and so on, in case of a greater number of panels, to the foot of the last diagonal each way, inclining outward from the lower nodes, these figures are obviously, the co-efficients of w' to express the weights contributed by the material of the structure, to the stresses of diagonals extending upward and outward from the points to which the figures respectively refer.

FIG. 12 A.



Again, we have seen [XL], that a certain condition of the movable load, tends to throw $1w''$ upon *bn*, and another condition of such load, tends to throw $3w''$ upon *cm.* But, since, as we now see, the weight of structure tends to throw a constant weight of $2w'$ upon *oc*, which is antagonistic to *bn*, the actual maximum weight upon *bn*, is $1w''—2w'$, which will always be a negative quantity, in practice; whence *bn* must always be inactive, and may be dispensed with.

The maximum weight upon *cm*, as modified by weight of structure, is in like manner reduced to $3w''—w'$, which will in practice, be either negative, or of quite small amount. Hence, we have the following rule: For the absolute maximum stresses of diagonals (in case of parallel-chord trusses with verticals), we add the effects of weight of structure to the maximum effects due to variable load, where both fall upon the same, and subtract the former, in cases where the two forces fall upon counter, or antagonistic diagonals.

In case of parallel-chord trusses *without* verticals, we add the effects of constant and variable load upon each diagonal, when alike, i. e., when both tensile or both compressive, and subtract the former when the effects are alike.

DOUBLE CANCELATED TRUSSES.

LV. The use of chords in a truss being to sustain the horizontal action (whether of thrust or tension) of the oblique members, it follows that the aggregate stress of chords, is equal to the aggregate horizontal action of all the diagonals acting in either direction. And, the horizontal action being obviously as the number and horizontal reach directly, and as the vertical reach inversely; also, the length of truss being as the number and horizontal reach of diagonals, while the vertical reach is as the depth of truss, it follows that the stress of chords is directly as the length and inversely as the depth of truss, other conditions being the same.

Hence, if the depth of truss be so reduced as to make the ratio of length to depth indefinitely large, the stress and required material of chords, become indefinitely large. On the contrary, if the depth be indefinitely great, although the stress of chords be ever so small the length and required material for diagonals and verticals must be indefinitely large. It is manifest, then, that between these two extremes there is a practical optimum,— a certain ratio of length to depth of truss, which, though it may vary somewhat with circumstances, will give the best possible results as to economy of material in the truss. This matter will be taken into consideration hereafter, and is referred to here, to show the expediency of generally increasing the depth, with increase of lengths in the truss.

Now, in trusses of considerable length, and, consequently, depth, it becomes expedient, in order to avoid too great a width of panel (horizontally), or an inclination of diagonals too steep for economy of material in those members, to extend them horizontally across two or more panels, or spaces between consecutive nodes of the chords. In such cases, the truss may be called double or treble cancelated, according as the diagonals cross two or three panels.

LVI. To estimate the stresses of the members of double cancelated trusses with vertical members, a slight modification of the process already described, [XL, &c.], is required, as follows:

Having placed the numbers 1, 2, 3, &c., over the nodes of the upper chord, as seen in Fig. 18, place under each odd number, the sum of all the odd numbers in the first series, up to and including the one under which the sum is placed; and the same with respect to the even numbers. Then, the second series of figures may be used in precisely the same manner as that explained with reference to Fig. 12, to determine the weight sustained by, and the maximum stress produced upon, each diagonal and vertical, by equal weights upon all or any of the nodes of either chord.

For example; supposing the truss to have tension diagonals and thrust verticals; take the diagonal havits lower end under 5 (upper series), and its upper end under 7. This diagonal may be represented by 5/7, while 5\7 may indicate its antagonist, and so of other diagonals. Then, as we see 9 (the sum of 1+3+5), in the second series, over the lower end of 5/7, and, as the diagram represents a truss of 16 panels, we know that the diagonal in question is liable to a maximum weight of $\frac{9}{16}w, = 9w''$. This amount is to be diminished, of course, by the weight due from weight of structure to the counter diagonal.

Again, the diagonal 9/11 sustains as a maximum from variable load, $25w''$; which will require to be *increased* on account of weight of structure, since the latter, in this case, acts upon the main, and not upon the counter diagonal, as in case of 5/7.

Now, to obtain the effects of weight of structure and uniform load, the truss having even panels, we place $\frac{1}{2}$ under the centre node of the lower chord, because half of the weight w', which is supposed to be concentrated at that point, tends to act on each of the diagonals rising from that point.

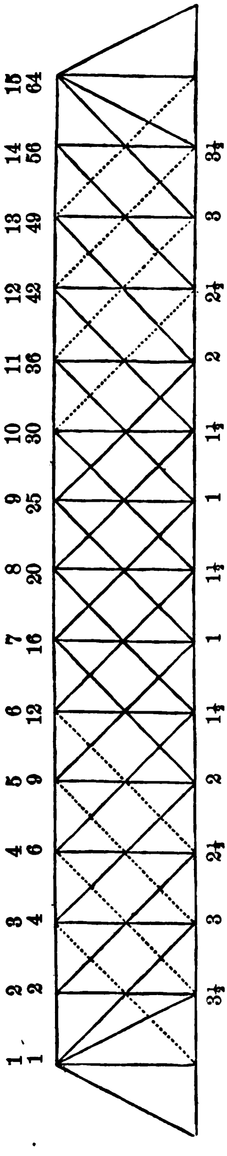


FIG. 18.

At the next node from the centre, each way, the figure 1 is set, because, of the weights (w'), concentrated at those points, each bears upon its nearest abutment (the truss being uniformly loaded), through the diagonals running upward and outward from those points. If this be not so, each must transmit a part of its amount past the centre, through the antagonistic diagonals 7╱9 and 7╲9, which is contrary to statical law.

Then we put 1½, 2½, 3½, etc., under alternate nodes from the centre, and 1, 2, 3, etc., under alternates beginning at the first on each side of the centre; as shown in diagram Fig. 18.

These figures form the co-efficients of w', to indicate the weights acting, or tending to act, upon the diagonals running upward and outward from these numbers respectively, arising from weight of structure, and also, the co-efficients for $(w+w')$, to express the load tending to act on diagonals, arising from both superstructure and movable weight, when the truss is fully loaded. For illustration;

the diagonal 5/7 we have seen to be liable to a maximum stress of $9w''$ from variable load, and, as we have the figure 1 at the foot of 5\7, it shows that the weight due to the latter on account of structure is $1w'$, which must be subtracted from $9w''$ to obtain the *actual* maximum to which 5/7 is liable; which is $9w''-w'$.

If w' be equal to or greater than $9w''$, then 5/7 is subject to no action, and may be dispensed with. As to the advantage of introducing counter diagonals, merely for the purpose of *stiffening* the truss, the results of my investigations will be given in a subsequent part of this work.

The maximum weight sustained by any thrust upright, is manifestly equal to the greatest weight borne by either diagonal connected with it at the upper end, since any weight borne by 3/5, for instance, being transferred to the antagonist of 5\7, thereby diminishes by a like amount, the maximum action of the latter. Whence the upright at 5, can receive no more load from the two diagonals, than the maximum load of one, and this relation holds in general.

The reason of adding alternate figures to form the second series over the diagram, will be obvious, when it is observed that there are two independent systems of uprights and diagonals; one of which includes the uprights under even numbers in the upper series, and the diagonals connecting therewith, and the other, the remaining uprights and diagonals. Now weight applied at the nodes of either of these systems, can only act upon members of that same system; that is, weight applied at nodes indicated by even numbers in the upper series can only act upon the first above named system of uprights and diagonals, and vice versa.

The main end braces are acted upon by both systems; so that to obtain the weight sustained by them, we must add the numbers 56 and 64 (and corresponding numbers in other cases), making in this case $120w''$ equal to $7\frac{1}{2}w$.

The uprights under 1 and 15, sustain each a tension equal to w, for variable load, and to $w+w'$, for weight of variable load and superstructure together; which obviously gives their greatest strain.

Having thus determined the greatest *weights* to which the several verticals and diagonal members are liable, we proceed as in former cases, to multiply those weights by lengths of diagonals, and divide the products by lengths of verticals, to obtain the stresses of diagonals; remembering to take into account the difference in length between those having a horizontal reach of only one panel, at and near the ends of the truss, and those that reach across two panels.

The mode of estimating the stresses upon the different portions of the chords, depending upon the horizontal action of diagonals, has been sufficiently explained. It is only necessary to observe that the end braces produce compression upon the upper, and tension upon the lower chord, through their whole lengths, equal to $\frac{1}{2}(w+w')$, multiplied by the number of nodes of the lower chord, and that product multiplied by $\frac{h}{v}$; and that each pair of intermediate diagonals analogously situated with respect to the ends of the truss, whether acting by thrust or tension, produce tension and thrust in like manner, upon the portions of the lower and upper chords, between their points of connection with the chords. Thus is generated a progressive and determinate increase of action upon succeeding portions of the chords from the ends to the centre of the truss.

In the case of a deck bridge, the weights sustained by thrust uprights, are respectively indicated by the figures over the diagram on the right hand half of the truss, prefixed to w'', for movable load, and the figures under the diagram prefixed to w', for weight of structure, being the same weight which gives the maximum stress to the diagonal running upward and outward from the foot of the upright. Tension verticals at the ends sustain no weight.

TRUSSES WITHOUT VERTICALS.

It will be seen upon a general view of the action of the different parts of a truss with parallel chords, that the diagonals (and verticals when used), form media through which weight acting upon the truss, is reflected back and forth between the upper and lower chords, until it comes finally to bear upon the abutment.

A weight applied at one of the nodes of the lower chord, of course, cannot be sustained by the tension of that chord, which acts only in horizontal directions; but is suspended by a tension piece, whether oblique or vertical, from a node in the upper chord. But the upper chord acting also horizontally, cannot sustain the weight. Consequently, a thrust member, either oblique or vertical, must meet the force at that point, to prevent the weight from pulling down the upper chord, and destroying the structure.

Hence, we see, that in all the cases we have considered, of trusses with parallel chords, the weight, whether applied at the upper or lower chord, acts alternately upon thrust and tension pieces, extending directly or obliquely from chord to chord.

With reference to Fig. 18, we have regarded the weight as transferred from tension diagonals to thrust verticals, and the contrary. But if we conceive the verticals to be removed, except the endmost, we have only to insert a thrust brace from the abutment to the second node (or the first from the angle), of the upper chord, and to so form and connect the other diagonals as to enable them to act by either tension of thrust, and we have a truss capable of sustaining weights applied at all, or any of the nodes of the upper and lower chords, in the same manner as the truss *with* verticals, represented in Fig. 18. In this condition, the truss will act upon the principles discussed with reference to Fig. 13. For this modification of the truss, see Fig. 19.

To estimate the strains upon the several parts of such a truss, due to weights w, w, etc., at the nodes of the lower chord; we may place the figures 1, 2, 3, etc., over the nodes of the upper chord, as was done in the case of Fig. 18. But, instead of adding alternate figures to form the second series, to be used as co-efficients of w'', for expressing the weights sustained by diagonals, we add every fourth figure; because it is only the weights at every fourth node, that act upon the same set of diagonals.

For instance; the weights at 1, 5, 9 and 13, act upon their peculiar set of 8 pieces (excluding the end braces, but including the tension vertical at 1), and none of the weights at the other nodes have any action upon those pieces; as is made obvious by an inspection of Fig. 19.

Again, the weight at 2, 6, 10 and 14, have their peculiar and independent set; and so of those at 3, 7, 11 and 15, and those at 4, 8 and 12. Therefore, in form-

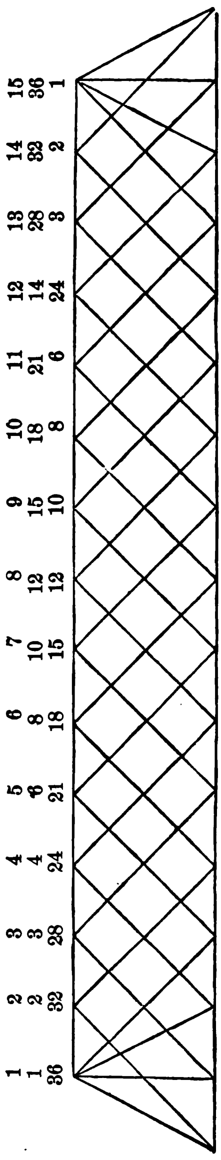


FIG. 19.

ing our second series of numbers, we place under each figure of the first series, the sum of that figure, added to every 4th figure preceding; that is, under 12, place the sum of 12, 8 and 4 = 24. Under 5, the sum of 5+1 =6. The four first figures, having no 4th preceding figures, are simply transferred, without addition or alteration.

These numbers in the second series, are the co-efficients of w' ($=w$ divided by the number of panels in the truss, being 16 in this case), to express the greatest weights acting by tension on each diagonal having its lower end under the number used, and the upper end under a higher number. Also the weight acting by thrust upon the diagonal meeting the former at the upper chord. The last, or highest number, determines the weight sustained by the tension vertical under the number, the vertical being a member of one of the four sets of alternate thrust and tension pieces connecting the two chords.

A third series of figures, formed by reversing the order of the second—placing the lowest number of the third under the highest of the second series, and *vice versa*, prefixed as before to w'', will show the weights sustained by thrust and tension of diagonals in the reversed order; i. e., whereas one series shows the amount of tension a particular diagonal is liable to, the reversed series shows the *thrust* the same piece must exert in a different condition of the load.

Thus we ascertain, as in the case of truss Fig. 13 [XLV], that nearly all of the diagonals are exposed to two kinds of action, thrust and tension; and it is only the preponderance of the larger over the smaller of these forces, which has place when the truss is fully loaded, and it is only this preponderance which is to be used as co-efficient to $(w+w')$ in estimating the stresses upon the different portions of the chords, and as co-efficient to w', in modifying the effects of the variable load upon diagonals, as affected by weight of structure. But it is to be remembered that the numbers over the diagram are to be divided by the number of panels, before being used before w and w', in the expression of stresses of members. Thus, we have, as the effect of variable load upon the diagonal 2/4 ..., $2w''$ $(=\frac{2}{16}w)$, as the greatest weight acting by tension, and $\frac{18}{16}w$, the greatest acting by thrust. Hence the weight upon this piece, due to weight of structure, is $(\frac{18-2}{16})w',=w'$, and it produces thrust or compression, because the thrust tendency is the greater. This weight (w'), added to $\frac{18}{16}w$, the greatest effect of variable load shows the maximum weight which can act by thrust upon that diagonal, to be $\frac{18}{16}w+w'$. We have, also, for the greatest weight acting by *tension* as modified by weight of structure, $\frac{2}{16}w - w'$, which is a negative quantity when w is less than $8w'$, as will usually be the case in practice; consequently that diagonal can seldom or never be exposed to the force of tension.

Again $\frac{16}{16}(w+w')\frac{h}{v}$, ($h$ and v representing horizontal and vertical reaches of the diagonal, as in previous discussions), is the amount contributed toward the maximum tension of the lower chord by the diagonal in question, not affecting, of course, that portion of the chord outside of the connection therewith, or a like portion at the opposite end.

LVIII. It is to be remembered that the tension or thrust of a diagonal, is always equal to the weight sustained, multiplied by the length, and divided by the vertical reach of the diagonal.

The method here under discussion for estimating stresses, seems to need no further illustration. But the question as to *decussation*, affects the case of Fig. 19, as well as that of Fig. 13. The two sets of diagonals which meet the upper and lower chords in the centre, have symmetrical halves on each side of the centre, and no action can pass the centre upon either, when they are uniformly loaded; whereas, the two sets to which 7╱9 and 7╲9 belong, have the half of one on either side of the centre, a counter part to the half of the other set on the opposite side; and the diagonals 7╱9 and 7╲9, will act or not, according as their opposite points of connection with upper and lower chords, are carried farther apart, or the contrary. Now, as the points 9 and 7, upper chord, are depressed by the change in one vertical and 3 diagonals, while the opposite points at the lower chord are depressed by the change in 3 diagonals only, we might naturally expect to find greater depression in the upper than the lower points; though this does not follow as a matter of necessity, since the less number of members, by being more nearly under a maximum stress, might give greater depression than the greater number, under less stress, as compared with their maximum. Now, the vertical at 1, being under maximum weight, gives depression = E; (adopting the notation used with reference to Fig. 13 [XLIX.]). The two diagonals 1╱3 and 3╲5 being under $\frac{20}{21}$ maximum, give depression equal to $\frac{20}{21}\times 4$E (making $h=v=1$), = 3.81E; while the diagonal 5╱7, under $\frac{4}{10}$ maximum, gives depression = 0.4×2E, = 0.8E, making a total depression of point 7, upper chord, = 5.61E. Again, the diagonal 1╲3, under maximum stress, gives depression = 2E, while 3╱5 and 5╲7, under $\frac{12}{15}$ maxm u m stress, give depression = $\frac{4}{5}\times$ 4E, = 3.2E, making a total for the point 7, lower chord, equal to 5.2E, which is *less* by 0.41E, than the depression of the opposite point in the upper chord, whereas it should be greater by 0.8E, in order to give to 7╲9 and 7╱9, the tension assigned to them by the decussation theory.

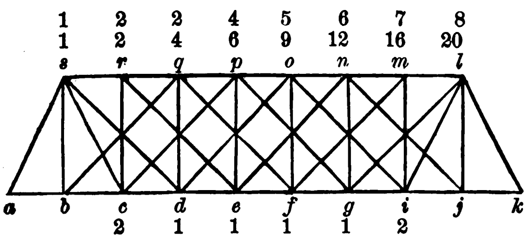
But we must not conclude from this fact, that there is *no* decussation in this case. For, if we assume that 7╲9 is inactive under the full load, it follows that 5╱7 is also inactive, and that 1╱3 × 3╲5 sustain only $\frac{16}{21}$ maximum stress, producing $\frac{64}{21}$ E, = 3.05 E, which added to 1E for the vertical at 1, makes 4.05 E, = depression at point 7, upper chord; while the 3 diagonals contributing to depression of the opposite point in lower chord, are under maximum stress, producing depression = 6 E. Hence, we see, that upon this hypothesis, the distance between these two points, measuring the vertical reach of the diagonal, is increased by $(6-4.05)E = 1.95E$. This can not be, without producing tension upon diagonals 7╲9 and 7╱9. Since then these members can not be entirely without action, and as previously shown, they can not have as much action as the decussation theory assigns to them, it follows, in this case, that they must *act*, but with less intensity than the theory assigns them.

In this case, as well as in that of Fig. 13, the result would be changed somewhat, by taking into the calculation the weight of structure, which would change to a small extent, the relation between the maximum stresses of diagonals, and the stresses they sustain under a full load. For the stress due to weight of structure, is constant, and that due to variable load, is greater, upon most of the diagonals, under certain conditions of a partial, than under a full load. Hence, while 5╲7 sustains (under full load), only $\frac{12}{15}$ maximum upon that part of the material provided for variable load, it sustains a *full* maximum upon the part provided to sustain weight of structure. It is easy enough to take these things all into account, in estimating the amount of decussation in special cases. Still, it is doubtful whether any better practical rule can be adopted, than the one previously given, [XLVIII]; namely, to estimate stresses upon both hypotheses, and take the highest estimate for each part.

DECUSSATION IN TRUSSES WITH VERTICALS.

LIX. In trusses of this class with odd panels, and diagonals crossing two panels, as in Fig. 20, it will be seen, on subjecting them to analysis, such as was explained with reference to Fig. 18 [XVI], that, while in trusses of even panels, the figures in the second line over the diagram, indicate the *maximum* stresses of diagonals, and those *under* the diagram, the stresses under uniform load (which are generally less than the maximum under partial loads), in case of the truss with odd panels, the bottom figures show, for certain diagonals, greater stresses for the full, than the upper figures give as the maximum for partial loads. Thus, in Fig. 20, the number 16 over *m*, indicates $16w''$ $(=\frac{16}{9}w)$, as the maximum weight for *il*, while the figure 2 under the point *i*, indicates that *il* sustains $2w$ $(=18w'')$, under the *full* load. It should be remarked here, that the figure 1 under the first two nodes on either side of the centre, and the figure 2 under the next, are thus

FIG. 20.



placed upon the assumption that all the weight on either side of the centre, is made to act on its nearest abutment. This would necessarily be so, if *en* and *fq* were removed or relaxed. But, with those members in place, and properly adjusted, there may be a decussation of forces through them, whereby a portion of the weights at *e* and *f*, may be made to bear upon the more remote abutments. Now, as the maximum on *en* is $6w''$ and that of its antagonist only $4w''$, the latter is not sufficient to neutralize the former entirely, but leaves a balance of $2w''$ which *may* be transmitted through *en* to *gl*, as an offset for a like amount transmitted through *fq* to *ds*. If this be so, then *fm* and *er* do not sustain the full weight of $1w$, but only $7w''$, which, being transmitted to *il*, makes, with the weight w ($=9w''$), applied directly at *i*, $16w''$, as indicated by the figures over the diagram, instead of $2w$ ($= 18w''$), as the figure 2 under the point *i* would indicate.

Now, whether the two diagonals *en* and *fq*, being apparently, in a state of partial antagonism, do in whole or in part neutralize the tendency of each other to transmit weight past the centre each way under a uniform load of the truss, is not quite obvious, and it may be proper to estimate stresses under both hypotheses, and take the highest estimate for each part of the truss.

It will be seen that *il* and *cs* are the only diagonals in Fig. 20, which show greater stress with a full than a partial load, upon the non-decussation hypothesis. But all the diagonals undergo different stresses, with the uniform load, as viewed under the different theories, and consequently, their effects upon the chords are different. The end brace *as*, sustains $4\ (w+w') = 4W$ substituting W for $w+w'$), under either theory, and the tension of *ac* equals $4w\frac{h}{v}$ (making $h=ab$, and $v = bs$). *cs* sustains 2W, or $\frac{16}{9}W$, whence *cd* sustains either $6W\frac{h}{v}$ or $5\frac{7}{9}W\frac{h}{v}$. Again, *ds* sustains W, or $1\frac{2}{9}W$, the former without, and the latter with decussation. This diagonal having a horizontal reach of $2h$, adds $2W\frac{h}{v}$ or $2\frac{4}{9}W\frac{h}{v}$ to tension of chord, making $8W\frac{h}{v}$, or $8\frac{2}{9}W\frac{h}{v}$, as the tension of *de*. For *er*, we have W without decussation, making a tension of $10W\frac{h}{v}$ for *ef*; while with decussation, *er* sustains $\frac{7}{9}W$, from which we subtract $\frac{2}{9}W$, for opposite action of *en*, leaving $\frac{5}{9}W$

giving horizontal pull $=1\frac{1}{9}W\frac{h}{v}$ to be added to $8\frac{2}{9}W\frac{h}{v}$ making $9\frac{3}{9}W\frac{h}{v}$ = tension of *ef*.

Upon the non-decussation hypothesis, *s r* and *m l*, of the upper chord sustain thrust equal to $8W\frac{h}{v}$, and the remainder of the chord, $10W\frac{h}{v}$. By the other hypothesis, *sr* and *ml* sustain $8\frac{2}{9}W\frac{h}{v}$, *rq* and *nm* sustain $9\frac{7}{9}W\frac{h}{v}$, and the other 3 sections, $10\frac{2}{9}W\frac{h}{v}$

LX. We may derive some more light upon this subject, by considering the conditions resulting from the elasticity of materials. Supposing the upper and lower chords to be so proportioned as to be uniformly contracted or extended under a uniform load of the truss, this does not require or imply any appreciable difference in lengths of diagonals. But the stress upon chords being produced by the action of diagonals, the latter, when, as here supposed, acting by tension, necessarily undergo extension, by which means, the panels (except the centre one), are changed from their original form of rectangles, to that of oblique trapezoids. For instance, the figure *g j l n* becomes longer diagonally from *g* to *l*, than from *n* to *j*, whence the point *g* falls lower than it would do, if the diagonal suffered no change.

Suppose then the truss to be fully loaded, and the diagonals *il*, *gl* and *fm*, to be each exposed to the same stress to the square inch of cross-section. In that case, *il* and *gl* suffer extension proportionally to their respective lengths, thereby causing depression of the points *i* and *g* respectively as the *squares* of those lengths. [See note in section XLIX.] Hence, the point *g* is depressed more than the point *j*, by the extension of diagonals, by as much as the square of *gl* exceeds the square of *il*, or as 8 to 5; assuming diagonals to incline at 45°. The panel *gm*, must therefore be oblique, and the distance *gm*, greater than *ni*. Again, the point *f* suffering the same depression from the extension of *fm*, as the point *g* suffers from that of *gl*, and a still further depression from the compression of *mi*, and the extension of *il*, it follows that the panel *fn* must also be oblique, and the distance *fn*, greater than the distance *og*.

Now, the obliquity of both of the panels *gm* and *fn*, manifestly contributes to the excess of distance *fm*, over *oi*. On the contrary, the centre panel *eo* has no obliquity due to extension of diagonals, or compression of uprights; since there is no cause for obliquity in one direction, more than the other. It seems to follow that *en*, crossing one oblique panel, must undergo extension; but not so much as *fm*, which crosses *two*.

Now, *fm* and *en* being equal in length, the weight sustained by each, is manifestly as the cross-section and extension combined; and as the former, *fm*, should be the larger in the ratio of 9 to 6, or as their maximum stresses; if we allow their extensions to be as 2 to 1, the greater for *fm*, the relative weights sustained would be as 18 to 6, or as 6 to 2. Our decussation theory gave their relative stresses as $7w''$ to $2w''$. This is not a wide discrepancy, seeing that the above computation is based in part upon a mere approximate data.

We may conclude then, that in cases like the one under consideration, decussation does actually take place. Still it obviously depends upon conditions which are not of the most determinate character. For, if *en* and *fq*, be relaxed or removed, under a full load of the truss, decussation can not take place, for the same obliquity of the two panels next to the centre one, which produces the tendency toward *tension* of *en* and *fq*, on the contrary, tends to *relax do* and *gp*, through which latter alone decussation could take place, in the absence of the former.

On the other hand, if *en* and *fq* be sufficiently strong, they may be strained to such a pitch as to bear all the weights at *e* and *f*, and leave *fm* and *er* entirely inactive. Hence, there is an uncertainty as to the action of these diagonals, which may be best obviated by estimating stresses upon both theories, and taking the highest estimates; as recommended with reference to trusses without verticals, and as previously suggested with reference to the case in hand.

In view of preceding facts and principles, it may be advisable to avoid the odd panel in trusses with verticals, when practicable without incurring more important disadvantages in other respects.

DECK BRIDGES.

LXI. Are those having the movable load applied at the nodes of the upper, instead of the lower chord, as generally assumed in preceding analyses.

It will readily be seen, on a brief contemplation of Figures 12 and 13, for instance, that weights applied at the upper chord, act directly upon compression members, either erect or oblique, as the case may be; and are thence transferred to tension members at the lower chord; according to the general principle, that weight applied at the upper end of a member, always acts by compression, and that which is applied at the lower end, by tension.

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In the case of truss Fig. 12, the action of tension diagonals is precisely the same, whether the weight be applied at the upper or lower chord. But the compression verticals, in the deck bridge, sustain as their maximum, the weights indicated by the figures immediately above them respectively, from the centre toward the right hand; and these weights, of course, are equal to those acting upon the diagonals respectively meeting the verticals at the lower chord; and consequently, greater than when the weight is applied at the lower chord. For illustration, in Fig. 12, as the truss of a deck bridge, the vertical *fk* sustains $15w''$, the same as *fj*, whereas, in the case of a "Through bridge" (with load applied at the lower chord), *fk* sustains only $10w''$ communicated to it through *ek*.

In the deck bridge also, the tension verticals *bo* and *jg* are essentially inactive, merely sustaining a small portion of the lower chord. The chords suffers the same stress in both through, and deck bridges.

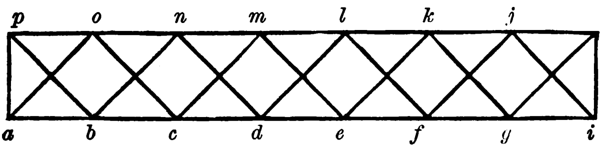
LXII. Load applied at the upper chord of truss Fig. 13, acts by thrust directly upon the diagonals meeting at the upper chord, and the maximum weight (from movable load), sustained by diagonals meeting at one of the upper nodes, are indicated by the two figures immediately over the node; the larger figure referring to the diagonal running toward the nearer abutment; e. g., the numbers 4 and 6 over the point *m*, signify $6w''$ = greatest weight borne by *mc*, and $4w''$ = the greatest borne by *me*.

It is obvious also, that the maximum thrust of any diagonal, equals the maximum tension of the diagonal meeting the former at the lower chord; that is, maximum thrust of *mc*, is equal to $6w''\frac{D}{v}$, = maximum tension of *co*. The maximum thrust of *bn* being equal to $9w''\frac{D}{v}$, the maximum tension of *bo*, equals $9w''$. This is an extra weight thrown upon the point *o*, in consequence of the vertical *bo*, being turned out of its regular direction of a diagonal in the position of *bp*,* in order to throw its load upon *oa*, whereby *op* and *pa* are rendered unnecessary. The weight borne by *oa*, therefore, instead of being $12w''$, as indicated by the figure 12 at *o*, is $12w''+9w''$, $= 21w''$, $= 3w$.

The figure 1 over *o*, denotes the tendency of $1w''$ to act upon *oc*, by thrust, by which tendency the tension of *oc*, under a full load of the truss, is reduced to $5w''\frac{D}{v}$.

LXIII. If Fig. 12 be assumed to represent a truss with tension verticals and thrust diagonals, the figures over the upper nodes, prefixed to w'' indicate the weights tending to act by compression upon the diagonals descending toward the right from the nodes respectively; which weight is transferred to the vertical meeting the diagonal at the lower chord. This constitutes the maximum load of the vertical, in case of a deck bridge. Otherwise, the maximum stress of verticals is shown by the figures immediately over

FIG. 13A.



* The point *p*, not seen in Fig. 13, is assumed to be at the intersection of a vertical line through the point *a*, with the upper chord-produced. The arrangement above alluded to, gives the truss a reclangular, instead of a trapezoidal form of outline, which involves no more action upon material, 'though it increases the number of members in the truss. [See Fig. 13A.]

them, prefixed to w'', provided, that in this case, the maximum stress of a vertical can never be less than $w.$ = the weight applied immediately at its lower end.

RATIO OF LENGTH TO DEPTH OF TRUSS.

LXIV. Having explained and illustrated, it is hoped intelligibly, methods by which may be computed the stresses of the various parts composing most of the combinations of members capable of being used in bridge trusses, with a view to giving to each part its due proportions, it may be proper to give attention to the *general* proportions of trusses, and such other considerations as may affect the efficient, and economical application of materials in bridge construction.

The ratio of length to depth of truss is susceptible very great range, and it is obvious that some certain medium, in this respect, will generally give more advantageous results, than any considerable deviation toward either extreme. For, it will be observed, that in the expressions we have derived for the amount of action open *chords*, $\frac{1}{v}$ appears as a factor; v representing the depth of truss, between centres of chords. Hence, the smaller the value of v, the greater the stresses of chords, so that when v=0, these stresses become infinite, and the chords require an infinite amount of material; in other words, the case is impossible. On the other hand, if v be infinitely great, though the stress of chords be reduced to nothing, the verticals and diagonals being infinitely long, and sustaining a definite weight, also require an infinite amount of material.

Now, between these two impracticable extremes where shall we look for the most advantageous ratio?

It can not be the arithmetical mean, for there is no such mean between $v = 0$, and $v =$ infinity. Undoubtedly, we shall be unable to do more than answer this question approximately; and that, only with reference to specific cases; for the ratio suitable for one length of span, and in one set of circumstances, will often be found quite unsuitable under different circumstances.

We have seen that the material required in chords, is in general, inversely as the depth of truss, or as $\frac{1}{v}$. Also, that the material for verticals and diagonals, increases with increase in the value of v; though not in a determinate ratio. But assuming the latter classes of members, including the main end braces of the Trapezoidal truss, to increase in the ratio at which v increases, while the chords diminish at the same rate, we might reasonably assume, that the minimum amount of action upon materials would occur when the amount of action upon chords were just equal to that upon all other parts of the truss.

By recurrence to the analysis of truss Fig. 12, [XLIII], we find amount of action upon chords, represented by $56\frac{h^2}{v}$M, and that upon all other parts, by $\left(16\frac{h^2}{v} + 22.57v\right)$ M. Here, h is equal to $\frac{1}{7}$ part of the length of truss, while v is variable; and, by making these two co-efficients of M equal, and deducing thence the value of v, we have the depth of a 7 panel truss in which the amount of action upon chords, equals that of all other parts. Thus, putting $56\frac{h^2}{v} = 16\frac{h^2}{v} + 22.57v$, substracting $16\frac{h^2}{v}$, and multiplying by v, we have $40h^2 = 22.57v^2$; whence $v = \sqrt{\left(\frac{40h^2}{22.57}\right)}$, $=1.34h$ nearly. This gives length to depth of truss, as 5.2 to 1.

Again, referring to analysis of truss Fig. 10, we find action upon chords represented by $20\frac{h^2}{v}$M, and action upon other parts, by $\left(8\frac{h^2}{v} + 11.2v\right)$ M. To make these quantities equal, requires that $v = 1.03h$, and that the length of truss be equal to $\frac{5}{1.03}$ times its depth, or nearly 5 to 1.

From this last case, we may infer as a *probability*, that a ratio of length to depth as 5 to 1, is the most economical for a truss of 5 panels, other things the same. We know, moreover, that by making $v = \frac{1}{2}h$, in the same truss, we double the amount of action upon chords — making it equal to the aggregate upon all parts with the ratio of 5 to 1, while the action, and consequently, the material of the other parts is *probably* reduced one-half. Hence, a ratio of length to depth as 10 to 1, probably increases the aggregate amount of action by some 25 per cent, over what takes place with a ratio of 5 to 1. We may therefore unhesitatingly conclude, that whether the ratio of 5 to 1 be too small or not, the ratio of 10 to 1 is much too large.

Referring again to the 7 panel truss, it appears above, that a ratio of 5.2 to 1 indicates the same amount of action upon chords, as on all other parts. But we can not with certainty infer that the *absolute* amount of action upon the truss, is less with $v=1.34h$, than with $v=h$; in which case length is to depth as 7 to 1. In fact, if we estimate the absolute amount of action, assuming these two values of v successively, we shall find no essential difference in the results. Hence, if other conditions were the same in both cases, it would follow that the ratios of 5.2 to 1 and 7 to 1 were equally favorable to economy, and that there is a better ratio still, between the two; probably, about 6 to 1.

But the conditions are *not* the same in the two cases, aside from the different values of v. For, while with $v=h$, the diagonals incline at 45°, in the other case, their inclination from the vertical is considerably less, being only about 37°. This, we shall see hereafter, is a less favorable inclination for diagonals acting by tension, than 45°; and, since the ratio of 5.2 to 1 shows an equality as to economy, with the ratio of 7 to 1, with the more favorable conditions on the side of the latter, it would seem at least, highly probable that the ratio of 5.2 to 1 is the more near approximation to the desired *optimum.*

Now, after much thought and investigation, with some considerable experience in planning and constructing truss bridges, I can give no better practical rule as to the proper depth of a truss of a given length, than to adopt that ratio between 7 to 1 and 5 to 1, which will best accommodate the desired length of panel (or value of h), and afford the best, and most economical inclination of diagonals; matters to which attention will shortly be directed.

It is not supposed, however, that these limits of ratio will not frequently be exceeded, particularly in the adoption of a greater ratio than 7 to 1. In case of the very long spans dared and achieved in this age of rail roads and locomotion, engineers may recoil from the towering altitudes of 50 or 60 feet depth of truss which some of the long spans now occasionally constructed would require, perhaps more in deference to European precedent, and from an instinct of conservatism, than from regard to economy, and a true appreciation of the real merits of the question. But for important bridges for heavy burthens, a ratio greater than 8 to 1 can not be regarded as commendable, except in rare and peculiar circumstances.

INCLINATION OF DIAGONALS.

LXV. We have seen the absolute importance of oblique members in bridge trusses, and we have also seen the excellence, in point of theoretical economy, of the trapezoidal truss, with parallel chords connected by diagonal members, with or without verticals. Now, since there is an endless variety in the positions which a diagonal member may assume, it becomes an important question, what degree of inclination these members should have, to give the most economical and satisfactory results.

The inclination may be increased till it reaches a horizontal position, or diminished till it becomes a vertical; when, in either case, the member ceases to be a diagonal, and becomes incapable of performing the office of effecting a horizontal transfer of vertical pressure.

The greatest efficiency of material used in diagonals, is manifestly, when the weight sustained by a given quantity of material, multiplied by the horizontal reach, gives the largest product; and, when the member acts by tension, the weight capable of being sustained by a given amount of material, is as the cross-section directly, and as the rate of strain inversely. But the rate of strain, or stress produced by a given weight, is as the weight multiplied by the length of diagonal (D), and divided by the vertical (v), or as $\frac{D}{v}$, while the cross-section is inversely as D, or as $\frac{1}{D}$ Hence, the weight is as $\frac{1}{D} \div \frac{D}{v}, = \frac{v}{D^2}$.

Now, representing the horizontal reach by h, the efficiency of the material must be as $\frac{vh}{D^2}$ equal to $\frac{vh}{v^2+h^2}$. Then, making v, constant, and dividing by v, the expression becomes, $\frac{h}{v^2+h^2}$, still being proportional to the efficiency of material. Consequently, that value of h, which gives the largest value to the expression $\frac{h}{v^2+h^2}$, will indicate the inclination at which the diagonal will act with the greatest efficiency.

This value of h, is found by differentiating the function $\frac{h}{v^2+h^2}$, (h being the variable), and putting the differential equal to 0: by which process we obtain: $\mathrm{d}\left(\frac{h}{v^2+h^2}\right)=\frac{(v^2+h^2)\mathrm{d}h-2h^2\mathrm{d}h}{(v^2+h^2)^2}=0$, whence canceling the denominator, $v^2\mathrm{d}h + h^2\mathrm{d}h = 2h^2\mathrm{d}h$, and $h^2\mathrm{d}h = v^2\mathrm{d}h$.* Then, dividing by $\mathrm{d}h$, and extracting the square root, we have $h=v$; thus showing that an inclination of 45° is the most advantageous for tension diagonals as far as relates to those members alone.

THRUST DIAGONALS.

LXVI. The efficiency of material in a thrust brace, is directly as the useful effect produced by the member, and inversely as the amount of material required in it.

Now, the useful effect, as in the previous case, is as the weight sustained and the horizontal reach, while the amount of material, depends not only upon the stress and length, but also upon the *ratio of length to diameter*, which affects the power of resistance.

Theoretically, the power of resistance is as the cube of the diameter (d), divided by the square of the length ($=v^2\times h^2$), a rule which is not sustained by experience,

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* d(Roman), before a variable, or the function of a variable, denotes the differential of such variable or function.

except in case of long slim pieces which break by lateral deflection, under a comparatively small compressive force. We will, however, use the rule for the present occasion.

The efficiency of the material then, will be as the power of resistance and the horizontal reach directly, and as the stress produced by a given weight, inversely; which stress is as $\frac{\sqrt{v^2+h^2}}{v}$. Whence we have $\frac{d^2h}{v^2+h^2} \div \frac{\sqrt{v^2+h^2}}{v} \left(=\frac{d^2vh}{(v^2+h^2)^{\frac{3}{2}}}\right)$ proportional to the efficiency of material in a thrust brace. Making $d^2v=1$, the last expression becomes $\frac{h}{(v^2+h^2)^{\frac{3}{2}}}$, and the value of h which gives the greatest value to this function, will indicate the inclination at which a thrust brace will act with the greatest efficiency, as it regards the brace alone. Differentiating, and putting the result equal to 0, we have:

$$d\left(\frac{h}{(v^2+h^2)^{\frac{3}{2}}}\right)=\frac{dh\,(v^2+h^2)^{\frac{3}{2}}-\frac{3}{2}h\,(v^2+h^2)^{\frac{1}{2}}\times 2h\,dh}{(v^2+h^2)^3}=0;$$ whence,

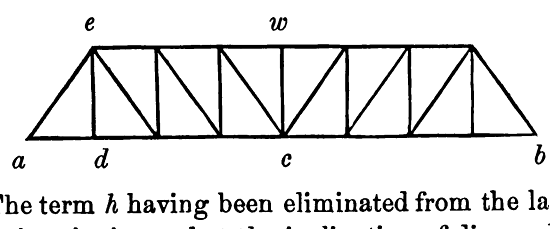
multiplying by the denominator $(v^2+h^2)^3$, we obtain $dh\,(v^2+h^2)^{\frac{3}{2}} = \frac{3}{2}h\,(v^2+h^2)^{\frac{1}{2}} \times 2h\,dh$, and, dividing by $\sqrt{(v^2+h^2)}dh$, we have $v^2+h^2 = \frac{3}{2}h\times 2h = 3h^2$ whence, $v^2 = 3h^2-h^2 = 2h^2$, and by evolution, $v = h\sqrt{2}$, and $h = \frac{v}{\sqrt{2}} = 0.7072v$.

If we deduce the value of the expression $\frac{h}{(v^2+h^2)^{\frac{3}{2}}}$, (which is equal to the horizontal reach divided by the cube of the length of brace), putting $h=v$ and $h=\frac{1}{2}v$ successively, we find the degree of efficiency less than the maximum, as above determined, by about 9 per cent in the former, and 8 per cent in the latter case; showing that considerable deviations may be made in the inclinations of thrust braces without much detriment to efficiency of material in braces, when required by other considerations; which will often be the case, as will be seen hereafter.

EFFECTS OF INCLINATION OF DIAGONALS UPON STRESS OF CHORDS AND VERTICALS.

LXVII. The comparative effects of different positions of diagonals upon the chords, may be illustrated with reference to Fig. 21. It is manifest that a given weight w on the centre of this truss, will produce a vertical pressure equal to $\frac{1}{2}w$ at each of the points a and b, and that each oblique member between a and w, will sustain a weight equal to $\frac{1}{2}w$; and will exert each a horizontal action upon the upper and lower chords, equal to $\frac{1}{2}w\frac{h}{v}$. Hence, the stress of chords in the centre, will equal $\frac{1}{2}w\frac{h}{v}\times n$, in which n represents the number of oblique members between a and w, or between a and c. But n equals $\frac{ac}{h}$ whence $\frac{1}{2}w\frac{h}{v}n = \frac{1}{2}w\frac{h}{v}\times\frac{ac}{h} = \frac{1}{2}w\frac{ac}{v}$.

FIG. 21.



The term h having been eliminated from the last expression, it shows that the inclination of diagonals has no effect upon the stress of chords in the centre, produced by weight in the centre of the truss; and by similar reasoning it is shown that the same is true in relation to other parts of the chords, or to weight at any other points in the length of the truss; the only difference being that the shorter the panels, or the smaller the value of h, the shorter the intervals at which the increments in the stress of chords are added, and the less the magnitude of such increments, in the same proportion. Hence, in general, there is no difference in the stresses of chords, whether the diagonals have one inclination or another.

With regard to the effect upon verticals, that part of their stress which they receive through diagonals, is equal to the weight sustained by those diagonals, and is the same for a given weight, whatever be their inclination. On the vertical wc, the pressure is received directly from the weight. But on the next adjacent vertical, on either side, one-half of the same pressure is received through to the intervening diagonal, and transmitted to the next, and so on to the end.

Consequently the aggregate action of verticals, produced by the weight w, is equal to $w + \frac{1}{2}wn$, taking n for the number of verticals receiving their stress through the medium of diagonals, and which is equal to the whole number less 3, when the number is odd, and the verticals act by thrust, as assumed in the case of Fig. 21. If the weight be applied at the lower chord, the whole action of verticals is communicated through diagonals, the latter acting by tension.

Hence the aggregate action of verticals increases and diminishes with their number, and economy as regards those members, would require the diagonals to incline at a greater angle with the vertical than that which is most favorable as to the diagonals themselves.

We have seen, however [LXVI] that by placing the diagonals at 45° when they act by thrust, we lose about 9 per cent in economy of those members, and we now learn that such an arrangement increases the economy in verticals to a considerable extent by diminishing their number; the actual amount depending somewhat upon the number, and not deducible by a general rule.

