Design of a Concrete Bowstring-Arch Bridge, Including Analysis of Theory

Type of structure uncommon in United States used on Oregon state highway—Foundation and climate determining factors—Temporary hinge used in arch ribs—Tied arch analyzed for load, temperature and shrinkage stresses

By CONDE B. McCULLOUGH
Bridge Engineer, Oregon State Highway Commission

RARELY USED in the United States, in comparison to its applications in Europe, a bowstring-arch design of reinforced concrete was used, because of foundation and climatic conditions, for the Wilson River bridge being built by the Oregon state highway commission. The structure consists of a pair of arch ribs for a 120-ft. span carrying a suspended through-roadway deck with arch reactions taken by the reinforced-concrete floor slab acting as a tie, thus converting the structure into a completely closed ring supported on vertical piers. The structural elements, which differ from standard arch-design practice, and the analysis of the tied-arch theory are described in the following.

Location of the bridge near the coast, where corrosion of steel is severe, dictated a concrete structure in accordance with the practice of the highway department for such conditions of coast exposure. The economy of concrete in this position, other considerations being equal, is due primarily to maintenance cost of painting steel structures. The type of concrete structure to be used was influenced by foundation conditions, which rendered abutments capable of sustaining arch thrust uneconomical, and the relation of high-water level to roadway grade, which eliminated deck-girder construction for the 120-ft. span, even if such construction had been economical.

Design Features—The axial rise of 36 ft. for the arch was based on an economic balance between lateral thrust components, which influenced the design of the tie and concrete yardage in the ribs. In addition, a rather high rise allows a more rigid system of bracing. The curve of the arch is not a simple second-degree parabola frequently used for this type of loading, but the exact locus of computed points on the equilibrium polygon. This type of curve has been employed by the writer's office on several jobs and has been termed the "multi-parabolic axis," because its equation consists of the algebraic sum of parabolas of different degrees from the second degree to that represented by the number of panel points in the half span.

The ribs are 3 ft. 6 in. wide by 2 ft. 8 in. thick at the crown and 3 ft. 6 in. thick at the springing. Arch reinforcing consists of sixteen $1\frac{1}{8}$-in. square bars per rib. Additional $1\frac{1}{4}$-in. square bars at the ends of the ribs are fanned out diagonally into the floor to transmit the thrust back into this 12-in. slab, which is used as the tie member. Arrangement of this diagonal tie steel is shown in the accompanying detail. The transverse component of this tension is taken by transverse tie steel of 1-in. bars (see same detail). This scheme of transferring arch thrust into the tie, of course, is only one of many possible methods. For example, the tension rods could be placed in a beam along the curb instead of spreading them out into the slab. Preliminary studies, however, indicated that the method used was somewhat more economical of material.

The hangers, which are spaced at 12-ft. intervals, were designed as a relatively thin rectangular section, with a thickness of only 5 in. parallel to the axis of the roadway. This was done to avoid secondary bending stresses induced by angular distortion of the arch rib. Also, the hanger tops were articulated by means of asphaltic felt. Placement of hanger concrete will be deferred until the arch and deck falsework have been released and the hanger rods are under full dead load. In view of the experience with hanger members of this character, such a precaution appears necessary in order to avoid the formation of horizontal circumferential cracks.