We shall not, however, err greatly in assuming, that with an inclination of 45°, for thrust diagonals in conjunction with tension verticals, the loss upon the former is quite made up by saving in the latter, and that a less inclination in this case, should be regarded as very questionable practice.

In case of tension diagonals and vertical struts, a saving in material may undoubtedly be made by making the horizontal greater than the vertical reach of the diagonal, whenever such a course is found consistent with a proper regard to just proportions of the truss in other respects; such as width of panel, depth of truss, etc.

THE WIDTH OF PANEL.

LXVIII. Which we have represented in our formulæ by h, has only been hitherto considered as to its relations to v, representing the depth of truss.

With regard to the best *absolute* value of h, the question is affected by the relative expense of floor joists, and the extra amount of material and labor in forming connections at the nodes of the chords; as well as, in some cases, the lengths of sections in the upper chord. The latter requires support laterally and vertically at intervals of moderate length, depending upon the absolute stress, which, other things the same, governs the cross-section.

The upper chord usually, of whatever material, has a cross-section so large as to exclude all danger of breaking by lateral deflection, in sections of 10 to 14 feet; and, as there will seldom be occasion for exceeding these lengths in cancelated trusses, the increased expense of joists for wide panels, and the expense of extra connections in narrow ones, are the principal considerations affecting the absolute value of h, as an element of economy.

The transverse beams, supposed to be located at the nodes between adjacent panels, may, of course, be proportioned to the width of panel, so as to require essentially the same material in all cases. But the joists, or track stringers of rail road bridges, the depth being proportional to the length between supports, have a supporting power as their cross-sections; and since the load, at a given weight to the lineal foot, is directly as the length, it follows, that to support the same load per foot, as bridge joists are required to do, the cross-section should be as the length. The expense of joists and stringers, therefore, is directly as the width of panel.*

On the contrary, the expense of connections will be as the number of panels, nearly, and consequently, inversely as their width, or inversely as the

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* The thickness of joist most economical for a short reach would be liable to buckle with greater length and depth. Hence joists require increase of thickness with increase of length and depth. The thickness should be as the depth, and the cross-section, as the square of the depth (d).

Upon this basis, the required material for joists, increases at a greater ratio than the increase in width of panels. The supporting power of a joist or beam of a given form of section, or a given ratio of depth to thickness, is as the cube of the depth directly, and the length (l) inversely; or, as $\frac{d^3}{l}$. If there be two joists of depths respectively as d and x, and lengths as l and nl, their supporting powers P, P′, for load similarly applied, will be as $\frac{d^3}{l}$ to $\frac{x^3}{nl}$. But the power should be as the load; in other words, as the length of joists. Hence we have the proportion, $\frac{d^3}{l} : \frac{x^3}{nl} :: l : nl$, whence, $nd^3 = \frac{x^3}{n}$ and $x = dn^{\frac{2}{3}}$. Now n is as the length of joists, and the depth, therefore, is as the $\frac{2}{3}$ power of the length, and the cross-section, and consequently the required material, as the $\frac{4}{3}$ power of the length. Hence, if m represent the material for joists with panels of a given width, the material for panels twice as wide, will be represented by $m \times \sqrt[3]{2^4} = \sqrt[3]{16} = m2.52$. But this is rather anticipating the subject of lateral, or transverse strength of beams.

length of joists. Hence, if we could find the point where the cost of connections (consisting of extra material in the lappings of parts, connecting pins, screws and nuts, and enlarged sections at the ends of members, together with the extra labor in forming the connections), becomes equal to the whole cost of material in joists or stringers; that would seem to indicate the proper width of panel, or value of h, as far as depends upon these elements.

But aside from the fact that our data upon this question are so few and so imperfect, that it would be mere charlatanism to attempt to reduce the matter to a mathematical formula, the occasions would be so rare which would admit of the application of such formula, without incurring disadvantages in other respects, such as improper inclination of diagonals, unsuitable ratio of length to depth of truss, &c., that no attempt will be here made to give any thing more definite upon this point, than to refer to the best precedents and practice of the times; which seem to confine the range of width of panel mostly within the limits of 8 and 15 feet.

Within these limits, and seldom reaching either extreme, plans may be adapted to any of the ordinary lengths of span, by adopting the single or double cancelated trusses, Figs. 12 and 13, or 18 and 19, or the arch truss Fig. 11, (which unquestionably contain the essential principles and combinations of the best trusses in use), according to length of span, the purposes of the bridges respectively, or the taste and judgment of engineers and builders.

ARCH BRIDGES.

LXIX. An arch bridge may be distinguished from an *Arch Truss Bridge*, by the fact that in the former, the bridge and its load are sustained by one or more arches *without chords;* and, consequently, requiring external means to withstand the horizontal thrust or action of the arches at either end; which means are afforded by heavy abutments and piers, in case of erect arches, and by towers and anchorage in the earth, in case of inverted, or suspension arches.

It is not the purpose of this work to treat elaborately of either of these forms of bridging, as the author's experience and investigations have been mostly confined to truss bridge construction. But as some of the largest bridge enterprises and achievements of the age are designed upon the principles here referred to, a brief notice of the subject, and some of the conditions affecting the use of these classes of bridges, may be regarded as desirable in a work of this kind.

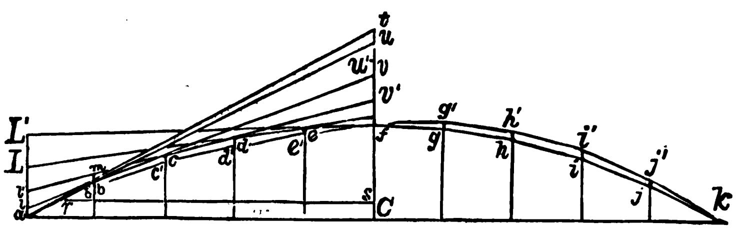
Suspension, or inverted arch bridges of very great spans, have long been in use, both in this and foreign countries; and the capabilities of that system have been pretty thoroughly tested experimentally and practically.

But bridges supported by erect metallic arches, have hitherto been confined to structures of moderate span. Within a few years, however, the magnificent enterprise of spanning the Mississippi at St. Louis by three noble stretches of about 500 feet each, supported each by four arched ribs of cast steel, has been undertaken and is understood to be in rapid process of execution. The interest naturally felt in the progress and final result of this grand enterprise, by students and practitioners in the engineering profession, will perhaps aid in rendering the following brief, and somewhat superficial discussion acceptable.

LXX. An erect arch subjected to the action of weight, or vertical pressure, is in a condition of unstable equilibrium; and can only stand while the weight is so distributed that all the forces acting at each point of its length, are in equilibrio. To illustrate this, we may assume the arch to be composed of short straight segments meeting and forming certain angles with one another, and the weights applied at the angular points.

A weight at *c*, Fig. 22, for instance, acts vertically, and, if *dc* be produced till it meet the vertical drawn

FIG. 22.



through *b* in *m*, then the triangle *bcm* has its sides respectively parallel with the directions of three forces acting at the point *c*; namely, the weight at the point *c*, the thrust of the segment *bc*, and that of *dc*. Hence, if these three forces be to one another as the sides of said triangle,—that is, if the weight (*w*) : thrust of *bc* : thrust of *dc* : : *bm* : *bc* : *cm*, then they are in equilibrio. If *w* be greater than is indicated by this proportion, the point *c* will be depressed, *bcd* approaching nearer and nearer to a straight line, and becoming less and less able to support the weight, and a collapse must result.

If *w* be *less* than the above proportion indicates, it will be unable to withstand the upward tendency of the point *c*, due to the thrust of *bc* and *dc* (or, to the preponderance of the vertical thrust of *bc*, over that of *dc*), the point *c* will rise, the upward tendency becoming greater and greater, and the result will be a collapse, as before. The same reasoning, and the same inference, apply to any other angular point, as at *c*. It is, therefore, only in theory that such a thing as an equilibrated erect arch, can exist. The arch is here considered as a geometrical line without breadth or thickness.

It is this property of instability, in the Erect Arch, that the diagonals in the *Arch Truss*, [Figs. 5 and 11] are designed to obviate, and to enable the arch to retain its form and stability under a variable load.

LXXI. Still, in theory, an arch may be in equilibrio with any given distribution of load, whenever the points *a*, *b*, *c*, etc., are so situated that the sides of the triangle *bcm*, for instance, formed by a vertical with lines respectively coinciding or parallel with the two segments meeting at *c*, are proportional to the 3 forces acting at *c*, as above stated, and so at the other angular points of the figure.

To construct an equilibrated arch adapted to a given distribution of load, consisting of determinate weights at given horizontal intervals between the extremities of the arch, we may proceed as follows:

Draw a horizontal line representing the chord *ak*, and upon the vertical *Cft*, erected from its centre, take *Cf* equal to the required versed sine, or depth of the arch at the centre. Also, take *ft*= *Cf*, and erect verticals upon the chord, at all the points at which the load is applied, and join *a* and *t*.

Then, if the load be uniformly distributed (horizontally) upon the arch, we have seen, [LII], the arch should be a parabola, to which of course, *at* is tangent at the extremity, *a*. But, regarding *ab* as tangent to the curve at *r*, half way between *a* and *b* (horizontally), we seek the abscissa *fs*, which is to $Cf :: rs^2 : aC^2$. Then, taking the distance of *fu*=*fs*, *au* is tangent to the curve at *r*, and coincides with the first segment (*ab*) of the arch. (These segments are supposed to be so short, that the tangent and curve may be regarded as essentially coinciding, for the length of a single segment).

Now, the thrust of *ab*, is to the whole weight bearing at *a*, as *ru* to *us*; and, erecting the vertical *al*, such that *al* : *ab* : : weight at *b* : thrust of *ab*, and drawing the straight line *lbc*, cutting the second vertical in *c*, we have *bc* for the second segment of the arch, being in the line of *lb*, which represents the resultant of the two forces acting at *b*; namely the weight at *b*, and the thrust of *ab*.

In like manner, take *bm*, representing the weight at *c*, and the straight line *mcd*, meeting the third vertical in *d*, gives *cd* as the third segment of the arch.

Repeat the same operation for each of the succeeding segments *de*, *ef*, &c., till the arch is completed, and it is obvious that the forces acting at each of the several angular points *b*, *c*, *d*, &c., are in equilibrio; and that the arch throughout is, theoretically, in a state of equilibrium.

We may vary this process so as to secure greater accuracy of construction, in the following manner :

Producing *lbc* till it meets *Ct* in *v*, we see that *abl* and *ubv* are similar triangles, and *al* : *uv* : : horizontal distance of *l* : horizontal distance of *v*, from the point *b*. Hence, we may take the point *v* instead of the point *l*, by which to establish the position of the line *bc*, and thereby secure greater relative accuracy of measurement.

So may we also take *vv'*, or *ll'* instead of *bm*, to determine the line *cd*. By this means we multiply the small spaces *al*, *bm*, &c., and diminish the amount of error in measurement, and if the angular points, or nodes be at uniform horizontal distances, the process is very simple.

LXXII. We have assumed, in describing the arch *a*, *b*, *c*, *d*, &c., a uniform distribution of load, horizontally. But the general process is obviously the same for an unequal distribution, after locating the first segment *ab*; which we may do by first ascertaining the amount of bearing at *a*, due to the load of the arch. This will be to the whole load, as the distance of the centre of gravity of load from *k*, horizontally, to the whole chord *ak*. For instance, if the centre of gravity be half way between *C* and *k*, one quarter of the load bears at *a*. The weight bearing at *a*, whatever it be, may be represented by A; and supposing it to exert the same horizontal thrust at *a* as half the load (W), would do when uniformly distributed, we take *u'* in *ft*, so that ½ W : A : : *uC*: *u'C*.* Then *au'* gives the direction of *ab'*, and we proceed in the same manner

* We may assume any amount of horizontal thrust, and the greater the assumed thrust, with a given load, the less will be the depth of the arch, and vice versa. It is proposed here to construct an equilibrated arch *a, b, c, d,* &c., of about the same rise at the crown, as the normal curve, *a, b c, d,* &c., has.

as before using the weights given for the several nodes of the arch, to determine the points c', d', &c. These being connected by straight lines, we have an equilibrated arch adapted to the given distribution of load.

LXXIII. But of course, this arch will not stand under any other disposition of the load. To obviate this difficulty, and to construct an arch which will stand under a variable load, without the chord and counter-bracing of the arch truss, the device has been adopted, of constructing the arch of such vertical width that all the equilibrated arches or curves, required by all possible distributions of load; may be embraced within the width of the arched rib. Then, if there be sufficient material to oppose and withstand the forces liable to act in the lines of said several equilibrated curves, complete *vertical* stability must result.

The proper width, or depth of the arched rib, will depend upon the length and versed sine of the arch, as well as the amount and distribution of load; and the material will act most efficiently, when mostly disposed in the outer and inner edges, or members of the rib, and connected, either by a full, or an open web, to distribute the action between the outer and inner members, according as the resultant line of action approaches the one or the other of those members.

The normal form of the arch should be such as to be in equilibrio under a uniform load,* and hence it will be parabolic, as to the movable load, and the weight of road-way, and catenarian, as to the weight

* The method above explained, for describing an equilibrated arch, is applicable to all cases where the load, both constant and variable acting on the several parts of the arch, is known, whether it be the normal curve, adapted to a full load, or a distorted curve, suited to an irregular distribution of load.

of arches (as far as they are uniform in section), and should approach the one or the other form, according to the weight of arches, as compared with the other weight to be supported thereby.

The distance between outer and inner members, or the width of web, reckoned from centre to centre of those members, should be such that no condition of unequal and partial load, could throw greater action at any point of either member, than the extreme uniform load would throw upon both.

Let us suppose that the curve *a*, *b*, *c*, *d*, etc., be centrally between the two members and that *dd'*, and *hh'* be the greatest vertical departures, inward and outward, of any equilibrated curve, from the normal curve *a*, *b*, *c*, etc.

Let us further suppose that the thrust of the arch at the points *d* and *h*, be $\frac{3}{4}$ as great under the load acting in the curve of greatest departure from the normal as the extreme uniform load produces at those points. Then, if the outer and inner members of the rib, be placed at a distance of six times the greatest departure of the distorted from the primary, or central curve, one member will be twice as far from the line of action (at the point of greatest departure), as the other, and the latter will sustain two-thirds of the action, equal to one-half the action of the full load, and the same as in the latter case.

If the width of web be less than six times the greatest aberration of the distorted curve, the action, under the suppositions above, will be greater upon one member than that due to a full uniform load; a condition altogether to be avoided.

A few trials at constructing curves adapted to assumed possible distributions of load, may determine satisfactorily what condition gives the curve of greatest distortion and the greatest departure from the normal; and the amount of action under that condition, can be readily calculated with sufficient nearness, whence the proper width of web may be deduced.

LXXIV. The points of the equilibrated curve may be located by calculation, and perhaps with as much ease, and greater accuracy than by construction.

Suppose Fig. 22 to have a vertical depth, *Cf*, equal to one of ten equal sections of the chord *ak*. Having found the length of *fu*, in the manner already explained [LXXI], it is known that for a uniform load at each angle, the vertical reaches of the several segments, begining at the centre, are as the odd numbers, 1, 3, 5, 7 and 9; and, if we conceive *Cf* to be divided into 25 equal parts (25 being the sum of these numbers), each of these parts will be equal to $0.04 Cf$, or $.04v$; and this factor, multiplied by the numbers 1, 3, 5, &c., give the vertical reaches of the respective straight segments, which vertical reaches being substracted successively from v, and successive remainders, show the several verticals to be as follows: At the centre, f, vertical $=$ $Cf = v$. At e, vertical $= v - .04v = .96v$. At d, vertical $= (.96 - .12)v = .84v$; at c, vertical $= (.84 - .2)v = .64v$, and at b, vertical $= (.64 - .28)v, = .36v$. This establishes the normal curve for uniform load.

Now, supposing the weight of structure to be equal to $1w$ at each of the angles of the arch, and also, that a movable load of a like weight, w, be acting at each of the five points f, g, h, i, j; the permanent weight of structure gives a bearing of $4.5w$ at a, and the movable weights at f, g, h, &c., give respectively $.5w$, $.4w$, $.3w$,

.2*w*, and .1*w*, together, equal to 1.5*w*; making the whole bearing at *a*, equal to 6*w*, which is $\frac{1}{7}$th less than if the same weight were distributed uniformly.

Then taking $Cu' = \frac{6}{7} Cu$, and drawing the line au', (not shown in the diagram), we have the inclination of ab', the first segment of the required curve, which gives the same horizontal thrust at *a*, as the normal curve would exert under the same load uniformly distributed. We find *fu* (=*fs*), by the proportion.

$Cf : fs :: Ca^2 (= \overline{5v^2}) : sr^2 (= \overline{4.5v^2} :: 25v^2 : 20.25v^2 :: v : .81v$; and, reducing 1.81*v* (= *Cu*), by one-seventh, we obtain $Cu' = 1.5514v$. This length is to aC :: A (=6*w*), : horizontal thrust of ab'; that is (making *v*=1), 1.5514 : 5 :: 6*w* : $\frac{30w}{1.5514} = 19.33w$, = horizontal thrust *ab*.

Now, if this thrust be represented by $\frac{1}{5}aC$, = *v*=1, then *w* will be represented by a space equal to $\frac{1}{19.33}$, = .05173, which is equal to the vertical departure (D), of $b'c'$ from $ab'u'$. Knowing the value of this departure which, of course, is directly as *v*, and inversely as *aC*), we can locate the points c', d', e' and f', by their vertical distances from au', as follows: The vertical at b', is evidently equal to $\frac{1}{5}\times1.5514$, = .31026; consequently, the vertical at $c' = 2\times.31026 - .05173 = .56879$. Vertical at $d' = 3\times.31026 - 3\times.05173 = .77559$. Vertical $e' = 4\times.31026 - 6\times.05173 = .93068$, and the vertical at f', equals $5\times.31026 - 10\times.05173 = 1.034$, showing that the new curve crosses the normal, between *e* and *f*, and f' is above *f*, but not shown.

Then, if each of the segments $b'c'$, $c'd'$, &c., be produced to meet the indefinite vertical drawn through *a*, they will evidently cut that line at intervals of D, 2D, 3D and 4D together, equal to 10 D, = .5173. Then, the weight at f being equal to $2w$, it follows that $f'g'$ makes twice the deflection from $e'f'$ that the latter makes from $d'e'$, that is, equal to 2D in the horizontal distance of $1v$, or 1, or 10D (= .5173), in the distance aC, or 5. Hence, $f'g'$ produced, cuts the vertical at a, twice as high as $e'f'$ cuts it, or, at a point 1.0346 above a; being just as high as the point f'; except a small difference resulting probably from omitted fractions. This shows that $f'g'$ is horizontal, and tangent to the curve at its vertex.

It follows that all the weight at f', and at the left of that point, is brought to bear at a, aud all that at g', and on the right thereof, bears at k. This affords a check upon our work thus far, as we already knew that the bearing at a was equal to $6w$, and we now see that this is made up of $1w$ at each of the four points b', c', d', e', and $2w$ at f'. If $f'g'$ were not horizontal the arch could not be in equilibrio under the assumed condition of load.

Now, as we manifestly have for the 4 remaining segments, a vertical reach for each, as the weights they respectively sustain; i. e., equal respectively to 2D, 4D, 6D, and 8D; making 20D (= Cf'); altogether, we have only to subtract these quantities successively from Cf' (=1.0346), to obtain the lengths of verticals at h', i', j'; as follows:

$$1.0346 - 2 \times .05173 = .93114 = \text{vert. at } h'$$
$$.93114 - 4 \times .05173 = .72422 = \text{ " " } i'$$
$$.72422 - 6 \times .05173 = .41384 = \text{ " " } j'$$
$$.41384 - 8 \times .05173 = 0 \quad \text{ " " } k$$

The differences between these lengths of verticals, and those of the normal curve at the same points, show the aberrations vertically, of the distorted, from the normal curve, as below.

	Nor.	Dist.	Below.	Above.
bb′=	.36—	.31026=	.04974	"
cc′=	.64—	.56879=	.07121	"
dd′=	.84—	.77559=	.06441	"
ee′=	.96—	.93068=	.02952	"
	Dist.	Nor.		
ff′=	1.034 —	1.00=		.034
gg′=	1.034 —	.96=		.07446
hh′=	.93114=	.84=		.09114
ii′=	.72422—	.64=		.08422
jj′=	.41384—	.36=		.05384

LXXV. From this exhibit, we perceive that the greatest vertical aberration externally for the condition of load here assumed, is at hh', and equals $.091v$, and the greatest internally, at cc' (or a little to the right of these points in both cases), equal to $.071v$, traversing a zone equal in width to $.162v$. nearly $\frac{1}{6}$ of the versed sine of the normal curve.

Now, we have seen that the horizontal thrust of the arch for a gross load of $14w$, equals $19.23w$, with the assumed proportion of versed sine to span, as 1 to 10, whether upon the normal or the distorted curve; and, the thrust being evidently as the gross load, other things the same, it follows that, with the full gross load of $18w$, or $2w$ at each angle, the thrust would be to $19.33w$ as 9 to 7. Hence the load, as above assumed, produces $\frac{7}{9}$, or $77\frac{7}{10}$ per cent of the maximum thrust under the full uniform load.

The uniform load being supposed to act equally upon the outer and inner members of the rib, the action of 50 per cent is due to each; and, in order that neither member, at the nearest approach to the equilibrated curve, may be subjected to greater stress than under the greatest uniform load, the web should be so wide that (assuming the outward and inward aberrations to be each equal to the mean of $.081v$, and putting $x=$ width of web), $x : \frac{1}{2}x+.081v :: 77.7 : 50$. Whence $50x = 38.88x+77.7\times.081v$; and $x=.557v$.

But this value of x being equal to the distance vertically across the web between c and d, or between h and i, is greater than the distance square across, about in the ratio of distance from a to f, to the line aC, in this case as $\sqrt{26} : 5$. The actual width of web, therefore, is only $.545v$, still considerably more than half the versed sine Cf.

The condition of load here supposed, may or may not be the one requiring the greatest distortion of the equilibrated curve. The case has been assumed to illustrate this discussion, as it seemed likely to be near the condition requiring the greatest width of web; and I leave this part of the subject, without attempting a more general and determinate solution of the question.

LXXVI. The movable load has been taken as only equal to the weight of superstructure, upon the supposition that this style of bridging would seldom be adopted, except for very considerable lengths of span, where the weight of superstructure is relatively greater than in case of short spans.

This double arch, as here under consideration, consisting of an outer and an inner curved member, connected by a web, in order to act most efficiently should be so adjusted that the outer and inner members may be subjected to equal action under a full maximum, uniform load. Hence, the normal and equilibrated curves, representing the line of the resultant of forecs acting upon the arch, have been assumed as terminating at each end, at points centrally between the extremities of the outer and inner curved members.

It might seem possible that the distorted curve adapted to the above assumed condition of load, might so fall as to recross the normal between the points of greatest departure and the ends, and thus diminish the extent of aberration, and the necessary width of web. If the curve *a*, *b*′, *c*′, etc., be turned upon its centre, by raising the end at *a*, by ⅔*rds* of the greatest departure, that is, by $\frac{2}{3}\times.081v,=.054v$, the aberration half way between *a* and *f*, where it is at or near its maximum point, would be reduced by $.027v$, and become $.054v$ just the same as at the end. The other end would drop to the same extent, and would reduce the outward aberration in the same degree. This, of conrse, would be the least possible extent of aberration; and if we could rely upon the resultant stress following this curve in such a position, it would enable us to diminish the width of web to $.364v$.

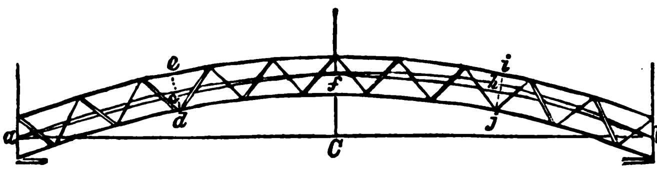
But there seems to be no obvious reason why we should assume the equilibrated curve to take the position just described, rather than one with the left end below *a*, and the other above *k*, thus increasing instead of diminishing the aberration. Hence, in the case of an arch ribbed bridge, liable to a movable load equal to the weight of structure, foot for foot, upon the whole or any part of its length, if the web of the ribs be less than 36–100th, of the versed sine (*Cf* Fig 22), *certainly*, and if less than 54–100ths *probably*, the material in the principal members is liable to greater strain in some parts, under a partial, than under the extreme load;

which would be decidedly an unfavorable condition, with regard to economy.

LXXVII. The operation of the web in distributing the action upon the outer and inner curved members of the rib, and transferring it from one to the other, may be understood by the diagram Fig. 23, exhibiting said curved members, connected by a web consisting of a simple system of diagonals, capable of acting by thrust or tension as may be required.

The normal curve is represented parallel with, and midway between the curved members; and the equili-

FIG. 23



brated curve is represented as crossing the normal near *f*, meeting it again at *a* and *k*, at the ends; and having its greatest aberrations at *c* and *h*. It is manifest that the action of the outer member at *i*, is to that of the inner one at *j*, as *jh* to *ih* (inversely as their distances from the distorted curve), and that the action upon the outer diminishes, while that of the inner one increases each way from *i* and *j*, until the action upon the two becomes equal at the meeting of the curves at *k*, and at the crossing point near *f*. Hence the diagonals leaning toward the point *i* must act by thrust, while those leaning *from j*, act by tension. On the contrary at *d*, where the greatest compression is upon the inner member, and diminishes each way, the diagonals leaning from *e*, act by thrust, while those leaning toward c, act by tension. The tension diagonals are represented by single, and the thrust diagonals, by double lines.

But the action changes more or less with every change in the position of the load, and if the load were reversed upon the two halves of the arch, each diagonal here represented as acting by thrust, would then act by tension, and vice versa.

Now, assuming that $dc = \frac{1}{2}ce$, and that the action upon the inner member at this point equals twice that of the outer one, it follows, since the action should become equal upon the two at a, that $\frac{1}{6}$ of the whole thrust of the rib must be transferred from the inner to the outer member between c and a, by the thrust and pull of diagonals, exerted in the direction of the normal curve; the action accumulating and increasing upon successive diagonals each way from c, and in like manner from h.

The action of diagonals is still further affected by the transfer of the action of load, from the outer to the inner member; the load being first applied directly to the outer curved member. Hence it becomes a somewhat complicated problem to determine the maximum action of diagonals; especially as the complication becomes increased by taking into account the

EFFECTS OF TEMPERATURE.

LXXVIII. The expansion and contraction of metallic arches without chords, the ends remaining fixed as to position and distance asunder, must obviously cause the intermediate portions to rise and fall with the increase and decrease of temperature.

The outer and inner members, if parallel, being similar concentric arcs, will rise and fall, by the same changes of temperature, proportionally to their respective radii;* the outer one undergoing the greater vertical change, whence, it must follow that in warm weather the outer, and in cold weather the inner member sustains the greater relative compression, a result for which there appears to be no obvious remedy, except by balancing the end bearings upon pivots at *a* and *k*; which would allow the two curved members to adjust themselves to an equal action upon the two. Or, if the curves be formed upon the same radius, and of equal length, they would rise and fall alike, and the distance across the web vertically, would be the same at all parts of the arch.

In this case, as in all others, of the arched rib, the depression of the arch, whether from reduction of temperature, or the action of load, would be attended by increased thrust action, or diminished tension action upon diagonals *less* inclined from the vertical position, and the reverse of action, upon those *more* inclined.

The absolute rise or fall of an arch, resulting from a given change of temperature, may, without essential error, be regarded as proportional to the change in the length of a circular arch of the same span and depth (from chord to vertex), within the limits of change produced by temperature; and, may be found by the following process:

Divide the square of the half chord by the depth of arch (v), add the divisor to the quotient, and half the sum equals the radius. Divide the half-chord by the radius, to get the natural sine of half the arc; find in the table of natural sines, the angle corresponding

* The curves not being supposed to be circular arcs, it is not strictly correct to speak of their radii, but the meaning will be comprehended.

with the sine thus found, and double that angle, for the number of degrees in the arc. Multiply the number of degrees (reducing minutes and seconds to the decimal of a degree), by .01745329 (= length of a degree, radius being equal to 1), for the length of the arc.

Then, in the same manner, find the length of an arc upon the same chord, and with a depth (v') one or two per cent greater or less than v; and, the difference in length of arcs thus found, is to the difference between v and v', as the change in length of arch due to the given change of temperature, to the rise or fall of the arch, resulting from such change.

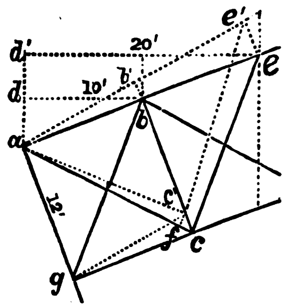
By applying this rule to a specific case, we can the better appreciate the importance of the effects of change of temperature upon this species of arched ribs. If we assume an arch of 500 feet chord and 50 feet depth, $=v$, we find the length of arc to be 513.25 *ft.* The length of an arc of the same span, and a depth (v') = 51 *ft.*, is 513.715 *ft.*, the difference being 0.465 *ft.* The expansion of steel for a change of 110° Fahrenheit, is .0007271 × length (513.25), = .37318. We have, therefore, .465 : 1*ft.* : : .37318 : .8025*ft.* = rise or fall of the arch in the centre, resulting from a change of 110°.

Regarding this rise in the centre as the abscissa of a parabola, and the half chord as the corresponding ordinate, the rise at any other point of the curve is equal to the difference between .8025, and the abscissa answering to the ordinate of the given point.

Suppose the point be 10 feet from the end, and the ordinate, of course, 240*ft.*, we have, $250^2 : 240^2 :: .8025 : .7395$ = abscissa for the given point; whence, the rise at that point, equals .8025—.7395 = .063*ft.*

Let Fig. 24 represent the end portion of the arch, *abe* the upper, and *gc* the lower member, *ag* and *bc* the width of web,$=12'$. *ab*, with a horizontal reach of $10'$, equals $10.75'$. Then, *bg* being regarded as a rectangle, the diagonal $ac=16.1$ft and the temperature being raised $110°$, the points *b* and *c* rise to *b′* and *c′*, *bb′* being equal to the vertical rise multiplied by the cosine of the angle *abd*, i. e., equal to $.062 \times$ cosine *abd*. This angle is a little over 22°, and its cosine about .93 whence $bb'=.058$ft,$=cc'$. Joining *a* with *c′*, and drawing *c′f* at right angles with *ac*, and *ac′* (as these lines are essentially parallel), we have $cf,=cc' \times \sin. acb = cc' \times \frac{ab}{ac} = .038$ft,$=$ the contraction required to take place in the length of the diagonal *ac*, to accommodate a change of 110° in temperature.

FIG. 24.



In the mean time the point *e* rises to *e′*, the distance *ee′* being equal to .1147, so that *c′e′* is extended about the same as *ac′* is contracted; a change equal to what would be produced by a force of 70,000 lbs to the square inch of cross section.

If the normal length of the diagonals be adjusted for a medium temperature, the change would be half the above amount each way, or equal to that produced by 35,000 lbs to the inch.

Succeeding diagonals toward the centre would be affected in a similar manner, though in a less degree; and the consequence must be an accumulation of thrust or compression upon the inner member toward the centre, and the outer one toward the ends, upon a rise of temperature, and the reverse on a fall below the normal point.

THE WIDTH OF WEB.

LXXXIX. For an arch of 500 ft. chord, and 50 foot depth. We have seen that, with a load as assumed [LXXIV], with reference to Fig. 22, the aggregate aberration outward and inward, traverses a zone of $.162v$, equal in this case, to $.162 \times 50 = 8.1$ ft. If the web, therefore, be 8.1 feet wide between centres of curved members, the equilibrated curve will reach the centre of said members at the points of greatest aberration, both ways, and the whole thrust at these points, will fall upon a single member, producing as we have already seen, 77 $\frac{7}{10}$ per cent of the amount of thrust due to a maximum uniform load; being over 55 per cent more stress under a partial than under a full load.

Again, suppose the web to be 12 feet wide. The distorted curve would approach within two feet of the outer and inner curved members, throwing upon one member at one point, and upon the opposite member at another point, almost 30 per cent more action than what is produced by the full maximum load. It was shown moreover [LXXV] that nothing short of $.545v = .545 \times 50 = 27.25$ feet width of web, could be relied on to give as small a stress upon the curved members in this particular case of a partial, as that produced by a full maximum load.

This would be an inconvenient, and an expensive width of web, and probably a less width would be preferable, even with a greater occasional stress upon the curved members which might be enlarged in section in parts liable to the greater stress. But I shall not undertake at this time, to determine the exact optimum.

Finally, considering the difficulty of securing the most efficient thrust action of the curved members of the arch, the serious disturbances as to the action of the diagonals composing the web system, occasioned by changes of the temperature, together with the extra weight and strength of piers and abutments to withstand the horizontal thrust of the arches, it seems reasonable to conclude that the erect metallic arch bridge will only be adopted under rare and peculiar circumstances; and that in such cases, the plans should be subjected to especial examination and investigation.

Truss bridges possess the advantage of having all the forces in operation, except the vertical action of weight, and the opposite resistance of the end supports resisted by means of members contained within the structures themselves, and composed of materials of so nearly uniform expansibility by heat, that no important disturbance in the relations of the different members, can be produced by changes of temperature. Plans, also, may be so arranged as to secure a near approximation to uniform maximum stress upon all the parts; at least, to a much greater degree than seems practicable in the case of the arch without chords.

BRIDGE MATERIALS.

LXXX. Having discussed the general principles and relative characters and merits of different plans and forms of bridge trusses, and their proper proportions, particular and general, the question as to the best materials for the purposes of bridge construction may properly be considered.

We have seen that the materials of a bridge truss are principally subjected to two kinds of action, that of tension, and that of compression. Lateral, or transverse action should be avoided in the principal parts and members of the truss.

It is obvious then, that those materials best calculated to resist these kinds of force respectively, should, when practicable without sacrifice of economy, be employed in the situations where those forces are respectively exerted. For instance, when the diagonals act by tension, the upper chord (or the arch, in case of the arch truss), and the verticals, should be composed of the material best adapted to the sustaining of a compressive force, while the lower chord and the diagonals, should be of the best material for sustaining tension.

Wood and iron are the only materials that have been employed in the construction of bridge superstructures to an extent worthy of notice; and it seems reasonable to conclude that on these we must place our dependence.

Cast iron resists a greater compressive force than any other substance whose cost will admit of its being used as a building material. Steel has a greater power of resistance, but its cost precludes its employment as a material for building purposes.* Wrought iron resists compression nearly equally with cast iron. But its cost is twice as great, which gives the cast iron a decided advantage.

On the other hand, wrought iron resists a tensile force nearly four times as well as cast iron, and 12 or 15 times as well as wood, bulk for bulk.

Not only are these the strongest materials, but they are also the most durable. In fact, with proper precautions, they may be regarded as almost imperishable.

It would seem then, that wrought iron for tension, and cast iron for compression, were the *best* materials that could be employed in building bridges. But wood, though greatly inferior in strength and durability, is much cheaper and lighter, so that, making up with quantity for want of strength, and by frequent renewals, its want of durability, it has hitherto been almost universally used in this country for bridge building; and, in the scarcity of means, and the unsettled state of things in a new country, where improvements are necessarily, to a great extent, of a temporary character, this is undoubtedly the most economical material for the purpose.

But it is believed that the state of things has now assumed that degree of settled permanency in many parts of this country, and available means have accumulated to that extent which renders it consistent with true economy to give a character of greater permanence to our improvements; and, in the erection of important works, to have more reference to durability, even at the cost of a greater present outlay. In this view

* This remark, made originally some twenty-five years ago, may require some modification at the present time, when steel is being employed extensively for rail way track, and in some important arch and suspension bridges; but not in truss bridges, to the writer's knowledge.

of the subject, it seems highly probable that one of the channels in which this tendency of things will develop itself, will be in the extensive employment of iron in the construction of important bridges. With this impression, I proceed to some general comparisons as to the relative cost and economy of wood and iron as materials for bridges.

LXXXI. The power of cast iron to resist compression, equals some twenty times that of wood; consequently, it will only require one twentieth as much of the former to withstand a given force, provided it can be put into a form in which its liability to flexure, and yielding laterally, is not greater than that of wood. This may be accomplished in part, by giving the iron a hollow form, so as to make the diameter of the pieces approximate to an equality with twenty times the same amount of wood, which must generally be used in a simple rectangular, or cylindrical form of section.

Assuming, then, that a cubic foot of cast iron will do the same work as 15 cubic feet of wood (after making allowance for the necessarily smaller diameter of the iron), we can institute a comparison which would seem, upon the surface, to show the relative economy of the two materials.

A cubic foot of cast iron, manufactured for the work will cost about $13.00. 15 cubic feet of wood in a bridge will cost, say $6.00. Whence it appears that the cast iron is more than twice as expensive, in the first outlay, for sustaining a compressive force, as wood.

Again a cubic foot of wrought iron in the work, say 450 ℔ at 7½cts.=$34.00.

Wood is about $\frac{1}{18}$ as strong as iron. But about one-half of its fibres must be separated in order that the other half may be so connected in the structure, as to be available to their full strength, acting by tension. Hence, it will take some 30 feet to equal one of iron; for which it will cost, say $12; showing a difference of a little less than three to one; making the average for both kinds of iron, reckoning equal quantities of each, about 2.6 to 1.

To offset against this, we have the superior durability of the iron, which, as before observed, may be regarded as imperishable; whereas, wood requires frequent renewals, at a cost each time, equal to the first outlay. Now, the first cost of the iron is sufficient to provide for the first cost of the wood, and nearly two renewals. Besides this, money, though an inanimate substance, is, nevertheless, in these usurious times, made to be exceedingly prolific; insomuch, that with good management, it is found to double itself once in ten or twelve years, according to the hardness of *face* in the lender, or of *fortune* in the borrower.

Assuming 5 per cent per annum as the net income of money invested, the term of time in which the $1\frac{60}{100}$ dollars saved in the wooden structure, will require to produce one dollar for renewal, will show the time that wood ought to last, to be equal with iron in economy,

One dollar and sixty cents at compound interest will yield, at 5 per cent, one dollar in a little less than ten years. Therefore, if an imperishable iron structure cost 2.6 times as much as one of wood, and the latter last but ten years, and money will net 5 per cent, compound interest, the two materials are nearly upon a par as to economy.

Experience has shown that wooden bridges, unprotected by roofing and siding, seldom last with safety over eight years, or thereabouts; and, the more there be expended to increase the durability, the less surplus capital will be left to be invested toward renewals.

LXXXII. But the above comparison is too superficial and general to be entitled to a great deal of confidence, except, perhaps, as it regards the sustaining of a given weight by a simple post, or suspending it by a bar or rod of iron or wood. In the complicated assemblage of pieces forming the superstructure of a bridge, there are numerous other facts and considerations which materially vary the results. First, there is a difficulty in connecting pieces of timber in such a manner that every part may be proportioned to the strength required of it, to the same extent as can be done with iron. Second, it is frequently necessary to use considerable quantities of iron in bolts and fastenings for putting together a structure of wood requiring great stability. Third, wood soon loses a portion of its strength by partial decay, and consequently, requires additional strength in the beginning, that it may be safe for a time after decay has commenced.

Hence, but little can be predicated upon the simple general comparison of wood and iron as to strength and cost, relative to the comparative economy of the two materials for bridge building.

It is only by comparing the results of actual experience, or, where this has not been had, by comparing the results of detailed estimates, upon well matured plans, founded on well established principles, that a satisfactory conclusion can be arrived at.

With regard to wooden bridges, much experience has been had, and the reasonable presumption is, that a good degree of economy has been attained in their construction. But the idea of building *iron* bridges in this country, is of recent date, and but little has been experimentally proved in relation, to their cost and qualities.

LXXXIII. This much, however, my own experience has demonstrated. Having received Letters Patent for an "Iron Truss Bridge," upon the arch truss plan, and constructed two bridges thereon, over the Enlarged Erie Canal (of 72 and 80 feet spans), one of which has been in use for six years, it may be regarded as a demonstrated fact, that bridges may be sustained by iron trusses. It has also been shown that the cost of the above class of bridges, is only about 25 per cent more than the same class of bridges of wood, as *heretofore built*, under the most favorable circumstances, upon the Erie Canal. That the iron portion, constituting some three-fourths of the whole, as regards expense, in the iron bridge, gives fair promise of enduring for ages, while the wooden structure can only be relied on to last eight or ten years.

Upon these facts, experimentally established, I found the following comparison:

A common road bridge of 72*ft.* span (the usual length for the enlarged Erie Canal), will cost, with iron trusses:

For 7,000 lbs. of cast iron at 3cts.,	$210.
" 6,000 " " wrought iron, manufactured for the work, at 7cts.,	420.
" Timber, labor and painting,	230.
" Superintendence and profit,	80.
Whole first cost,	$940.
$175 will renew the perishable part once in 9 years, to produce which, at 5 per cent compound interest will require capital of,	320.
Total for a perpetual maintenance,	$1,260.

With wooden trusses, fastened with iron for timber, labor, paint and profit,	$550.
" 2,000 lbs. of iron fastenings,...............	150.
Whole first cost,............................	$700.
(Some have cost $1000, or $12,000, and taken 3 to 4 thousand pounds of iron)..........	
To renew $550 worth of perishable material once in 9 years, will require, at 5 per cent, compound interest,..................	$1,000.
Total for perpetual maintenance,......	$1,700.

The reason of the apparent difference between this result, and that arrived at from the general comparison of the cost, &c., of wood and iron, is, that the bridges here referred to, have been constructed with a very large amount of iron fastenings, and with large quantities of casing and painting for protection and appearance. Were the comparison confined strictly to the expense of timber work, in the sustaining parts of the trusses, the result would be found not to differ so essentially from that of the general comparison.

The above estimate of $700, for the first cost of a 72 foot wooden bridge, though considerably below the average cost of canal bridges of that description, is nevertheless believed to be greatly above the minimum for which bridges may be built, dispensing with the parts which are not essential to strength.

It is probable that bridges may be built for $500, as about the minimum, of equal strength and convenience, and nearly the same durability, as those hitherto built upon the Erie Canal Enlargement at a cost of from 800 to 1,000 dollars. Upon this supposition, which may be regarded as an extreme case in favor of wood, the comparison will stand thus:

First cost of wooden structure, ...	$500
Capital invested at 5 per cent to produce $500 once in 9 years for renewal,......................	909
Total for perpetual maintenance,.................	$1409
The same for iron structure, as above, ...	1260
Balance in favor of the iron bridge,................	$149

Finally, since theoretical calculation and general comparison show a *probable* advantage, for a long term of time, and experience, as far as it has gone, shows a *decided* advantage in favor of iron, it would seem very unwise to discard the latter, without at least a fair trial of its merits. If in the first essays at iron bridge building, the iron bridge has competed so successfully with wooden bridges, improved by the experience of ages, may not the most satisfactory results be anticipated from an equal degree of experience in the construction and use of iron bridges?

LXXXIV. Presuming the affirmative to be the only rational answer to the above question, I have arranged the details of plans for carrying into practice the preceding principles and suggestions in the construction of rail road bridges of iron.

I have also made careful detailed estimates of the expense of bridges of different dimensions and in different circumstances, some of the more general results of which I will here state.

In proportioning the parts of a rail road bridge, I have assumed that it may be exposed to a load of 2,000lbs. per foot run, for the whole, or any part of its length, in addition to its own weight; and in case of tension, have allowed one square inch cross section of wrought iron for every 10,000 lbs. of the maximum strain produced upon every part by such weights, acting by dead pressure. In case of thrust, or crushing force, I have allowed one square inch cross section of cast iron, for every 12,000lbs. acting on pieces (mostly in the form of hollow cylinders), of a length equal to 18 diameters, and a greater amount of material, where the ratio of length to diameter is greater; always having regard to practicability, as well as theoretical proportions, in adjusting the dimensions of the part.

My estimates, made upon these bases, have fully satisfied me that a bridge of 100 feet span, with track upon the top (with wooden cross-beams), will cost about $2,000, or $20 per foot, assuming the present prices of iron (1846), in ordinary circumstances. If the track pass near the bottom of the trusses, the expense will be increased by two or three dollars a foot.

For a span of 140 feet, by a liberal detailed estimate I make, in round numbers, a cost of $4,000. For 70 feet, I estimate a cost of 9 to 10 hundred dollars, according to circumstances.

Thus it will be seen that actual estimate makes the cost of a single stretch of any length, very nearly as the square of the length, as should be expected from the nature of the case. Hence, knowing the cost of a span of any given length, we readily deduce that of a span of any other length, in similar circumstances, with reliable certainty.

Now, although my investigations have forced the conviction upon me, that where strong and durable bridges are required, iron should be preferred in their construction, still there is a multitude of cases where wooden structures should be preferred; especially in sections of country comparatively new, where timber is plenty and capital scarce; and where improvements must necessarily be of a more temporary character.

With this view of the subject, I have given considerable attention to the details of wooden bridges; and, with a good deal of investigation and experiment, have arranged plans which are confidently believed to possess important advantages over the plans generally in use.

The preceding few pages have been transcribed from the author's original and first essay upon bridge building; and are introduced here, not on account of any practical value they may possess in the present state of progress in the science of bridge construction. But they may possess some little interest as marking about the starting point of the construction and use of Iron Truss Bridges.

If the estimates above exhibited, of the cost of iron bridges, appear small and inadequate, under the lights furnished by the experience of a quarter of a century, much allowance may be claimed on account of the change of times and circumstances within the period in question. And, when it is borne in mind that the author actually contracted for, and built iron railroad bridges of 40 and 50 feet span, for $10, and of 146 feet for $30 per foot, the estimates above given may not seem entirely preposterous, although much higher prices are obtained for bridges of like dimensions at the present day.

PRACTICAL DETAILS.

LXXXV. In preceding pages I have endeavored to give a short and comprehensive general view of the subject, and to ascertain and point out the best general plans and proportions, for the main longitudinal trusses, or side frames of bridges, and the relative stresses of their several parts.

The side trusses may be regarded as vastly the most important parts of the structure, and the strength and sufficiency of these being secured, there is much less difficulty in arranging the remaining parts, the forces to which they are exposed being much less than those acting upon the trusses. I propose now to enter more into details, and give such practical explanations and specifications as to the strength of materials, the methods of connecting the several parts or pieces, both in the main trusses, and other parts of the structure, illustrated by the necessary plans and diagrams, as, it is hoped, will enable the young engineer and practical builder to proceed with judgment and confidence in this important branch of the profession.

IRON BRIDGES.

STRENGTH OF IRON.

LXXXVI. Iron has the power of resisting mechanical forces in several different ways. It may resist forces that tend to stretch it asunder, or forces which tend to compress and crush it; the former producing what is sometimes called a *positive*, and the latter, a *negative* strain. It may also be exposed to, and resist forces tending to produce rupture by extending one side of the piece, and compressing the opposite side; as where a bar of iron supported at the ends, is made to sustain a weight in the middle, which tends to stretch the lower, and compress the upper part. This is called a *lateral*, or transverse strain.

Iron may likewise be acted upon by forces tending to force it asunder laterally, in the manner of the action of a pair of shears. This is called a *shear* strain; and though less important than either of the preceding cases, it will frequently have place in bridge work, partially at least, in the action of rivets, and connecting pins.

With regard to the simple positive and negative strength of iron it is only necessary for me to state in this place, as the result of a multitude of experiments, that a bar of good wrought iron one inch square, will sustain a positive strain of about 60,000lbs. on the average; and a negative strain, in pieces not exceeding about twice the least diameter, of 70 or 80 thousand pounds. But in both cases, the metal yields permanently with much less stress than the amounts here indicated; and hence, as well as for other considerations, it can never be safely exposed in practice, to more than a small proportion of these stresses, say from $\frac{1}{8}$ to $\frac{1}{4}$.

Cast iron resists a positive strain of 15,000 to 30,000lbs. to the square inch, but usually, not over 18,000. But it is seldom relied on to sustain this kind of action especially in bridge work, wrought iron being much better adapted to the purpose. On rare occasions, it may perhaps safely be exposed to a strain of 3,000 to 4,000lbs. to the square inch, but should not be used under tension strain, when wrought iron can be conveniently substituted.

Cast iron, however, is capable of resisting a much greater negative strain than wrought iron; its power of resistance in this respect, being from 80,000 to

140,000lbs.; seldom less than 100.000 to the square inch, in pieces not exceeding in length, twice the least diameter.

But in pieces of such dimensions as must frequently be employed in bridge work, fracture would take place by lateral deflection, under a much smaller force than what would crush the material. It is therefore necessary to take into account the length and diameter, as well as the cross-section, in order to determine the amount of compression which a piece of cast iron, or any other material may be relied on to sustain.

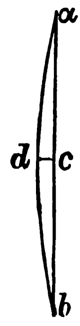
LXXXVII. The cause of lateral deflection resulting from forces applied at the ends, and tending to crush a long piece in the direction of its length, is supposed to be a want of uniformity in the material, and a want of such an adjust of the forces that the line joining the centres of pressure at the two ends, may pass through the centre of resistance in all parts of the piece.

These elements are liable to considerable variation, and can not be very closely estimated in any case. Therefore the absolute power of resistance for a piece of considerable length, can not be deduced by calculation from the simple positive and negative strength of the material, but resort must be had to direct experiment upon the subject; and, even wide discrepancies should naturally be expected in the results of experiment, unless the lengths of pieces experimented upon, be very considerable.

In respect to pieces, however, having their lengths equal to twenty or more times their diameters, a somewhat remarkable degree of uniformity is found in their powers of negative resistance, and the following formula, deduced theoretically, though not fully sustained by experiment, may be useful in determining approximately the relative powers for pieces of similar cross-sections, but different dimensions. The power of resistance (R), is as the cube of the diameter (d), directly and as the square of the length (l), inversely, that is, R is as $\frac{d^3}{l^2}$.

The reason of this formula may be illustrated with reference to Fig. 25, in which adb represents a post loaded at a, so as to bend it into a curve, of the half of which cd is the versed sine. It is obvious that in this condition, the convex side of the post is exposed to tension (or at least, to less compression than the other side), and the concave side to compression; also, that the effect of the load at a, toward breaking the post at d, is as the versed sine cd, which is as the square of ab. But the power of the post to resist rupture transversely, is manifestly as the cross-section of the post (i. e., as the square of the diameter), multiplied by the diameter. Hence, the power is as the cube of the diameter. Now, the ability of the post to sustain the load at a, is directly as the power to resist rupture, just determined, and inversely as the mechanical advantage with which the load acts, above seen to be as the square of the length of the post. Hence, the formula.

FIG. 25.



We shall see as we progress, the relation which this formula seems to bear to the results of experiment.

The following list of experiments made by the author some 25 years ago, though few in number, and upon a somewhat diminutive scale, nevertheless, may afford some light as to the law governing the resisting power of cast iron in pieces of different lengths, as compared with their diameters. It may at least enable us the better to appreciate the better lights since shed upon the subject.

LXXXVIII. EXPERIMENTS UPON THE NEGATIVE STRENGTH OF CAST IRON, IN LONG PIECES.

Ends, flat cones or pyramids.

No. of Ex'pts.	Form of section.	Inches. Diam.	Inches. Length	Wt. in lbs.	Wt. borne.	Breaking wt.	Remarks.
1	Cylinder	$\frac{5}{16}$	9.	0.16	990	1002	Broke $\frac{1}{10}$ in. from centre.
2	"	"	"	"	978	990	Broke $\frac{1}{3}$ in. from centre.
3	Square	$\frac{1}{4}$	"	0.15	803	854	Deflected cornerwise, and flew out without breaking.
4	"	"	"	"	914	938	Broke in half a minute not cornerwise, $\frac{1}{8}$ inch from centre.
5	Cylinder	$\frac{5}{16}$	7.1	0.126	1417	1437	Broke in 3 seconds, $\frac{1}{40}$ in. from centre.
6	"	"	"	"	1377	1397	Broke $\frac{15}{16}$ in. from centre.
"	"	"	"	"			Piece flattened by flask not shutting true, and had been straightened with the hammer where it broke.
7	"	"	4.5		2580	2580	Broke in 1 minute into 4 pieces of nearly equal lengths.
							Piece of same as last experiment.
8	"	"	4.5		3218	3218	Broke in $\frac{1}{3}$ minute into 3 pieces in centre, and 1 in. from centre.
9	Square	$\frac{1}{4}$	4.5		2813	2838	Broke in $\frac{1}{4}$ minute, $\frac{1}{16}$ in. from centre, deflected parallel with sides.

From experiments 7 and 8, in the above table, it appears that cast iron will sustain at the extreme, in cylindrical pieces whose lengths equal about 14½ diameters, a negative strain of 41,000 to 51,000lbs to the square inch, say an average of 46,000. Square bars, according to experiment 9, length equal to 18 diameters (or widths of side), will sustain about 45,000lbs to the square inch.

Now, a hollow cylinder of a thickness not exceeding about $\frac{1}{18}$ of the diameter, according to calculation, has a stiffness transversely, about 50 per cent greater to the square inch than a solid square bar whose side equals in width the diameter of the cylinder. Hence, a hollow cylinder of a length equal to 18 times its diameter, should sustain a negative strain of 67,500 lbs. to the square inch. But it should be observed, however, that direct experiments upon the transverse strength of the pieces used in the experiments leading to the results and conclusions above stated, as to negative strength, showed them to possess uncommon strength transversely, even to from 30 to 50 per cent greater than the fair average transverse strength of cast iron; as will be seen hereafter. It is therefore not considered proper to estimate the strength of hollow cylinders of the proportions above stated at more than 45,000 or 46,000 lbs. to the square inch.

The hollow cylinder is undoubtedly the form best adapted to the sustaining of a negative strain, having equal stiffness in all directions. It is therefore highly desirable that the power of that form of pieces to resist compression, with different lengths, should be ascertained by a careful and extensive series of experiments. But until that shall have been done, and the results made known, I shall assume the above estimate upon the subject, as probably not very far from the truth; subject, however, to correction, whenever the facts and evidences shall be obtained, upon which the correction can be founded.*

In the mean time, since we know not the exact ratio between the greatest safe practical stress, and the ab-

* Since the original writing of this paragraph (25 years ago), extentensive experiments and investigations have been made, in the direction

solute strength of iron, and therefore should in practice keep considerably within the limits of *probable* safety, it becomes a matter of less importance to know the exact absolute strength; though this, of course, is desirable.

LXXXIX. Having decided upon a measure of strength for pieces of a given length, we may properly endeavor to ascertain the rate of variation for different lengths as compared with the diameters.

It is seen in the table, [LXXXVIII] that two cylindrical pieces of 9 inches in length, bore the one 990, and the other 978lbs., giving a mean of 984 pounds.

Now, by the formula $\frac{d^n}{l^2}$, the same cylinders reduced to 4.5 inches, should sustain four times as much, or 3936lbs. But, by experiments 7 and 8, we find that they bore only 2,580, and 3,218, a mean of 2,899 pounds. Whence it appears that, the diameter being the same, the strength diminishes faster than the length increases, but not so fast as the *square* of the length increases; being about half way between the two. In fact, if we examine the results of these experiments throughout, we find that the weights borne by pieces of like cross-sections, whether round or square, were very nearly the arithmetical mean between the results obtained by considering them to be inversely as the simple length, and as the square of the length, successively.

For illustration; take experiments 1 and 5. If the piece 9 inches long bore 990 lbs., taking the strength

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here indicated, and ingenious and convenient formulæ deduced upon the subject involved, which might perhaps, be profitably substituted for the writer's own crude deductions in this behalf. But, as previously remarked on other occasions, the latter may possess interest as affording a monument upon the line of the march of progress.

to be inversely as the length, we have this proportion $\frac{1}{9}:\frac{1}{7.1}::990:1,255$. Then, taking the strength to be inversely as the *square* of the length, we have : $\frac{1}{81}:\frac{1}{50.41}::990:1,591$. Taking the mean of these results, we find $(1,255 + 1,591), \div 2 = 1423$. This is the weight which, according to the rule, the piece in experiment 5 should have borne, and it varies only 6lbs. (less than ½ of one per cent), from what it actually did bear.

Again, take experiments 1 and 8; in which the lengths were as 2 to 1. Supposing the weights to be inversely as the lengths, and as the squares of the lengths successively, and taking the mean of the results, we have $(1,980+3,960) \div 2=2,970$, which is 248lbs. less than the weight borne in experiment 8. But it is also 390lbs. *greater* than that borne in experiment 7, by a piece of similar form and dimensions, but an inferior specimen. It does not seem, therefore, that the rule is widely at fault.

The same rule applied to experiments 4 and 9, lengths being also as 2 to 1, gives 2,784 lbs. as the bearing weight, and 2,814 as *breaking* weight for No. 9; the former varying 71lbs. and the latter 24lbs. from the weights shown in the table. Now, if we observe that the one broke in a quarter of a minute, and the other endured half a minute, it is no extravagance to assume that if No. 9 had been loaded with 24lbs. less, it would have stood ¼ of a minute longer, giving a result in precise accordance with the rule.

From what precedes, it is believed that the following may be adopted as a safe practical rule for determining the power of resistance to compression, for pieces of similar cross-sections, after knowing from experiment, the power of a piece of given dimensions, and similar cross section.

Rule: *Make the power of resistance as* $\frac{D^3}{L^2}$, *and as* $\frac{D^3}{L}$ *successively, and take the mean of the results thus obtained, as the true result;* D representing the diameter (or width of side, in square pieces), and L, the length of the piece.

This rule will be probably apply without material error, to pieces of lengths from 15 to 40 times as great as their diameters, and perhaps for greater lengths; although, in bridge building, greater lengths will seldom be employed.* But, as the length is reduced to 8 or 10 diameters, or less, it is manifest that the power of resistance increases at a less rate than that given in the rule. For, we see by the table of experiments, that a square piece of a length equal to 18 diameters (experiment 9), bore at the rate of 45,000lbs. to the square inch, which is nearly one-half of the average crushing weight of cast iron, and one-third that of the strongest iron. But according to the rule, a piece of half that length, or equal to 9 diameters, should sustain 135,000lbs. which is about the maximum for cast iron; whereas, experiment shows that the power of resistance increases with reduction of length, down to about 2 diameters. It may, therefore, be recommended to apply the rule above given, to hollow cylindrical, and square pieces above 15, and to solid cylinders, above 12 diameters. From those lengths down to 2 diameters, it cannot lead to material error to estimate an increase of power proportionate to diminution of length, according to the differences between the weights, or resisting powers determined as above, for square pieces and hollow cylinders of 15, and solid cylinders of 12 diameters in length, and the absolute crushing

* It is probable that for greater lengths than 40 diameters, the formula $\frac{D^3}{L^2}$ alone, would be more nearly sustained than in case of smaller lengths.

weight of the iron; that is, if a square piece whose length equals 15 diameters bear *m* pounds, and the crushing weight for pieces of 2 diameters be *n* pounds to obtain the resistance (R), of a piece of (15—*a*), diameters in length, take $m + \frac{a}{13}(n-m)=R$.

XC. It has already been remarked that in practice, materials should be exposed to much less strain than their absolute strength is capable of sustaining for a short time. This fact is universally recognized, and the reasons for it, are perhaps, sufficiently obvious; still it may be proper to mention a few of them in this place.

First, there is a great want of uniformity in the quality and strength of materials of the same kind, and no degree of precaution can always guard against the employment of those containing defective portions possessing less than the average strength.

Again, when materials are exposed to a strain, although it be but a small part of what they can ultimately bear, a change is produced in the arrangement of their particles, from which they are frequently unable fully to recover; and whence they generally become weakened, especially if they be repeatedly exposed to such process. Hence, it often happens that a piece is broken with a smaller strain, than it has previously borne without apparent injury.

Now, there is no means of estimating exactly the allowance necessary to be made on account either of these facts, as well as, probably, many others. Consequently, we can not determine with certainty, how much of a given material may be relied on to sustain with safety a given force. We should therefore, incline toward the side of safety, the more strongly, in proportion as the consequences of a failure would be the more disastrous. The breaking of a bridge is liable, in most cases, to be a serious affair, involving hazard to life and limb, as well as destruction of property. Hence, they should be constructed of such strength as to render failure quite out of the range of probability, if not absolutely impossible.

XCI. Good wrought iron bars, will not undergo permanent change of form under a tensile strain of less than from 20,000 to 30,000 pounds to the square inch; and though they will not actually be torn asunder with a stress below 50 or 60 thousand, and often more, to the inch, any elongation would certainly be deleterious to the work containing them, even if not dangerous from liability to fracture. Hence, it is certainly not advisable to expose the material to a stress beyond the lowest limit of complete elasticity.

In the original predecessor of this work, the traditional allowance of 15,000lbs. to the square inch, was adopted as the tensile stress to which wrought iron might safely be exposed, and beyond which it was deemed improper to rely upon it. No evidences or arguments since that time, have induced a change of opinion in this respect. But in the case of a bridge, there is variety and uncertainty as to the exact amount of load, as well as in relation to the limit of safe strain for the material; and while it seemed *probable* that the load of a single track rail road bridge would never exceed 2,000lbs. to the lineal foot upon any part of its length, still, seeing that rail roads were comparatively a new institution, and *iron* bridges for rail roads almost unheard of, especially in this country, it was deemed wise, in recommending their introduction, to so adjust their proportions as to meet almost any possible contingencies.

This could be accomplished either by assuming a greater possible load for the bridge, or a lower limit to the stress of materials with the smaller load, with the same ultimate result. And, perhaps the former would have been the more consistent course, as avoiding the seeming absurdity of the assumption that iron could safely stand a strain of 15,000lbs. in a common bridge, but only 10,000lb in a rail road bridge; and the no less seeming absurdity of assuming that the same material could stand 50 per cent more strain in a bridge composed partly of wood, than in one entirely constructed of iron. Now, instances in great numbers could be pointed out, of rail road bridges of wood and iron, where 2,000lbs. to the lineal foot would produce a stress considerably exceeding 15,000 to the inch upon certain bolts of wrought iron.*

* The author had occasion several years ago to refer to the following instances in corroboration of the statement above made, in this wise " The best evidence that exists as to the capacity of a material to bear a strain with safety, is derived from experience as to the strain it has been exposed to in works, and conditions similar to those in which it is proposed to employ it, and where it has by long usage, proved itself adequate to the labor required of it. If wrought iron, for example, has been used in railroad bridges for a great number of years, in numerous and repeated instances, where a given load, in addition to the weight of structure, would produce upon it a tension of 15,000lbs. to the square inch, and has withstood such usage without cases of failure not caused by manifest defects in the quality of material, or by casualties which such structures are not expected to be proof against; it may be fairly assumed to be reasonably safe and reliable in other railroad bridges where a similar gross load can not produce a greater stress; and much more so, where a like load can only produce a stress one-half, or two-thirds as great.

Now, it is provided in the plan herewith presented, that a load of 2,000lbs. to the lineal foot upon each pair of rails, on the whole, or any part of the length of the bridge, can not produce upon any part of the wrought iron work in the trusses, a tension exceeding 10,000lbs. to the square inch; and, to show that such provision is eminently safe and liberal, I proceed to give some examples of what the same material is liable to with the same load in other structures, where long and severe usage has fully proved its sufficiency.

And yet, it was deemed expedient by the author of this work, in the outset of the introduction of iron rail road bridges, to provide that 2,000lbs. to the foot upon each pair of tracks, should not give a stress exceeding 10,000lb to the square inch upon any part of the wrought iron work, not from a conviction that the material was unsafe under a stress of 15,000lbs. but to provide against the possible contingency of its being sometimes exposed to greater stress than that produced by a dead weight of 2,000lb. to the lineal foot.

XCII. The use of cast iron to sustain a tensile strain, should undoubtedly be avoided, as a general

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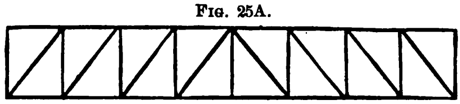
To begin with an instance near at hand; the bridge from the island to the main shore on the Hudson River rail road at East Albany, has, in one of its stretches, trusses 48 feet long, in 8 panels. It is a double track bridge with three trusses, of which the middle one sustains one-half of the two pairs of tracks, and of the loads passing over them.

The truss is composed of top and bottom chords, and thrust braces of timber, and vertical suspension bolts of wrought iron, in pairs; and it is at once obvious that $\frac{7}{8}$ of the weight of the tracks and their loads (or, of the half bearing upon the centre truss), is concentrated on the two pairs of suspension rods located 6 feet from each end. [See diagram.]

The weight of middle truss, and other parts of the structure sustained by it, probably exceeds 16,000 lbs., of which $\frac{7}{8}$, or 14,000 lbs. bear

FIG. 25A.



upon the endmost suspension bolts. Add 2,000 lbs. per foot for $\frac{7}{8}$ of one pair of tracks, or rails, and it makes 56,000lbs. upon the suspension bolts in question, with only one track loaded. These bolts are 4 in number, and $1\frac{3}{8}''$ in diameter; and, allowing $\frac{3}{16}''$ to be cut away by screw thread, the aggregate net, available cross section of the four, is equal to 4.43 square inches; whence the tension, with only one track loaded, is 12,641 lbs. to the square inch, and 22,120 lbs. to the inch with both tracks loaded.

2. The bridge leading into the freight house of the Boston rail road, at East Albany, is a "Howe bridge," and acts upon the same principle as the one just spoken of. It is a double track bridge with two trusses, having 8 panels of 10′8″, and is a heavy covered bridge. Allowing 64 tons for weight of superstructure, or 56,000 lbs. for the portion sustained by the endmost bolts of each truss, and 2,000 lbs per foot upon one track, of which $\frac{7}{8}$ at least, bears on one truss, giving

rule; and, if on certain occasions it should be liable to that kind of action to a small extent, the stress should probably not be allowed to exceed 3,000 to 4,000 pounds to the square inch.

When exposed to compression, in pieces of such length as to break by lateral deflection, it is believed it may be safely loaded to one-third of its absolute capacity. If a long piece exposed to a negative strain have a defective part, it does not diminish its power of resistance to the same extent as when it acts by tension. The power of negative resistance being, in a measure, inversely as the deflection produced by a

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100,000 lbs. on the end bolts, we have 156,000lbs. sustained by 6 bolts of 1½″ diameter, containing 8.1 square inches, besides screw thread. This is a strain of 19,259 lbs. to the square inch with one track, and 25,432 lbs. with both tracks loaded with 2,000 lbs. to the lineal foot.

3. The East bridge over the creek in the sonth part of Troy, is a double track covered bridge with three trusses, having 8 panels of 12′8″ each, or 88.66 ft sustained by the endmost suspension bolts. Say, of weight of structure bearing on end bolts of middle truss, 35,000 lbs. and of load upon one track 88,666, making 123,666 lbs. on 4 bolts of 1½″ diameter and two of 1¾″ diameter, having a net cross-section of about 7.65 square inches. Hence the stress must be 16,156 lbs. to the inch, with one track loaded, and 27,750 lbs., with 2,000 lbs. to the foot upon each track.

4. The West bridge over the same stream, a few rods below the last mentioned, has three trusses containing 9 panels of 10½ ft. each in length. It is a high truss bridge with roof and siding.

For weight of superstructure on endmost bolts of middle truss, say 28,000 lbs. and for load on one track, 84,000, making 112,000 lbs. on 4 bolts of 1½″ containing a net section of 5.41 square inches, giving a tension of 20,702 lbs. to the inch for one track, and 36,229 lbs. for both tracks loaded with 2,000 lbs. to the lineal foot.

5. The bridge across the Erie canal near Canastota, on the N Y. C. R. R., is a double tack bridge with 2 trusses, which have 9 panels of 10 feet. If the superstructure be estimated to weigh 40 tons, it gives a little over 35,000 Ibs. on the end bolts of each truss. Add ⅘ of 80 tons for 2,000 lbs. per lineal foot upon one track, and it gives 141,666 lbs. on 4 bolts of 1½″ diameter, and 5.41 square inches of net cross-section; equal to 26,173 lbs. to the inch, with one track, and 36,044 lbs. with both tracks loaded."

All these cases are stated from personal examination by the author, except the last, which was reported to him from authority considered reliable. The cases were not *selected*, but taken as the most accessible, and conveniemt for the author's observation. And still, he can not help regarding them as *remarkable*, and somewhat *exceptional* cases given weight, and the deflection depending on the stiffness of the piece throughout its whole length, the power is manifestly only diminished as the amount of defect, multiplied by the ratio of length of the defective part, to the whole length; that is, if the piece be defective so as to lose one-fourth of its stiffness, for that part of its length to which is due one-tenth part of the deflection, the deflection will only be increased by $\frac{1}{4} \times \frac{1}{10} = \frac{1}{40}$, and the power of resistance is diminished in the same ratio; whereas the power of positive resistance would be diminished by $\frac{1}{4}$.

The effect of negative strain, moreover, is believed not to be so deleterious to the strength of iron, as that of positive, or tension strain; though I can refer to no particular facts or evidences in coroboration of the opinion.

Upon the whole, I am inclined to estimate the power of cast iron to resist compression (as against the tension of wrought iron at 15,000lbs. to the inch), in pieces of lengths equal to 18 diameters, for hollow cylinders, at 15,000lbs. for solid cylinders, at 8,000, and solid square pieces, at 10,000lbs. to the square inch of cross-section.

There are other forms of section for cast iron members of bridges, which it will frequently be convenient and economical to employ where lateral stiffness, as well as longitudinal resistance is required, among which may be named, the cruciform +, the T, and the H form.

The former of these, with equal leaves, probably possesses about the same resistance to the square inch, as a solid square which will just contain the figure. For, though it is not so stiff to resist a simple lateral force *diagonally* of the including square, as parallel with its sides, and would be broken by tearing asunder the flange, or leaf upon the convex side, still when under longitudinal compression, the tension upon that leaf would be somewhat relieved.

The T and H section will usually be employed where greater stiffness is required in particular directions, and if proportioned with judgment, will usually possess about the same power to the inch, as the including solid square, or parallelopiped.

XCIII. Having determined (approximately, at least), the safe strain for pieces of a certain length, and the ratio of variation in power, depending upon change of length, we readily deduce the safe strain for pieces of similar action, with any given dimensions.

The following table, exhibiting the negative power of resistance to the square inch of cross-section, for hollow and solid cast iron cylinders, and solid square pieces (under which class may be included the + T and H formed sections, under proper conditions), calculated for length of from 2 to 60 diameters, is intended to show the safe practical rate of strain for the material, being about one-third of its absolute strength, in columns headed $\frac{1}{3}$, and one-fourth of the absolute, in those headed $\frac{1}{4}$; the former to be used against wrought iron at 15,000, and the latter, where wrought iron is estimated to sustain 10,000lbs. to the square inch.

This is the author's original table, slightly modified, with the addition of two columns showing corresponding weights at $\frac{1}{3}$ and $\frac{1}{4}$ of the absolute strength, as calculated by "Gordon's formula," deduced from Hodgkinson's, experiments upon cast iron hollow pillars; which is regarded as the best authority upon the subject at the present day. Also, two corresponding columns for wrought iron hollow pillars, according to the same authority.

The Gordon formulæ are:

for cast iron, $S = 80,000\text{lb.} \div (1 + .0025\frac{l^2}{d^2})$,

for wrought iron, $S = 36,000\text{lbs.} \div (1 + .00033\frac{l^2}{d^2})$.

S representing absolute strength per square inch of section, l, the length, and d, the diameter of column, both referring to the same unit of length. Or making $d = 1$, we have $\frac{l^2}{d^2} = l^2$.

The table of negative resistances, presents a scale of numbers so adjusted as to touch at certain points established by experiment, and running in consistent gradations from one to another of such points.

The columns for cast iron hollow cylinders, are the only ones referring to the same class of pieces, and exhibiting the difference in results, arising from difference in the mode of calculation. The Gordon formula is supposed to give results agreeing with those of experiment, for lengths included within the range em braced by the experiments from which the formula was deduced. Within that range, those results may be presumed to be more reliable (being founded on trials of the same kind of pieces as those to which they refer), than those in the author's original table, based upon trials of solid cylinders and parallelopipeds.

Taking the 4th and 6th columns, it will be seen that the numbers agree at some point between the lengths of 18 and 20 diameters; the numbers above that point, being the larger in column 6, while, below that point, they are larger in column 4, down to about 50 diameters, where they come together and cross again, and those in 6, are thenceforward the larger. But the differences are small, for the range of lengths principally employed in bridge work.

Power of negative resistance to the square inch, in pounds.

Hollow Pillars, by Gordon's Formulæ.					Of cast iron, as by the author's original table.						
Length in diameters,	Wrought iron.		Cast iron.		Hollow Cylinders.		Solid cylinders.		Square, +, T, and H sections.		Length in diameters.
	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	
2	11,984	8,988	26,400	19,800	33,333	25,000	33,333	25,000	33,333	25,000	2
4	11,936	8,952	25,641	19,231	31,132	23,349	29,666	22,250	30,277	22,710	4
6	11,859	8,894	24,464	18,348	28,932	21,699	26,000	19,500	27,223	20,419	6
8	11,751	8,813	22,988	17,241	26,731	20,048	22,333	16,750	24,169	18,128	8
10	11,616	8,712	21,333	16,000	24,531	18,398	18,666	14,000	21,115	15,837	10
12	11,455	8,591	19,608	14,706	22,330	16,748	15,000	11,250	18,061	13,547	12
14	11,271	8,453	17,897	13,423	20,130	15,098	11,755	8,817	15,007	11,256	14
16	11,065	8,298	16,260	12,195	17,930	13,448	9,562	7,172	11,953	8,965	16
18	10,841	8,131	14,733	11,050	15,000	11,250	8,000	6,000	10,000	7,500	18
20	10,600	7,950	13,333	10,000	12,825	9,619	6,840	5,130	8,550	6,413	20
22	10,347	7,760	12,066	9,049	11,156	8,367	5,950	4,463	7,438	5,579	22
24	10,083	7,564	10,927	8,195	9,844	7,383	5,250	3,938	6,562	4,922	24
26	9,811	7,358	9,913	7,435	8,787	6,590	4,688	3,515	5,858	4,398	26
28	9,533	7,150	9,009	6,757	7,921	5,941	4,224	3,168	5,280	3,939	28
30	9,254	6,942	8,205	6,154	7,200	5,400	3,840	2,880	4,800	3,600	30
32	8,969	6,727	7,490	5,618	6,592	4,944	3,515	2,636	4,395	3,297	32
34	8,686	6,515	6,855	5,141	6,072	4,554	3,238	2,429	4,048	3,036	34
36	8,405	6,304	6,289	4,717	5,625	4,219	3,000	2,250	3,750	2,813	36
38	8,127	6,095	5,784	4,338	5,235	3,924	2,792	2,094	3,490	2,618	38
40	7,853	5,890	5,333	4,000	4,893	3,669	2,610	1,958	3,262	2,447	40
45	7,193	5,395	4,397	3,298	4,200	3,150	2,240	1,680	2,800	2,100	45
50	6,575	4,931	3,678	2,759	3,672	2,754	1,952	1,464	2,448	1,836	50
55	6,005	4,504	3,113	2,335	3,257	2,443	1,737	1,303	2,171	1,628	55
60	5,484	4,113	2,666	2,000	2,925	2,194	1,560	1,170	1,950	1,463	60

One obvious reason of the more rapid increase of numbers in the 6th column, for lengths under 15 or 16 diameters, is, that in the latter, the crushing weight for the iron is assumed at 100,000lbs. to the square inch, whereas, by the Gordon formula it is limited at 80,000lbs, and that formula can give no result greater than that limit, even when $l=0$. Now, if 80,000lbs. was less than the actual crushing load for the kind of iron used in Hodgkinson's experiments (from which the Gordon formula is understood to have been derived), it must follow that Gordon's formula gives results smaller than the true ones, for short pieces. This is probably the case, and, although Mr. Gordon's formula is very simple and ingenious, sliding smoothly and plausibly from one extreme in length to the other, it unquestionably gives closer approximations to correct results for the ordinary range of lengths, than when applied to the very short pieces.

The numbers in the table are deduced upon the supposition that the thrust members in a bridge, will not act with less advantage than when bearing upon a pivot at each end of the axis of the pieces respectively; and it is not deemed proper to assume that, in consequence of having flat end bearings, the piece in any case can sustain a greater stress than is indicated by the numbers in the table.

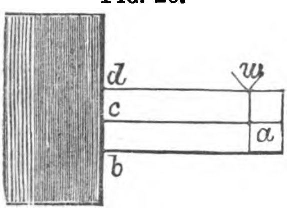
It will be observed that, in order to obtain the absolute strength of a piece, we should multiply its corresponding number in the table, by the denominator of the fraction ($\frac{1}{3}$ or $\frac{1}{4}$) at the head of the column.

LATERAL, OR TRANSVERSE STRENGTH.

XCIV. The transverse strength of bars or beams, would seem to be deducible from the positive strength of the material, in the following manner:

Let *ab*, Fig. 26, represent a portion of a rectangular beam or bar, projecting from a wall in which it is firmly fixed. If a weight be applied at *w*, the upper part of the beam will be extended, and the lower, compressed; and, where these portions meet, is what is called the *neutral plane*. Experiment shows that this plane, in rectangular beams, is central between the upper and lower surfaces; or at least, very nearly so, for all elastic substances, until they approach rupture.

FIG. 26.



The tendency of the weight at *w*, then, is to produce rotation about the point *c* (or, the line of intersection of neutral plane and face of wall) and the cohesion of the upper portion *cd*, and the repulsion of the lower part, *cb*, tend to resist rotation. Now, to determine the amount of this resistance, which is the measure of transverse strength, we will first consider the upper portion; and it is obvious that, at every part of the cross-section, the resistance to rotation is as the resistance to extension, multipled by the distance of the part above the neutral plane. But the resistance to extension, by the law of elasticity, is as the degree, or amount of extension, which is determined by the distance from the neutral plane; parts at 2 inches from this plane, or the centre of motion, being extended twice as much as those at one inch, and resisting twice as much.

Then, denoting the distance from this plane by the variable quantity x, the resistance to extension by any part, equals x multiplied by a certain constant (s), and may be denoted by sx, while the resistance to rotation about c, equals sx^2.

Again, representing the horizontal breadth or thickness of the beam by t, we have $t.dx$ to represent the differential of the section (in its state of increase from c toward d), and $s.t.x^2dx$, the differential of resistance. Then, integrating, and making $x = cd = h$, we have the whole resistance to rotation, of the part above the neutral plane, equal to $\frac{1}{3}\ s.t.h^3 = \frac{1}{3}t.h \times h \times s.h$. But $s.h$ becomes equal to the positive strength of the material when $x = cd = h$, and $t.h =$ the area of section above the neutral plane. Therefore the power of this part to resist rotation, is equal to $\frac{1}{3}$ of the area, multiplied by half the depth of the beam, and by the positive strength of the material; in case the negative strength exceed the positive.

Now, it is obvious that the part below the neutral plane exerts exactly the same amount of resistance to rotation, as the part above. Therefore the whole power of resistance to rotation about c, in other words, the resistance to rupture, is equal to $\frac{1}{3}$ of the whole cross-section, multiplied by $\frac{1}{2}$ the depth of beam, and by the positive, or cohesive strength of the material; that is equal to $\frac{1}{3}\ C.t.D \times \frac{1}{2}D, = \frac{1}{6}\ C,t.D^2$; in which expression, D represents the depth (db), and C, the cohesive power of the material.*

* Another mode of illustrating this case, is the following: It being obvious that the resistance to rotation about c, by each lamina from the neutral plane outward, is as the extension it undergoes, and the leverage upon which it acts, such resistance must increase outward in a duplicate

If we wish to determine the greatest weight (W), which the beam is capable of bearing when applied at any horizontal distance (L) from *c* or *d*, we institute the equation, $W.L = \frac{1}{6} C.t.D^2$; whence we have:

$$W = \frac{C.t.D^2}{6L}.$$

This formula applies to all projecting rectangular beams, when the force (W), acts parallel with the sides, and L represents the nearest, or perpendicular distance of the fulcrum *c*, from the line in which the force has its action; provided, that if the material have greater power to resist tension than compression, *C* is to be taken as representing the repulsive, instead of the cohesive power.

XCV. This formula is deduced on the supposition that the material is perfectly elastic, so as to suffer no permanent change of form until the strain produces actual rupture. There are few substances if any, and certainly wood and iron are not such, that fulfill this condition so nearly but that considerable discrepancies are found between the deductions of theory, and the results of experiment. Indeed in the case of cast

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ratio to the increase of distance from that plane, and decrease in a like ratio, inward. Hence. if we represent the resistance of the outer lamia by the base of a pyramid having its apex at the neutral plane, and its base coinciding with said outer lamina, the resistance of any other lamina will be represented by the section of the pryamid made by snch lamina, or a lamina of the pryamid at the point of intersection, of the same (indefinitely small) thickness as the lamina of the beam in question; and the sum of resistances of all the laminæ of the beam, will be represented by the sum of laminæ of the pyramid; and will bear the same ratio to what the resistance of all those laminæ of the beam would be, if all were acting at the distance of the outermost lamina, as the solidity of the pyramid bears to a prism of like base aud attitude; that is, in the ratio of 1 to 3. But the resistance of the outer lamina, equals the absolute strength of material (C), multiplied by half the depth of beam. Hence, the resistance of the half beam equals $C \times \frac{1}{3}$ cross-section $\times$ depth of the half beam; being the same result as above obtained by integration.

iron, experiment shows the transverse strength to be fully twice as great as it is made to appear by the above formula.

If in the expression $\frac{C.t.D^2}{6L}$, we make L=D, it may be reduced to $\frac{1}{6}$ *C.t.D*; showing that the power of a projecting rectangular beam to sustain weight at a distance from the fulcrum equal to the depth of the beam, is only one-sixth as great as the positive (or negative, in case that be the smaller), strength of the material. This is a convenient way of expressing transverse strength, viz: as equal to a force of so many pounds to the square inch of cross-section, the force being understood as acting upon a leverage equal to the breadth of the beam in the direction of the acting force.

If we call 18,000lbs. to the square inch, the positive strength of cast iron, we may call the transverse strength (according to the above deduction), $\frac{1}{6}$ 18,000 =3,000lbs.; meaning that a bar one inch square will sustain upon its projecting end, 3,000lbs. at 1 inch from the fulcrum, and proportionally less, as the distance is greater.

Now, experiment shows that it will sustain twice this amount, and frequently more, so that we may in reality, reckon the transverse strength of cast iron at about 6,000lbs. to the square inch.

I know of nothing to which to attribute this great discrepancy between theory and experiment, except a want of complete elasticity in the material, and perhaps, also to the assumption of too low an estimate (18,000) lbs. for the co-hesive power of cast iron.

Cast iron, when exposed to a transverse strain, suffers extension on one side, and compression on the other; and the power of resistance to both these effects, increases very nearly as the amount of extension or compression, until a certain point or maximum is reached, and after passing this point, the power diminishes. Now, it is reasonable to suppose, in fact we can hardly suppose the contrary, that for a certain interval on each side of the maximum point, the power of resistance remains nearly stationary. But this stationary interval is reached on the positive, much sooner than on the negative side, and the inevitable consequence must be, that the neutral plane is transferred further from the positive side, so as to preserve the equilibrium between the resistance to extension and the resistance to compression. Hence, the amount of resistance on the positive side is increased, both by the increased area of section exposed to tension, and increased leverage, or distance from the neutral plane.

Moreover, a greater portion of the fibres (so to speak), of extension, act with their full power; since, while the outside portion is passing through what we have called the stationary interval, successive portions toward the neutral plane, are reaching and approaching that interval. Hence, some considerable proportion of all the fibres of extension, may act with their maximum power; whereas, if the material were perfectly elastic up to the point of actual rupture, only the outside fibres farthest from the neutral plane, could act with absolute power, and all other parts, only in the ratio of their respective distances from said plane. To illustrate, suppose the extreme positive side, when extended one inch, reach the stationary interval, which is one inch more. It follows that when the outside has passed to the other limit of that interval, one-half of the positive portion of the bar, will be within the the range of that interval, and act with its maximum power, producing one third more resistance to extension than the same fibres could afford if the body were perfectly elastic, up to the point of rupture. I know of no more plausible manner of explaining the observed discrepancy between experiment and calculation upon the subject.

But, having well authenticated direct experimental evidence as to the transverse strength of cast iron, we may safely be guided thereby; and, though it would be a satisfaction to find a complete agreement between the results of *direct* experiments, and the deductions from those that are indirect, still, where such agreement is not found, the direct evidence should have the preference. We may, therefore, regard the transverse strength of cast iron in pieces with rectangular sections, as equal to 6,000lbs. to the square inch, upon a leverage equal to the width of the piece in the direction of the force.

Wrought iron has something over three times the positive strength of cast iron, on the average; and if we consider its transverse strength to be in the same ratio to that of cast iron, its transverse strength would be about 20,000 pounds. That is, the projecting end of a bar of wrought iron one inch square, should sustain, at one inch from the fulcrum, a weight of 20,000lbs But it becomes permanently bent with about one-third of that weight, and therefore, in practice it should not be exposed to more than 4,000 to 5,000lbs. as we should manifestly keep within the elastic limit, as well in case of a transverse, as a direct tensile strain. It may, therefore, be recommended to estimate the transverse strength of wrought iron at 5,000lbs. as against a tensile strain of 15,000lbs. to the inch, upon the same material, and 3,500 to 4,000 transverse, against 10,000, tensile strain.

Cast iron having an average absolute transverse strength of 6,000lbs. should not in practice, be exposed to over from 1,000 to 1,500lbs. to the square inch, according to the circumstances in which it is used.

XCVI. Representing by A the area by D the depth and by L the length (from fulcrum to weight) of a projecting rectangular beam, the safe load, according to the above assumptions, equals 5,000 $\frac{A.D}{L}$ for wrought, and 1,500 $\frac{AD}{L}$ for cast iron.

If the beam be supported at the ends and loaded in the middle, using the same symbols, the safe load is four times as much; that is, 20,000 $\frac{AD}{L}$ for wrought, and 6,000 $\frac{AD}{L}$ for cast iron. This follows from the fact that the lifting force at each end, equals only one-half of the load, and acts upon a leverage equal to $\frac{1}{2}$L, hence it takes 4 times the weight to produce the same stress on the beam.

If the load be equally distributed over the length of the beam, the safe load is twice as much as when it is concentrated at the end of the projecting beam, and in the middle of the beam supported at the ends. For, in the former case, each part of the weight produces stress at the fulcrum in proportion to its distance therefrom, and the average distance of the whole load, being only half as great, the stress is only half as much as when the whole load is at the end.

In the latter case, the beam being regarded as fixed in the centre, the lifting force at the end, tends to produce a strain in the centre, measured by the force multiplied by its distance from the centre, in other words, by the *moment* of the force with respect to the centre. On the contrary, the load tends to produce a strain in the opposite direction, according to the distance of each part of the load from the centre. This, as just seen in the case of the projecting beam, is equal to half of that produced by the lifting force at the end. Hence the effect of the weight neutralizes one-half of tendency of the lifting force at the end, to produce stress in the centre of the beam.

Upon the same principle is based the following rule for determining the stress at any given point in the length of a beam, however the load may be distributed. Take the moment with respect to the given point, of all the forces on either side of said point, tending to deflect the beam in one direction, at the given point, and all the forces on the same side, tending to deflect it in the opposite direction, and the difference in the sums of those opposite moments, is the measure of the stress at the point in question.

As an illustration, if the cross-beam of a rail road bridge support tracks 5′ apart, the beam being 15′ between supports, and a weight W bear equally upon the two tracks, each end support lifts $\frac{1}{2}W$, and the moment with respect to the nearest track, is $\frac{1}{2}W \times 5 = 2.5W$. There being no force acting in the opposite direction between the end and the weight upon the rail, 2.5W (upon a leverage of 1 foot), is the measure of stress of the beam at the rail.