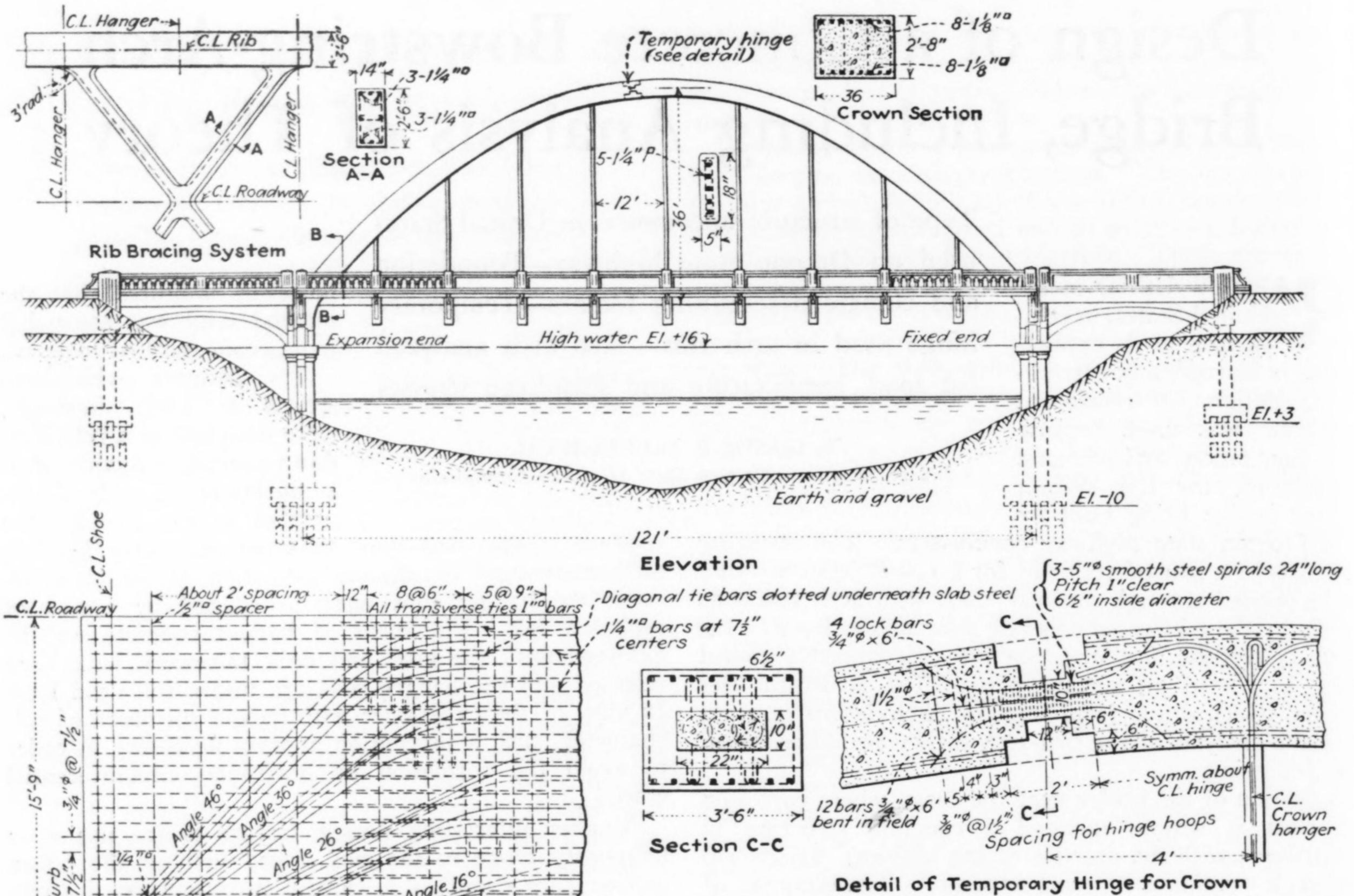
Another interesting design feature is the use of a temporary hinge in the ribs placed just off the crown section (see detail). A careful investigation of stress distribution indicated a distinct economy resulting from use of this hinge, which causes the arch to act as a single-hinged arch under dead load. The hinge will be keyed in solid after the rib has received full dead load, thus providing an unarticulated structure under live load.

To determine a safe working stress for the crown hinge sections, tests were made on full-size hooped columns. The results of these tests indicate the high compressive strength developed in a tightly hooped column of this character.

Test No.	Longitudinal Bars	Spiral Hooping	Compressive Strength, Lb. Per Sq. In. Minimum	Maximum
1	6-$\frac{7}{8}$-in.	$\frac{1}{8}$-in.	13,500	13,900
2	6-$\frac{3}{4}$ "	$\frac{1}{4}$ "	14,200	14,800
3	6-$\frac{1}{2}$ "	$\frac{3}{8}$ "	11,000	12,000
4	6-$\frac{1}{2}$ "	$\frac{1}{4}$ "	13,300	14,400

Tied-Arch Theory—The basic theory of the tied arch is quite similar to that for the ordinary fixed-arch span, but it has distinctive elements. A development of the theory follows.