If we seek the stress in the middle of the beam $7\frac{1}{2}'$ from the end, and $2\frac{1}{2}'$ from the rail, the upward force at the end is $\frac{1}{2}W$, the same as before, and the moment $\frac{1}{2}W \times 7\frac{1}{2} = 3.75W$; while the moment of the weight on the rail, is $\frac{1}{2}W \times 2\frac{1}{2}, = 1.25W$. Hence, the stress in the centre $= (3.75—1.25)W = 2.5W$, the same as at the rail.

Taking the moment of the end lift with respect to the off rail, we have $\frac{1}{2}W \times 10$, $= 5W$, while the moment of weight on the near track, with respect to the off track, is $\frac{1}{2}W \times 5$, $= 2\frac{1}{2}W$, acting in the opposite direction. Hence, stress at the off track, is equal to $(5—2.5)W = 2\frac{1}{2}W$.

Again, assuming a point at 2′ from the end, the moment of the lift at the farther end, is $\frac{1}{2}$ W × 13′ =6.5W. The sum of moments of weights upon the two rails is, $\frac{1}{2}$ W × 8 + $\frac{1}{2}$ W × 3 =5.5W in opposition to the effects of the end lift. The stress of the beam, therefore, at the given point, is (6.5 —5.5)W, = W ; being the same result as if we had taken the moment at the near end, = $\frac{1}{2}$W × 2, = W, with no opposite force on the same side of the given point.

Hence, we see that the stress is the same at all points between the rails, while it obviously diminishes from the rail to the end, in proportion as the distance of successive points from the end diminishes. Therefore, the beam having a uniform depth, in order that the strain be uniform on all parts, the thickness should taper uniformly from the rail, to an edge at the supporting points. If the thickness be uniform (the cross-section being rectangular), the depth may diminish as the square root of distance from the support diminishes; that is, may have a parabolic form, This follows from the fact that the stress at different points in the length is as the distance from the support, and the power of resistance, as the area multiplied by the depth, in other words, as the square of the depth, the area being simply as the depth.

XCVII. Iron beams of a rectangular section will seldom be used in bridge work, the material acting more effectually in a web and flange form, as in the I beam, with about half the material in the flanges. In this kind of beam, the web may be estimated as a rectangular beam — say at 5,000lbs to the inch (on a leverage equal to the depth of the beam), while the flanges may be estimated at 15,000lbs upon a leverage of half the depth of beam, less half the thickness of flange, thus: for a beam 12″ deep, web ½″ thick, flanges 2″ wide and ¾″ in average thickness on each side of the web, we have 6 square inches of web section at 5,000 = 30,000lbs. plus, 6 inches of flange section at 15,000 × leverage of 5,625″, equal to 42,187lbs. on a leverage of the depth of beam, making a total of 72,187lb. =6,015lbs. to the inch upon the whole section.

Hence, it is deemed safe to estimate the working strength per square inch of wrought iron I beams, in the above proportions of web and flanges, at 6,000 $\frac{D}{L}$lbs. for projecting ends, and 24,000$\frac{D}{L}$for beams supported at the ends, and loaded in the middle; and double those amounts of distributed load. For instance; a 12″ I beam of 12 square inches in section, and 16′ long, between bearings, is good for 24,000 × $\frac{12}{16}$ = 18,000lbs. in the middle, 36,000 distributed uniformly, and 26,180lbs. upon two rails 5 feet apart, or 5.5 feet from end supports.

XCVIII. One of the cases in which wrought iron is frequently exposed to transverse strain, is in the use of cylindrical pins for connecting the other parts of bridge work. In such cases, the forces will act with a certain leverage which can be nearly determined. The power of a round pin to sustain a transverse force acting on a leverage equal to the diameter, may be assumed at about $\frac{1}{10}$ less to the square inch, than that of a square bar upon a leverage equal to its width of side. Hence, A representing the area of section, D, the diameter, and L, the length, the safe stress $= 4{,}500\frac{AD}{L}$ acting on a projecting end, and $18{,}000\frac{AD}{L}$ acting in the middle between two outside bearings; that is, a 1″ pin with centres of outside bearings 6″ apart, will bear in the middle, $18{,}000 \times .\frac{785}{6} = 2{,}355$lbs. If the pin connect an eye 1″ thick, between one of ½″ thick on each side, the length (L) between centres of outside pieces, will be 1½″; whence, a 1″ pin will bear 4 times as much as in the preceding case; or $2{,}355 \times 4 = 9{,}420$lbs. The tensile strength of a 1″ round rod, being 11,775lbs. being equal to the cross-section (0.785), $\times$ 15,000lbs. it shows that the strength of the rod is greater than that of the pin, in the condition here assumed, in the proportion of 11,775 to 9,420. Therefore, the stiffness of the pin being as the cube of the diameter, in order to find the diameter (x), of a pin for connecting 1″ bars by eyes and connecting straps, we have this proportion $9{,}420 : 11{,}775 :: 1^3 : x^3$, whence $x = 1.077$ = to about $\frac{1}{13}$ larger than the diameter of the rods to be connected, and in this proportion for any size of round rods connected by straps and pins or bolts.

But if the eyes and straps be drilled, so as to fit the pin through the whole thickness, the action approaches the shear strain, and the pin should have about ¾ the area of section of the bars to be connected. The author would recommend, however, for general practice, that connecting pins be considered as acting by transverse stiffness, upon the lever principle, as above discussed.

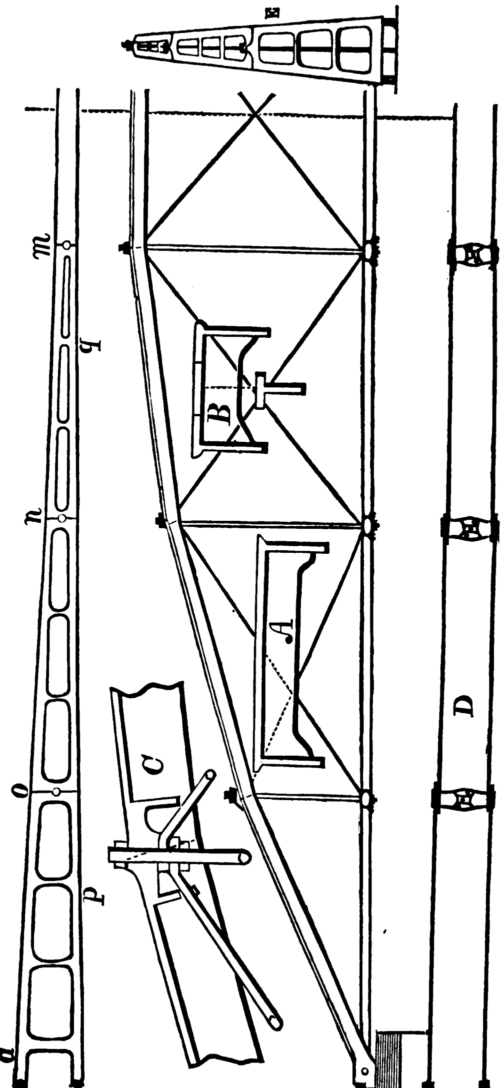


FIG. 27.

ARCH TRUSS BRIDGES.

XCIX. The general form in outline of the Arch Truss, may be seen in Figs. 8 and 11.

The forms of the different members, and the modes of connecting them to form the complete structure, are many, and a minute description of each possible variety, in this respect, even if such a thing can be regarded as practicable, will not be undertaken on this occasion.

The arch may be of cast or wrought iron in various forms of section. The following form of cast iron arch has been extensively used in the state of New York, with uniform success and satisfaction. The arch is composed of cast iron sections, equal in number to the number of panels in the truss; an odd number being deemed preferable. In Fig. 27, *a o n m* presents a top view of the arch, and D, a top view of the chord, from end to centre; and A and B, enlarged cross-sections at *p* and *q*, adjacent to the cross-bars to be described below, and which also appear in the figure. Each piece consists of two side portions of an ⌉ formed section, connected at the ends, and at 2 or 3 intermediate points, by cross-bars of a T formed section for the intermediates, and at the ends, with sections as seen at *C*, where a view of the arch connection is shown, as it would appear if cut vertically and longitudinally through the centre, and the near half removed.

The width of the side plates of arch castings (from the top), should be about $\frac{1}{20}$ of the length of pieces, with an average thickness of from $\frac{1}{10}$ to $\frac{1}{8}$ of the width. The top plate, about the same thickness (or a trifle less, to prevent a tendency in the piece to become hollow backed in cooling), and a width, a little over one-half that of the side plate.

The resisting power may be estimated as in the table of negative resistances under the head of square, &c., pieces, calling the width of side plates the diameter, and using the column under $\frac{1}{8}$, for trusses supporting 12 feet or more width of flooring, and the column headed $\frac{1}{4}$, in case of trusses supporting a width of 10 feet or less, to each truss.

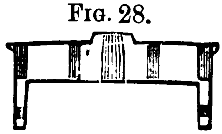
The intermediate cross bars should have about the same thickness of plate as the side portions, a depth, about $\frac{3}{5}$ that of side plates, and top plate not less than $\frac{3}{4}$ as wide as the top plate of side portions.

End cross-bars should have a top width of about $\frac{7}{8}$ the width of side plates, and cross-section sufficient to sustain a whole gross panel load for the truss, by transverse resistance. If it have a depth equal to $\frac{1}{5}$ of its length, and a form of section as strong as a rectangular bar, it will safely sustain 1,000 to 1,200lbs. to the inch; and it is recommended to allow one inch of section in each end cross-bar to every 1,000lbs, sustained at the joint. Then, there being two cross bars together, the point will be doubly secure.

Semicircular notches in the ends of contiguous arch pieces, form a vertical circular hole at the joint, for the passage of the vertical member.

When the side plates are thin, the thickness should be increased for a few inches from the end, to afford a suitable bearing surface at the joint; and the ends of arch pieces should be fitted (usually by planing), to a proper bevel to form a fair joint. The joints, however, are sometimes formed by cutting taper key seats (as seen in Fig. 28),

FIG. 28.

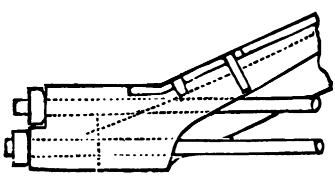


in one of the contiguous ends, to admit wrought iron wedges about an inch wide, and in sufficient number to give a bearing upon wedges, equal to at least one-half the section of iron in the chord. This method has answered well in a large number of bridges, and is convenient for adjusting the arch in line; but the planed ends form much the more workmanlike joint.

The centre arch piece has usually a full top plate over the whole width of the piece.

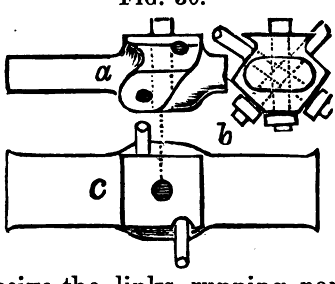
The endmost section, or foot piece of the arch, connects with the chord by means of horizontal holes in the feet to receive the ends of an open end link of the chord, which is secured by screws and nuts as shown in Fig. 29, representing an inside view of the foot of one branch of the arch.

FIG. 29.



C. The chord is composed of two long links of round or square iron to each panel, connected by cast iron connecting blocks at points vertically under the arch joints. The form of these blocks is represented in Fig. 30. They diminish in length from the endmost to the centremost, the former being long enough to receive the links running parallel from the connection with the arch, and the next block, being shorter by twice the diameter of the link iron; the ends of links toward the centre of the truss, going next the ends of

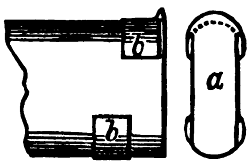
FIG. 30.



connecting blocks, and outside of the ends pointing toward abutments; and, the members of each pair of links being parallel with one another. [D, Fig. 27.]

The connecting block has an oblong section where it receives the links, being rounded on the sides to fit the semicircular ends of links. There should be an accurate fit between these parts, to effect which, perhaps, the best plan is to ream the ends of links, and turn the bearings of blocks to a uniform size. For this purpose, the block is cast with extra metal to be turned off at the bearings, and with the portion between bearings a little thinner vertically, than the turned portions, as shown in Fig. 30A, in which *a* is a section and *bb* are the bearing surfaces.

FIG. 30A.



The vertical thickness of the block where it receives the links, should be at least 1½ times the diameter of the link iron, and the cross-section multiplied by the width of block, and divided by diameter of link iron, should give a quotient about 13 times as great as the cross-section of both sides of the link.

The middle portion of the block is cast with the proper size and form for the upright and diagonal members to pass through in the required directions, and is provided with suitable facets for the bearings of nuts. The least cross-section through all or any of the holes, should be at least one-quarter greater than the section at the link bearings. In Fig. 30, *a*, *b* and *c* respectively represent a side, end and top view of the cast iron connecting block.

The oblong section of the connecting block was adopted to obtain greater transverse strength in the direction of the strain. But it has recently occurred to the author, that perhaps, after all, a circular section of block would have the advantage, inasmuch as it would not require so short a bend at the ends of links; whence they could the better adapt themselves to the block, and would not require so great a disturbance in the condition of fibres or particles of the iron in forming the bends. With a diameter of block equal to 3 times that of the link iron (in case of round iron), it is believed that good iron would suffer the bend without material deterioration, or greater liability to break the ends, than in other parts of the link, especially if welded in the straight part.

The enlarged central portion of the connecting block has upon its upper side, a flat surface rising a little above the links, to afford a beam seat for the cross-beams of the bridge to rest upon; which, in case of wooden beams, should present a bearing surface of 30 to 40 square inches.

CI. The upright is made of round wrought iron, $1\frac{3}{4}$ to 2 inches in diameter, for bridges from 60 to 100 feet in length, when designed for common road purposes. The upper end is furnished with a screw nut, and a ring or collar welded on at a sufficient distance below the nut to allow the arch castings, and eyes of diagonals to come between nut and collar.

The lower end is turned or swaged down to a diameter $\frac{1}{4}''$ or $\frac{3}{8}''$ less than the body of the rod, for a length sufficient to reach through the connecting block, and receive a nut on the end. This is to form a shoulder at the upper side of the block, to act in case of a thrust action of the upright.

The two longest uprights have usually been made double and divergent from the collar downward, the branches being of iron from ¼″ to ⅜″ smaller in diameter than the single uprights, and passing through the connecting block near the links, either inside or outside, as deemed most appropriate, with a thin nut above, and a common nut below the block; also a cast iron washer above the upper nuts, for the beam to rest on, instead of resting upon the central part of the block. The object is to give lateral steadiness to the arch, and diminish its vibration.

A better effect in the same direction is produced by connecting the upper ends of those uprights across from truss to truss, in case of long bridges. For this purpose, the upright may extend a little above the arch, when necessary to give head-way (or, perhaps better still, the arch itself might rise higher above the chord, thus diminishing the action upon both arch and chord), and a light cast or wrought iron strut introduced, to counteract the tendency to vibration of the arches, arising from the spring of the beams. As the the two trusses naturally tend to vibrate in opposition to each other, it is suggested whether simple ties of ¾″ iron, would not so break the regularities of the vibrations as to prevent their increase to an objectionable extent. The rigid strut, however, would be more effective, being capable of acting in both directions; and, if thrown into the form of a graceful arch, it would be ornamental withal.

CII. The diagonals are round rods, with an eye at the upper, and a screw and nut at the lower end of each; the screw portion being about ¼″ larger in diameter than the plain part of the rod. Two eyes of diagonals go upon each upright (except the endmost); that of the rod running downward toward the centre going above the other, the better to prevent interference with the cross-bars of arch pieces, as will be understood by reference to C, Fig. 27.

The eyes lie in horizontal positions, the rod in each case being bent to the required pitch to meet the connecting block. The bend should be as near as may be to the eye, without preventing a fair bearing of the eye upon the collar, or the subjacent eye. Care should be taken to have fullness and strength in the neck of the eye, that it may withstand the indirect strain at that point.

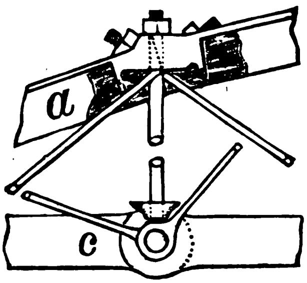
The proper sizes for diagonals and chords should be such, in common road and street bridges, as to afford at least one square inch of cross-section for each 15,000lbs. of stress produced by the greatest load to which such bridges are liable, which in the author's opinion, should be estimated at about 100lbs. to each square foot of bridge flooring, exclusive of weight of structure; a rule which he originally adopted, and has adhered to in practice with most satisfactory results. Many bridges have been constructed with lighter proportions than this rule would require, some of which have endured, while others have failed.

It is true that ordinary road bridges are seldom exposed to 100lbs. to the foot of floor surface, but it is nevertheless, deemed expedient to provide for such a contingency.

The modes of estimating stresses of different parts of the truss, have been fully discussed in preceding pages, [XXVII &c.], and it seems unnecessary in this place, to specify more particularly the dimensions of the several members of the truss.

CIII. Another devise for the connections of diagonals at the arch, is to replace the bent eye of the diagonal by a straight end with screw and nut, and to have oblique holes cast in the ends of arch pieces for diagonals to pass through, on each side of the upright. [See Fig. 30B]. The diagonals may be single, or in pairs. The latter plan is preferable, as giving a better balanced action; especially in case of rail road bridges, which are subject to greater action upon diagonals. This plan obviates a degree of lateral strain upon uprights, resulting from the eye connection. In this case, the upright should have a shoulder bearing on the under side of arch castings, to sustain the thrust action.

FIG. 30B.



CIV. It will be seen that these trusses, having a width of base equal to about one-fourth of the height, will support themselves laterally, without any assistance from one another, or from other parts of the structure, wherefore the flooring, including cross beams, may be entirely of wood, and may be renewed at pleasure, without any disturbance of the iron work; a property peculiar to this kind of truss.

The original design, therefore, was to use wooden cross-beams, formed in two pieces, as by slitting an ordinary beam vertically, bolting the parts together, and boring at the ends for the uprights, so that they may be conveniently put in and removed whenever they require renewal. Diagonal braces of wood, or what is much better, tie rods of iron with an eye at each end, and a swivel or turn buckle adjustment near one end, a pair between each two consecutive beams, to which they are bolted near the uprights, are required to prevent a lateral swinging or *swaying* of the bridge; whence these members are usually called sway braces, or sway rods. In the end panels the sway rods are attached to the feet of the arch.

Upon the cross-beams, longitudinal joists are placed to support the floor plank, a thing so simple, and so generally understood as to require no further description or illustration in this place. More or less casings and finishings of wood work outside of the road way, are usually added, according to circumstances, or the taste of the builder.

CV. The rise of the arch above the chord, will admit of a considerable degree of variation. A pitch of 24 to 26 degrees for the end arch pieces, it will seldom be advisable to exceed in either direction. That pitch divided by the whole number of joints in the arch (6, for a seven panel truss), gives the angle of deflection at the joint, equal, of course, to twice the angle of bevel for ends of arch pieces.

This, however, does not produce an arch in equilibrio under a uniform load, which, as we have seen, [XXVII and LII] requires a parabolic curve, while equal deflections produce a curve between the parabola and the circular arc; not departing from the former, however, widely enough to be of material moment for ordinary spans. The effect is only the throwing of a trifle more action upon certain diagonals; for which the convenience of uniform bevels, is, perhaps, an adequate offset.

CYLINDRICAL ARCHES.

CVI. Arch Truss bridges have been constructed with cylindrical arch castings, in connection with uprights, dividing and diverging downward from the arch to the beams, thus serving to give lateral support to the arch, and preserve it in line.

This form of arch castings was supposed at one time, to possess sufficient advantage over that already before described, and which is commonly known as the *Independent Arch*, to warrant its adoption, inasmuch as it is the stronger form to withstand compressive strain. But it is also more expensive in the manufacture, to an extent perhaps sufficient to balance any practicable saving in weight of metal. Hence, the Independent Arch has acquired decidedly the greater popularity, to which its just title can scarcely be questioned. Further detail, therefore, as to the mode of constructing the cylindrical arch bridge will not be here recited.

IRON BEAMS FOR BRIDGES.

CVII. It is now over thirty years since the writer's attention was first directed to the subject of Iron Truss Bridges; a period which may be said to comprise the history of the use of iron as the sole or principal material in the main supporting members of those useful structures.

At that time, there was one *Iron Truss Bridge* in use in the state of New York, and only one, to the writer's knowledge, either in this state, or in the world, though the fact may be otherwise.

That bridge, though possessing merit as the result of a first effort, did not prove a complete success, having failed, and, being rebuilt, failed a second time, many years ago; so that, at the present time, a certain Iron Truss Bridge built by the author of this work in 1841 and 42, upon the Arch Truss plan, essentially as described in the last few preceding pages, is believed to be the oldest Iron *Truss* bridge in use in this country, if not in the world.

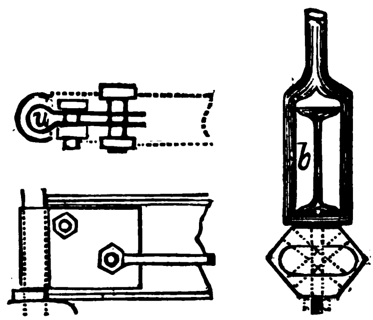
At that time, it was not thought advisable to attempt more than the construction of Iron Trusses, to be used in connection with wooden beams, joist, &c.; which latter portions could be renewed as required, with comparatively little trouble, and at much less cost, than the interest upon the extra expense of iron beams would amount to within the lifetime of wooden beams. But as the public mind seems now to have become convinced, not only of the safety and expediency of the use of iron for the *trusses*, but also for the *beams* of bridges, it becomes a question of interest to determine the best manner of constructing and inserting such beams.

CVIII. Four general plans of iron beams have been used successfully; namely, the cast iron web and flange beam, the wrought iron skeleton, the composite (wrought and cast iron), and the solid wrougth iron rolled web and flange, or I beam. These may all be used with good results, in particular cases, and under modifications adapted to respective circumstances.

For general use, however, I regard the solid rolled I beam as entitled to a decided preference; and, without discussing relative merits in this place, I propose simply, at this time, to suggest plans for adapting the last named beam to the Whipple Arch Truss, thus making the plan about all that can be hoped to be attained, as a cheap, substantial and durable iron bridge for general use, for spans varying from 40 to, — perhaps 125 feet.

For bridges 16 to 18 feet wide in the clear, and panels ten or eleven feet long, a 9 inch beam, weighing 30lbs. to the foot, is in good proportion; and when side walks are not required, the beams may be cut with square ends, just long enough to go between the uprights of opposite trusses, and provided with a fixture at each end, formed of a plate of iron about ⅝″ thick, 7″ wide, and about 2′ long, bent in the form of a jews-harp bow. The loop, or bow (*u*, Fig. 31), is to encir-

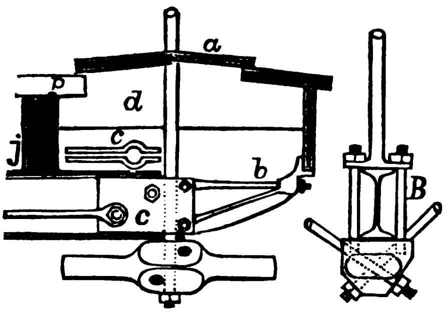
FIG. 31.



cle the upright, and the straight sides, to receive the vertical web of the beam between them, and to be fastened thereto, by two bolts and nuts. One of these should be 1¼″ in diameter, and long enough to receive the eye of a lateral diagonal tie, or sway rod (to prevent swaying or swinging), under both head and nut, and placed about 5½″ from end of beam, and 2½″ above lower edge of plate. The other bolt may be 1⅛″ or 1¼″, and placed with its centre 1½″ from end of beam, and from upper edge of plate. The thread of the screw should not run into the plate, even if a washer be required in order to fetch the work together.

A convenient modification of this fixture is to have it made in two pieces with two $\frac{5}{8}''$ bolts outside of the upright, as seen at *c*, Fig. 32. This affords a con-

FIG. 32.



venient means of attaching a light bracket (*b*), to sustain face plank and coping (*a*), over the chords, such as are commonly used in this kind of bridge. It also enables iron beams to be inserted in bridges originally built with wooden beams.

The connecting block in this case, should have an elevated ring around the upright, for the eye of the fixture to bear upon, to keep the beam from bearing altogether upon the inside of the upright, and producing unequal strain.

CIX. Another suggestion is, to form a stirrup in the upright just above the connecting block for the beam to pass through and rest in: as seen at *b*, Fig. 31. This will admit of projecting beams to support side walks.

The stirrup may be formed of iron 1″ by 2″ or $2\frac{1}{4}''$, according to the character of bridge. The iron should be upset, so as to give sufficient width and strength at the bottom of the stirrup to allow a 1¼″ stem to be screwed in, to pass through and support the connecting block. This stem may extend above the bottom of the stirrup, about ⅝″, a hole being made in the under side of the beam to receive that projection. The thread of the projecting part of the screw, which enters the beam, should be turned or chipped off. This plan may be used in bridges either with or without side walks.

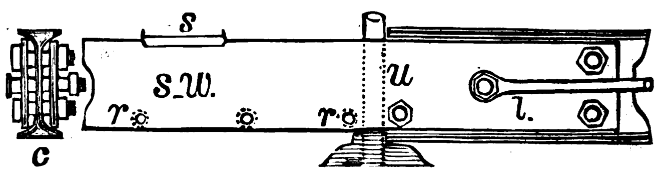
Again, the upright may terminate in a flange at the top of the beam, and bolts screwed or cast in the top of the block, or running through the block with head or nut below, one on each side of the beam, and connecting with the flange of the upright, as shown at B, Fig. 32.

In the case of double uprights, the beam being cut to go between the inner branches, the fixture plates should lap about 20″ upon the beam, and extend so as to clasp both branches of the upright.

CX. To introduce the solid wrought beam in bridges with sidewalks originally constructed for wooden beams, the following plan is suggested.

Let the beam be cut, say 1″ shorter than the space between opposite uprights. Then, take for each end

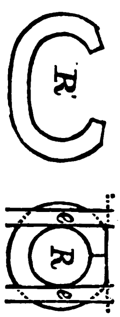
FIG. 33.



of the beam, two plates ½″ thick and 7½″ wide, or, wide enough to fill the space between the flanges of the beam at 1″ from the centre, so that one being placed on each side, they will be kept far enough apart to admit the upright between them. The plates should be long enough to lap 20″ upon the beam, and extend to outside of side walk. They may be bolted with two 1″ bolts near the end of the lap, and one near the end of the beam by the upright; as seen under the letter *u* in Fig. 33. A $1\frac{1}{4}''$ bolt in the centre of depth, and 7 or 8 inches from the upright, will serve both to aid in holding the plates in place, and to connect the sway rods *l*. These plates should not be cut by bolt or rivet holes in the upper part, except at considerable distance from the upright *u*.

Small bolts or rivets, *r r*, etc., should be inserted at intervals of 9 or 10 inches, near the lower edge, with thimbles to stay the extension plates apart, leaving a space equal to the diameter of the upright. In Fig. 33, *s-w* is a part of the extension for supporting side walk; *s*, a cast iron saddle weighing about 4lbs. for joist bearings, and *c*, a cross-section through the splice.

FIG. 34.



To afford a proper bearing upon the connecting block, it is proposed to use a wrought iron ring (R. Fig. 34), high enough to throw the whole weight upon the extension plates *ee*, and $\frac{3}{4}''$ to 1″ in width, except on the side next the beam proper, where it is to be clipped or drawn down to $\frac{1}{2}''$. This, however, is not an essential point. In case of bridges already erected, the ring will have to be left open as at R′, and when used, heated and closed around the upright.

CXI. The Link Chord, composed of a set of links to each panel, connected by pins or connecting blocks (the latter affording also points of attachment for verticals, diagonals, &c.), both for Arch and Trapezoidal trusses, was originally adopted by the author, as the readiest means of putting the requisite amount of chord material in a manageable form, both as it regards manufacturing the parts, and erecting the structure. This form renders the whole section available for sustaining tension, avoiding any loss in rivet or bolt holes for forming connections.

The experience of more than a quarter of a century, during which time many hundreds of bridges with link chords have been constructed, and used in almost all conceivable conditions, (in many cases, undoubtedly, the links having been but imperfectly manufactured and fitted to the connecting blocks), with a degree of success and satisfaction seldom exceeded, may reasonably be regarded as fairly establishing the efficiency and safety of this mode of construction, when proper care is used in the performance of the work.

Continued and successful usage in a multitude of instances, is regarded as a better criterion as to the reliability of a plan of construction, than a small number of isolated tests, however severe; and such usage the link chord has been subjected to.

CXII. The theoretical questions to be considered in this case, would seem to be, as to the possible deterioration of the cohesive strength of the iron, produced in forming the bends at the ends of links — the indirect, or lateral strain in those parts, resulting from imperfection of the fitting to the connecting block or pin, and, the imperfection of the weldings, both as it regards complete cohesion, and the tendency to crystallization under the welding heat, not being fully destroyed by subsequent hammering and working.

The whole process of the manufacture and refinement of iron, is based upon the principle that disconnected pieces of iron brought in contact under intense heat, but without complete fusion, and subjected to violent compression, as by hammering or rolling, will unite, and become a single piece or mass.

Every bar of refined iron found in the iron market, is composed of half a dozen or more parts, which were once separate and disconnected. Those having been "fagoted," or placed in juxtaposition, and submitted to a welding heat, and passed repeatedly between ponderous rollers, or subjected to the blows of heavy hammers, are united and drawn into bars of required sizes and forms for use.

These masses, taken from the furnace and suffered to cool without hammering or rolling, would be found more or less crystaline and brittle. But the latter operations prevent such a result, and the iron becomes more or less soft and flexible, even in a cold state.

Iron which has undergone the uniform process of rolling, is generally of uniform quality and strength throughout the whole piece; and, as far as it can be used in that state, without re-heating and re-working, it may be regarded as somewhat more reliable than when it has been forged and welded into different and more complex forms.

The high temperature required in welding, demands experience and judgment in determining the proper time to "strike," that is, when the metal is hot enough to adhere firmly, but not overheated to burning. Moreover, though the hammering required to bring the parts together and reduce them to proper form and size, may prevent crystalization immediately at the welded point, still on either side are portions which may have been heated so as to change the arrangement of particles, and not subjected to sufficient hammering to counteract the deteriorating tendency. Hence, a break is more liable to take place a little on one side, than immediately through the welded part.

To obviate this liability, the parts to be welded should be enlarged by upsetting several inches from the end, so as to admit of re-drawing under the hammer a little beyond where the intense heat has reached.

But theory aside for the moment, although the avoidance of welding in work to be exposed to great stress is desirable, it is nevertheless a fact established by large experience, that welded parts will bear as great a strain as takes place in well proportioned bridge work, with as much certainty as ever has been realized in any department of the means of locomotion.

Danger lurks everywhere at all times. In railroad travel, boilers burst, rails break, wheels and axles break, etc., etc., but the failure of a weld in bridge work is rare indeed, and very few authenticated cases can be referred to.

I would, however, prefer a weld in the straight part rather than in the end of a link, unless made with an excess of section around the bend. Whether a bend around a pin of 1½ or 2 times the diameter of the link iron is more liable to break than the straight sides of the link, I can refer to no reliable authority to determine. The longitudinal strain is no greater in the bended, than in the straight parts, if well fitted to the pin. But of course, it can not be expected to have a fit so close as to ensure a firm pressure quite round the semi-circle. Hence the bearing is mainly on the back side of the pin, until by a yielding to compression, and by a slight bending of the link end, a pressure is produced all around.

This slight bending, good iron will undergo without having its strength impaired, when in its normal condition. But this condition is disturbed in the process of bending, the outside portion being extended, and the inside compressed, whereby the stiffness of the part is increased. In the outside portion the power of resisting extension is increased, while that of the inside portion is possibly diminished; and, whether the aggregate resistance to extension is increased or diminished, experiment alone can determine; and, undoubtedly, the more soft and flexible the iron, the better can it adapt itself to a bearing upon the pin. Hence, it should be allowed to cool gradually from a full red heat, after the shaping is finished.

Hence, also, the necessity of extra section in welded ends, which, being less flexible, must obtain bearing surface by compression and yielding of contiguous parts, rather than by bending, and consequently, must undergo greater transverse strain in the end of the link.

CXIII. A link formed of wire $\frac{1}{8}''$ in diameter, formed to a pin $\frac{7}{32}''$ in diameter at one end, and brazed with a long lap at the other, suffered a permanent stretch in the straight part, of one per C. of its length, with no apparent injury at the ends. Other analogous experiments have shown similar results, namely, that the straight portions will yield before the bended portions.

Now the same degree of disturbance in the metal takes place in a small, as in a large rod, bent to a curve whose radius has the same ratio to the diameter of the rod. Hence, it is difficult to avoid the conclusion that rods of soft and flexible iron, such as ought to be used for tension members in bridge work, bent to a proper fit upon connecting pins of diameter about twice that of the rods, and formed into links by welding in the straight parts, are quite safe under any stress within the limits adopted in bridge work.

But it seems to be more convenient to form the weld at one end of the link, if not both, and such has been the usual practice; and, as before remarked, if a surplus of metal section quite around the bend be secured, and the work well performed, this plan can scarcely be regarded as faulty, especially, in view of the long, varied, and successful usage of such vast numbers of links made in this manner.

Now, although the link chord is very simple, efficient, and convenient to make and manage, there are available alternative devices, some of which will be here described.

THE EYE-BAR CHORD.

CXIV. This is composed of two or more single rods, of oblong, square, or round section to each panel; connected by cylindrical pins passing through strong eyes at each end of the chord bars.

This plan until recently, has involved quite as much welding as the link chord; the eyes having been formed in separate pieces, and welded to the body of the rod. But within a few years a process has been devised by the Phœnix Iron Co., of Pennsylvania, for upsetting and forming eyes upon rolled bars. A mold or die gives the desired form and size to the head, and aside from the fact that a violent disturbance of the normal condition of the iron is produced in the vicinity of the head, there can be no question as to the excellence of the work produced; and it is undoubtedly, perfectly reliable, under any stress to which it is admissible to expose the material in bridge work.

Figure 30B represents the joint of an eye-plate chord at *c*, adapted to the arch truss. Upright and diagonals have each an eye to receive the connecting pin at the lower end. The upright has a washer above the eye to form a beam seat above the eyes of the chord plates. Perhaps the washer should be in the form of a saddle or stool, with downward projections bearing upon the pin outside of the diagonals; or, perhaps inside, in case the diagonals be in pairs, as before suggested. [CIII.]

SIZE OF CONNECTING PIN.

CXV. Considering the average bearing upon the pin, to be at the centre of thickness of the eye, or link end, as the case may be, the thickness of the eye indicates the leverage upon which opposite links act, when side by side upon either end of the pin. Estimating the strength of the pin, then, at 4,500lbs. to the square inch of section, with a leverage equal to the diameter of the pin [see XCVIII,] we obtain the proper diameter of the pin as follows:

Let a=area of section in link or chord bar.

t=thickness of eye=leverage of action.

x=diameter of pin, in inches.

Then, $.7854x^2$=area of pin section; and this multiplied by $4,500\frac{x}{t}, = \frac{3534.30}{t}x^3$, equal to the resisting power of the pin; while $15,000a$=the power of the link; and putting these two expressions equal to one another, and deducing the value of x, we have the required diameter of the pin, $\sqrt[3]{4.244a.t}$ inches,$=x$.

CXVI. If a=4 square inches, and t=1.5 inches, then $a.t = 6$, and $x = \sqrt[3]{6\times4.244} = 2.94$ inches. This diameter of pin is required to withstand the action of the chord alone, which is the only stress upon the pin when the chord is at maximum tension. But when the diagonals running in the same direction horizontally, with the inside links, are brought into action, they act in conjunction with the links in producing stress on the pin.

Now, the greatest stress upon bn, Fig. 11 [see XXXIV] occurs when the point b alone is loaded, and the links ab sustain $\frac{6}{7}$ of their maximum stress from movable load, and bn sustains $5w''$, giving a horizontal pull of about $6.5w''$, the amount varying with depth of truss. Again, besides the $6w''$ bearing at the point a, in virtue of the movable weight (w), at b, we have $3w'$ due to weight of structure, also bearing at a; and assuming $3w'$ to be equal to $1w$, or $7w''$ the whole pressure at a, equals $13w''$, when the horizontal pull of bn equals $6.5w''$.

The tension of ab, in the usual proportion of arch trusses, equals about $2\frac{1}{4}$ times the bearing at a, whence the stress of ab with the point b alone under load, equals $13w''\times2.25, = 29.25w''$. Deducting from this, $6.5w''$ for horizontal pull of bn, it leaves $22.75w''$ = stress of bc. Then, assuming the diagonal to act in the centre of the pin, and the length of pin between centres of bearing of outside links to be 27″, we find the stress at the centre of the pin, by taking the moments with respect to the centre, of the action of the two links at either end of the pin. The difference of these moments, the forces being opposite, is the moment of the force producing stress at the centre of the pin; in other words, it is the force acting transversely upon the pin, at a leverage of 1 inch, the inch being our unit of length.

We found the pull of $ab = 29.25w''$, or $14.625w''$ at each end of the pin, which multiplied by distance from centre (13.5″) gives a moment $= 197.4375w''$, while for bc, the moment is $\frac{1}{2}\times22.75\times12'' = 136.50w''$; and the difference $= 60.9375w''$ = stress in centre of pin, upon a leverage of 1″.

Assigning such a value to w'' as will give the assumed stress of 15,000lbs. to the inch upon ab with the truss fully loaded, with a bearing at a, of $21w''$ for movable, and $7w''$ ($= 3w'$), of weight of structure, we find a stress of $28\times2\frac{1}{4}$ ($=$ 63)$w'' = 8\times15,0000$lbs. $= 120,000$lbs; whence $w'' = 1,905$lbs. which, being substituted in the above amount of $60.9375w''$ gives the stress in pounds at the centre of the pin, on a leverage of 1″, equal to 116,086lbs.

We have seen [XCVIII] that the resisting power of a projecting pin equals $4,500\frac{\text{AD}}{\text{L}}$, which in this case, equals 4,500AD (L being $= 1$), equal to $4,500\times.7854x^3$. Then, making this expression $= 116,088$lbs. we have $x = 3.2''$; being 0.26″ larger than is required to withstand the action of chord alone, at its maximum stress, as already shown [CXVI.].

By similar process we find very nearly the same results with respect to the shorter pins toward the centre of the truss. For, although the maximum action of diagonals takes place under greater stress upon chords, the difference is balanced by diminution in length of pins toward the centre of the truss.

Should this mode of connection be adopted, the preceding illustrations and examples, it is hoped, will enable the proper proportions of connecting pins to be determined for trusses of whatever dimensions.

A RIVETED PLATE-CHORD.

CXVII. May be formed of flat plates as long as may be conveniently managed, connected by splicing plates of a little more than half the thickness of the chord plates, one upon each side, riveted or bolted with such a distribution of rivets, &c., as may not weaken the plates by more than the width of one rivet hole.

The area of rivet section should be at least $\frac{5}{8}$ to $\frac{3}{4}$ as great as the net section of the chord plate, on each side of the joint; and, *go*, Fig. 34½ denoting the splicing plate, the distance *cd*, from the joint to the centre of the first rivet hole, should be at least twice the diameter of the rivet (depending somewhat upon the size of rivet and thickness of plate, as well as the soundness of grain in the iron). The succeeding rivets, *a*, *e*, *f*, &c., should be placed alternately on opposite sides of the centre, so that the oblique distance *ac* (= O), may equal the transverse distance (= T), + the diameter of whole (= H). Then, representing the longitudinal distance *bc*, by L, we have $T+H = O$, and $(T+A)^2 = O^2, = T^2+L^2 = T^2+2TH+H^2$; whence L $= \sqrt{2T.H + H^2}$.

If the plates be 6″ wide, and T = 3½″ (which is regarded as in good proportion, the above formula gives L = 2½″ very nearly, for a ¾″ hole. Then, 5″ being allowed for the space *ce*, and 2″ each for *cd* and *eg*, the splice plates would have a length of 20½″, and $\frac{7}{8}$ of the whole section of chord plates would be available for tension; since an oblique section through two holes, would quite equal a direct transverse section through one hole.

The amount of rivet section above given is estimated upon the assumption that each rivet must be sheared off in two places; and that it will resist, those shearings, each, with about $\frac{3}{4}$ of the force required to pull the rivet asunder by direct longitudinal strain.

It is obvious that the two rivets *e* and *f*, Fig. 34$\frac{1}{2}$, sustaining a portion of the stress of the chord plate, relieve in the same degree the stress upon the portion between those rivets and the joint, or end of the plate; whence it is not necessary to preserve the same section in the portion thus relieved, as in other portions of the plate. Therefore the rivets *a* and *c*, nearer to the joint, may be larger than *e* and *f*, when the section of plates requires more rivet section; provided always, that the least net section of splice plates, have as great an area as the chord plate has through only one of the smallest rivets. For instance, four $\frac{3}{4}''$ rivets are sufficient for plates $6'' \times \frac{1}{2}''$. But plates $6'' \times \frac{5}{8}''$ require more rivet section — say $\frac{3}{4}''$ for *e* and *f*, and $\frac{7}{8}''$ for *a* and *c*; while, the same for the former and 1″ rivets for the latter, give about the required section for plates $6'' \times \frac{3}{4}''$. This leaves in each case, the same proportion of net available section of plates.

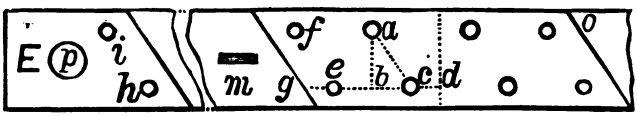
Moreover, if rivets *a* and *c* be placed opposite to each other, and *f* be removed to *a*, the rivets being $\frac{3}{4}''$ and 1″ respectively. Then, the smaller rivets sustaining over $\frac{1}{3}$ of the stress, while the others sustain less than $\frac{2}{3}$, the latter may cut off $\frac{1}{3}$ of the net section (which is, in this case $\frac{3}{4}''$ less than the whole width of plate), and still leave enough to sustain more than their own legitimate share of the stress.

This may be done by one rivet or two, placed opposite *c*; and thus the length of splice plates may be shortened to 15$\frac{1}{2}$ inches, instead of 20$\frac{1}{2}$, as represented in the diagram. But, as in this case, the long plate has a net width of 5$\frac{1}{4}''$ and the splice plates, only 4″

the latter require 37½ per C. more thickness than the former, so as to nearly or quite balance the saving in length.

As to the proportions of parts, in this kind of work, I would suggest that the thickness of plates be from ⅛th to $\frac{1}{12}$th of their width, and the diameter of rivets, from 1 to 1½ times the thickness of plates. If plates be very wide and thin, they may be liable to be strained unevenly, and if very narrow, an unnecessary proportion of section is lost in rivet holes.

FIG. 34½.



CXVIII. The end connections of plate chords of this kind, may be effected by riveting on side plates at the ends, as seen at E, Fig. 34½, so as to give a thickness that will allow about ⅓ of the width of plate to be cut away by a hole for the connecting pin P, either round or oblong with square ends for adjusting keys or wedges.

Or, the side plates may be omitted, and two key-holes made in the middle of the plate, one for a key having a thickness equal to the diameter of the smaller rivets, and far enough from the end to admit of another hole nigher to the end, with about 2″ between the holes. This may, if necessary, have twice the width of the other hole, and should leave at least twice the width of hole, between hole and end.

The width of the wider hole, + twice that of the other, should equal about half the width of the plate; and the keys should be driven to an equal bearing before the work be subjected to use.

The connecting blocks used with this chord, sustaining only the horizontal action of diagonals, may be considerably lighter than those used with the links, especially in arch trusses. In order to transfer the horizontal action of diagonals to the chords, mortises may be made in the plates, as seen at *m* Fig. 31, not wider than the smallest rivets used in splicing, to receive tenons of wrought iron cast in the block.

As to the merits of the *riveted plate*, as compared with the *link* chord, it may be assumed that two splices are sufficient for any truss not exceeding 100′ long, and that the weight of splicing plates and rivets will equal 4 or 5 feet extra length of plates, say 6 per cent upon a chord 80′ long. To this we have to add about 14 per cent for extra section to compensate for rivet holes, making 20 per cent of iron lost in forming connections.

Links require about half as much extra material, to be taken up in bends, lappings, and enlargement of section at the ends; showing about 10 per cent less iron for the link, than for the plate chord. This would amount to about 400lbs. for two trusses of 80′, with links of $1\frac{1}{2}''$ round iron. But this may be nearly or quite balanced by 500 or 600lbs. of castings, which may be saved in weight of connecting blocks.

The economy of material being so nearly equal in the two chords, their relative merits must depend mostly upon the comparative cost of manufacture, and the relative efficiency of the chords in use. It is deemed far from improbable that the riveted plate chord might be found, on fair and thorough trial, to be worthy of extensive use in arch trusses, in place of the link chord. The fact that in the plate chord, the iron is used in its original condition, as it comes from the rollers, is certainly favorable.

BRIDGES WITH PARALLEL CHORDS.

CXIX. These may be constructed with or without vertical members, and in form, either rectangular, with vertical end posts, or trapezoidal, having inclined end members, or king braces, as exhibited in Figs. 12, 13, 18 and 19.

TRAPEZOIDAL TRUSS BRIDGE, WITH TENSION DIAGONALS AND COMPRESSION VERTICALS.

For short spans, less than 70 or 80 feet long, the simple cancel, as in Fig. 12, will generally be used, with trusses too low to admit of connection between upper chords, except in case of deck bridges.

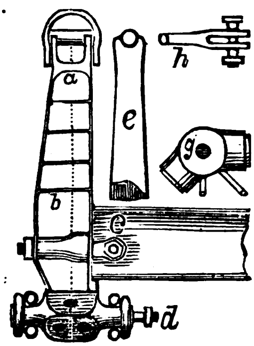
The same plan of lower chords composed of links and cast iron connecting blocks, may be used, as already described for the arch truss. The connecting blocks are shorter, and may be cast in connection with the upright, or the latter may be in a separate piece. In the latter case, the block should have a suitable seat to receive the upright, and keep it in place.

As the upper chord depends upon the stiffness of the beam and upright for lateral support to keep it in line, the upright should be firmly attached to the beam, and at right angles therewith.

There is no means of estimating exactly the transverse force which the chord may exert upon the upright. But if the ends of chord segments be properly squared and fitted, the lateral tendency will be quite small. It is recommended, that each upright have a transverse strength sufficient to withstand a force of 1,000lbs. acting at the upper chord; that it have a web and flange form of section, with a width of web at the connection with the beam, not *less* than $\frac{1}{15}$ of the distance of upper chord from the beam.

Fig. 35 will serve to illustrate the modes of connection for most of the members of a bridge of the kind under consideration. That part of the upright between *a* and *b*, is contracted in length. Otherwise, the parts are represented in nearly correct proportions. At *c*, is represented the connection of the upright with the end of the beam, by means of a double eye and bolt, as shown at *h*. This receives the web of the beam, to which it is secured by the transverse bolt, which should be long enough to receive the eye of a sway rod under both head and nut. The stem of this fixture extends through the upright at its widest part (whence it may taper in both directions), and is secured by a nut upon a screw of about 1½″ in diameter. The beam should rest with its lower flange upon a small projection cast upon the upright, and not hang upon the connecting fixture.

FIG. 35.



If so preferred, the sway rods may be connected by a screw and nut cast in the end of the connecting block, as seen at *d*. This plan has been used, but the connection by the bolt at *c* is deemed preferable.

The outer and inner flanges of the upright at the top, being increased to nearly an inch in thickness, according to size of bridge, and extending 3 or 4 inches above the web, terminate in semicircular concaves to receive the pin connecting the diagonals with the upper chord. A full view of the flange at the top of the upright, with the pin resting in the concave, is shown at *e*.

A heavy cross-bar from flange to flange at *a*, and light cross-bars at intervals of 16 to 18 inches from *a* to *b*, serve to support the flanges, and stiffen the piece.

The diagonals are formed with eyes to receive the connecting pin at the upper end, and screws and nuts to connect with the block at the lower chord, in the same manner as in the arch truss.

The main diagonals, those inclining outward from the centre of the truss, should be in pairs, and in size, proportioned to the stress they are liable to, as determined by the process fully described in sections XXXIX, &c.

The links acting in conjunction, horizontally, with the main diagonals, should go on next the end of the connecting block, as that arrangement obviously produces less stress upon the block.

The upper chord, usually formed of hollow cylinders, has openings in the underside at the joints, for uprights and diagonals to enter, where they connect by means of the transverse pin already mentioned. The cylinders should have an extra thickness for 3 or 4 inches from the ends, and a strong collar around the opening, to restore the loss of strength occasioned by the opening; and the ends should be squared in a lathe, to secure a perfect joint and a straight chord.

If it be required to give a cambre to the truss, the ends of cylinders should be slighly beveled at the ends, making the under side a trifle shorter. This is easily effected by throwing the end opposite the one being turned, out of centre more or less, according to the cambre required. An 8 panel truss requires an excentricity equal to $\frac{1}{16}$ of the requiredr ise in the centre of the truss. For any even number of panels, make a series of odd numbers, 1, 3, 5, &c., to a number of terms equal to half the number of panels; add the terms of the series, and divide the required cambre by the sum, and the quotient equals the required excentricity to give the proper bevel.

For an odd number of panels, take as many even numbers 2, 4, 6, &c., as equal half the greatest even number of panels; add the terms and divide as before for the excentricity. For illustration, for 8 panels, the four odd numbers $1+3+5+7=16$, whence the excentricity should be $\frac{1}{16}$ of the cambre, as above stated. For a 7 panel truss the three even numbers $2+4+6=12$. Hence the excentricity should be $\frac{1}{12}$ of the cambre. The reason for this rule will be obvious without more particular demonstration.

At the obtuse angles of the truss, a hollow elbow is inserted (*g*, Fig. 35), reaching about 10 inches each way from the angular point, at the centre of the connecting pin, with an opening in the under side for upright and diagonals to enter, where they are fastened by a pin or bolt, as at the intermediate joints; the cylinders meeting the elbow, being shortened by as much as the elbow extends from the angle, either way.

The vertical member connecting with the elbow, is exposed to tension only, sustaining a weight equal to the gross panel load of the truss. It may be composed of two wrought iron suspension rods, united in a single eye at the top, and diverging downward to a connection with the beam and connecting block; or, it may be of cast iron, like the intermediates, with wrought iron eye plates, in place of the cast iron flanges with concaves as seen at *e*. These should be fastened by efficient means to the cast iron part of the upright; which latter should have a cross-section nowhere less than one square inch to each 2,000lbs. of the gross panel load. A complete wrought iron connection from beam to elbow, however, is to be preferred.

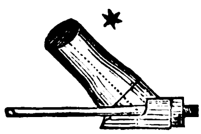
The thickness of web and flanges of the uprights, should be from ⅜ to ½ inch, and the cross-section of upper chord cylinders should be about 20 per C. greater than that of the portion of bottom chord forming the opposite side of the oblique parallelogram included between consecutive main diagonals and included sections of chords; as *d e k l*, Fig. 12.

The upright should be so formed as to bring the centres of upper and lower chords in the same vertical plane.

Sway rods in this class of bridges, should be about ¾″ in diameter, with a turn buckle near one end for adjustment, and an eye at each end, for connection with the bolt at *c*. The screw working in the turn buckle is cut upon the short piece, which should be ¼″ larger in diameter than the long piece which has no screw upon it.

The lower chords, king braces, and sway rods of the endmost panels, connect with cast iron foot pieces upon the abutments, as represented in Fig. 36. The portion of lower chord in the end panels, usually consists of single rods, instead of links, with an oblong eye at one end to receive the connecting block, and a screw and nut for connection with the foot piece (Fig. 36), at the other end.

FIG. 36.



This plan of construction will generally yield precedence to the Arch Truss plan, for short spans, except for deck bridges upon rail roads, in which case the structure will be secured laterally, by × ties, or sway rods between beams, and between king braces at the ends; no × bracing being required between lower chords.

Low trusses constructed in the manner above described, have been used satisfactorily for supporting the outside of wide side walks; answering the purposes of a protection railing at the same time. For this purpose, the uprights are only 5 or 6 feet long, so as to bring the upper chord about 4 feet above the flooring. The first instance of this kind was in the case of the canal bridge on Genesee street in Utica, built 18 or 20 years ago, and repeatedly copied since.

CXX. Bridges from 80 to 100 feet for common roads may be constructed with single canceled trusses, 13 to 14 feet high; in which case the panels will require to be wide (horizontally) in order to avoid an inclination of diagonals too steep for good economy.

But for railroad purposes, the trusses require a depth of about 20 feet to afford sufficient head room under the top connections, unless the beams be suspended below the bottom chords. Hence, the

Double Cancelated Truss

should be adopted for "through bridges" of spans exceeding 70 or 80 feet.

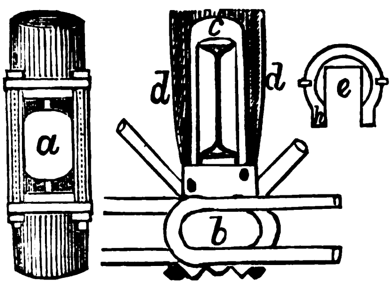
Figures 18 and 20 exhibit in outline, the general character of the double cancelated trapezoidal truss bridge; and, it is only necessary in this place, to describe feasible modes of forming and connecting the various members; which may be done essentially as described in the preceding section, with such modifications as follow.

Cast Iron Uprights

are composed of two or more pieces. When of two pieces, they may be connected by flanges and bolts at the centre, where they should have a diameter of about $\frac{1}{30}$ of the length, and a cross-section determined by the maximum stress, and the power of resistance of the material, as indicated in the table [XCIII.]

The upright may taper from the centre to either end to a diameter of 5 to 6 inches, internally. The lower end is to stand upon a properly formed seat (*h* Fig. 37), upon the connecting block of the lower chord, and may have an opening at the bottom, upon the inner side, where the beam may enter and rest upon a seat (*e*), inside of the upright, upon the connecting block. The strength destroyed by this cutting the post should be restored by additional metal in a band or collar (*c*, Fig. 37), around the opening, and, if necessary, by the wing flanges *d d*, extending 6 or 8 inches above the opening. To avoid too much cutting of the post, the flanges of the beams may be reduced to 3 or $3\frac{1}{2}$ inches in width. The post and beam seat upon the connecting block may be elevated 3 or 4 inches above the links, as may be required, so as to allow sway rods to pass through with simple screws and nuts for adjustment; thus dispensing with turnbuckles.

FIG. 37.



Holes should be cast in the central part of the post, for diagonals to pass obliquely through. Or, what is perhaps better, the connecting bolts may be lengthened so as to permit the insertion of an open box, or frame, between the flanges, as seen at *a*, Fig. 37. This intermediate piece should be so constructed as to close the ends of the hollow pieces meeting it, and prevent the water from getting inside.

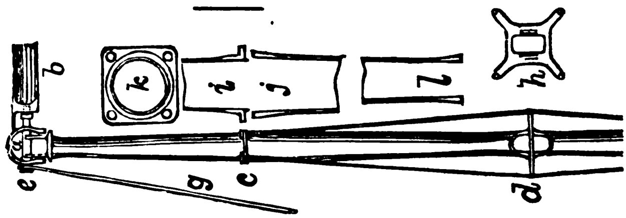
The top end of the upright is forked, with concaves for the connecting pin to rest in, as described in the last section, and as seen at *a*, Fig. 38. The cap piece of the post may be cast separate, or in connection with the upper half of the column. Both plans have been satisfactorily used. All joints, when practicable, should be accurately fitted by turning or planing.

This plan of a cast iron upright, composed of two principal parts, with or without the centre piece, is perhaps as good as any for general use; the principal disadvantage being the difficulty of giving a sufficient diameter in the middle for stiffness, without two much reducing the thickness of metal, or increasing the amount of cross-section beyond the proper theoretical proportions.

To obviate this difficulty, the device adopted in the original model of the Trapezoidal bridge, was that of using truss-rods, or stiffening rods, to secure the post against lateral deflection, after the mannner shown in Fig. 38.

In the case of using stiffening rods for the uprights, it may be recommended to form each half of the column in two pieces, somewhat in the manner above described for the whole one, without stiffeners; making the piece forming the end portion about $\frac{1}{4}$th shorter than the other, with a strong flange at the larger end, to afford attachments for the stiffening rods.

FIG. 38.



In Fig. 38, *a c d* exhibits the upper half of the upright; *h*, the stretcher at *d*; *k*, the flange at *c* (enlarged), and *i*, *j*, enlarged sections of the two ends forming the joint at *c*. The piece running toward the centre has no flange at *c*, but has an increase of thickness for a short distance from the joint, as shown at *j*, and a diameter about ½″ larger than the abutting piece, which latter has a small burr entering the former ¼″ or ½″ to keep the ends in place. At *d*, each of the pieces meeting at that point, has a bi-furcation, so as to form an opening for diagonals to pass through, at the same time passing through the stretcher *h*.

The lower half of the upright is the same as the upper, except the end, which is squared to fit a flat bearing upon the connecting block. An enlarged vertical section of the lower end is shown at *l*, Fig. 38. See also Fig. 37, where is shown the arrangement for the beam to enter the opening in the lower part of the upright, as described a few pages back.

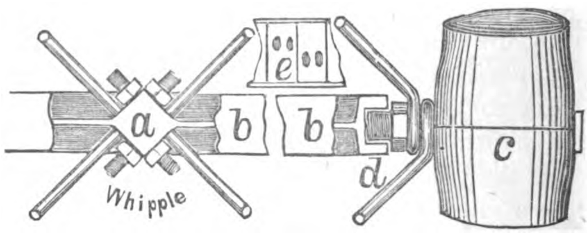
Floor beams of wood or iron may be suspended below the chords by bolts passing down through the connecting blocks, or, wooden beams may be in two parts, resting upon flanges cast upon the upright about 3″ above the lower end; the beam timbers being hollowed out upon the insides, so as to embrace the upright, in part, leaving a space of 2 or 3 inches between, and secured in place by bolts and separating blocks.

The mode of inserting iron beams by means of openings in the uprights, has already been explained. Lateral × ties, or sway-rods may be inserted by bolting to the beams (Figs. 31 and 33), attaching to the inner end of connecting blocks, as at *d*, Fig. 35, or by passing through the block between the links and the post and beam seat, in the manner referred to two pages back.

Diagonal ties of wrought iron, and transverse struts of wrought or cast iron, are also required between the upper chords, to keep them in line. Cast iron cross-struts may have the web and flange form of section, with shallow sockets at the ends, to admit the connecting bolts at the upper chord to enter, after passing through eyes upon the upper sway-rods and nuts to hold them in place. These sway-rods require turnbuckles for adjustment, when they extend across one panel only. But if the bridge be wide between trusses, the rod may extend only from the end of one cross-strut to the centre of the next, where it may pass through the strut, and receive a nut on the end. Thus, four rods meeting at the centre of the strut, each having its appropriate hole to pass through, all as near to one another as practicable, with sufficient space for nuts to turn (see *a* and *e*, Fig. 39), it forms a convenient arrangement for adjusting the rods to a proper tension, at the same time affording lateral steadiness to the cross-strut.

The end-most struts, however, should have no rods connecting with them in the centre, as they can have no antagonist rods on the opposite sides to prevent the springing of the struts. The end panels should have two full diagonals with turn-buckles, and two half diagonals connecting with the centre of next strut.

FIG. 39.



In Fig. 39, *a* shows the middle of the cross-strut, with the upper flange removed; *c*, a joint of the upper chord, where the connecting bolt passes transversely, receiving eyes of sway-rods, and nut, and entering the end of the strut at *d*; the upper part of the strut being removed, down to the socket. The bolt bears upon a slight swell in the bottom of the socket, to ensure a central thrust: (see also, *b*, Fig. 38). At *e* is presented a side view of the centre of the strut, showing the arrangement of the holes.

A similar device has been used with good effect for giving lateral support to posts or thrust uprights, of the web and flange form, so proportioned as to have greater stiffness transversely than lengthwise of the truss.

It has been demonstrated that the weight sustained by these posts, increases toward the ends of the truss, while the tension of counter diagonals runs out to nothing, a little way from the centre of the truss. For instance, 4/6 Fig. 18, sustains $6w''—1\frac{1}{2}w'$, which is a negative quantity whenever w is less than $4w'$, that is, when the greatest movable load is less than four times the weight of structure, as is usually the case. But instead of dispensing with that member, and other counters on the left, they may be made in two pieces each, of $\frac{5}{8}''$ or $\frac{3}{4}''$ iron, connecting with the upright at the crossing by screws and nuts, in the manner above described; thus preventing the uprights from deflecting lengthwise of the truss, where the greatest weights act upon them, and where otherwise, they would require to be heavier.

GENERAL TRANSVERSE SUPPORT.

CXXI. The system of cross-struts and diagonal ties serves to preserve the upper chords in line, but does not prevent the whole structure from swaying bodily to the right or left; a result which would be fatal to the structure.

In the arch truss Fig. 27, the width of base at the bearings upon abutments, resulting from the peculiar form of the arch, affords the required stability in this respect.

In case of the trapezoidal truss, when high, various devices have been resorted to for producing the same results. For deck bridges, cross tying between king braces at the ends, is an easy and efficient means of accomplishing the object. For through bridges, guys from the connecting bolt at the elbow of the obtuse angle, anchored in the abutment, may be employed. But this requires extra length of abutments and piers, and the effects of change of temperature, are, to tighten and slacken the guys, so as to impair their efficiency.

To obviate the latter objection, double acting guys (acting by thrust and tension), applied at one side only of the bridge, have been employed; the effect of temperature being only to very slightly sway the bridge laterally, but not so as to be detrimental to stability. This also, requires 5 or 6 feet more length of pier, than what is necessary to bear the vertical pressure.

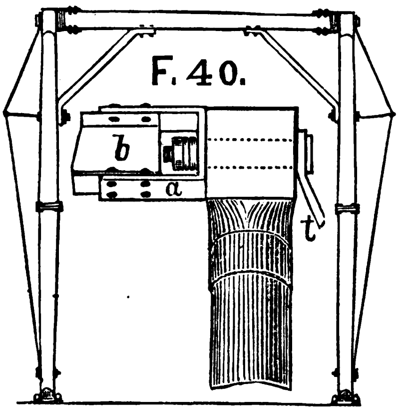
Again, the king braces have been made with two branches diverging from the elbow to a base of 2 or 3 feet in width, according to height of truss. This plan has been used in a large number of bridges, with satisfactory results. But it contracts to a small degree, the available width of bridge; not, however, so as to produce material inconvenience.

Another device is, the introduction of two or more long beams, extending 5 or 6 feet outside of the trusses, say at the first thrust uprights from the ends (as over Figs. 3½, 3½, Fig. 18), with guys extending from the connecting bolt at the upper chord, to the ends of said long beams (see *g* Fig. 38).

Arches may also be introduced at the ends of the bridge, attached to the king braces, say a quarter of the way down from the top, and with the connecting bolt at the elbow. These may be made with a full, or an open-work web, and flanges of 2¼ or 2½ inch angle iron upon both sides of the web, at the top, and around the arch, and either angle iron or plain flat bars, along the sides next the king braces.

A web of $\frac{3}{16}''$ plates placed edge to edge, and battened upon both sides with plates of the same about 4″ wide, riveted alternately on each side of the seam, with angle iron, etc., as above, riveted once in 6″, forms a stiff and substantial arch for the purpose under consideration, such as have been used effectively in a bridge of 160ft. span.

Moreover, simple arch braces extending from the king brace to a stiff and substantial cross beam from elbow to elbow (see Fig. 40), will effect nearly the same result as the arch. In both cases, a considerable degree of lateral stress is liable to be thrown upon the king braces, which accordingly should be strong, or supported by truss rods, and struts opposite the feet of the arch or braces.



Whether the truss rods be used or not, it is advisable that the connection with the king brace be made by means of a bolt running through the whole diameter of the king brace, with nut or shoulder bearing externally and internally upon both sides, to counteract any tendency to collapse.

Fig. 40 presents an end view of a bridge, showing arch braces, with truss rods to sustain the thrust of arch braces against king braces. The internal figure gives an enlarged view of the connection at the elbow. A strap *a* (about $\frac{3}{4}'' \times 5''$), bent twice at right angles, is riveted or bolted to the flanges of an I beam (about 9″ deep), leaving a space of about 4 inches from the end of the I beam, for eyes of two sway rods and a nut upon the large connecting bolt. This bolt in large bridges being from 3 to 4 inches in diameter through the elbow, is reduced to 2 or $2\frac{1}{4}$ inches in the part projecting through the strap above mentioned, and the eyes of sway rods.

The truss rods may not be necessary (with substantial king braces), for spans not exceeding 150 feet. But they will add to the security, in all cases of railroad bridges having cast iron king braces. These members being over twice the length of the cylinders in the upper chord, are usually cast in two pieces, and connected by bolts and flanges in the middle, where they have a diameter of about $\frac{1}{20}$ of the length of brace, and taper to the size of the upper chord at the ends.

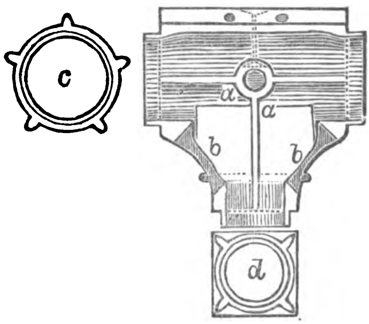
CXXII. WROUGHT IRON THRUST MEMBERS.

The trapezoidal bridge, as described in detail in the preceding section, and as originally intended, is a wrought and cast iron bridge. But it will readily be seen that with slight modification of detail, it is easily adapted to the use of wrought iron upper chord, vertical posts, and main end braces; which latter, for convenience, have been designated in this work, as king braces.

All of these members may be in the form of the patent wrought iron column of the Phœnix Iron Co. of Pennsylvania, formed of flanged segments, united by riveting; or of rectangular wrought iron trunks, as well as various other forms of section.

For the Phœnix column, a cast iron connecting piece may be inserted at the joints of the upper chord, with ends formed to enter the squared ends of the chord cylinders, and receive them against a shoulder of the connecting piece. This piece may have an opening in the under side to receive the diagonals and uprights, where they are secured by a transverse connecting bolt, in the same manner as at the joint of the cast iron chord cylinders, as before described. In this case the upright may have a cast iron top piece, formed as seen in Figs. 35 and 38 upon the top of cast iron uprights. A separate top piece has sometimes been used with cast iron verticals.

FIG. 41.



The connecting piece may also be formed as indicated in Fig. 41, with a downward branch like process to meet and receive the squared end of the vertical in the same manner as the horizontal part connects with chord cylinders. In this case the connecting piece must have openings as at *b b* Fig. 41, for the eyes diagonals to enter.

Fig. 41, shows an inside view of the joint piece, as it would appear if cut vertically and longitudinally, and the near half removed. The horizontal part consists of a cylindrical shell a little thicker than the wrought iron chord cylinder, with ribs upon the outside corresponding with those of the wrought cylinders, and as shown in end view *c*. Upon the inside, the ring and flanges *a a*, project inward, leaving usually a space of about 5 inches (according to dimensions of bridge), for eyes of diagonals. These are to ease the lateral strain of the connecting bolt or pin.

The process meeting the vertical, may be rectangular in horizontal section, composed of two parallel flat plates, in form as may suit the taste of the designer, united by two irregular plates formed to the profile of the parallel plates. The openings for diagonals, are, of course, through the irregular plates. These are drawn in at the bottom so as to form a square with the parallel sides, large enough to cover the flanges of the 4 segment column selected for the upright. See Fig. 41.

The inside of the square *d*, is filled in to form a hollow round, about an inch less in diameter than the hollow of the column, that it may have a ring or collar (represented by the inner white ring around *d*), projecting about 2 inches beyond the shoulder into the wrought iron column.

On the top of the joint piece may be an arrangement of oblique holes for the attachment of lateral × ties, and on the inside, facing the opposite truss, an abutting seat for the cross-strut, which may be in the form of a 6″ I beam, or such other form as may be preferred.

The foot of the post may stand upon a properly formed seat upon the connecting block of the lower chord, with an opening to receive the beam, in the same manner as described for the cast iron post. See Fig. 37.

It will be necessary for diagonals to pass through the centres of uprights, and for that purpose 10 or 12 inches in length, as may be necessary, may be left out of two opposite segments, and the strength thus lost, restored by additional metal, in such form as may be found convenient and efficient. Or, a cast-iron middle piece may be inserted in the upright.

In the case of an upper chord of rectangular trunks, and uprights of other than a cylindrical form, the joint piece will be correspondingly modified.

The position of diagonals may be reversed, connecting by an eye with a wrought cylindrical connecting pin at the lower chord, and by screw and nut with the joint piece of the upper chord. This involves merely a question of practical economy and convenience.

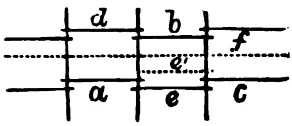
Sometimes, also, the connection is made by an eye at both ends of the diagonal, depending upon accuracy, as to length, in the manufacture, for the proper adjustment of parts. It is also practicable to provide means of adjustment in the length of vertical members.

CXXIII. But, to enumerate all the changes, and peculiarities of detail admissible in the construction of the Trapezoidal Truss Bridge, even if practicable, could hardly be regarded as expedient in this place. The essential requisities are, to provide material enough of good quality in all parts, to withstand the forces to which they are respectively liable, with efficient connections of parts, by the most direct and simple means, and with such an arrangement and adjustment as may produce the most uniform degree of strain upon all parts of each member. For instance, ecah section of the lower chord is usually composed of several bars, and it is important that each should sustain its proportionate share of the stress.

In the link chord composed of two links to each panel, if the links be properly fitted, the two sides of each must act very nearly alike, while the connecting block acts as a sort of balance beam to equalize the tension of links acting upon its two ends; and, if the two links of a pair vary slightly in length, the connecting block still secures equality of stress upon the two. The same is the case with regard to a chord composed of two eye bars instead of links, to each panel.

But the serious mistake is sometimes committed, of putting the two links or bars upon the same side of those in the succeeding panel, as in Fig 42; where it is obvious that the inside links (a, b, c), are exposed to more action than d, e, f.

FIG. 42.



For, if the inside links be 3″, and the outside ones 4″ from centre of pin, since a and b tend to turn the pin in one direction about its centre, and d and e in the opposite direction, the forces being in equilebirio — the *moments* (with respect to the centre), of forces tending in one direction, must be equal to those of forces tending in the opposite direction. Hence, representing the stresses of the several links by the letters designating them respectively on the diagram, we have $3 \times (a + b) = 4 \times (d + e)$, whence, $a + b = \frac{4}{3}(d + e)$; showing $\frac{1}{3}$ more stress upon the inside than upon the outside links.

On the contrary, if the link e, be removed to e' upon the inside of a, then d and e' act in one direction, and a and b in the other; and, assuming as before, the inside links to be 3″, and the outside ones 4″ from centre of pin, we have $4a + 3b = 4d + 3e'$. But $a + d = b + e'$, and if the force be communicated at the ends, equally upon the two sides of the chord, giving equal stress upon a and d, for instance, the tendency is to an even balanced action throughout the length of chord.

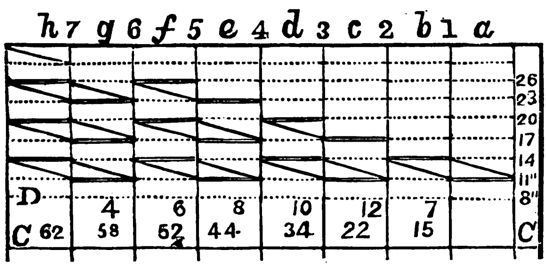
Hence the two links of each panel should always act upon the connecting block or pin, at equal distances from centre of pin.

MULTIPLEX CHORDS.

CXXIV. In very long or heavy bridges, the required amount of chord section in the middle portion of the truss, is so great, that it is deemed expedient to introduce more than two links or eye bars to the panel. This is sometimes done by alternating them upon the connecting pin, increasing the number and sizes according to the increase of stress from panel to panel toward the centre.

This mode of construction, unless the bars be arranged and proportioned with almost impracticable care and nicety, is liable to be attended by an accumulation of lateral strain upon the connecting pin, beyond what it can bear without bending, or springing so much as to materially disturb the equality of stress upon the links, or chord bars.

FIG. 43.



To illustrate this subject, let Fig. 43, represent one quarter of the chord of a 16 panel bridge. The line CC may denote the central axis of the chord running through the centres of connecting pins; D, at a distance of, say 8″ from C, the line in which the diagonals act upon pins, and the other parallel lines at intervals of 3″ from D, and from one another (see Figs.

on right band of diagram), the centres of thickness of links, at which points the action of respective links is supposed to be concentrated upon the pins. Also, let *a*, *b*, *c*, etc., represent the panels of the chords.

Now, if 15W, or 15, represent the stress upon the chord in the two first panels, *a* and *b*, that of the succeeding panels to the centre, will be as 22, 34, 44, 52, 58 and 62 (see lower figures in diagram), and the diagonals (producing increments of action upon chord), will have a horizontal action represented by 7 in panel *b*, by 12 in panel *c*, and so on by 10, 8, 6, 4. These being added successively to 15, produce the numbers just stated for the chord in the several panels.

The first three panels, *a*,*b* and *c*, require only one link upon each side, as indicated by the oblique black lines. The 4th panel, *d*, may have 2 links on a side, and the most favorable position for them, as regards action upon connecting pins, will be as shown, diverging from the central axis, so as to bring the end toward the abutment, nearest to the main diagonal connecting with the same pin.

The first pin, connecting *a* and *b*, having two equal forces acting in opposition, but at different distances from the centre line *C*, we take the moments of these forces with respect to that line; which are, for *a*, $15\times14=210$, and for *b*, $15\times11=165$. The difference (45) between these moments, equals the moment of the resultant, or the lateral stress of the pin, exerted on a leverage of 1″.

Assuming the value of W, our unit of stress (and always understood as annexed to the figures denoting stress), to represent 5,000lbs. we have for stress of pin in this case, $45 \times 5{,}000 \div L$. The L, being 1″ may be omitted in the expression.

Then, making x=diameter of pin, its resisting power= $\frac{AD}{L}\times 4{,}500$ (see [XCVIII])= $.785x^2 \times x \times 4{,}500 \div 1''=$ $3{,}532.5x^3$; and putting this equal to $45\times 5{,}000$ (the stress above found), we obtain $x=4''$ (very nearly),=required diameter of pin.

At the next pin we take the moments of one link $15\times 14''=210$, and one diagonal, $7\times 8''=56$, making 266 in one direction, against that of one link, $22\times 11''=$ 242. Hence the resultant moment = 24, and $24\times 5{,}000=3{,}532.5x^3$, gives the required diameter of pin *in the centre*, $x=3\frac{1}{4}''$, nearly. But this is the general stress in the portion of pin between diagonals, and may be greater or less than at certain points where forces are applied. For instance, if the aggregate moments of forces in opposite directions be equal, the resultant moment is nothing, and the middle portion of the pin, between diagonals has no stress, and might be cut out and removed, as far as strength of chord is concerned. In the case in hand, the moment of link *b*, with respect to link *c*, equals $15''\times 3=45$=stress of pin at centre of *c*. Hence the required diameter at this point is found by the equation $45\times 5{,}000=3{,}532.5\times x^3$, whence $x=4''$, the same as pin No. 1.

At the next pin, if we add another link, making 2 links sustaining 34W, at an average of 14″ from centre, giving a moment of 476, against one link, 22×14, + one diagonal $12\times 8=404$, we obtain a resultant moment of 72; whence, $72\times 5{,}000=3{,}532.5x^3$, and $x=$ 4.67 inches, = required central diameter of pin, and as will be readily seen on trial, the greatest required at any point.

Again, assuming at the 4th pin 2 links and 1 diagonal against two links, we have for the former, $34\times 17''+10\times 8=658$, and for the latter, $44\times 14=$

616, whence the resultant moment is 42. Therefore the equation $42 \times 5{,}000 = 3{,}532.5x^3$, gives $x = 3.9$ inches, = required diameter in centre, while for the outside link on this pin, the stress, 17, multiplied by 3 shows a moment of 51. Hence, $x = \sqrt[3]{\left(\frac{51 \times 5{,}000}{3{,}532.5}\right)} = 4.16$ inches = required diameter at that point.

At the 5th pin, there are 3 links, against 2 links and one diagonal, giving moments for the latter, $44 \times 17 + 8 \times 8 = 812$, and for the former, $52 \times 17 = 884$; whence the resultant moment = 72, and $x = \sqrt[3]{\left(\frac{72 \times 5{,}000}{3{,}532.5}\right)} = 4.67$ inches.

The moments at pin No. 6, are, for 3 links, 52×20, + (for diagonal) $6 \times 8 = 1088$, in one direction, and for 3 links, $58 \times 17 = 986$, giving a resultant of 102; whence, $x = \sqrt[3]{\left(\frac{102 \times 5{,}000}{3{,}532.5}\right)} = 5.24$.

Lastly, adding another link at the 7th pin, the moments are 58×20 + (for diagonal) 4×8, = 1,192, against $62 \times 20'' = 1240$, whence the resultant is 48, and $x = \sqrt[3]{\left(\frac{48 \times 5{,}000}{3{,}532.5}\right)} = 4.08$.

In this case the eyes, or link-ends are supposed to be bored in the direction of the pin, a little obliquely to the direction of the link, so as to bear through the whole thickness, as long as the pins remain perfectly straight. But the pins having a degree of elasticity, and considerable length, must yield to the action of links, springing more or less in the direction of the greater sum of moments. It will be seen, moreover, that in each case, the consecutive ends entering the outside link, as 3 and 4, 5 and 6, &c., are always sprung toward one another; the inevitable result of which must be, a relief or relaxation of the outside link, whence it must sustain a less degree of strain than its fellows located farther from the ends of the pins.

Now, as a 12 foot link, under a stress of 10,000lbs. to the inch is extended *less* than $\frac{5}{1,000}$ of a foot, a slight springing of connecting pins would relax the outside links materially, especially when the pins tend to spring toward one another.

Again, if the links run parallel with the centre of chord, and at right angles with the connecting pins, as indicated by the double black lines (Fig. 43), the moments of forces upon — pin No. 5, for instance, will be — for 3 links acting toward the right hand, $44 \times 17 +$ (for diagonal) $8 \times 8 = 812$, against 3 links acting toward the left, with moments equal to $52 \times 20 = 1,040$, showing a difference of 228 ; whence $x = \sqrt[3]{\left(\frac{228 \times 5,000}{3,532.5}\right)} =$ 6.85 inches = required diameter of pin at the centre.

At pin No. 6, are 3 links with a combined moment of 52×20, + (for diagonal), 6×8, $= 1076$, against 3 links with a combined moment of $58 \times 17 = 986$, showing a difference of 90; consequently, $x = \sqrt[3]{\left(\frac{90 \times 5,000}{3,532.5}\right)} =$ 5.03 inches = required diameter of pin.

Such would be the result as to stress and required diameter of pin, provided the pin remain perfectly straight. It is true that the spring of the pin in the direction of the greater moment, or sum of moments, will, in practice, produce an obliquity in its direction through the eyes, which will throw the centres of bearing upon the pin, nigher to the adjacent sides of the eyes, and thus reduce the difference of opposite moments, and consequently, the stress upon the pin. But such relief to the pin must be attended with a disturbance of the central and uniform strain of the chord bar; the strain being brought near one side of the bar. Moreover, as this can only result from actual springing of the pin, there must inevitably be a degree of relaxation of the outside link, whenever the pins at its two ends are deflected toward one another. On the contrary, an outside link or bar connecting with two pins springing *from* one another, is necessarily subjected to greater strain than those nigher the centres of pins, in the same panel.

In this case, the forces tend to spring the pins toward one another at the ends, whence the outside link must suffer more or less relaxation.

It seems unnecessary to carry these examples further. The above results show a decided advantage in the oblique position of links, diverging toward the centre of the span, so as to have the inside link opposed to the diagonal.

The arrangement of links, or eye bars, here assumed, and the amount of stress assigned to them, are no exaggeration upon what has been put in practice. But the preceding calculations must be sufficient to demonstrate the exceptionable character of such practice. Two links upon a side (4 to the panel), after two or three panels next the end, so thin as not to occupy an unnecessary length of pin — each taking hold of the pin outside of the succeeding one toward the centre of the truss, may be admissible. But a greater number, in the opinion of the author, for reasons already given, is not to be recommended.

DOUBLE CHORD.

CXXV. To obviate the difficulty attending the use of the multiplex chord, consisting of many links in a panel, we may make use of what may be distinguished as a *Double Chord.*

We have seen [LVI], that in double cancelated trusses with vertical members, there are two independent sets of diagonals and verticals, which have no interchange of action between one another. Now, each of these sets may have its own lower chord, also acting independently, each of the other, but uniting at the same point at the foot of the king brace, which is common to both sets of web members.

In such case, the two chords (which we may call *sub-chords*), may be one above the other, and composed of links or eye-bars, extending horizontally across two panels; the links or bars of one sub-chord connecting opposite the centre of those in the other, and the uprights in one set, being as much longer than those in the other, as the distance, vertically, between the upper and lower sub-chords.

By this means, about one-half of the extra material in chord connections would be saved; and a more uniform stress upon the chord bars secured, than would be practicable, even with 4 links acting upon one connecting pin.

DETACHED, AND CONCRETE PLANS OF CONSTRUCTION.

CXXVI. In the plan of Trapezoidal truss had under consideration in the last few preceding sections, the several members are formed in separate pieces, to be erected in place, and connected by screws, bolts, connecting pins, &c., as the parts of wooden bridges and building frames are erected, after being framed and prepared, each for its particular place.

There is another mode of construction, in which members and parts of members are permanently riveted together *in place;* or, in case of small bridges, the whole structure is permanently put together at the manufactory, and transported by water or rail to the place of erection and use. The former of these may be called the *detached*, and the latter, the *concrete* mode of construction.

The detached plan is probably the best adapted to wrought and cast iron bridges, and also, at least, equally adapted to bridges entirely, or essentially constructed of wrought iron, when vertical thrust uprights are employed.

But it can hardly be regarded as advisable to construct iron bridges with independent members, without thrust verticals. For, although as we have seen, [XLVI,] the latter plan shows a trifle less action upon the material than the plan *with* verticals, the oblique thrust members in the web, are 40 or 50 per cent longer (according to inclination), as well as being in greater number, and sustaining less average action to the piece.

The 7 panel truss, Fig. 12, has 4 compression verticals, liable to an average action of $8w''$; while truss Fig. 13, has not less than 6 diagonals, liable to an average compression of $4w''\sqrt{2}$ (when the inclination is 45°), equal to $5.65w''$. In the mean time, these members being over 40 per cent longer, and sustaining only about the same aggregate amount of action, can not be so economically proportioned to perform their required labor, when acting independently, as the fewer and shorter uprights.

Still, the Trapezoid with individual members is practicable, probably with about the same economy of material without verticals as with them; and, if it be deemed expedient to adopt the former, the modes of forming and connecting the various parts may be so nearly like those already described for the latter, that particular specifications will not be given in this place.

The essential conditions to be observed, are, besides proportioning the parts to the kind and degree of strain to which they may be exposed, to see that the forms of diagonals liable to compressive action, be made capable of withstanding such action, according to the table of negative resistances [XCIII]; and, that those liable to a change of action from tension to compression, and the contrary, be formed and connected in such manner as to enable them to act in both directions.

CXXVII. In the concrete, or rivet work plan of construction, the Trapezoid without verticals may, it is thought, be generally adopted with advantage. Upon this branch of the subject, however, but little of detail will be attempted at this time, the author having had very little direct practical experience in the premises.

The first point to be attended to, of course, as in all cases of bridge construction, is, to arrange the general outline and proportions of the truss; that is, the number of panels, and depth of truss suitable for the particular case in hand. This being done, the amount and kind of force, whether thrust or tension, to which each part is liable, should be determined; for which purpose, the value of w, and of w' (the variable and constant panel load for the truss), must be assumed, or estimated according to the best data at command; when the stresses of the several parts are readily obtained by process already explained; [XLIV, &c.].

We are then prepared to assign the requisite cross-section to each part, and to adopt a suitable form of bar, or combination of bars and plates, for each member. Thrust members will usually (if long), be formed of several parts, mostly flat plates, angle iron, T iron, and channel iron, united by riveting in such form of cross-section as may give the largest diameter practicable without too much attenuation of the thickness of material, a point upon which no certain rules can be given.

Flat plates, when connected by riveting at the edges, may be of a width of 30 to 40 times the thickness perhaps, without liability to "buckle" under reasonable compression. When riveted along the centre, a width of 12 to 20 times the thickness, will be in better proportion.

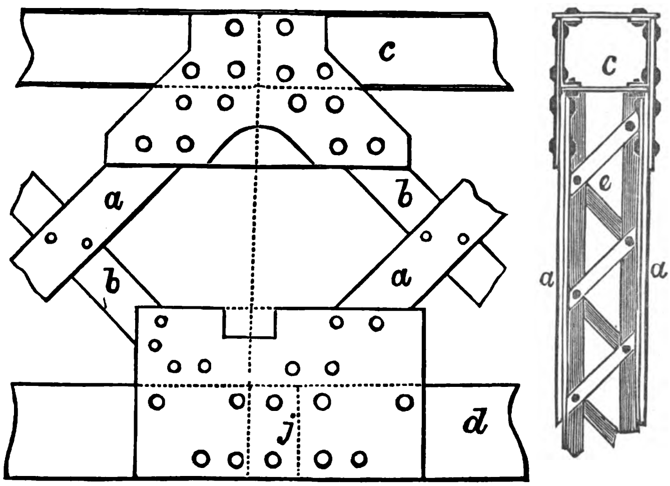
UPPER CHORD.

CXXVIII. A good upper chord may be made in rectangular, or box form, of flat plates and angle iron; or, for small bridges, of channel iron, with flanges either inward or outward, upon the two vertical sides, with flat plates upon upper and under sides; the upper riveted, and the lower one either riveted, or put on with screws, tapped into the lower flanges of the channel bars.

The upper plate, when flanges turn inward, may project half an inch, or an inch, and the lower one, come even with the sides. The channel bars should meet at the nodes, or connecting points, and a splice plate covering the joint may project below the chord far enough to form a connection with diagonals by riveting. (Fig. 44).

Diagonals acting by tension only, may be plain flat bars of width from 8 to 10 times the thickness. Those acting by thrust principally, may be of T iron with short diagonal bars riveted to the mid rib, (*e* Fig. 44), giving a width corresponding with that of the upper chord, or with the space between tension diagonals, so that the latter may be riveted to the cross-plate of the T iron at the crossings, to give lateral support to the thrust members. Angle iron may also be used instead of T iron, in these members.

FIG. 44.



Diagonals acting by both thrust and tension, should be formed and connected with reference to the forces they are liable to.

For small bridges, small plain I bars may be used for thrust diagonals with advantage.

In all cases of tension, rivets should be so arranged when practicable, as to leave all the section available, except the diameter of a single rivet hole; that is, no section through two or more holes, including the one farthest from the end, should have less area than a square section through one hole. [CXVII, Fig. 31.]

In Fig. 44, *a*, *a*, &c., represent tension diagonals, of plain flat bars, with cross-section proportioned to the stress in each case; *b*,*b* — thrust diagonals of T iron and short diagonal plates, as seen at *e*; *c*, *c*, the upper, and *d*, the lower chord; the dotted line *j*, shows the meeting of lower chord plates, about 4 inches toward the abutment from the point of meeting of the several centres of chord and diagonals. The side plates of upper chord may meet at the centre of the node, or connecting point.

The upper splice plates are of irregular form (or, they may be cut on a regular slant from upper to lower angle), but such as to cut without waste of iron. They may be clipped out upon the under side, as by the curved line, or not, as may be preferred.

The lower splice plates may be rectangular, and of such length and width as to admit of a sufficient number of rivets, properly arranged, to be equal in strength to the net section of chord plate and diagonals.

It is scarcely necessary to repeat, that rivet section connecting two thicknesses of plate only, should exceed the net section of plate by as much as the direct tensile strength exceeds the shear-strength of iron.