If the arch-rib tie (Fig. 2) is severed at any section (2–2) and if each of the severed ends is assumed connected to a rigid bracket terminating at some point O (later taken as the elastic center of the system), it is apparent that the structure may be maintained in equilibrium by the introduction, at point O, of three unknown force components, X, Y and Z, acting against each bracket as indicated. These three so-called redundant components may be so determined as to maintain the structure in exactly the same stress and strain condition as that which obtained with the unsevered tie. The original tied arch has therefore been replaced by a simple curved beam b–c with two interior cantilevered stubs, each terminating in a rigid bracket whose terminal point O is under the action of three redundant force components X, Y and Z. The value of these redundants may be readily determined from the obvious



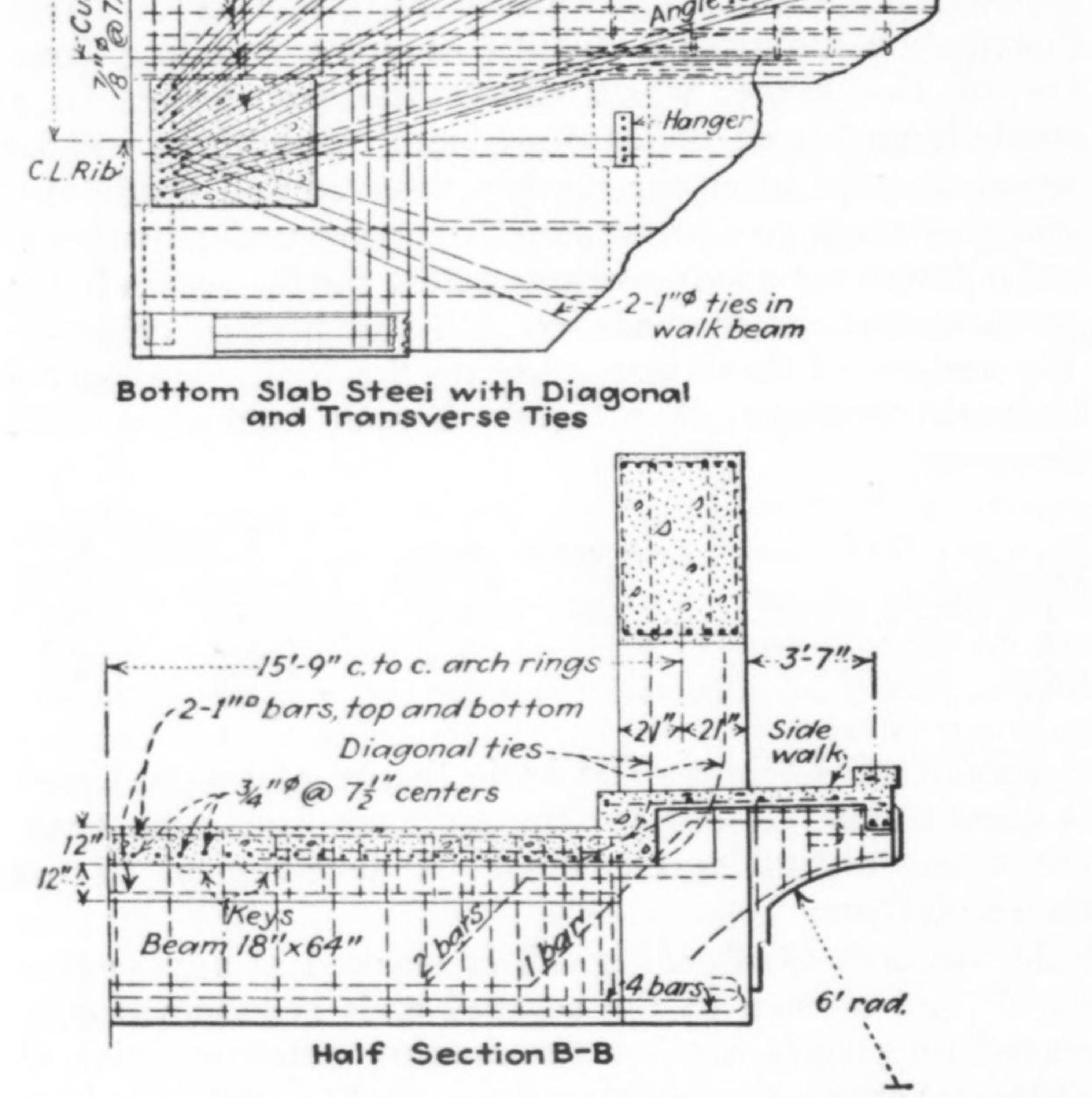


Fig. 1—Details of Wilson River bowstring concrete arch bridge

fact that points a and d have no displacement, either angular or linear, relative to each other.

If Δ_x, Δ_y and Δ_z represent the displacement components of point O along the direction of action of the three respective redundants, we may write:

$$\Delta_x = 0, \; \Delta_y = 0 \text{ and } \Delta_z = 0 \qquad (1)$$

But $$\Delta_x = \Delta_{ox} + X\delta_{xx} + Y\delta_{yx} \times Z\delta_{zx} \qquad (2)$$

wherein Δ_{ox} represents that portion of the displacement caused solely by the external loads, and δ_{xx}, δ_{yx} and δ_{zx} represent the displacement components (measured in the direction of the redundant X) that would be caused by the forces X = unity, Y = unity, and Z = unity, respectively, all forces being assumed as acting on the severed or statically determinate rib.