LOWER CHORD.

CXXIX. The following plan of a flat plate bottom chord adapted to a connection of diagonals by connecting pins, is transcribed from the author's former work; and, by widening the splice plates, as in Fig. 44, is equally adapted to the concrete mode of construction; *i. e.*, by rivet work.

The plan contemplates each half-chord as composed of two courses of plates (except near the ends), spliced alternately, one at each node so as to "break joints." The two half chords are to be placed at such distance apart as to accommodate the connections with diagonals, and with uprights, when used in connection with uprights.

For a 16 panel truss, as arranged in Figures 18 and 19. Suppose $w = 12m$ (m representing 1,000lbs.); $w' = 4m$, and $W = 16m, = w + w'$; — diagonals (except the steep ones), inclining 45°.

The end brace, then, sustaining $7\frac{1}{2}W = 120m$, [LVI], produces tension equal to $60m$, upon the first and second section of chord, in Fig. 18, the proportions for which will be here considered. Allowing then, $10m$ to the square inch, each half chord requires a plate of about 8″ by $\frac{7}{16}$″, up to the second node from the end.

This plate may extend — say within 8″ of the centre of the connecting pin at the 2d node, where it may be connected with a $\frac{5}{8}$″ plate, by two splice-plates about 27″ long (see A. Fig. 45), with a net section equal to the $\frac{7}{16}$″ plate, or, say $\frac{1}{4}$″ thick. Fig. 45, exhibits a disposition of rivet and pin holes, at A, so arranged as to preserve the full section of plates, less the diameter of a single 1″ rivet hole.

Or, the splice-plates may be 7″ shorter, and $\frac{1}{8}$ thicker, and the two rivets next the joint (j), on either side, opposite one another, as at BB, Fig. 45; thus giving the same section (of splice-plates), through two opposite rivets in the thicker, as through one rivet in thinner and longer splice plates. In this case, the joint should be $4\frac{1}{2}$″ from centre of connecting pin (p), and a little more, when the rivets exceed 1″ in diameter.

At the third node, an increase of section is required, and a $\frac{5}{8}$″ plate may be added on the inside, lapping 9 or 10 inches back of the pin, with a $\frac{1}{4}$″ splice plate of the B pattern to balance the extra inch in width required for opposite rivet holes, and a 2″ pin hole.

The inside plate continuing past the next, or 4th node, the $\frac{5}{8}$″ outside plate may be met by, and spliced to a $\frac{7}{8}$″ plate, in either of the modes indicated by A and

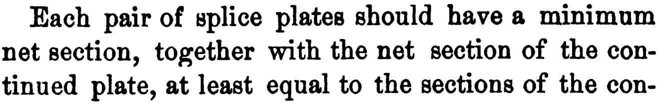
B, Fig. 45. On plan B, the outside splice plate should be at least ½″ thick, and the inside one, $\frac{5}{16}$″. In this, as in other cases where a thinner plate meets a thicker one, the former is to be furred out to the thickness of the latter.

At the 5th node, the outside plate may continue, while the inside one is succeeded by a ⅝″ plate, with a ⅜″ splice-plate inside, and one of $\frac{3}{16}$″ thickness upon the outside; splice-plates in all cases being intended to be upon the outside, and not between the two courses of plates forming the half chord.

The same general process being continued, each course being spliced at alternate nodes, and breaking joints with one another, we introduce in the outside course, a 1″ plate from the 6th node to the centre of the chord, and a ¾″ plate from the 7th node, past the centre to the 9th node, and so on, with a reversed order of succession to the other end of the chord.

The two 1″ plates in the outside course, should meet at the centre connecting pin, and all other joints should be a few inches from the pin, on the side toward the end of the chord, as in diagram, Fig. 45.

FIG. 45.



Each pair of splice plates should have a minimum net section, together with the net section of the continued plate, at least equal to the sections of the continued, and the thinner spliced plate, through one of the smaller rivets used in the splice; and the relative thickness of the two splice plates should, as nearly as practicable, be *inversely* as the respective distances of their centres from the centre of the spliced plate.

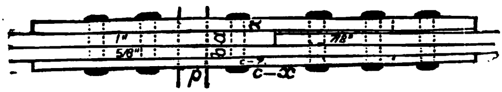
For illustration; at the 6th node, the continuous plate is $\frac{5}{8}''$, and the thinner spliced plate $\frac{7}{8}''$, making in the two, a thickness of $1\frac{1}{2}''$, by $7''$ for the net width; giving a section of $10\frac{1}{2}$ square inches. This splice requiring $1\frac{1}{4}''$ rivets next the joint, to give the necessary rivet section, the net width of splice plates and continuous plate through two opposite $1\frac{1}{4}''$ rivets, is only $5\frac{1}{2}''$. Consequently, the aggregate thickness required to give $10\frac{1}{2}$ square inches, is about $1.91''$; and, deducting $0.625''$ for the continuous plate, we have $1.285''$ for thickness of the two splice-plates.

Then, representing thickness of spliced plate by a (disregarding the furring plate, or including it in the quantity a), that of the continuous plate by b, that of the two splice-plates by c, and that of the thicker one by x; we form the following equation, as will be obvious on reference to Fig. 46, which is an edge view of splice at node 6.

$x \times \frac{1}{2}(a+x) = (c-x) \times (b+\frac{1}{2}(a+c-x))$; whence, the formula $x = c \times (a+2b+c) \div 2(a+b+c)$.

This formula applied to the case represented in Fig. 46, gives $x = 0.7804''$, and $c-x = 0.5046''$.

FIG. 46.



The letter a in the diagram shows the splicing of a $1''$ with a $\frac{7}{8}''$ plate, the thickness being equalized by a furring plate.

Figure 46 gives also, a *general* idea of the splices proposed for this kind of chord, in case of the adoption of the short splice plates and opposite rivets, as seen at BB, Fig. 45. *p* indicates the connecting pin (which, in the concrete plan of construction should be replaced by two opposite rivets, as seen in Fig. 44), having a cross-section in the parts passing through the chord plates, about equal to that of one of the two main diagonals connecting with each pin respectively, at the several nodes.

The body of the pin between chord plates, should have lateral stiffness enough to withstand the stress produced by diagonals horizontally, estimated upon the principles of the lever, which will be greater as the distance of diagonals from chord plates is greater, and the contrary. If the bearing of the upright upon the pin be between the diagonals and the chord plates, as by a bi-furcation like that at the upper chord (see *a* Fig. 38) the body of the pin will usually require a section about equal to that of the *two* main diagonals connected with it. But this is no certain rule.

The ends of the connecting pin should extend through the chord plates so as to receive a thin nut upon each end, and also the eyes of sway rods upon the inside end, in case that mode of connection be adopted for those parts.

In the case of trusses without verticals constructed in rivet work, the best balanced action will be secured by connecting diagonals between the splice plates, by means of rivets through both, thus bringing each diagonal bar directly over each half chord, and producing uniform stress, as nearly as is practicable. When diagonal bars do not fill the space between splice-plates, the deficiency may be made up by furring plates, or thimble rings.

Tension diagonals will usually require from 25 to 33 per cent of extra section to make up the loss in rivet holes. In thrust diagonals, no allowance need generally be made for rivet holes, as rivets properly distributed, will not impair the efficiency of the member in withstanding compression.

With regard to the relative merits of this kind of lower chord, it requires, in the proportions above assumed, namely, 8″ width of plates and 1″ diameter of the smaller rivets, about 14 per cent of extra section on account of rivet holes, through the whole length. For splice plates and rivets, at least an equal amount should be allowed, making 28 per cent for waste material, over and above the net available length and cross-section. The corresponding waste in the link chord, and in the eye-plate chord [CXIV], can scarcely exceed 10 per cent, when the connections are made with wrought iron pins.

Hence, the advantage as to economy of material, seems decidedly in favor of the latter plans; and the cost of manufacture can hardly be estimated in favor of the former. If the riveted chord, then, have any claim to favor and preference, it is mostly owing to the fact, that being manufactured cold, it escapes the deteriorating effects frequently resulting to iron in the process of forging and welding, and the risk of flaws, and imperfect cohesion of the welded surfaces.

How far this consideration should be regarded as an offset, or an overbalance to 15 or 20 per cent, of material lost in rivet holes and splices, further experience and observation alone can probably determine.

SWAY BRACING.

CXXX. The primary and essential purpose of a bridge is, to withstand vertical forces which are certain, and, to a large extent, determinate in amount. We can estimate nearly the weight of a train of rail road cars, a drove of cattle, or a crowd of people; and the amount of material required to sustain them.

But the lateral, or transverse forces to which a bridge superstructure is liable, are of a casual nature, depending upon conditions of which we have only a vague and general knowledge; and, can not predetermine their effects with any considerable degree of certainty.

We know full well from experience, that it is always expedient to provide every bridge superstructure with means of support against transverse horizontal forces; and we introduce certain parts and members for that express purpose. These have been frequently alluded to heretofore in this work, under the designation of *sway-rods, lateral ties, or lateral braces.* But no attempt has ever been made, to the author's knowledge, to point out the proper sizes and proportions of such members, upon any determinate principles or data.

In this respect, reliance has mostly been placed upon "judgment," and general observation as to precedent and common practice; as was the case in fact, with regard to bridge construction generally, until within the last twenty-five or thirty years. Within this period, and since the extensive use of iron in bridge construction has been introduced, more attention has been given to scientific principles, in adjusting the proportions of the several parts and members designed to withstand the effects of vertical pressure.

The modern bridge builder, if he has been properly educated for his business, having arranged the outline of his truss, makes his computations, and marks upon each line of his diagram, so many thousand pounds of tension upon this, so many tons of compression upon that, and so much shear strain, or lateral strain upon each rivet, connecting pin, or beam, and assigns to each place a member containing such an amount, and such a kind of material, as experience has proved to be sufficient to sustain the given stress with safety.

Thus far, his course is scientific and sensible. But in arranging his system for securing lateral stability and steadiness, science can lend him but little assistance.

He knows the wind will blow against the side of his structure; but whether with a maximum force of one hundred pounds, or as many thousands, he has no means of knowing with any considerable degree of certainty, or probability.

He knows, furthermore, that every deviation from a straight line by a body passing over and upon a bridge, even to changing the weight of a pedestrian from one foot to the other (unless his steps be directly in front of one another, and this could hardly form an exception), is attended by more or less tendency to lateral swaying of the structure.

Every inequality in the line of a rail road track, laterally or vertically, unless both rails have precisely the same vertical deviation, produces a transverse motion in the centre of gravity of the load, and consequently a lateral sway in the structure. The passage of a carriage wheel over a stick or a pebble, raising one wheel above the opposite one, changes the centre of gravity of the load to the right or left, and impels the structure in the opposite direction.

These are some of the external causes generating transverse action, and motion of the structure. But in addition to these, the upper chord itself, acting by thrust, is, at best, in unstable equilibrio, and liable at all times to exert more or less transverse action, and, if not kept in line by an efficient system of transverse bracing or tying, will lose its equilibrium, and be deprived of the power of performing its appropriate functions in the structure.

Now, these disturbing lateral forces are quite small, compared with the vertical action upon the trusses; and, the vertical strength of the truss does not necessarily imply any power of resistance transversely; the tendency of the lower chord to preserve a straight line, being essentially balanced by that of upper chord or arch to buckle laterally;* provided the chords be so dependent upon one another that both must sway to the right or left at the same time.

Hence, it is always expedient to provide some especial means for counteracting these lateral forces, which is usually done by the introduction of a system of horizontal diagonal ties or braces (small iron rods in iron, and the same, or timber braces, in wooden bridges), below the track or platform, in the horizontal panels formed by consecutive beams, and the chords of opposite trusses. Also, when trusses are sufficiently high, diagonals and cross-struts are introduced between upper chords, to prevent lateral buckling.

No attempt will here be made to assign specific stresses as liable to occur in sway rods or braces, based upon calculations from the uncertain and indeterminate elements upon which the lateral action upon

* The only truss known to the author, not liable to this lateral buckling, is the Whipple Independent arch truss, shown in Fig. 27.

bridges depends. But, judging from experience and observation, it may be recommended that iron sway-rods be made of iron not less than $\frac{5}{8}$ inch in diameter, for bridges of five panels or under, $\frac{3}{4}$ inch from six to ten panels, inclusive. For twelve and fourteen panels, $\frac{3}{4}$ inch for ten middle panels, and $\frac{7}{8}$ inch for the rest; and, for sixteen the same as last above, with the addition of a pair of 1 inch rods in the end panels.

These are the *least* dimensions recommended (in all cases exclusive of screw thread), for ordinary bridges with panels not much exceeding 10 feet. For panels approaching or exceeding 12 feet, $\frac{1}{8}$ inch may properly be added to the above specified diameters generally.

If upper sway rods connect in the middle of cross-struts, with a longitudinal reach across two panels, [see cxx, and Figs. 38 and 39], they may safely be made smaller than when they cross one panel only.

The action of wind is nearly a uniform pressure from end to end of the structure, and causes much the same progressive increase of stress upon sway-rods, as the weight of structure and uniform load produces upon diagonals in the trusses — a fact which was recognized in assigning larger sway-rods at and near the ends of long bridges. But the casual impulses resulting from unevenness in track or platform, giving slight lateral movement to passing loads, and acting at single points here and there, this way and that, do not produce an accumulation of effect toward the ends. Hence, as it regards withstanding the latter forces, no variation in sizes of sway-rods is required.

CXXXI. Sway-rods acting by tension would obviously draw the opposite chords toward one another, but for the resistance of transverse beams or struts, while they also exert a longitudinal action upon the chords, thereby increasing or diminishing the stress upon chords, due to the action of structure and load. Chords, however, are usually proportioned without provision for increase of stress liable to accrue from action of sway-rods; and, from the small sizes of the latter, as compared with the former, and the obliquity of their action, seldom expending more than half their direct stress upon the chords longitudinally, this small action may be neglected, as forming one of the contingencies for which a large surplus of material is always provided in chords, over what is actually required to withstand the effects of any *probable* vertical action.

Certain modes of inserting and connecting sway-rods have been previously alluded to, sometimes with the beams by means of eyes and bolts [CVIII, Figs. 31 and 33], and sometimes more directly with the chords [CXIX, Fig. 35, *d*, and Fig. 39, *d*.]

The best connection is that which gives the nearest approximation to central and uniform action upon all parts of the chord, and also of the beam or strut. The plan described in section CXX, and seen in Fig. 37, when admissible, affords a good connection for bottom sway rods.

Undoubtedly there may be better devices for the purpose under consideration, as well as for other details, than any that have occurred to the author. But such as are herein described have mostly been put in successful practice, and are thought not to be seriously faulty.

COMPARISON OF DIFFERENT PLANS OF IRON TRUSS BRIDGES.

CXXXII. It is the purpose of this chapter to canvass the relative merits of most of the several systems of IRON BRIDGE TRUSSING, which have claimed and received more or less of public notice and approval during the last few years; and of which the distinctive principles have already been discussed in preceding pages; though not in the precise combinations here about to be presented.

We may take the number, lengths and stresses (the latter governing principally the required cross-sections), of the several long pieces or members of the truss, in the manner employed in the fore part of this work, as affording a near criterion of the comparative cost and economy of the bridges respectively. Then, after reference to such peculiarities as may seem advantageous or otherwise, leave the reader to his own conclusions in regard to the relative merits.

THE BOLLMAN TRUSS, FIG. 47,

Is founded upon the general principle discussed in sections XXII and XXIII, with oblique tension rods, and a thrust upper chord, in place of the thrust braces and tension *lower* chord as represented in Fig. 9.

Let Fig. 47, represent a truss 15′ high, and 100′ long; or, in the proportion of 1 to $6\frac{2}{3}$. Also, let w represent the maximum variable load for each of the points *c*, *d*, *e*, etc., and w′ (say, $= \frac{1}{3}$w), the permanent weight of one panel of superstructure, supposed to be constantly bearing at each of said points. Then making W = w × w′, we have $\frac{7}{8}$W = weight sustained by *ac*.

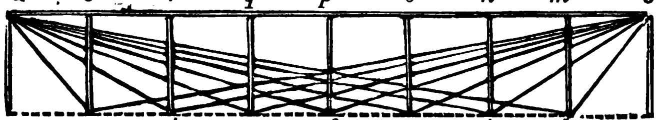
Now, we have seen [VII], that the stress upon an oblique in such case, equals the weight sustained, multiplied by the length, and divided by the vertical reach of the oblique; and, assuming that the member requires a cross-section proportional to the stress, it follows that (making $ab = 1$), the amount of *material* required in ac, will be as the weight it sustains, multiplied by the square of its length. Hence, the material required in ac, must be as $\frac{7}{8}\,W \times ac^2$. Then, diminishing bc until ac coincides with ab, $W \times ab^2$ becomes W, which is still proportional to the material required in ac (which has now become $= ab, = 1$), and, being replaced by M, representing the *actual* material required to sustain the weight W, with a length equal to ab (our unit of length), in a vertical position, we have only to substitute $M \times ac^2$ for $W \times ac^2$, to know the actual material necessary to sustain the weight W (at a given stress per square inch of cross-section), with any length and position, retaining the same vertical reach, equal to unity.

It must be obvious, therefore, that M, with the coefficient used before W, to express the weights respectively sustained by the several oblique rods in truss 47, will, when multiplied by the squares of the respective lengths of those obliques, show the amount of material required in their construction, under the conditions above expressed.

Let $m = \frac{1}{8}M$, and $h = bc$. Then, we manifestly have, for material in the 14 obliques of the truss in question $7m(h^2+1) + 6m(4h^2+1) + 5m(9h^2+1) + 4m(16h^2+1) + 3m(25h^2+1) + 2m(36h^2+1) + 1m(49h^2+1) = (336h^2+28)m$, for those meeting at a, and a like amount for those meeting at l; making a total of $(672h^2+56)m$. But $h^2 = 0.694$, which substituted in the last expression, gives $522.368m, = 65.296M$.

FIG. 47.

BOLLMAN TRUSS.



The thrust of the chord *al*, equals the horizontal action of the 7 obliques connected with either end. Making then x $= \frac{1}{8}$W, and $h = bc, = \frac{1}{8}bk$, it is obvious that each oblique carries weight equal to x $\times$ the number of panels not crossed by it, while its horizontal reach equals $h \times$ the number of panels it *does* cross. Hence, the horizontal *action* of each oblique, equals $hx \times$ the product of the numbers of panels at the right and left respectively, of the lower end of the oblique.

The compressive force acting from end to end, upon *al*, then, must be equal to hx $(7, + 2\times6, + 3\times5, + 4^2 + 5\times3, + 6\times2, + 7), = 84h$x, $= 10\frac{1}{2}$W$\times0.833, = 8.75$W.

Multiplying stress by length, and substituting M, we have 8.75×6.66M $= 58\frac{1}{3}$M = material required in *al*, at a given stress per square inch of cross-section; M being the amount required for a unit of length (*ab*), to sustain the unit of weight (W), at the same rate of stress.

Add 7M for two end posts, with length equal to 1 and bearing weight equal to 7W, and we obtain $65\frac{1}{3}$M as a total for thrust material in long pieces, not including 7 intermediate uprights, not properly to be classified with other parts, as their action is merely incidental, except that of supporting the weight of upper chord.

The parts above considered, mainly determine the character of the truss as to economy of material.

Other parts, such as short bolts, nuts, connecting pins, &c., although just as essential, are comparatively, of small amount and cost, except the intermediate uprights, which will be referred to hereafter.

If the truss be used in a deck bridge, and the end posts be replaced with masonry, the intermediates will sustain the same weight as the ends sustain in a *through* bridge, thus giving the same representative of material as above found.

THE FINCK TRUSS, FIG. 48,

CXXXIII. Possesses several of the characteristics which distinguish the Bollman plan. Both dispense with the bottom chord, which is common to most, if not all other plans of truss, for both iron and wooden bridges. Both also employ a pair of tension obliques acting in horizontal antagonism to each other, at each of the supporting points *c*, *d*, *e*, &c. But while in the one, the members of each pair of obliques are of equal length and tension, in the other, the pairs consist of unequal members (except at the centre), as the diagrams will sufficiently illustrate.

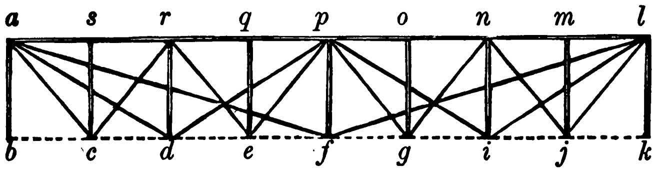
It will readily be seen that Fig. 48 exhibits three classes of obliques, consisting respectively of 2, 4, and 8 members to the class. Supposing a truss of the same dimensions and proportions, and subjected to the same load, as in case of Fig. 47, and using the same notation, as far as applicable; it is manifest that each of the 8 short obliques, sustains $\frac{1}{2}$W. The 4 next longer sustain upon each, a weight equal to W – one half directly, and the other, through the short obliques and uprights. The two long obliques sustain 2W each, being the half of 1W, received directly at *f*, and 1 and 2 respectively through the upright, from members of the other classes, meeting at the point p.

The material required for all the obliques, then, (ab being $= 1$, and $bc = h$), is $\overline{8 \times \frac{1}{2} (h^2 + 1) + 4 \times 1 (4h^2 + 1) + 2 \times 2 (16h^2 + 1)}$M, being the number of pieces in each class multiplied by co-efficients of W in weights sustained, and by squares of length respectively, and the sum of products multiplied by M.

Subsituting in the above expression the value of h^2, (0.694), and, reducing and adding terms, we derive material in obliques = 70.296 M.

FIG. 48.

FINCK TRUSS.



The compression upon the chord $a\,l$, is equal to the horizontal action of one member of each class of obliques, communicated at each end; that is, equal to $(\frac{1}{2} h + 2h + 8h)$W, $= 10\frac{1}{2} h$ W; and, multiplying by length (= 6.66), and substituting 0.833 for h, and M for W, we have $(10.5 \times .833 \times 6.66)$M $= 58.\frac{1}{3}$ M, to represent the material required in al; — the same as in case of Fig. 47.

The uprights of the Finck truss obviously sustain 12W, namely, $3\frac{1}{2}$ at each end, 3 in the middle, and 1 at each of the quarterings, r and n. But, in comparing this with the Bollman truss, it seems fair to offset 6 uprights, not including the end and centre ones, in the Finck, against 7 in the Bollman truss not estimated; hus leaving 10M for uprights in the former, making a total of $68\frac{1}{3}$M, for compression material, excepting the 6 intermediate uprights, excluded as above.

Both of the above considered trusses exhibit a beautiful simplicity, and facility of comprehension in principle, and they will be left for the present, for a discussion of the

POST TRUSS.

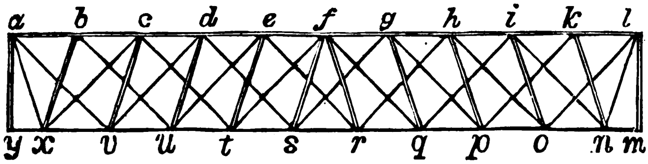
CXXXIV. This, like the two preceding plans, is designated by the name of its distinguished designer and publisher, S. S. Post, Esqr., of Jersey City.

Fig. 49 gives a general view of the only specimen of this truss which the author has had an opportunity of examining. It is a sort of compromise between the trusses represented by Figs. 18 and 19, of which the object sought appears to have been, to obtain a nearer approximation to the most economical angle of inclination for both thrust and tension members (between chord and chord), by inclining the latter at an angle of 45°, and the former at a less angle with the vertical. These are both favorable conditions, considered alone and by themselves, as we have already seen [LXV and LXVI]; and it is proposed to compare the economy of this particular arrangement, with that of a truss having vertical posts, with oblique tension diagonals; as well as with other plans, preceding and succeeding.

Assuming the same length and depth of truss, and the same load, both constant and variable, as in the preceding cases, acting at the points x, v, u, &c., let w represent the greatest variable load for the length of one panel, and w' the weight of superstructure bearing upon one truss, for the same length, supposed to be concentrated at the nodes of the lower chord, and assumed to be equal to $\frac{1}{3}w$. Also, let 1 equal the vertical depth of truss (between centres of chords), and let tension diagonals incline 45°, and posts lean 1 horizontally to 3 vertically; the space between posts being two-thirds of the depth of truss.

FIG. 49.

Mr. Post's Truss



Then, omitting counter ties up to tf, from the left, as neutralized by weight of structure ; we see that the weight at x, being only $\frac{3}{4}$ as great as at the other nodes, on account of the short space xy, $3w \div 80$ (or $3w''$, substituting for the occasion, w'' for $w \div 80$), represents the proportionate part of that weight, tending to bear upon the abutment at m; and this, with $12w''$ for weight at v, and $20w''$ for weight at u, $+ 28w''$ for weight at t, makes $63w''$ accumulated upon tf, when x, v, u and t alone are loaded.

Now, the action upon this truss is less certain and determinate than where the thrust pieces are vertical, or inclined equally with the tension pieces. But supposing that the weight of superstructure at s, or at s and r together, neutralizes, or reflects back a part equal to w', or $\frac{1}{3}80w''$, $= 27w''$ nearly, of this $63w''$, we have a balance of $36\frac{1}{3}w''$, as the maximum weight for tf.

Then, whether this $63w''$* which must go to the

* This full amount $63w''$ is used here; for, although it is assumed that only a part of it is transmitted through tf, the balance is restored from weight of structure which otherwise would pass to the abutment at y.

abutment at m, in virtue of the loads at x, v, u and t, is transferred through fs to sg, or through fr to rh; or whether it is divided equally or unequally between the two, is not quite obvious. But assuming, as what might seem probable, that it is transferred in equal portions to sg and rh, in that case, sg sustains as a maximum, $36w''$ for weight at s, + half of $63w''$, making — say $67w''$; supposing that sg and re sustain none of the weight of structure; which, though probably not strictly true, will not materially affect the result.

Again, (we are now considering the nodes at the lower chord as being loaded successively from left to right), the weight at r gives $44w''$ to rh, in addition to, say $32w''$ tending to be transmitted from tf, and w', or $27w''$ for structure, making $103w''$.

For maximum weight on iq, there is due to movable weight at q, $52w''$, + $67w''$ from sg, $+27w''$ on account of weight of structure, making $146w''$; while pk sustains $(60+103+27)w''$, $= 190w''$, and ol sustains $(68+146+27)w'' = 241''$. The maximum weight upon nl, is made up of that of pk + $\frac{3}{4}$ $(w+w')$ at n, $= 270w''$.

Having thus determined the maximum weights which these diagonals are respectively required to sustain, disregarding some small matters of uncertainty, of little practical importance, we find the sum of these maxima, for the 6 pieces parallel with ue on the right, to be $783w''$, $= 9.7875w$. Then, multiplying by 2 (the square of the common length, ay being $= 1$), and substituting M′ for w (as M was substituted for W in the preceding cases), we derive 19.575M′ = material required for the 6 pieces in question. Add to the last amount 3.7M′ for the steep diagonal nl (being the square of length by weight sustained, and w changed to M′);

and we have the whole material for tension obliques in the half truss: which doubled, exhibits for that class of members in the whole truss, 46.55M′; omitting 6 counter ties, not required to sustain structure or load, and the value of which will be considered (in general) hereafter under the head of counter bracing.

The short section *mn* of the lower chord, has no determinate action. The section *no* has a tension equal to $\frac{1}{3}$ of the weight acting on *nl* and *kn*, under a full load of the truss, equal to $\frac{1}{3}$ the weights upon *r*, *p* and *n*, for *nl*, and $\frac{1}{3}$ of those at *r* and *p* for *kn*; the whole equal to $\frac{1}{3} \times 2\frac{3}{4}\,(w+w')+\frac{1}{3} \times 2\,(w+w'), = 2.11w$.

To this, the diagonal *ol* adds at *o*, $2\,(w+w')$, and *io* adds $\frac{1}{3}\,(w+w')$, making $5.22w$ = tension of *op*; while a like addition at *p*, for the action of *pk* and *hp*, shows $8.33w$ for *pq*. Again, *qi* adds at *q*, $w+w'$, equal to $1.33w$, while *rh* contributes a like amount at *r*; making for *qr* and *rs* respectively, a tension of $9.66w$, and $11w$, restoring neglected fractions.

It is probable that a small decussation of forces through *re* and *sg*, under a full load of the truss, would modify these stresses slightly, but not so as to produce a material difference in the final results of the present discussion.

Summing up the stresses thus determined for different portion of the lower chord, counting like strains upon corresponding sections, and deducing the required material (as above done with regard to diagonals), remembering that the length of sections equals $\frac{2}{3}$ of unity, we obtain 41.1M′ = material required in lower chord. This added to 46.55M′, the amount above determined for obliques, gives tension material for the whole truss, equals to 87.65M′.

Now, it is manifest that the quantity here represented by M′, has the same ratio to that denoted by M in the estimates of material for trusses Fig. 47 and Fig. 48, as the weight w in the former case has to the weight W in the latter. But W was used to express $\frac{1}{8}$ of the *gross* load of the truss, while w represents only $\frac{1}{10}$ of the *variable*, assumed to be equal to $\frac{3}{4}$ of the *gross* load. Therefore $w : W :: \frac{3}{4}\times\frac{1}{10} : \frac{1}{8}$; whence, $w = 0.6W$; and $M' = .6M$. This equivalent substituted in the expression $87.65M'$, gives $52.59M$ = tension material for the post truss.

The maximum weights sustained by the thrust braces, equal respectively those borne by the tension rods communicating such weights, and for the 5 pieces on either side of the centre, the amount is equal to $w'' \times (36+67+103+146+190) = 6.77w$, which doubled, gives $13.54w$ for the whole of that class of members. This aggregate weight, multiplied by the square of the common length of pieces (1.11), with w changed to M′ produces $15.02M'$, $= 9.01M$.

The end section (kl) of the upper chord, sustains compression equal to the weight upon ol and $\frac{1}{3}$ of that upon nl, under a full load of the truss, $= 2\,(w+w')$, $+\frac{1}{3}\times2\frac{2}{3}\,(w+w')$, $= 3.88w$. Add $2\,(w+w')$ for weight on pk, and $\frac{1}{3}$ of that amount for that on kn, and it makes a compression of $7.44w$ upon ki.

Again, adding $w+w'$ ($= 1.33w$) for action of qi, and $\frac{1}{3}$ of the same for that of io, makes $9.22w$ for compression of ih, while a like addition for action of rh and hp, makes $10.99w$ = compression of hg and gf. We may call the last stress $11w$, as some fractions have been neglected.

The above amounts of stress upon the several sections of the half chord, added together and doubled to represent the whole chord, and multiplied by the length of section ($\frac{3}{5}$), produce $56.72w, = 34.03W$; whence, material for top chord $= 34M$; very nearly.

The two end posts obviously sustain the gross load of the truss (deducting what comes upon one half of the short spaces *mn* and *xy*), which equals $9\frac{1}{2}(w + w'), = 12.66w$; and, the length being 1, the material equals $12.66M' = 7.6M$.

Summing up the amounts thus determined, of material for the several classes of thrust pieces, we have:

For Braces, or inclined posts, ...	9.01M.
" Upper Chord, ...	34.00M.
" End Posts, ...	7.60M.
Total, for Thrust, ...	50.61M.
" " Tension, ...	52.58M.

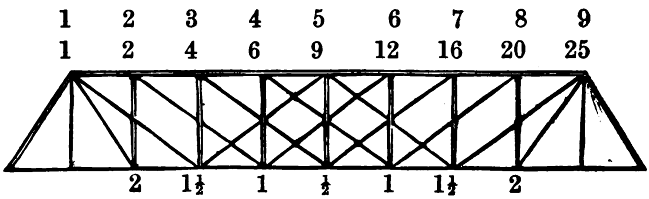
WHIPPLE'S TRAPEZOIDAL TRUSS.

CXXXV. The distinctive characteristics of this plan are, an Upper Chord made shorter than the Lower, by the width of one panel at each end, giving to the truss a Trapezoidal form — dispensing with non-essential members, and proportioning the several parts in strict accordance with the maximum stresses to which they are respectively liable; principles and devices first promulgated in the original edition of this work, and applied by its author in the construction of trusses with parallel chords, with or without vertical members.

Truss Fig. 50 has vertical posts and tension diagonals; and, using w and w' to denote the same quantities as in the last preceding case, and pursuing the method explained with reference to Fig. 18, [LVI], we have the maximum load for 3/5 equal to $4w'' - \frac{1}{2}w'$

(making $w'' = w$ divided by the number of panels $= 0.1w$), $= .4w - \frac{1}{8}w$, since $w' = \frac{1}{3}w$. For 4/6, we have $.6w$, without increase or diminution on account of structure; while, for the 3 next diagonals on the right, we have successively, $9w + \frac{1}{2}w'$, $1.2w + w'$ and $1.6w + 1\frac{1}{2}w'$, making altogether $3.7w + 3w'$, $= 4.7w$; showing for the 5 pieces, $5.53w$. This being doubled and multiplied by square of length (2.775), and w changed to M′, gives material for 10 long diagonals = 30.69M′.

FIG. 50.



The two steep diagonals togther, sustain 4 $(w + w')$, $= 5\frac{1}{3}w$, which, multiplied by square of length (1.44), produces material = 7.68M′; while the two tension uprights manifestly require 2⅔M′. We have consequently, material for the system of tension obliques and verticals = 41.03M′.

The end brace obviously sustains $4\frac{1}{2}(w + w')$, and exerts a horizontal stress $= 4w$ (two-thirds of the weight borne), upon the two first sections of the lower chord. The steep tension oblique adds ⅜ of weight borne, making $5.76w$ for the next section, while the two succeeding diagonals toward the centre, adding $1\frac{1}{3}$ times the weights borne successively (under a full load of the truss, of course), give $8.42w$ and $10.19w$, for tension of second and first sections from centre, respectively. Then, adding, doubling, and multiplying by length of section, we obtain, material for lower chord = 43.16M′.

Add to this the amount for diagonal system as above found, and we have the whole amount of tension material for the truss = 84.18M′ = 5.05M.

The maximum weights sustained by obliques, and by them transferred to 7 thrust verticals, being in the aggregate = $6.62w$, the length of members being unity, need only the substitution of M′, to express the required material for said verticals; which, reduced to terms of M, equals 3.97M.

The first and second sections of the upper chord, obviously sustain the same action respectively, as the fourth and fifth of the lower chord while the 4 middle sections of the former, receive the additional action of diagonals 3╲5╱7 (upper figures), under full load. Hence we cipher up, material for upper chord = 32.6M.

The end braces, sustaining $9(w+w') = 12w$, with a length whose square is 1.44, obviously require material = (12×1.44)M′ = 10.37M.

The truss, then, requires thrust material, for upper chord, 32.6M, for end braces, 10.37M, and for uprights, 3.97M; making a total for the truss, of 46.09M.

Tension material as above, total 50.50M.

TRUSS WITHOUT VERTICALS.

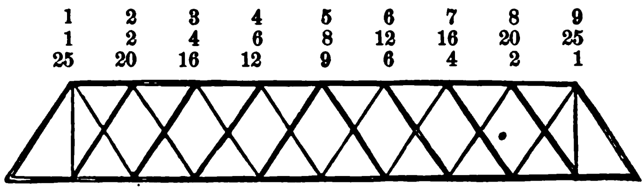
CXXXVI. Assuming a truss (Fig. 51), of same length, depth, and number of panels, and same load, variable and constant, as in the two cases last considered, with diagonals crossing one panel only, we have nearly the Isometric Truss,* adopted by Messrs. Steele and McDonald.

Arranging the numbers over the diagram, as in Fig. 51, and using the process explained [XLVII, Fig. 19], it

* In the Isometric, the diagonals incline at 30°, while in Fig. 51 they incline nearly 34°.

will be seen that either end brace, and the obliques parallel therewith, are liable to maximum weights as follows, proceding from end to end.

FIG. 51.



		Compression.	Tension.
End Brace	$4.5(w+w') =$	$6.000\,w$.†	
Oblique No. 1	....	2.100 "	
" " 2	....	1.533 "	
" " 3	....	1.066 "	.233 w
" " 4	....	1.600 "	.600 "
" " 5	....	.233 "	1.066 "
" " 6	....		1.533 "
" " 7	....		2.100 "
" " 8	....		2.666 "
Totals		$11.533w$	$8.200w$.

Then, doubling for the two sets, multiplying by square of length (1.44), and changing w to M′, we have, to represent material....for compression 33.215M, tension 23.616M′.

The end brace, sustaining $4.5\ (w + w'), = 6w$, exerts a tension of $4w$ upon the end section of the lower chord. The next brace sustains $1\frac{1}{2}\ (w + w') = 2w$, making a tension of $5.333w$ for the second section. The tension and thrust diagonals meeting the chord

† The small thrust action which the movable load tends to throw upon 6, 7 and 8, and the small tension upon 1 and 2, are neutralized by weight of structure.

at the next node, sustain together (under a full load of the truss), 3 ($w + w' = 4w$, adding $\frac{2}{3}$ of which, gives $8w$ = tension of the 3d section, while $2\frac{2}{3}w$ borne by the obliques meeting at the next node, makes a tension upon the 4th section equal to $9.777w$; and $1\frac{1}{3}w$ at the next node (the tension diagonal only, being in action, under a full load), gives for tension of the 5th section, $10.666w$.

Adding the stresses of the several sections of the half-chord, doubling, multiplying by the common length ($\frac{2}{3}$), and changing w to M′ shows material for lower chord = 50.37M,.

The end section of the upper chord sustains thrust equal to $\frac{2}{3} \times$ (weight on end brace, ($= 6w$), + weight on tension oblique meeting said brace), $= \frac{2}{3}\ 8.666w = 5.77w$.

The two obliques meeting at the first node from the end, sustain together $4w$, adding $2.666w$ to the above, and making a compression of $8.444w$ upon the second section; while succeeding diagonals make the stresses of the 3d and 4th sections, $10.222w$, and $11.1w$ respectively; whence, by process already employed and described, we derive:

Material for upper chord =........	47.392M′	= 28.435M
Add for end braces,..................	17.28 "	= 10.368"
" " other obliques, ...	15.935"	= 9.561"
Total for compression material,	80.607M′	= 48.364M
Tension, chord,............ 50.37M′		
Obliques, ... 20.616		
Verticals,................... 5	78.986M′	= 47.391M.
Grand Total,		95.755M.

THE ARCH TRUSS.

CXXXVII. A parabolic Arch Truss of the same length, depth and load as allowed in the five preceding cases, and having 9 panels, will compare, as to representative of amount of material, as follows :

Let $w_{,}$ represent the variable, and $w_{,,} = \frac{1}{3}w_{,}$, the permanent panel-load. Then, taking the greatest depth of truss (15*f*.), as the unit of length, as before, the length of chord will be 6.666, and the verticals respectively 1, 0.9, 0.7, and 0.4.

The length of panel (11.111*f*.), being divided by 15*f*. (the unit), gives 0 .74074. Hence, tension of chord = $4\,(w_{,} + w_{,,}) \times \frac{.74074}{.4}, = 1\frac{1}{3} \times 7.4074w_{,}$, which, multiplied by length of chord (= 6.666), and $w_{,}$, changed to $M_{,}$, gives representative of material = $9.8765 \times 6\frac{2}{3}M_{,}$ = $65.843M_{,}$; in which $M_{,}$ is the unit of material, proportional to the unit of length (15′,) × unit of stress, $w_{,}$.

The maximum tension of diagonals, as determined instrumentally by process explained [XXVII, &c.,] varies from $1.11w_{,}$ to $1\frac{1}{3}w_{,}$; and, taking the highest, multiplying by the aggregate length (15.4), and changing $w_{,}$ to $M_{,}$, we obtain material = $20.52M_{,}$.

The verticals sustain tension, each, = $1\frac{1}{3}w_{,}$, with an aggregate length of 6, giving material = $8M_{,}$; making a total of tension material = $94.376M_{,}$.

The horizontal thrust of the arch, must be in all parts the same as the tension of the chord (at the maximum under full load), and it is manifest that the material for each segment, must be to that of the middle segment, as the squares of respective lengths to unity; that is, equal to material in said middle segment, multiplied by squares of respective lengths.

But the representative for the middle piece equals $\frac{1}{8}$th that of the lower chord, $= 7.316\text{M}_{,}$. Hence, this amount multiplied by the sum of squares of all the others, +1 for the middle segment, found to be 9.058+1, = 10.058, gives, to represent material for the whole arch, $73.584\text{M}_{,}$.

Then, the vertical members are liable to be exposed to compressive action, represented by the small amount of $2.058\text{M}_{,}$, which added to the above, gives a total of compression material, equal to $75.642\text{M}_{,}$.

Now, the factor $\text{M}_{,}$, here used, is to the factor M used in the preceding cases, manifestly, as $\frac{2}{3}\times\frac{1}{8}$, to $\frac{1}{8}$, as $\frac{1}{12}:\frac{1}{8}$; whence, $12\text{M}_{,} = 8\text{M}$; and we reduce the coefficients of $\text{M}_{,}$, by $\frac{1}{3}$, and change $\text{M}_{,}$ to M, to bring the last results to the same standard measure as in the preceding.

Effecting these changes, we have, for tension material, Chord 43.895M, + Diagonals 13.689M + Verticals 5.333M, equal to a total of 62.917M. For compression, Arch, 49.056+Verticals, 1.372, = 50.428M.

SYNOPSIS OF PRECEDING DEDUCTIONS.

The following tabulated statement may promote the convenience of comparison :

Trusses.	Material required expressed in Ms.					
Designated.	Tension total.	Compression. Chord.	Ends.	Posts, &c.	Comp. Total.	Grand Total.
Bollman, .	65.296	58.333	7.000	*	65.333	130.629
Finck, ...	70.296	58.333	7.	3.000	68.333	138.629
Post, ...	52.590	34.	7.6	9.01	50.610	103.200
Whipple, .	50.500	32.6	10.37	3.97	46.94	97.44
Isometric, .	47.391	28.435	10.368	†9.561	48.364	95.755
Arch, ...	62.917	‡49.056		1.372	50.428	113.345

* Actual, but not a determinate quantity. † Thrust Diagonals. ‡ Arch.

CXXXVIII. The figures in this table are to be understood in all cases as prefixed to the quantity M, which, as far as relates to tension material, represents a determinate amount of *wrought iron ;* while, as it relates to compression material, M represents an amount of *cast or wrought* iron, varying as the forms and proportions of parts vary. But, in the present discussion M may be assumed to have a uniform value in expressions relating to material under the heading of chords; and of ends, whether oblique or vertical.

The quantities under the head posts, require in general, probable twice as high a value for M, as that required for the other classes of thrust members, as it regards all but the first named truss, while the first is not represented in that column at all, although the parts there referred to are as indispensible, practically, and require nearly as much material as corresponding parts in the other plans.

With regard to plan No 2 (the Finck), 6 posts actually required (two of which, at the quarterings, sustain determinate weight equal to W each), are also omitted in the table, to place this plan upon an equal footing with the preceding one.

There is also a consideration with regard to the effects of load upon these two trusses, especially the first, which render it *partially* necessary to use diagonal ties, or "panel rods" in the several panels; and such have usually been introduced wherever such bridges have been constructed.

As any one pair of suspension rods in the Bollman truss may be under full load, while the others are without load, the loaded node would, in such case, be depressed, while that on either side would retain nearly its normal position. Thus would result an obliquity in panels adjacent to the loaded point, and consequently, a tendency to kink in the upper chord, by opening the joint above the loaded point upon the under side, and the next joint either way, upon the upper side. Hence the compression of certain chord segments would be thrown upon the extreme upper side at one end, and the lower side at the other end. This would be decidedly an unfavorable condition, which the panel rods are used to obviate by distributing the load of loaded points over adjacent, and more remote parts of the truss. Otherwise, the bridge would act under a passing load, somewhat in the manner of a pontoon bridge.

By estimating a reasonable amount of material for posts and panel ties, the figures in the table, opposite the first two trusses would be materially increased.

Hence, it must be obvious that the necessary material for the two above named trusses, is not so fully represented in the table, as in the case of the other four; with regard to which — assigning proper values to M in the different columns of the table, and assuming the members to adhere to one another as firmly as the different portions of each cohere among themselves, a complete truss would be formed in either case (of dimensions as above assumed), sufficient to be used in a bridge required to bear a gross load equal to 4 times the weight of superstructure; provided the proper ratio of safe variable load to weight of structure be as 3 to 1; as is nearly the case with regard to a 100 foot bridge.*

---

* M, in the preceding table, represents a piece of iron, 15′ long sufficient to sustain with safety, a weight W, equal to $\frac{1}{8}$ of the gross maximum load for one truss of a 100ft. bridge. Allowing 1,000lbs. to the lineal foot for movable, and 333lbs. for permanent load, W, represents $\frac{1}{8} \times$ 133,333lbs.= 16,666lbs. Then, reckoning the safe stress of

In such case, the results already obtained, would show the relative cost of the several trusses (excepting the first two), with almost absolute exactness.

But, as the parts of a truss can not be so connected and welded into a single piece, without enlargements at the joinings, by any skill or process now in use, we have to include as an item of cost, in all plans, a considerable amount of material above and beyond the net lengths and cross-sections, as here before determined with regard to the trusses under discussion, required for the lapping of parts, screws and nuts, eyes and pins, &c., to form the connections of the different members with one another.

With regard to the trusses under comparison, no obvious reason presents itself, why any one should require a percentage of allowance for connections materially greater than another. Leaving out the two first, as perhaps already sufficiently discussed, the others consist of about the same number of necessary members, and with the exception of the arch truss, admit of nearly the same forms and connections of parts. The Isometric, or Trapezoid without verticals, presents the fewest lines in the diagram; but some six of those lines represent both tension and thrust members, either separate or combined, which probably complicates the

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iron (thrust or tension), at — say 10,000lbs. to the inch of cross-section, it takes $1\frac{5}{8}$ square inches to sustain the weight W; being about $5\frac{5}{8}$lbs. to the foot, or $84\frac{3}{8}$lbs. for 15′. This, increased by — say, 20 per c. for extra material in connections, gives the practical value of M; which, multiplied by the co-efficient of M in the table, produces approximately, the respective weights of trusses.

Now, $1\frac{1}{5} \times 84.37 = 101\frac{1}{4}$lbs. which multiplied by 113.345, the co-efficient for the Arch truss, gives for the weight of that truss, 11.476lbs. Add for 10 feet width of platform (with wooden beams), — say 5,000ft. *b. m.* of timber and plank, equal to about 20,000 lbs., and we have 31.476lbs. to represent the permanent load of the truss. But we have assumed a truss proportioned to sustain with safety 133,333lb., which is a little more than 4 times the weight of structure here above estimated as supported by the truss.

details of connection, quite as much as the extra three members in truss No. 4. The Post truss presents the larger number of acting members, even omitting six counter ties seen in the diagram, with apparently no advantage as to modes of connection. Both the Post and the Isometric have 10 members represented in the 4th column of the table, whereas the Whipple truss has only 7, and these the shortest of all; and, as the material in these parts manifestly acts at a disadvantage, they being comparatively long and slim, and sustaining slight action, any excess in their number, would seem to be unfavorable to economy.

It is believed, however, that the Post truss would be improved in economy by reducing it to a trapezoidal contour, as, for instance, by removing the parts outside of *bx* and *kn* (Fig. 49), and changing the tension pieces *av* and *ol* for others connecting *b* with *v*, and *o* with *k*; thus converting the figure to a trapezoid very similar to that of Fig. 50; and, by striking out one panel from the latter, and arranging parts as in Fig. 20, except as to inclination, the relative merits of inclined and vertical posts, as represented in these two plans may be fairly tested.

Analysis of trusses modified as just indicated, show tension material slightly in preponderance with the vertical, and thrust material a little the greater with inclined posts; the average being about *one per cent* greater in the case of vertical posts.

This balance, though trifling in amount, is upon the side where it was to be lookod for, in view of the result of investigations had with reference to Figures 12 and 13 [XXXIX—XLVI], as well as the case of the Isometric. Both the Post truss and the Isometric, as to principle of action, may be classed with Fig. 13, where weight is transferred from oblique to oblique, and not from oblique to vertical, and the contrary. The same may be said of truss Fig. 15, sometimes called the Triangular, in which verticals are used merely to transfer the action of weight from the point of application to the connections of the obliques; after which, the weight has no action upon verticals.

Now finally, we see by table of results, that if the Post truss be changed to the trapezoidal form, as above suggested, it will occupy a position, as to amount of material, or more strictly speaking, the amount of *action* upon material, between Fig. 50 and Fig. 51; which latter differ from one another less than 2 per cent; a difference, which would undoubtedly be increased somewhat, under different general proportions of trusses. For instance, while Fig. 50, shows an inclination of diagonals used in connection with verticals, probably nearly approaching the *optimum*, Fig. 51, though superior to the true Isometric (with angles of 60°), in the greater inclination of its obliques, would give still better results with an inclination of about 40°.

CXXXIX. On the whole, we must look to other quarters than the amount of action upon material, for plausible ground upon which to found a decided preference for either of the three plans in question. A difference of two or three per C., and even more, may easily result from greater or less facility of constructing and erecting the structure, while a regard for appearance may also be worthy of consideration. Hence, Engineers and builders will adopt one or another plan, according to individual taste and judgment, and the one who carries out the principles of either system with the greatest skill, and the best materials and workmanship, will probably produce the best bridge.

Judging from the preceding tabulated statement, the arch truss seems, *prima facie*, to labor under a somewhat formidable disadvantage in the fact that it shows an amount of action upon material 10 or 15 per cent. greater than the three preceding plans just especially referred to. But for the light of experience, we might be led to discard the plan without a trial.

But, having chanced to be the first plan of iron Truss successfully put in use, and having had its capabilities fully tried and demonstrated, before any formidable competitor appeared in the field, it could not be dislodged from its position, until a rival plan could not only theoretically, but also practically demonstrate its superior claim to public favor.

The result has been such as to show that even a very considerable excess of action upon material, may be overbalanced by more advantageous action of *thrust material*, and greater simplicity and facility of construction; insomuch that the Whipple Patent Arch Truss, with trifling modifications from the original pattern, has competed successfully with all other plans, for the class of structures it was originally designed and recommended for (common bridges of 50 to 100 feet), during more than a quarter of a century, which has been fruitful in efforts at improvement in iron bridge construction.

COUNTER BRACING.

The elasticity of solid materials, is manifested in bridge trusses, by their downward deflection under load, and the recovery of their previous form and position on the removal of the load.

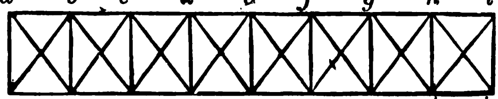
This arises principally, from the temporary elongation of parts exposed to tension, and the contraction of those exposed to compression, according to laws and principles supposed to be understood.

The deflection of trusses within the usual limits, when properly proportioned, is not essentially detrimental to their safety and durability; but rather enables them the better to resist sudden impulses,—except in case of a regular succession of impulses, at intervals corresponding with those of the natural vibrations of the structure, or with some multiple or even division thereof; a result frequently noticeable, and sometimes, to a degree somewhat unpleasant to the eye, as well as suggestive of danger. Hence, great emphasis is often employed, in expressing the supposed advantages of "counter bracing," as a means of *stiffening* trusses, and preventing, or diminishing their vibration.

What is technically called "counter-bracing," as applied to bridge trusses, is the introduction of a set of diagonal, or oblique pieces or members, to act in antagonism to the main diagonals, whether acting by tension or thrust, which contribute toward sustaining the weight of structure and load; the object being, to retain in the truss when unloaded, more or less of the deflection produced by the load, when the truss *is* loaded.

My object at the present time is, to exhibit the process and results of my investigations as to the theory and effects of this counter-bracing, as usually practiced in bridge building, and to state the conclusions arrived at, as to the value of counter-braces, towards effecting the object proposed.

FIG. 52.



I assume a truss (see Fig. 52) composed of horizontal chords (of equal lengths), at top and bottom, vertical posts, and diagonal tension rods, inclined at 45°, or at any other given inclination,—the truss being uniformly loaded from end to end, and so proportioned that all of the above named parts, in that condition of the load, shall undergo an amount of extension or compression, proportional to the respective lengths of parts, multiplied by a constant factor (E), equal to the elastic change effected in a length equal to that of the uprights between centres of chords, which is assumed as the unit of length for the occasion. Then, let L represent the length of truss, P, the number of panels, H, equal to $L \div P$, the horizontal reach of diagonals, and D (equal to $2LE$), the difference in length, occasioned by extension of lower, and compression of upper chord.

Now, assuming no change in lengths of diagonals and verticals, it is manifest that the chords assume, in these circumstances, the forms of two similar and concentric arcs of circles, of which the difference in length is to the mean length, as the difference of radii is to the mean radius, R.

But the difference of radii manifestly equals the distance between chords, equal to 1. Using, then, the representative signs before adopted, we have

$D : L :: 1 : R$; whence ...... $R = L \div D$.

Now, the depression at the centre of the truss, is evidently equal to the versed sine of half the arc made by the chords, and is found with sufficient nearness, by the equation . . Dep. $= (\frac{1}{2}L)^2 \div 2R, = \frac{1}{8}L^2 \div R$. Then, substituting $L \div D$ for R, we have dep.$= \frac{1}{8}L^2 \div (L \div D), = \frac{1}{8}DL$.

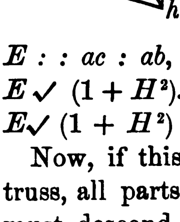
Hence, if the length of truss equal 8 times the depth, or 8, the deflection due to this cause, will equal the difference in length of the two chords, produced by their extension and compression.

Again, if length equal 6, then, dep.$= \frac{1}{8}D \times 6, = 3D \div 4, = 9E$.

The depression resulting from extension of diagonals, may be illustrated as follows.

If the points a and b of a rectangular panel $abcd$ (Fig. 53), be fixed, and ac be extended by an addition equal to eh to its length, produced by the action of weight at c, either directly, or through the upright dc; the points d and c will fall to g and h, and the very small triangle ceh (eh representing only the elastic stretch of ac), will be essentially similar to abc; whence, $ch : ac :: eh : ab$, and $eh : E :: ac : ab, :: \surd(1 + H^2) : 1$... Therefore, $eh = E\surd(1 + H^2)$. But $ch : \surd(1 + H^2) :: eh : 1, :: E\surd(1 + H^2) : 1$; consequently, . . $ch = E + EH^2$.

Fig. 53.



Now, if this represent one of the end panels of a truss, all parts of the truss between the end panels, must descend through a space equal to ch, in consequence of the extension of diagonals in the two end panels; and so for each succeeding pair of diagonals, to the centre of the truss. Therefore, the depression in the centre, due to the stretching of diagonals, must be equal to $\frac{1}{2}P \times (1 + H^2)E, = (\frac{1}{2}P + \frac{1}{2} PH^2)E.$

The depression in the centre of the truss, due to compression of uprights, is simply equal to the aggregate compression of all the uprights on either side of the centre one, and consequently, equal to $\frac{1}{2}PE.$ This amount added to that produced by extension of diagonals, as above determined, makes, for uprights and diagonals together, a depression equal to $(P + \frac{1}{2}PH^2)E.$

The value of this last expression, length and depth of truss being the same, varies slightly with variation in number and width of panels, but not so as to be a matter of practical importance.

Assuming $L = 8v, = 8$, we find that,

$P = 2$, and $P = 16$, make $(P + \frac{1}{2} PH^2)E = 18E.$
$P = 4$, and $P = 8$ " " $= 12E.$
$P = 6$, makes " $= 11\frac{1}{3}E.$

6 panels, therefore, seem to produce the least deflection.

The deflection resulting from changes in lengths of chords, has been shown to be equal to $\frac{1}{8}LD$; and, substituting PH for L, & $2PHE$ for D, we have.. $\frac{1}{8}LD = \frac{1}{4}P^2H^2E$, =deflection from change in chords.

The term E, then, with the following co-efficients, expresses the depression at the centre of the truss, resulting from all changes in length of parts, namely, for chords, $\frac{1}{4}P^2H^2$, for diagonals, ... $\frac{1}{2}P + \frac{1}{2}PH^2$, and, for uprights, $\frac{1}{2}P$.

Hence, the deflection of a truss, under the conditions here assumed, depends upon three simple elements, represented by the letters, P, H, & E; and is expressed in the following general formula;

Deflection $= (\frac{1}{4}P^2H^2, + \frac{1}{2}P + \frac{1}{2}PH^2, + \frac{1}{2}P) \times E.$

The parts of this aggregate co-efficient of E, referring respectively to chords, diagonals, and uprights, are separated and distinguished by commas.

The formula just given is equally applicable in case of thrust diagonals and tension verticals; as will be made obvious by a moment's examination of the principles involved.

Now, if the truss could be anchored down, by ties and anchorage absolutely unyielding, to the point of its utmost deflection under load; the load might be removed, and replaced, without any rising or falling of the truss,—the load and the anchors alternately retaining the deflection, and preserving a constant and uniform strain upon the truss.

The same effect is *partially* produced by counterbracing; and the object of the present investigation is, to determine, approximately, at least, to what extent this may be done, and what is the real advantage of counter-braces, in trusses with parallel chords; beyond where they are necessary, to counteract the effects of unequal variable load, upon the different parts of the truss.

We have seen that deflection results from three causes, all, of course, depending upon elasticity; namely; difference effected in lengths of,—first chords, second, diagonals, and third, uprights.

The theory of counter-bracing is, that by the introduction of antagonistic diagonals, the material is prevented from regaining its normal state on removal of the load; and consequently, that it yields to the reimposition of load, to much less extent than it would do, in the absence of counters.

As to the deflection due to the difference in lengths of chords, equal, as shown by the general formula one page back, to one-half of the whole, for a truss in which $L = 6H$, and to more than half, when L is greater than $6H$; the counter-diagonals have no tendency to retain or diminish that difference, or the deflection produced by it. The diagonals and counters, simply contract or extend (according as they act by tension or thrust), the two chords equally, without affecting the difference between the two.

On the contrary, the action of the counter diagonal tends to retain the tension (or thrust, in case of thrust diagonals), of the main in the same panel, and also, the compression (or tension), of uprights; and, in as far as that is accomplished, the deflection due to the elasticity of those parts, is retained, on removal of load from the truss.

Suppose, in a truss with tension diagonals, loaded and depressed as already explained, and all parts extended or contracted to the amount of $E \times$ respective lengths; a counter-diagonal to be inserted in each panel, crossing the mains, as shown in the diagram (Fig. 52), and of half the size of the latter, such being the usual proportion for counters.

Now, the counters being adjusted so as not to act while the load is on, but ready to act immediately, as the main diagonals begin to contract, then, the load being removed, the main will contract by its elasticity, opposed by the counter, until they come to an equilibrium; each sustaining the same amount of tension. Still, the aggregate extension of the two beyond the natural state, must be essentially the same as that of the one, under the load; the one gaining, just as fast as the other loses.

But the main, having a cross-section twice as great as the counter (chords and uprights retaining the same lengths), must lose two-thirds of its tension, while the latter is acquiring strain enough to withstand the remaining third. Hence, 2 thirds of the deflection due to extension of diagonals, is recovered, on removal of the load, while the counter-diagonal retains the other 1 third.

But the posts (the greater portion of them), do not remain stationary as to length, as above assumed; the main and counter diagonals together, exerting, obviously, only $\frac{2}{3}$ as much action upon them in the new condition, as the former exert under load, they are relieved of $\frac{1}{3}$ of their aggregate *stress* under load; but do not recover in the same degree, their original aggregate length; for the relief falls mostly upon the larger uprights, where the relative effects are less than the average.

To illustrate the case as to uprights — if equal weights act at the nodes of the lower chord (Fig. 52), the compressive action upon the posts at *p*, *q*, *r*, and *s*, is obviously as 1, 3, 5, 7, respectively, or as 3*n*, 9*n*, 15*n*, 21*n*. [See analysis of Fig. 12]. Then,— Counter-bracing, and removing load; *sa* is relieved of two-thirds of its stress, equal to 14*n*, while *bs* exerts a force of 7*n* upon *br*, making with 5*n* retained by *bq*, a total of 12*n* upon *br*, and showing a relief of 3*n*. Again; *cq* receives 5*n* through *cr*, and 3*n* through *cp*, = 8*n* in all, being a relief of 1*n*. But *dp*, receiving 3*n* through *dq*, and 1*n* retained by *do*, sustains 4*n*, being an *increase* of 1*n*.

Now, as these uprights are assumed to undergo the same contraction under load, $\frac{1}{4}$ of the deflection on account uprights, is due to each. Therefore, *sa* being relieved of 2-3ds of its action, restores . . 2-3ds of $\frac{1}{4}$, (= 16.6 per C.), of that deflection. In like manner, *br* restores 1-5th of $\frac{1}{4}$, = 5 per C., and *cq* 1-9th of $\frac{1}{4}$, or 2.8

per C., making, for the pieces together, 24.4 per C., restored. This is diminished by 1-3d of $\frac{1}{4}$, or 8.3 per C. (on account of increased compression upon *dp*), leaving a balance of 16.1 per C. only, of deflection from contraction of uprights, which is restored in spite of counter-diagonals, in the case under discussion.

Moreover, the main and counter diagonals, producing more or less effect of contraction upon the chords, according to the degree of inclination of the former, and the cross-sections of the latter, it may, perhaps, be reasonably assumed, that the contraction thus effected in the horizontal, is a full offset to the 16 per C. of expansion in the vertical sides of panels, as above shown; so that we may regard the whole deflection from uprights, as being retained by counter-diagonals.

To state the full result of the foregoing investigation then, we find in case of Fig. 52, which is a fair representative of the average of trusses; that counter-bracing, obviates all the deflection due to compression of uprights, together with $\frac{1}{3}$ of that resulting from extension of diagonals; and, making $H = 1$, in the formula for deflection (p. 267), we have — deflection saved by counter-diagonals, $= (\frac{1}{3} \times 8 + 4) \div 28$, = a little less than 24 per C. of the whole deflection. If $H = 0.75$ (truss 52), the result would be about $31\frac{1}{2}$ per C. saved.

But even these results are based upon conditions never occurring in practice. It has been assumed that all parts of the truss undergo equal degrees of change under a full load; which may be nearly true with respect to chords, but not to other parts. The maximum action upon *od* and *dp* (Fig. 52), requires those parts to be $2\frac{1}{2}$ times as great, as they need be under full load; while *pc* and *cq* require $\frac{1}{4}$ more, and, *qb* and *br*, 1-20th more cross-section at the maximum, than under a full load of the truss.

Now the deflection resulting from elasticity in these parts, being less in proportion as the parts are greater, the saving by counter-bracing, must be less in the same degree, as far as it relates to such parts. This at once reduces the above computations for deflection retained, from 31½ and 24, to 25 and 19 per C., for the two cases respectively; and, considering the increase of section required for uprights (in iron trusses), on account of great length and small diameter, as heretofore alluded to, it is deemed to have been fully demonstrated, that the effects of counter-diagonals, of half the size of the mains, are, to retain in the truss when unloaded, from one-sixth or less, to one-fourth of the deflection produced by a full movable load.

But it has been seen in the progress of our investigations as to the action of load upon the different parts of the truss, that counter-diagonals are required in one or two panels on either side of the centre, and there, they can not be safely omitted. But, beyond the point where the weight of structure acting on the mains, begins to overbalance the effects of unequal and variable load upon the counters, I do not consider the advantages of counter-diagonals to be sufficient to warrant their use.

In the case of rail road trains, gliding smoothly over bridges of ordinary spans, a quarter or a half of an inch more or less of deflection, is of slight importance, while, in bridges for ordinary carriage travel, the only objection to it is, that it slightly increases the degree of vibration produced by successive impulses, as of the trotting of animals, in time with the natural vibrations. Now, counter-bracing tends to shorten the intervals of the natural vibrations by diminishing their extent; but can not destroy the liability to vibration; and the alteration of interval produced, may as often bring the vibrations nigher in tone with the gait of a trotting horse, as otherwise. In certain cases the effect would be one way, and in others, the opposite; and in general, the only result would be, to diminish the extent of motion; by one quarter, or less.

Such is the result of the best reasoning and science that I have been able to bring to bear upon the subject of counter-bracing.

To find the actual maximum deflection of a truss it is only necessary to know the value of P and H, and to assign to E a value determined by the character of material, and the stress upon the several parts under full load.

In Fig. 52, if $H = 1 = 12\frac{1}{2}$ft., and the tension of wrought iron equal 15,000lbs. per square inch, the value of E for that material, will be about 0.0075 ft.; and this will apply to the lower chord, and the obliques, *ar* and *li*. But the average value of E' for diagonals of wrought iron, would be about 0.006ft.

For cast iron, 11,000lbs. to the square inch, requires about the same value for E, as 15,000 upon wrought I.; and, as that is a fair working rate of compression for cast iron in the upper chord, .0075ft. may be taken as the value of E for chords, in general. Uprights, for reasons heretofore explained, require a value for E'', not greater than .005ft.

The above values of E and H, substituted in the formula $(\frac{1}{4}\,P^2H^2, + \frac{1}{2}P + \frac{1}{2}PH^2, + \frac{1}{2}P,) \times E$, it becomes $\frac{1}{4}P^2H^2E + (\frac{1}{2}P + \frac{1}{2}PH^2)E' + \frac{1}{2}PE''$, equal to $\frac{1}{4} \times 64 \times .0075, + (4 + 4)\,.006, + 4 \times .005, =, 0.188$ft. $=$ about $2\frac{1}{4}$ inches. Hence, a well proportioned wrought and cast iron truss, one hundred feet long, by 12½ feet deep, may be depressed 2¼″ in the centre by a distributed load (including structure), with tension not exceeding 15, and thrust, not exceeding 11 thousand pounds to the square inch in cross-section of iron.

WOODEN BRIDGES.

STRENGTH OF TIMBER, &c.

CXL. The qualities of wood as a building material, have been extensively treated of by authors whose works have long been before the public, with a degree of ability and research to which the present writer can make no pretensions. He will therefore at this time, simply state the conclusions arrived at from reading and observation (coupled with some experimental research) with respect to the average absolute strength, positive, negative, transverse, and to resist splitting, in certain cases; of the timbers principally in use for building purposes; as also, the forces they will bear with safety under various circumstances; leaving it, of course, for others to adopt his views for their own practice, or to modify and correct them, according as their greater experience or better judgment may dictate.

At the same time, the author may be allowed to express his firm belief, that the views about to be presented, if fairly observed, will lead to the adoption or continuance of a safe and economical practice as to the proportioning of timber work in bridge construction.

Pine timber in this country is perhaps to be ranked as among the most valuable timber in use for building purposes; especially in bridge building. White oak, and some other varieties, are preferred for certain purposes, as being harder, stiffer, and especially better calculated to sustain a transverse action, whether tending to bend or crush it. But in what follows, reference will principally be had to the ordinary white pine of this country; and the deductions here made, may readily be modified so as to apply to other materials of known strength, when so required.

The absolute positive, or tensile strength of pine, may be stated at about 10,000lbs. to the square inch of cross-section. It might therefore seem to be safely reliable in practice, at 15 or 16 hundred pounds to the inch, upon that part of the section of which the fibres are not separated in forming connections with other parts of the structure. And so it probably would be, when new, sound, and straight grained. But timber in bridges, is usually more or less exposed to wetting and drying, and deterioration in strength,— especially as it regards *tension*. Moreover, in forming connections of parts and pieces in a structure, it is difficult to secure a uniform strain upon all the uncut fibres;— one side of the piece being often exposed to much greater stress than the other. In view of such facts, it is deemed advisable to seldom allow less than one square inch section of unbroken fibre to each 1,000lbs. of tensile strain.

NEGATIVE STRENGTH OF TIMBER.

CXLI. The ability of pine to resist compression in the direction of the length of piece, is from 4 to 5 thousand pounds to the square inch of section, and this varies but little, whether the pieces be of length equal to once, or five or six times the diameter. It moreover diminishes only about one-third with an increase of length up to 18 or 20 diameters.

Now, if we take about ⅛ of the absolute strength, say 800lbs. to the inch for a length of 6 diameters, and 560 for 18 diameters, and substract 40lbs. per inch for every increase of 2 diameters in length, between 6 and 18 diameters; and from 18 to 40 diameters, compute the quantities by the rule given [LXXXIX], in relation to negative resistance of cast iron, we shall form a table of negative resistances of timber, for a range of lengths which will cover the principal cases that will occur in bridge building, which the author feels confident in recommending for the adoption of engineers and practical bridge builders. If it be desired to extend the table to greater lengths than 40 diameters, the formula which makes the strength as the cube of the diameter divided by the square of the length, may properly be used.

The following brief table of negative resistance of timber, has been constructed in the manner above indicated, and exhibits at a single view, the number of pounds to the square inch of cross-section, which timbers of different lengths will bear with safety, at inter-

Table of Negative Resistance of Timber.

Diameters.	Pounds.	Diameters.	Pounds.	Diameters.	Pounds.
6	800	24	368	42	166
8	760	26	328	44	151
10	720	28	296	46	138
12	680	30	269	48	127
14	640	32	246	50	117
16	600	34	227	52	108
18	560	36	210	54	100
20	479	38	195	57	90
22	416	40	183	60	81

vals of 2 diameters in length, for all lengths between 6 and 60 diameters. The first column gives lengths in diameters, and the second, the number of pounds to the square inch, borne, with safety.

TRANSVERSE STRENGTH OF WOOD.

CXLII. Pine timber will bear a transverse strain of 1500 or 1600lbs. to the square inch of cross-section; that is, the projecting end of a beam will bear 1500lbs. for each square inch of its cross-section, applied at a distance from the fulcrum equal to the depth of the beam; the force acting parallel with the sides. In other words, a beam 1 inch square upon supports 2 inches apart, will sustain 3,000lbs. midway of supports, provided the timber be not split or crushed; as would certainly be the case with so short a leverage.

It will therefore be proper in practice, never to expose this material to a greater transverse strain than 250lbs. (upon a leverage of 1 diameter), to the square inch; and, to calculate the strength of a projecting beam, this quantity should be multiplied by the cross-section and the depth. and the product divided by the distance of the load from the fulcrum. [XCIV.]

For the safe load in the middle of a beam supported near the ends, take four times the above quantity (= 1,000lbs.), multiply by cross-section and depth, and divide by length between supports.

A beam will bear twice as much load uniformly distributed over its length, as when it is concentrated in the centre, in case the beam is supported at the ends, or at the end in the case of a projecting beam.

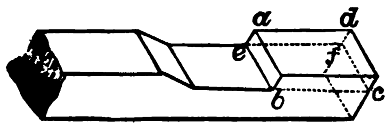
But these are familiar principles and need not be dwelt upon in this place.

CLEAVAGE.

CXLIII. In order that a piece of timber may act by tension, it is necessary that a portion of its fibres be separated, to form a heading for the stretching force to act against; and, that the strength of the piece may be made available for as great a part of its length as may be, without having the head split off, it becomes important to know the power of the material to resist such a result.

Let *ab* Fig. 54 represent a heading by means of which the stick is made to act by tension. Now, as the timber is incapable of supporting upon the ends of its fibres with safety, for a great length of time, a force of more than 8 or 10 hundred pounds to the square inch, the area *ab* should contain at least one square inch for each 1,000lbs. to be applied to it. And, if the head *ab* be too nigh the end of the stick, the part *abcd* will

FIG. 54.



split off, and be thrust over the end of the timber. It is found by experiment that to produce this effect upon timber of sound and straight grain, requires a force of nearly 600lbs. to the square inch of cleavage in the area *efcb*. It is therefore obviously necessary to safety, that the head *ab*, be at a distance from the end, equal to at least 10 times the depth (*ae*) of the head, that the area of cleavage may be sufficient to stand as great a force as the area of head can stand; i. e., there should be 10 inches of cleavage surface to one inch of head surface.

If the heading be formed in the central part of the stick, as by a mortice or pin hole, two cleavages must be made from the hole to the end in order that the part may be forced out. Hence, the hole need be only about five times the width of hole from the end; that is, an inch hole should be five inches, and a two inch hole, 10 inches from the end.

TRANSVERSE CRUSHING.

Timber is sometimes liable to be crushed by forces acting transversely to the direction of its fibres. If the pressure be applied to the whole side of the piece, it should not exceed 150, or at most 200lbs. to the square inch, in practice. If acting on one-half of the surface, it may perhaps, be 300lbs. to the inch, without yielding very injuriously; and, for a very small portion of surface, as under a bolt head or washer, a pressure of 500lbs. to the inch may be admissible. These limits are taken with reference to pine timber. Hard timbers, will bear, probably, 25 to 50 per C. more with safety.

CONNECTIONS OF TENSION PIECES, AND PROPORTIONATE AMOUNT OF AVAILABLE SECTION.

CXLIV. From what has been already said, it follows that for a piece to act with the best advantage by tension, if the connection be made all at one point in the length, one-half of the fibres require to be cut off, so as to form an area of heading equal to the cross-section of the remaining part of the stick; since it has been assumed that the power to resist tensile strain with safety, is the same as the power to resist compression upon the ends of fibres. But if several headings, or shoulders be made at different points, or distances from the end, a less portion of the fibres require to be separated.

If, upon a piece 4 inches thick, instead of one shoulder 2 inches deep at 20 inches from the end, we make two of one inch deep, each, the one at 10, and the other at 20 inches from the end, we have the same area of shoulder, and 50 per C, more fibres to act by tension; which may be made available by another shoulder at 30 inches from the end. Thus a greater proportion of the fibres, but a less proportion of the length is available.

In the same manner, if a piece be connected by pinning, requiring 2 pins of 2 inches in diameter, at 10 inches from the end, four 1 inch pins, two at 5, and two at 10 inches (if stiff enough), give the same shoulder surface, and require the cutting of only half as many fibres; and, two more pins at 15 inches from the end will give $\frac{3}{4}$ths of the whole area of section available for tension. In case the smaller pins be not stiff enough, they may be of an oblong section in the direction of the strain.

A still further reduction of depth of shoulder or width of pin, will make a still larger proportion of the fibres available, but not so much *length;* and, experience and judgment, with a little calculation, must dictate as to the proper medium in this respect. The theoretical limit is, when the shoulders are infinitely small, in which case, the whole cross-section becomes available. But, as the resistance to cleavage must be equal to the force of tension, it follows that the *loss* in available length, is proportional to the amount of cross-section available for tension.

In practice, it is usually not expedient to estimate more than one-half or two thirds of the whole section as available for tension. This reduces the safe practical strain for timbers sustaining tension, to from 500 to 700lbs. to the square inch, for the whole cross-section; and the proper point between these limits should be determined by the mode of forming the connections in specific cases.

PINS OF WOOD AND IRON, FOR CONNECTING TIMBERS IN BRIDGE WORK.

CXLV. Perhaps no more suitable place will occur for making a few general remarks upon the merits and use of pins for connecting pieces of timber.

While it is readily admitted that the plank lattice girder, put together exclusively with wooden pins, answered an excellent purpose in affording cheap and serviceable bridges in this country when timber was abundant, and the iron manufacture in its infancy, it is nevertheless believed that the use of wooden pins in bridge construction, is not destined to a long continuance. Where pins are required in wooden bridge work, it is thought that iron may be used with a decided advantage over wood — not in the lattice bridge of the usual form, composed of a great number of diagonals, and a legion of connecting pins; but in a modified form (as in Figures 13 and 19), with a greatly reduced number of pieces, and points of connection.

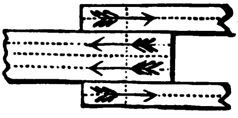
Wooden pins for the purpose under consideration, do not possess sufficient strength in proportion to the surface, unless made so large as to require too much cutting of the timber. Moreover, the action upon the pin tends to crush it laterally, in which direction the hardest timbers available for pins, scarcely offer as much resistance as the ends of fibres to which they are opposed.

Where pieces are connected with their fibres parallel, wooden pins or keys with cross-sections elongated in the direction of the grain, to give them the necessary strength, may be employed without too much cutting of the timber. But, as just remarked, the key is liable to yield before the cut ends of the fibres are taxed to their full capacity. It is therefore poorly adapted to the purpose in any case where great strength is required. Moreover, when the pieces to be connected are placed across one another, the hole will not admit of elongation without too much cutting of at least one of the pieces.

If it be required to connect a piece by a pin between two other pieces as seen in Fig, 55, upper diagram, the pin, as already seen, should be strong enough to bear as much strain as the opposed surface can sustain. Now, we have seen that this can scarcely be done by wooden pins. Still if sufficiently stiff, they may yield somewhat to compression, without material loss of strength.

Taking the transverse strength of pin timber at 300℔s. to the inch, with leverage equal to diameter, the expression $4 \times 300ad \div l$ (a representing the cross-section, d, the diameter, and l, the length of pin, between centres of outside bearings), gives the amount which the pin will bear in the middle.

Now, the two outside pieces, having each half the thickness of the centre one,* l must equal $1\frac{1}{2}$ times the thickness (t), of the middle piece; while the effect



* The outside bearings may be regarded as concentrated at the centres of thickness of the pieces, while the stress of the pin in the centre, is the same as if the force exerted by the middle timber, were concentrated half and half at the centres of the two halves of the piece; see diagram.

of the force exerted by said middle piece, is two-thirds of what the same force would produce, if concentrated in the middle of the pin, and consequently, the pin will bear 50 per C. more. Hence, we have $4 \times 1\frac{1}{2} \times 300ad \div 1\frac{1}{2}t, = 1200ad \div t, =$ strength of the pin.

But the opposed surface will bear $1{,}000td$; and putting this expression equal to the former, and deducing the value of d, in terms of t, it will show the smallest diameter of a wooden pin, strong enough to bear as much as the opposed surface. This equation gives $d = 1.03t$; † whence, it appears that the wooden pin should be 3 per C. greater in diameter than the thickness of the middle timber.

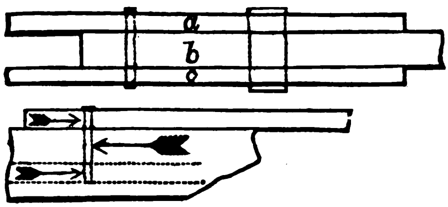
In the same manner, the strength of an iron pin in the same circumstances, is respresented by $4 \times 1\frac{1}{2} \times 5{,}000ad \div 1\frac{1}{2}t, = 20{,}000ad \div t$, which made equal to $1{,}000td$, gives $d = 0.252t$, hence, the most economical diameter for an iron pin in fastening one piece between two others, is about $\frac{1}{4}$th the thickness of the middle piece; i. e., taking the stiffness of a round pin at 5,000℔s. But reducing it to 4,500℔s. as proposed in another place [XCVIII], it gives $d = 0.266t$; whence, even upon this basis, it will be safe in practice to make $d = \frac{1}{4}t$, and the whole length of pin $= 2t$, so that it may extend into the outside pieces to the extent of half the thickness of the middle piece.

Since the outside pieces (Fig. 55), require half the thickness of the middle piece, and the pin requires a diameter equal to $\frac{1}{4}t = \frac{1}{2}$ the thickness of outside pieces, it follows that in pinning or spiking a plank or timber to the outside of a thicker piece, the pin or spike should

† Dividing the equation $1200ad \div t = 1{,}000td$, by $100d$ and multiplying by t, give $12a = 10t^2$. But $12a = 12 \times .7854d^2, = 9.4248d^2, = 10t^2$. whence, $d^2 = 1.061t^2$, and $d = \sqrt{1.061t^2} = 1.03t$.

have half the thickness of the piece attached, that it may not bend with less force than the ends of the severed fibres can bear; and should extend into the thicker timber at *least* 6 times its diameter. For, as

FIG. 55.



the inner portion of the pin or spike, must act upon the wood in the same direction as the part through the attached piece, it requires the same amount of surface to act upon, while the intermediate portion requires a surface equal to that acting upon the two end portions. And, even in this condition, the pressure is not uniform upon all parts of the length of the pin, since there is a neutral point, as represented by the upper dotted line (lower diagram, Fig. 55), where the pressure changes from one side to the other, and, near this point, must be very light in both directions. Hence, for the most perfect results, in such cases, the pin should probably enter the thicker timber to a distance of 7 or 8 times the diameter of the pin.

When the end bearings of the pin act transversely to the grain, they require at least 50 per C. more extent of bearing, or even twice as much, when practicable. At 50 per C. $l = 1\frac{3}{4}t$, and the effect of the pressure exerted by the middle piece, is $\frac{5}{7}$ths that of the same force at the centre of the pin. The equation for the proper diameter of the pin, then, is $4 \times \frac{7}{8} \times 5{,}000ad \div 1\frac{3}{4}t = 1{,}000td$; whence, $d = 0.283t$, and length of pin $= 2\frac{1}{2}t$.

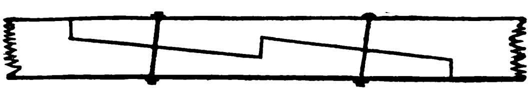
SPLICING.

CXLVI. The term *splicing*, as applied to timber work, may be defined to be the uniting of two pieces of timber by their end portions, so as to form (in figure) a continuous timber upon a straight axis.

The splicing of timber to withstand a thrust action, requires only the meeting of the squared ends of pieces; or, a half lap, formed by removing the half of each for a foot or two, more or less, from the end, and lapping the remaining halves, so as to have the extreme end of each, meet the shoulder of the other.

But the splicing of pieces to withstand tension, obviously requires a more complicated process; and, from what has already been said, [CXLIV,] it is clear that only a part of the absolute section can be made available to withstand a tensile strain.

FIG. 56.

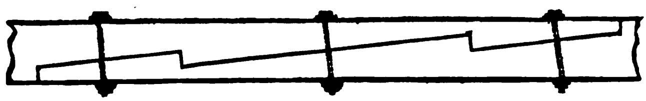


In Fig. 56, we have the profile of a lock splice, by which one-third of the section is available for tension; the depth of the locking being equal to one-third of the thickness of timber. Now, that the locking may not split off, we have seen that the lap should extend 10 times the depth of lock, each way, making a lap of 6⅔ times the thickness of the timbers.

By slanting the timber to a thickness at the end equal to that in the neck of the lock, we lose none of the cleavage required to split off the hook, while we gain in amount of section where it is required for bolt holes to secure the splicing. Otherwise, the bolt holes would reduce the available section below one-third of the whole.

It is proper to observe with regard to this splice, and also the succeeding one, that the power being applied upon the reversed shoulder, or hook, out of the line of the unbroken fibres which resist the power, the tendency is to throw the ends outward, and produce a degree of lateral action, which weakens the timber to a somewhat greater degree than in proportion to the amount of fibres severed.

FIG. 57.



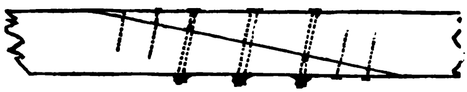
With a double lock splice, as in Fig. 57, one-half of the section is available. This requires a lap of 10 times the thickness of the timber.

By three lockings upon the same principle, $\frac{3}{5}$ of the fibres may be utilized for tension, with a lap of 12 thicknesses (or $12t$.), and, by a lap $13\frac{1}{3}t$, we make two-thirds of the fibres available. Finally, by a lap of $20t$. and an infinite number of lockings the whole cross-section would be available.

But this, of course, is a point not attainable in practice. From $\frac{1}{3}$ to $\frac{2}{3}$ — say an average of $\frac{1}{2}$, is as much as can be reckoned on, and about as much as can usually be made available for tension, at the end connections of a single timber.

Splicing may also be effected by a plain scarf, with bolting, pinning and spiking, as indicated in Fig. 58. With bolts, pins and spikes properly arranged and proportioned, a strong splice may be formed in this manner, with a less lap than what is required in the lock splice. In this case the fastenings should pass

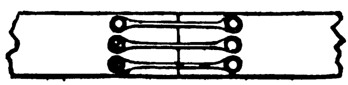
FIG. 58.



through at right angles with the plane of the joint, that they may not be slackened by a slight yielding of the timber to pressure, in the holes. This, however, is a device which will probably, seldom be resorted to in bridge construction.

Timbers may also be shackled together end to end by iron bolts and straps, as shewn in Fig. 59. The aggregate cross-section of straps should be about 1 square inch to each 10 to 15 thousand pounds of strain which the splice is intended to bear; and the diameter of bolts fastening the straps, about one-fifth of the thickness of timber, to secure the greatest effect for the amount of section destroyed in cutting the bolt hole.

FIG. 59



To connect two timbers 10×12 inches, so as make half of the fibres available for tension, we may take 6 straps 2 feet long from hole to hole, and containing a cross-section of about 1 square inch, each. Also 6 bolts of 2″ in diameter, and arrange the straps and bolts as shown in the figure, the straps being placed upon the 12″ sides. This will cost, say for 170lbs. of iron at 7cts., $11.90.

The expense of a double lock splice (Fig. 57), will

be about 7 cubic ft. of waste timber,......	$3.60
40lbs. of iron bolts, washers and plates,...	2.80
Labor in fitting the timbers, say,..............	1.
Total,...	$7.30.

showing the shackle connection to be from 4 to 5 dollars the more expensive.

CONSTRUCTION OF WOODEN TRUSSES.

CLVII. With a thorough comprehension of the power of timber to resist the various kinds of strain to which it may be liable in bridges, and other timber structures, and of the general principles of forming connections in timber work, as attempted to be explained and set forth in the last few preceding pages; and a knowledge of the general forms of arrangement for the several members in bridge trusses, or girders, and of the manner of computing the stresses to which the several parts are liable to be subjected, as treated of in the first 100 pages or so, of this work, the details of practical construction of wooden truss bridges may be intelligently entered upon.

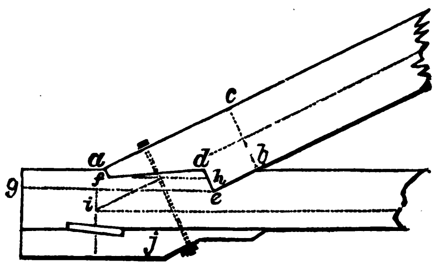
Nothing more elaborate will be here undertaken, than a reference to general forms of trussing suitable for wooden bridges of different spans, and a description of what seem to be the most feasible methods of forming connections at peculiar and specific points.

The method pursued will be, to proceed from the shorter spans, and more simple combinations, to structures of greater length, and requiring a greater number and a more complex arrangements of parts.

TWO PANEL TRUSSES.

CLVIII. The form presented in Fig. 3, with rafter braces *ad* and *dc*, and a tie or chord *ac*, together with an iron tension member *db* (in 1 or 2 pieces), is probably the best adapted to bridges from 20 to 25 feet in length. The braces should meet with a vertical joint at *d* (Fig. 3), and toe into the chord tie with two headings, and one or two small bolts, as in Fig. 60.

FIG. 60.



Assuming the brace to be capable of sustaining a thrust of 500lbs. to the inch of section, and the heading 1,000lbs, to the inch, the aggregate depth of heading, *af*, and *de*, should be one-half the depth *cb*, of the brace; and, the point *f*, should fall below the point *d*, by $\frac{1}{10}$ *ad*, so as to give a length of cleavage *fh*, = 10*af* or 10 *dh*. The shoulder *de*, then, should be,

(1),... $de = \frac{1}{2}cb - \frac{1}{10}ad, = \frac{1}{2}cb - \frac{1}{10}ab + \frac{1}{10}db$.

We here speak of *a d b* as a straight horizontal line, not shown. This is regarding *af* as equal to the vertical depth of cut at *af*; which will be sufficiently near the truth for our present purpose, provided the brace be not very steep.

But (2),...$de = db.$ sin. $dbe, = db.$ sin. cab.

and, putting this value of de equal to the one above, and changing vulgar to decimal fractions, we have,

(3), ..$db.\ \sin.\ cab = 0.5\,cb - 0.1ab + 0.1db$.

Then, transposing, and uniting co-efficients of db.

(4.)...$(\sin.\ cab - 0.1)\,db = 0.5cb - 0.1ab$, whence,

(5),...$db = \dfrac{0.5\,cb - 0.1\,ab}{\sin.\ cab - 0.1}$

Now, from equation (2) we derive $db = \dfrac{de}{\sin.\ cab}$, which value of db being substituted in equation (5), we have

(6), ... $\dfrac{de}{\sin.\ cab} = \dfrac{0.5\,cb - 0.1\,ab}{\sin.\ cab - 0.1}$, whence, multiplying by $\sin.\ cab$. we derive,

(7),... $de = \dfrac{0.5\,cb - 0.1\,ab}{1 - \dfrac{0.1}{\sin.\ cab}}$ Then, substituting for ab, its equal $\dfrac{cb}{\sin.\ cab}$ the last equation becomes,

(8),...$de = \dfrac{0.5\,cb - \dfrac{0.1\,cb}{\sin\ cab}}{1 - \dfrac{0.1}{\sin.\ cab}} = \dfrac{\left(0.5 - \dfrac{0.1}{\sin.\ cab}\right)cb.}{1 - \dfrac{0.1}{\sin.\ cab}}$

Making the angle $cab = 26°33\frac{1}{2}'$, which is regarded as a suitable inclination for the brace, being one, vertical, and two, horizontal reach, $\sin.\ cab = 0.447$, which substituted in (8), gives $de = .356\,cb$, and $af = .144cb$.

This, it will be recollected, is deduced upon the supposition that the brace will sustain a compression of 500lbs. to the inch, and no more; which will depend upon the length as compared with the least diameter. If the brace be capable of bearing with safety, more or less than 500lbs. to the inch, the heading, or butting surface should be more or less than half the area of cross-section, in like proportion. For, if unnecessarily large, it requires too much cutting of the chord, and if too small, the pressure upon abutting surfaces becomes too great.

With the inclination of brace above assumed, the compression upon the brace obviously equals the weight sustained multiplied by $\sqrt{5}$; and, for a rail road bridge, at $1\frac{1}{4}$ tons to the lineal foot, the weight upon each brace, will be 6,250lbs. $= \frac{1}{2}w$; or say, $\frac{1}{2}(w + w') =$ 7,500lbs. This by $\sqrt{5}$, gives 16,770lbs. = thrust of brace, while 15,000lbs. = tension of chord. Now, at 500lbs. to the square inch of gross section, the chord requires 30 square inches, and the brace $33\frac{1}{2}$ inches, being a little less than 6″ square. But the length of brace being about 11ft. or 22 diameters of a 6″ stick, we find by the table [CXLI], the brace is only capable of bearing 416lbs. to the inch. Hence, with a 6″ least diameter, a section of 40.3 inches, or nearly 6″ × 7″, becomes necessary. Still the butting surface required is only 16.77 square inches — a little less than $2\frac{1}{2}$″ depth (at right angles with the brace), by 7″ in width.

This $2\frac{1}{2}$ inches in depth may be divided between the two shoulders at *a* and *d*, in any manner that will leave a length of cleavage from *a* to the end of the chord equal to 10 *af*, or more strictly 10 *af* × cos. *cab*, which equals the vertical depth of cut at *f*. But the line *df*, should preserve a descent, equal to $\frac{1}{10}$th of its length.

The depth of shoulder being thus reduced from $\frac{1}{2}cb$ (= 3″ in this case), to $2\frac{1}{2}$″, *de* is diminished in the same degree, and from $.356cb$, becomes $\frac{5}{6} \times .356cb = .2966cb$; and, substituting 6″ for *cb*, we have $de = 1.78$ inches. In the meantime *af* becomes $\frac{5}{6} \times .144cb$, = .72″.

The vertical depth of cut for *de*, = 1.78″, is 1.78×cos. *cab*, = 1.78×.894, = 1.59″. Add to this the vertical cut at *f*, equal to .642″ and it makes 2.233″, = aggregate vertical depth of cut in the chord, whence the distance *eg* should be 22.33 inches, to afford the necessary resistance to cleavage.

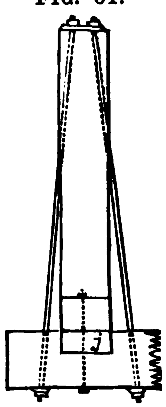
Now, we require in the chord 15 square inches of unsevered fibre, to withstand the horizontal thrust of the brace while we require, as seen above, 1.59× 7 = 11.13 inches to be cut away to form foothold for brace, making aggregate section of chord = 15+11.13 = 26.13 sqr. inches, equal to about 7″ × 3¾″, by strict computation.

Timbers so small, however, although capable of sustaining, without excessive stress, any action to which a bridge is legitimately exposed, is not to be recommended in practice, as the structures might be destroyed by casualties which would but slightly affect the large timbers required in heavier and longer structures.

The centre of bearing of the truss upon the abutment, should be directly under the point *i*, at the meeting of central axes of the brace, and the unsevered portion of the chord. Otherwise, an injurious lateral strain would result to the chord at its weakest point.

The transverse beam at the centre of the truss, may be placed above the chord or below, as preferred, and sustained by 2 suspension bolts descending divergently from a saddle, or double washer at the vertex of the braces, passing through the beam, and secured by nuts and washers upon the under side of the beam, as shown in Fig. 61. The divergence of bolts should be from ⅛th to ¼th their length, and the section of bolts, a trifle more than what is required simply to sustain the weight, as they may act unequally, in consequence of a small lateral tendency of the braces.

FIG. 61.



A small bolt should pass vertically through chord and beam, to preserve them in place. Also, a small bolster, or corbel block (*j*. Fig. 60 and 61), under the chord at the end, affords some protection at the weak point in the chord.

A pair of horizontal × braces in each panel, between beam and abutments, or plate timbers upon abutments, are required to produce lateral steadiness in the structure.

The idea of constructing the trusses of a rail road bridge, even of 20′ span, of 6″ timbers, to persons in the habit of seeing such bridges constructed with timbers 10 or 12 inches square, will undoubtedly suggest visions of catastrophe, courts and coroners; and, in view of liability to casualty, fretting at joints, and perhaps surface decay, it may be advisable to use in such structures, timbers somewhat larger than the above computations indicate as sufficient to withstand determinate forces.

But, as an instance of what strength may be obtained with very small timbers, properly proportioned and put together, it may be here stated that a model of a 20 feet truss, upon a scale of 1 to 12, constructed as above explained, of $\frac{1}{2}'' \times \frac{5}{12}''$ braces and chord, bore without material injury, 350lbs. at the centre, equivalent to 700lbs. distributed, and representing $700 \times 144 = 100{,}800$lbs. upon one truss, or over 100 net tons upon a 20 feet bridge; being some four times as much as a single track rail road bridge of that span is usually subjected to.

With regard to the proper size of transverse beam, the formula (see rule [CXLII]), $\frac{1{,}000a.d}{l} = W$, (a representing area of section, d, depth of beam, l, length between supports, and W, the load in the centre), gives $a = \frac{lW}{1{,}000d}$. Then, assuming $l = 15'$, $W = 7{,}500$lbs. (= 15,000lbs. distributed), and $d = 14''$; we have $a =$

$\frac{15 \times 7{,}500}{1{,}000 \times 1\frac{1}{6}'} = 96.4$ square inches; which divided by depth (d), in inches, gives thickness (t), $= 7$ inches nearly. Or the formula $t = \frac{lW}{1{,}000d^2}$ gives the required thickness directly. But in this case, l and d must express length and depth *in inches*, since the co-efficient of d (1,000) refers to square inches of section. Otherwise, the co-efficient must be multiplied by 144 to make it refer to the square foot of section; in which latter case the value of t will be obtained in feet.

In the case of beams to sustain rail road track, we may let l' = length of beam exclusive of the portion between rails, and W = weight upon the 2 rails. If $l' = 120''$ and $W = 25{,}000$ lbs., and $d = 14''$ the above formula becomes, $t = \frac{120 \times 25{,}000}{1{,}000 \times 14^2} = \frac{3{,}000{,}000}{196{,}000} = 15.3$ in.

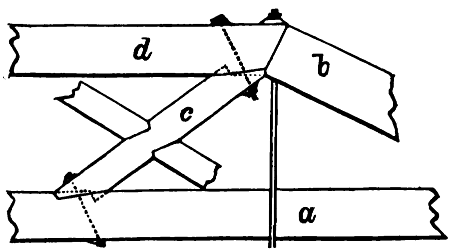
THREE PANEL TRUSS.

CLIX. A three panel truss bridge of wood may be constructed upon the plan shown in outline by Fig. 7. The main braces ab and $a'b'$ may connect with the chord in the same manner as in the two panel truss described in the last section, and illustrated by Fig. 60; while the upper end may be square, and the whole bevel to form the angle abb', given to the member bb'. Or, the bevel may be upon both members; in which case the saddle plates at b and b' should extend over the joint, so as to throw a part of the weight directly upon the brace. In case the bevel be all upon bb', the saddle need not bear upon the brace.

The counter braces in the middle panel may box into the chord and the horizontal bb', in the manner shown in Fig. 62, either by the black or the dotted lines; the upper end of the counter toeing against the end of the main brace, when the form of connection shown by the black line is used.

As the counter braces cross, or meet in the centre of the panel, one may be in two pieces thrusting into the other as at *c* Fig. 62; or one member may be in two full length pieces, and the other a single brace between the former, of such width vertically, as to possess the required cross-section; say $2\frac{1}{2}'' \times 6''$ for the outside, and 4×8 for the middle one, and the whole connected by a small transverse bolt at the crossing.

FIG. 62.



The stresses of the several parts of the truss may be determined in the manner explained in section XVIII, and the timbers proportioned accordingly, and in conformity to rules in relation to strength of timber [CXL and CXLI]. For a truss of 30 feet to carry a gross load of 15,000lbs. to the panel, with a horizontal reach of brace equal to twice the vertical — chord and "straining beam," (*bb'*, Fig. 7), should be 7" deep × 9" wide; main braces 8" × 9". Counter-braces being subject to only one-third of the movable panel load, may properly be 4 × 8 or 5 × 6, if one be severed at the crossing, or as above specified, if one member be in 2 full length pieces.

Two counter-braces might cross one another side by side, but this would not produce a well balanced action.

Bridges of this length of span are, moreover, often built with counter braces omitted, for common road purposes. But such practice is defective, unless extra depth of section be given to the lower chord, so that its stiffness may transfer a portion of weight over the quadrangular middle panel; and in no case is it advisable to dispense with counter braces in a rail road bridge of three panels.

Beams may be suspended by divergent bolts as in Fig. 61, and bolted to the chord; while horizontal × ties or braces, as may be preferred, in each panel will prevent lateral swaying of the structure.

The above is probably the simplest and best plan of wooden truss for bridges of 30 to 35 feet span.

FOUR AND SIX PANEL TRUSSES.

CLX. The same general arrangement, with the same kind of connections, in trusses of 4 or 6 panels, according to length of span, may be used with good effect for common road purposes, in any length up to 70 or 80 feet. In such cases, each panel should have one main brace, and counter braces may be entirely omitted; as the partial movable load is seldom so great as to neutralize the action of weight of structure upon the main braces.

In the 6 panel truss, the movable must exceed the permanent panel load upon the two beams next either end, with no movable load upon the other beams, in order to neutralize the constant tendency to action upon the central pair of main braces. This is obvious from the fact that the greatest tendency to tension action upon the latter, is $3w''$, $= \frac{1}{2}w$, while the permonent load gives a constant opposite tendency, equal to $\frac{1}{2}w'$. Should such cases occur, the transverse stiffness of

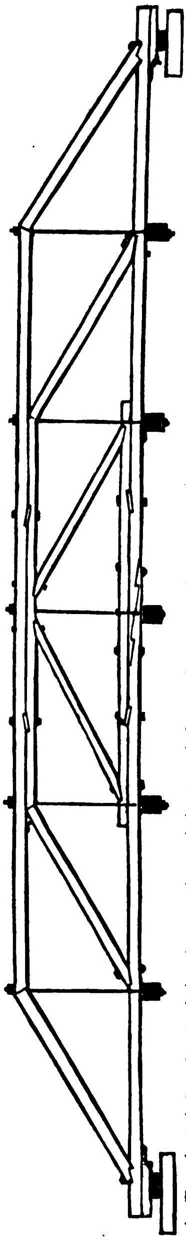


FIG. 63.

both upper and low chords must be overcome before a collapse could take place. In the case of iron trusses, the chords are supposed to have no lateral stiffness at the nodes; consequently, counterbraces, or ties, as the case may be, are always necessary in one or two panels each side of the centre.

Fig. 63 represents a Six Panel truss, as arranged and recommended by the author 16 or 18 years ago, and adopted by the Canal department of the State of New York, for farm and country road crossings over the State canals, upon which several hundreds of them are in use.

The arrangement of upper and lower chord timbers, and the divergent suspension rods, to maintain the erect position of trusses, as well as the assignment of correct proportions to all the parts throughout, are believed to have originated with the author of this work.

The lower and longer portion of the bottom chord, is usually in two pieces, spliced with double locking and bolting (see Fig. 57), over the centre beam. Transfer blocks are also inserted between upper and lower timbers, to transfer a part of the stress of the longer to the shorter portion, and thus diminish the strain at the splicing.

The long portion of the *upper* chord may also be in two pieces meeting with squared ends, or with a plain half lap, of a foot or so. Transfer blocks or packing pieces and bolts should likewise be inserted as indicated in the figure.

The dimensions of the several members, of course, will depend upon the length and depth of truss, and the load it is required to bear. It is seen by processes explained heretofore [XL and LIII], that the portion of chord under the triangular end panels, and also the endmost sections of the upper chord, are liable to action equal to $2\frac{1}{2}W\frac{h}{v}$, in which expression $W = w + w'$, $h =$ the horizontal, and $v =$ the vertical reach of braces. The next sections (top and bottom), are liable to $4W\frac{h}{v}$, and the lower chord under the two middle panels, to $4\frac{1}{2}W\frac{h}{v}$.

The end braces are liable to $2\frac{1}{2}W\sqrt{h^2 + v^2} \div v$, the next braces, to $(10w'' + 1\frac{1}{2}w')\sqrt{h^2 + v^2} \div v$, and the middle ones, to $(6w'' + \frac{1}{2}w')\sqrt{h^2 + v^2} \div v$; while the verticals are exposed to $2\frac{1}{2}W$ for the endmost, $1W$ for the middle, and $10w'' + 1\frac{1}{2}w'$ for the intermediates.

Now, we have only to assign specific values to w and w', and to h and v, in order to obtain the actual maximum stresses the several parts are liable to, from the general expressions just found.

Let $h = 12'$, and $v = 7'$; which, though not an economical proportion, as we have seen [LXIV], may be admissible for bridges of light burthens, giving a better appearance, and the structure being less top heavy.

The weight of a light superstructure of this description, is 18 or 20 tons — say, $w' = 3{,}000$ lbs. Then, assuming $w = 6{,}000$ lbs. which will be sufficient for the lighter class of private and country bridges. Then, $\frac{h}{v} = \frac{12}{7} = 1,714$, and $\sqrt{h^2 + v^2} \div v = 1.984$.

Substituting these values in the above expressions for stresses, we have $2\frac{1}{2} \times 9,000 \times 1.714, = 38,565, =$ tension of end section of bottom chord. For the next section, $4 \times 9,000 \times 1,714 = 61,704$lbs.; and, for the two middle sections, 69,417lbs.; while the compression of the two portions of the upper chord, is 38,565lbs., for the end, and 61,704lbs., for the middle sections.

The maximum compression of the three sets of braces, is 44,653 for the ends, 14,880 for the middle ones, and 28,760 for the intermediates.

The tension of suspension bolts, is, at the maximum, for the endmost 22,500, for the middle ones, 9,000 (= W), and for the intermediates 14,500.

The main portion of the lower chord, requires a lap at the splice, equal to 10 times its depth, [CXLVI.] Hence, the less depth, the less waste in splicing, and the more lateral stiffness of truss. But this also involves greater required section in the lighter braces, which become too thin, vertically, to act with advantage under compression.

There is no ready means of determining the exact optimum in the ratio of depth to width of timbers in this case; and we shall not err greatly by assuming a ratio of width to depth as 3 to 2, or as 4 to 3; neither to be rigidly adhered to.

The bottom chord may suffer tension in the second panel, equal to nearly 62,000lbs., requiring 62 inches of net section; while the second brace has a maximum horizontal thrust of nearly 25,000lbs., requiring the severing of 25 inches, whence this part of the chord should have a gross section of $62 + 25, = 87$ inches.

This amount may be furnished nearly, by a section of 8½″ × 10″, 8 × 11, or 7 × 12. Assuming the second, the end braces should be 8½ × 11, the next 7 × 11, and the middle ones, 5½ × 11.

We have seen above, that 62,000lbs. of tension, are communicated to the long timbers of the lower chord, while the splice at the middle is only good for 500lbs., to the inch of gross section, being 44,000lbs.; thus leaving a deficiency of 18,000lbs. to be sustained and made up by the upper timber. In the mean time, the middle braces exert about 8,000lbs. of horizontal action upon this piece, under a full load of the truss, and near 13,000lbs. at the maximum action of those braces. Hence that timber should have a mininum *net* section of 26 inches, + 18 inches to be severed for the insertion of transfer blocks. The timber should therefore be at least 4″ deep.

The transfer blocks should be 1⅝″ thick, in this case, and 15 or 16 inches long, and be well fitted in position as indicated in Fig. 63. This mode is preferable to that of using blocks twice as thick, and letting one-half into each timber by a square boxing; because it leaves a larger section of timber opposite the middle of the block where bolt-holes are required. Otherwise it would be necessary to provide additional gross section on account of bolt holes. The same reason applies in the case of braces toeing into chords, &c.; where the boxing, instead of being as deep at the heel as at the toe of the brace, should taper out to nothing at the heel. See black line at foot of counter brace *c*, Fig. 62.

This case has been given in pretty full detail, since the plan seems to merit, as it certainly enjoys, a high degree of popularity, for small bridges for ordinary use.

By increasing the depth to at least ⅛th of the length of truss, inserting counter braces in the two middle panels, and proportioning members to the respective strains to which they are liable; this plan is undoubtedly well adapted to rail road purposes in spans from 50 to 70 feet in length.

For greater spans than 70 feet for rail roads and 80′ for common roads, higher trusses, with top connections and lateral bracing or tying, should undoubtedly be adopted.

CLXI. The bridge usually designated as Beardsley's Bridge, is identical with the one shown in Fig. 63, modified by the substitution of iron bottom chords, composed of two parallel rods (to each truss) in 5 pieces or parts corresponding in size with the stresses of chords under respective panels. The middle and largest part extending under the two middle panels, and the others, each under one panel only.

These pieces or parts, being connected by turn-buckles, or screw couplings, pass through cast iron shoes, into and against which the several braces toe and thrust; the shoes being prevented from sliding outward upon the rods, by the couplings.

The shoe should in all cases be so formed and located that the axes of action of chord, brace and vertical, meet at the same point, as it regards the intermediates, while as to those upon the abutments, the axes of chord and brace should meet over the centre of bearing upon abutments.

This arrangement (understood to have been the suggestion and device of Mr. Geo. Heath), gives very satisfactory results, and the only practical question with regard to it, as compared with the one with wooden chords, seems to be merely one of economy and convenience. If suitable timbers for chords can be readily and reasonably obtained, it is thought to be quite as advantageous to use wooden chords.

THE HOWE BRIDGE.

CLXII. A very popular plan of wooden bridges, which has, in fact, superseded most others in New York and New England for rail-road purposes from the time of the introduction of the rail road system, is known as the *Howe Bridge*.

The trusses have upper and lower parallel chords, together with main and counter braces, of wood, tied vertically by wrought iron tension rods from chord to chord, the principle of action being the same as in the plan shown in Fig. 63.

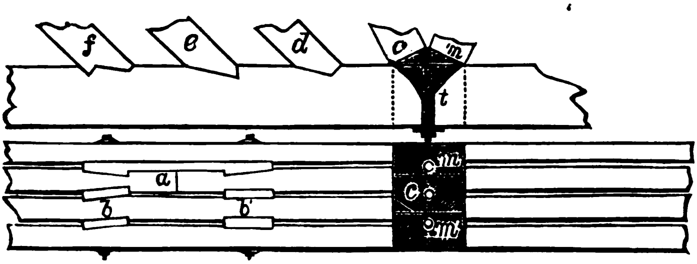
The braces act upon the chords and verticals through the medium of cast iron shoes or skewbacks, with ribs or flanges let into the chords to a sufficient depth to sustain the horizontal thrust of braces, and with tubes, or hollow processes, square externally, and having round holes to receive the vertical bolts. These tubes project downward through the lower, and upward through upper chord, between the courses of timber composing the chord, being boxed into the timber on each side of the tube, so as to leave about an inch between adjacent courses for ventilation; the tubes, extending through the chords, reach an iron plate upon the opposite side, which serves as a washer, or bearing for the nuts of the suspension bolts.

By this means the vertical action of braces is brought directly upon the verticals, without a transverse crushing action upon the chord timbers.

The chords are formed of 3 or 4 courses of timber side by side, with a depth equal to two or three times the thickness; the joints in the several courses being so distributed that no two courses may have a joint in the same panel when avoidable.

Fig. 64, represents a side view in the upper, and a top view in the lower diagram, of a portion of the bot-

FIG. 64.



tom chord. At *t* is represented a view of the tube of the skewback as it would appear with the outside chord timber removed; at *m m*, the seats of the main braces, and *c*, the seat of the counter brace. Over *a*, is a clamp, or lock piece, and *bb'* are transfer blocks, or packing pieces, to secure the joint, and transfer the strain from one to another of the chord timbers. The transfer blocks may be placed obliquely as at *b*, or straight, as at *b'*. The latter is the more usual, but the former leaves the greater section of timber at the point where the bolt holes occur.

The braces are usually placed with a horizontal about half as great as the vertical reach, and extending across one panel only. Counter braces used throughout, and the upper chord made of equal length with the lower, giving the truss a rectangular, instead of a Trapezoidal form.

Now, it is obvious that in a rectangular truss, as represented in Fig. 52, the end posts, and one panel-length of the upper chord at each end, as well as one counter-brace, are entirely useless, as it regards sustaining weight of structure and load. It will readily be seen, moreover, that no counter-braces except those of the two middle panels, in the 8 panel truss, Fig. 52, have any sustaining action, unless the variable exceed 4 times the permanent load of the truss.

It is furthermore manifest that there is a large amount of surplus material in the portions of lower chord toward the ends; the tension of that chord being in the several panels, proceding from the end (in the case of Fig. 52), as 3½, 6, 7½ and 8. Hence, over one-fifth of the material in a chord of uniform section, is in excess.

But the greatest sacrifice of economy in the Howe Bridge as *usually constructed*, results from the steep pitch of the braces. For, while, as was seen [LXVI], braces act with about the same economy at an inclination giving a horizontal reach equal to the vertical, as when the former equals only one-half of the latter, that is, with $h = v$ and $h = \frac{1}{2}v$, it was shown in the succeeding section, that the action upon verticals was nearly twice as great in the latter, as in the former case. For instance, suppose Fig. 18 to represent a 16 panel trnss, with thrust braces and tension verticals. Estimating successively the action upon verticals with diagonals crossing two panels, as in Fig. 18, and the same with diagonals crossing but one panel, we find the action over 85 per cent more in the latter than in the former case.

With regard to chords, the horizontal effect is essentially the same in both cases, while the vertical thrust of braces, being but little over half as great with the long, as with the short horizontal reach, may be sustained by the timber of the chord, thus obviating the necessity of tubes extending through the chord from the cast iron skewback; and furthermore, may enable the iron shoe to be dispensed with altogether, in many cases. Hence would result a still further saving in expense, as well as in weight of structure.

Take, for example a brace 10″ square, capable of resisting a tnrust of 50,000lbs. in the direction of its length, and a vertical pressure of 35,000lbs. when inclined at 45°. Whether the end be cut as at *d*, *e*, or *f* (Fig. 64), it covers a horizontal area of 141 square inches, giving a square inch for every 250lbs. of vertical pressure. This does not much, if any, exeeed the capacity of timber for resisting transverse crushing, as estimated in section CXLIII, when acting upon a portion of surface so limited with respect to the whole.

Perhaps, however, the propriety of dispensing with the iron shoe, should not be too strenuously urged. But there seems to be little excuse for incurring the sacrifice of iron required in suspension bolts in case of the steep braces, over what is required with the greater inclination. The interference of bolts with braces, when the latter reach across two panels, is perhaps the greatest obstacle in the way of adopting the latter arrangement; and this may be managed by either passing the bolts through the intervening braces (which does not materially impair their strength, when supported at intervals by counter-braces), or between main and counter braces, as may seem most favorable in respective cases,

In view of the above considerations, the author can not avoid regarding the usual *practicc* in the construction of Howe Bridges, as decidedly faulty.

TRAPEZOID WITHOUT VERTICALS.*

CLXIII. This form of truss, Figs. 13, 15 and 19, has been shown [XLIV, &c.], to be liable to a less amount of action upon materials, in sustaining a given load under like general conditions, than any of the other forms analyzed in this work; and this advantage may be made practically available in wooden bridge construction, by a system of chords and diagonals connected by transverse iron bolts and pins at the nodes of upper and lower chords.

The lower chords should be proportioned in their several parts, nearly in accordance with the stresses to which such parts are liable. This may be accomplished by a pair of parallel courses of timber of uniform section upon the outsides of the chord from end to end, placed at such distance asunder as to admit the ends of diagonals between them, and also, to admit of additional courses of chord timbers upon the inside of the former, to be introduced as required toward the centre, to give in each panel a section of chord, proportional to the computed strain for such part.

The pieces composing the several courses, may be spliced with the double lock, Fig. 57, usually with the centre of the splice at the nodes, or connecting points of chords with diagonals; no two splices in the same half-chord to occur at the same node.

The upper chord should be increased in section by enlargement of the *section* rather than the *number* of courses. Or, in some cases, timbers may taper in thickness toward the ends of chords, either upper or

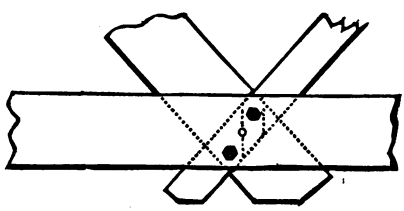
* The characteristic of this truss, is not that strictly speaking it has no vertical members, but that there is no general alternate transfer of weight from diagonals to verticals, and the contrary.

lower. For instance, if 5″ in thickness be sufficient in the end panel, and 7″ be required in the next, a timber extending over the width of two panels, 6″ at the smaller, and 8″ at the larger end, will answer the requirement with perhaps less waste of timber and labor than would suffice under a different arrangement. But such matters must be left to the judgment of the designer.

The upper chord acting by compression, the timbers may be connected by a half-lap of 1½ or 2 feet at the nodes, where the main connecting bolts will secure the ends.

The diagonals which act principally by compression (represented as the narrower ones in Figs. 65 and 66), may be in pairs, while those mostly exposed to tension (the wider ones), may be single, and placed between the former. Thus usually three pieces are united at each node.

FIG. 65.



In some cases where the thickness of diagonals exceeds the space between half-chords, the thrust diagonals may be shouldered to fit a boxing upon the inside of the chord; as by either of the vertical dotted lines, Fig. 65. Sometimes also, the boxing may extend through the whole depth of the chord, so as to require no cutting of the diagonal; and again, the thickness of the diagonals may be reduced in the parts between chords, and no cutting of chord timbers required.

When cutting of timbers becomes necessary for purposes as above, it should be in the parts where the greater surplus over the necessary net section occurs, whether in chord or diagonals. Every part should have a square inch of net available section for each 1,000lbs. of tension, and a square inch of bearing upon bolt, pin, or shoulder, for each 1,000lbs. of either tension or thrust to which the part is liable; and the bearing upon bolts and pins should be estimated as equal to the diameter multiplied by the length of hole through the piece; or, equal to the section of timber severed by the hole.

CLXIV. Fig. 66 is a general representation of the half of an 8 panel truss, suitable for a 100 foot common road bridge. Let $v = 14'$, = distance between centres of upper and lower chords, and $h = 12\frac{1}{2}'$, = horizontal width of panel. Then, assuming $w = 10{,}000$lbs. (= movable panel load), and $w' = 4{,}000$lbs. (= permanent panel load), we have $\frac{h}{v} = .893$ (nearly), and $D = 18.77'$ = length of diagonal; whence, $\frac{D}{v} = 1.34$; and, computing the stresses of the several parts and members by the process explained in sections [XLIV, &c.,], the maximum vertical pressure at a equals 49,000lbs. giving a longitudinal compression upon ai, equal to 65,660lbs., and a tension upon ab, equal to 43,750lbs.

For the double member ai, $8'' \times 9''$ timbers are sufficient; while $4'' \times 12''$ (in each half), would answer for ab. But to give greater transverse stiffness for supporting floor timbers, it is preferred to have the outside course of lower chord timbers 5×12 inches.

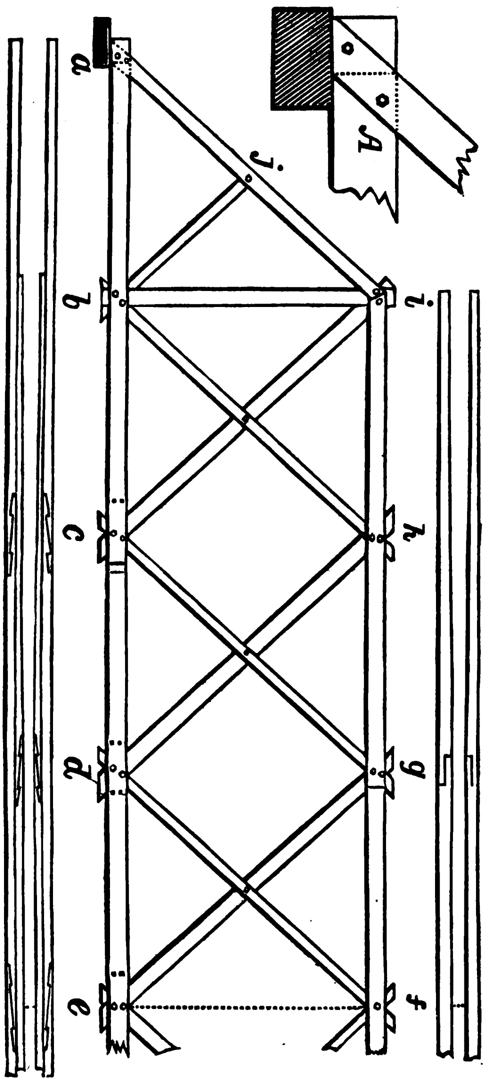


FIG. 66.

The piece *bj* having no office but to fill the space at *b*, and to give support to *a i*, may be of any convenient dimensions.

The maximum tension of other portions of the lower chord is, for *bc*, 56,250; for *cd*, 81,250, and for *de*, 93,750lbs. For the upper chord, we have compression of *ih*, *hg* and *gf*, 62,509lbs., 87,500lb., and 100,000lbs. respectively.

The diagonals and verticals are liable to maximum tension and compression as shown in the following statement; and may properly be of dimensions as marked opposite each in the right hand column below; in case of double members, the figures indicate the width and thickness of each.

Parts.	Tension, lbs.	Compression, lbs.	Cross-Section, inches.
bi	28,000		double 3 × 11
ci	28,140		single 5 × 12
dh	20,435		" 4 × 12
eg	12,730	670	" 3 × 11
fd	6,700	6,700	d. 3 × 6
yc	670	12,730	" 3 × 6
hb		20,425	" 3½ × 7
ia		65,660	" 8 × 9

The inside course of lower chord timbers may be — a 4″ × 12″ piece extending from *d* across the two middle panels of the truss, spliced at each end to a tapering piece 4 × 12 at *d*, and 2 × 12 at *b*; and consequently, 3 × 12 at *c*. Then, leaving a space of 8″ between half chords at *d* and *e*, we have 10″ at *c*, and 12″ at *b*.

Each half of the npper chord should be 8″ × 12″, in the two middle panels, and placed 9″ apart; connecting with a tapering piece each way, from 8 × 12 at *g*, to 6 × 12 at *i*; where the end should be beveled to a line bisecting the angle *aih*, and abut against a beveled shoulder upon the upper end of the king brace *ai*. The king brace is also cut away upon the inside, leaving only 1″ in thickness, to make up, with *bi* and *ic*, a thickness equal to the space (13″) between the half-chords at *i*.

The parts thus meeting at *i* are to be fastened by 2 transverse bolts of at least 2½″ in diameter. These afford the requisite square inch of bearing surface for each 1,000lbs. of pressure, with an unimportant deficiency for the member *ic*, which may be eked out with a 1″ pin through *bi* and *ic* only, if thought advisable, thus giving 55 square inches for vertical and diagonal together.

These members should extend at least 14″ beyond the centres of holes.

At *h* and *g*, the three diagonal pieces just fill the space between chord timbers, and require at *h*, two bolts and one plain pin of 1¾″ in diameter, and at *g*, the same number &c., of 1⅝″ diameter. The diagram shows only two bolts at each connection.

At the point *f*, where two pairs of braces meet, one pair may be cut off at the meeting, and a 4 by 6 inch piece introduced, lapping 2 feet between the cut pieces (reduced each ½ inch in thickness, inside, to the extent of the lap), and secured by 2 bolts and 1 pin of 1″ diameter; the upper end passing between the opposite braces, the latter being boxed ½″ inside, to afford room for the 4″ piece; and the whole secured by a single 1⅝″ or 1¾″ bolt through chord and braces.

The connections at the lower chord are somewhat more complicated, but involve little difficulty. The best connection at *a*, is made by cutting a vertical shoulder or heading ¾″ deep, upon both sides of the half chord, as shown by the vertical dotted line in the diagram A, Fig. 66; the brace being forked with counter shoulders upon the inside. This affords 36 square inches of shoulder surface, which, assisted by 2 bolts of 1″ diameter, give 50 square inches, to withstand less than 44,000lbs. The end of the brace is thus made to bear directly upon the abutment without any crushing action upon the chord.

At *b*, the space in the chord is 12″, while the verticals descending parallel, would occupy 11″. But giving a divergence of 2½″, and boxing ¾″ upon the inside of chord timbers, leaves a space of 6½″ between verticals at *b*. Then, boxing *bh* ½″ upon the inside at the crossing with *ic*, there will be a 3″ space between braces *bh* at *b*, and a thickness of 4½″ (of the pieces *bh*) between the verticals *bi*; also, a shoulder of 1¼″ upon the outside, which may be made to act vertically in a boxing upon the inside of *bi*, thus securing the requisite bearing surface for the thrust of *bh*. Thus arranged, the point should be fastened with 2 bolts and 1 pin of 1⅝″ diameter.

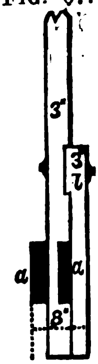
The piece *bj* will have 3″ in thickness at *b*, and will be furred out, if necessary, to fill the space at *j*.

The space at *c* is 10″; and, *cg* being shouldered ½″ at the upper side of the chord at *c*, and boxed ¼″ at the crossing with *hd*, the point *c* may be secured by 2 bolts and 1 pin of 1⅞″ or 2″ diameter.

A ½″ boxing of *df* at *d*, upon the inside, leaves a thickness of 9″, being 1″ greater than the space in the chord, and the pieces *df* therefore require a further reduction in thickness upon the outside between chord timbers, of ½″ upon each. The point *d*, requires 1¾″ bolts and pin.

The two single diagonals meeting at *e*, may be halved into one another at the crossing, and a 3×11 inch piece lapped and locked on to each, as shown by *a a* in Fig.67 ; thus serving to fill the space in the chord, and to restore strength to the diagonals. The lap pieces are to be reduced to 2½″ in thickness below the lock at *l*. Two 1½″ bolts are sufficient at the point *e*.

FIG. 67.



Transverse joists, or floor beams may be placed upon, or suspended below either the lower or upper chords. Sway bracing may be locked and bolted upon the upper chords, and iron × tie rods used at the lower chords ; the beam timbers being shouldered against the inside of cords, so as to strut them apart against the action of the ties.

Angle braces from the king brace *ai*, to a transverse beam from truss to truss at *i*, will aid in preserving the erect position of trusses. These braces should usually be lapped and bolted at the ends, so as to act by either tension or thrust

The preceding specifications, it is hoped, will serve to make the peculiarities of detail in the kind of truss under consideration, properly understood. It may be deemed advisable to adopt the rectangular, instead of the Trapezoidal form of outline for the truss, by extending the upper, to the same length with the lower chord, inserting vertical posts at the ends, and exchanging the double vertical *bi*, to a single diagonal meeting the upper chord and end post at their point of junction ; thus simplifying the connections at *b* and *i*.

This modification, unlike the case of the trapezoid *with* verticals, involves no increase in amount of action upon materials, though it increases the number of members, and changes the manner of distribution of the action.

MODULUS OF STRENGTH, FOR BRIDGE TRUSSES.

It is shown in preceding pages of this work, that, knowing by experiment the strength of the materials to be employed, we may calculate the necessary cross-section of each part of a bridge truss, in order that it may sustain a given load, with a given stress upon the materials.

It is sometimes, however, a satisfaction to have a confirmation of the correctness of our calculations, by direct experiment upon the same combination complete, which we propose to employ for actual use. For this purpose, instead of applying the test to a full sized structure, which would involve a great deal of labor and expense, the test may be applied to a model, made in the true proportions, upon any scale.

Now, it is obvious that with the same combination and arrangement of members, the stresses, whether positive, negative or transverse, produced upon the several parts by the acting forces, will be in proportion, throughout, to the weight sustained, whatever be the *length* of pieces; such stresses being determined by the positions and angles, and not by the lengths of pieces.

It is further manifest that the ability of parts to withstand the effects of the acting forces, must be as the cross-sections of parts respectively; and in *similar models*, the parts, being similar solid figures, have their cross-sections as the squares of the magnitude of scale upon which they are respectively constructed, while the bulk and weight of each corresponding part, and of the combinations complete, are as the *cubes* of the magnitude of scale.

Then, assuming two *similar* models, the scale of one being m times as great as that of the other, the weights which they will respectively bear, under the same stress of material, will be as W to Wm^2, while their respective weights will be as 1 to m^3.

Now, dividing the sustaining power of each by its own weight, the quotients are as W to $W\frac{m^2}{m^3}$ or as W to $\frac{W}{m}$. But the lengths being as L to Lm, if we multiply the quotients just found by respective lengths, we have WL for the one, and $Lm\ W \div m, = WL$ for the other; showing that the length of a model truss by the number of times its own weight which it can bear (with a given stress), is a constant quantity, whatever be the scale of such model.

Again, the quotients W, and $\frac{W}{m}$, multiplied by the lengths L and Lm, give the products WL, and $\frac{W}{m} \times Lm$, equal to WL. Hence, the product of a truss medal into the number of times its own weight which it is able to sustain, is also constant, whatever be the relative values of the two factors.

It follows, that making these two factors variable, and representing them by Q and L, the one increases at the same rate at which the other is diminished; and, when $Q = 1$, L must be equal to the greatest length at which a truss of the same plan and proportions, and under the same stress of materials, can sustain its own weight alone.

This length, as we have seen, is determined for a model upon any plan, constructed upon whatever scale, by multiplying the length of model by the number of times its own weight it is capable of sustaining.

This product may be called the MODULUS OF STRENGTH, and the plan of truss which gives the largest MODULUS, may fairly be regarded as the strongest plan.

The Modulus may refer either to the actual breaking load, as found by experiment, or to the load producing given rates of strain upon materials, as determined by calculation.

EXAMPLES.

(1). A bar of cast iron 1 inch square and 12″ between supports, will bear (at 6,000lbs. to the inch of section, upon a leverage equal to depth of beam), a distributed load of 4,000lbs. which divided by its weight, = say 3.12lbs. gives Q = 1250; and L being 1 foot, the Modulus = QL, = 1,250 feet.

(2). A beam of pine timber 12′ long and 6″ square, at 1,500lbs. to the inch upon a leverage equal to depth, as above, bears a distributed load of 18,000lbs. [CXLII.] For the weight, say 3 cubic feet at 36lbs. = 108lbs.; whence, $Q = \frac{18,000}{108} = 166.6$, which multiplied by L (= 12′) gives Modulus equal to 2,000 ft.

By reducing the length of the beam just considered, to 6 feet in length, retaining the same section, it would give a Modulus of 4,000 feet, instead of 5,000, as given in the Appendix to my former work; the difference arising from the assumption of a smaller specific gravity for pine in the latter case.

(3). The two panel model with chord and rafter braces, mentioned in the latter part of § [CLVIII], 20″ long, and weighing 0.18lb. supported a load equivalent to 3,885 times its weight, while $L = 1\frac{2}{3}$ feet; whence, $3,885 \times 1\frac{2}{3} = 6,475$ feet, = its Modulus.

(4). A model wooden truss 4 feet long, made many years ago by the author, on the plan of the truss Fig. 66, having 10 panels, and a depth equal to $\frac{1}{10}$ of its length, weighed 0.9lbs. and bore a distributed load of 600lbs. Hence, the modulus of the truss was $\frac{600}{0.9} \times 4 = 2{,}664$ feet, being more than half a mile.

The model was somewhat strained but not broken; and recovered its normal shape and condition on removal of the load. It was subsequently sent to the U. S. Patent Office.

These examples, however can not be taken as indices to the relative merits for general use, of the different forms of truss to which they refer. Each possesses qualities suited to special occasions.

(5). A model of a 6 feet Trapezoidal Iron Truss (*the first ever constructed*), weight a little less than three pounds, sustained 700lbs. distributed, without any appearance of overstraining; thus showing a modulus of $\frac{700}{3} \times 6 = 1{,}400$ feet, with an estimated stress upon the chord, at the rate of about 16,000lbs. to the square inch. The model represents a truss of 144 feet, upon a scale of ½ inch to the foot. The sustaining power of a full sized truss in the same proportions, would be $700 \times 24^2, = 403{,}200$lbs, while the weight of truss would equal $3 \times 24^3 = 41{,}472$lbs. Doubling this for two trusses, and adding, say 10,000lbs. for beams, &c., we have 92,944lbs. for the weight of a 144 feet bridge, capable of sustaining, at a stress of 16,000lbs. to the square inch upon the chords, over 356 net tons beside weight of structure.

CORRECTIONS.

Page 18, line 18 from top, $\left(\frac{h}{v} + v\right)$M, should be $\left(\frac{h^2}{v} + v\right)$M.

19, line 9 from bottom, $\frac{2h}{3}$ should be $\frac{2v}{3}$

20, line 3 from top, $v\sqrt{2}$ — should be $v\sqrt{2}$, . .

21, line 7 from top, $h = al$ should be $h = ab$.

22, line 2 from top, $3\frac{\sqrt{h^2} + \frac{1}{3}v^2}{v}w$, should be $3\frac{\sqrt{h^2 + \frac{1}{3}v^2}}{v}w$.

22, line 4 from top, contracted, should be counteracted.

29, line 2 from top, = stress, should be gives stress.

42, line 19 from top, $\times$ should be $+$

55, line 5 from top, reference letter y, should be g.

65, line 5 of Note, db′a, should be dba.

66, line 14 from top, $\frac{11.85}{12}$, should be $\frac{11.85}{12}$E.

73, line 1 below Diagram, four should be three.

88, line 13 from top, w' should be w''.

91, line 8 from bottom, $\times$ should be $+$.

110, line 2 from note, $\sqrt[3]{16}$, should be m$\sqrt[3]{16}$.

116, line 4 from bottom of note, a, b, c, d, &c., should be a', b' c'', d^r, &c.

119, line 2 from bottom $(.84 — 2)\,v$, should be $(.84 — .20)\,v$,

120, line 10 from top, $(\overline{4.5v}^2$, should be $(\overline{4.5v^2})$,

121, line 11 from top, = .84, should be — .84.

144, line 16 from top, adjust should be adjustment.

171, line 7 from top, $\frac{785}{6}$, should be $\frac{.785}{6}$.

196, line 13 from bottom, $(T + A)^2$, should be $(T + H)^2$.

198, line 1 from top, $37\frac{1}{2}$, should be $31\frac{1}{2}$.

199, line 6 from top, Fig. 31, should be Fig. $34\frac{1}{2}$.

252, line 5 from top, $9w$, should be $.9w$.

253, line 3 from top, 5.05M, should be 50.5M.

288, line 2 from top, \$3.60, should be \$3.50.

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