If point O for each bracket be taken as coincident with the elastic center of the system (the center of gravity of the elastic loads $G = \frac{ds}{EI}$) the term δ_{xz} vanishes and (since the arch is symmetrical) by taking the redundant axes X–X and Y–Y as horizontal and vertical the term δ_{xy} may also be made to vanish. (This fact is demonstrated in the writer's treatment of arch bridges in Hool & Kinne's "Reinforced Concrete and Masonry Structures.")

From equations (1) and (2), therefore, we may write:

$$\Delta_x = \Delta_{ox} + X\delta_{xx} = 0 \qquad (3)$$

or

$$X = \frac{-\Delta_{x}}{\delta_{xx}} \qquad (4)$$

in a similar manner

$$Y = \frac{-\Delta_{oy}}{\delta_{yy}} \qquad (5)$$

$$Z = \frac{-\Delta_{oz}}{\delta_{zz}} \qquad (6)$$

Equations (4), (5) and (6) are entirely general and hold for any condition of loading or for any other straining agency such as temperature variation and shrinkage. The numerator in each case represents the displacement component (measured along the direction of action of the particular redundant force considered) caused by the external straining agency (loads, temperature, etc.) acting upon the severed frame, while the denominator represents the corresponding displacement caused by a unit value of the redundnant force in question.

To transform these equations into terms representing the known loads and dimensions of the structure, we will consider the following cases:

Case I—Gravity loadings

If two auxiliary loads (X = unity) be applied at the bracket terminals, as indicated in Fig. 2, and if (n_x) and (m_x) represent, respectively, the axial thrust and bending moment at any section induced by this pair of loads, we may write (from the well-known formula for elastic displacements):

$$\Delta_{ox} = \Sigma M_o m_x G + \Sigma N_o n_x \frac{ds}{AE} \qquad (7)$$

$$\delta_{xx} = \Sigma (m_x)^2 G + \Sigma (n_x)^2 \frac{ds}{AE} \qquad (8)$$

where M_o and N_o represent moments and thrusts caused by the external load system acting upon the severed rib.

Whence:

$$X = -\left[\frac{\Sigma M_o m_x G + \Sigma N_o n_x \dfrac{ds}{AE}}{\Sigma (m_x)^2 G + \Sigma (n_x)^2 \dfrac{ds}{AE}}\right] \qquad (9)$$

In a similar manner:

$$Y = -\left[\frac{\Sigma M_o m_y G + \Sigma N_o n_y \dfrac{ds}{AE}}{\Sigma (m_y)^2 G + \Sigma (n_y)^2 \dfrac{ds}{AE}}\right] \qquad (10)$$

$$Z = -\left[\frac{\Sigma M_o m_z G + \Sigma N_o n_z \dfrac{ds}{AE}}{\Sigma (m_z)^2 G + \Sigma (n_z)^2 \dfrac{ds}{AE}}\right] \qquad (11)$$

But from Fig. 2, considering moments causing compression in the outside of the ring as positive and vice versa,

$$\left.\begin{array}{l} m_x = + y \\ m_y = + x \\ m_z = \text{unity} \end{array}\right\} \qquad (12)$$

Also, $n_x = -\cos\theta$, $n_y = \sin\theta$, $n_z = 0$, $n_x ds = -dx$, and $n_y ds = dy$.

Furthermore, the second term of the denominators of equations (9), (10) and (11) is very small and may be neglected without any material error· whence we may write:

$$X = -\left[\frac{\Sigma M_o G y - \Sigma N_o \dfrac{dx}{AE}}{\Sigma G y^2}\right] \qquad (13)$$

$$Y = -\left[\frac{\Sigma M_o G x + \Sigma N_o \dfrac{dy}{AE}}{\Sigma G x^2}\right] \qquad (14)$$

$$Z = -\left[\frac{\Sigma M_o G}{\Sigma G}\right] \qquad (15)$$

In equations (13), (14) and (15) the factors M_o and N_o, representing the external loads, must include not only the gravity loadings acting downward but also the shoe reactions acting upward. These latter are statically determinate the same as for any truss or beam, and may be easily calculated for any position of the unit load when deriving the influence lines for the redundants X, Y and Z.

If the bending in the tie due to arch-rib deflection be

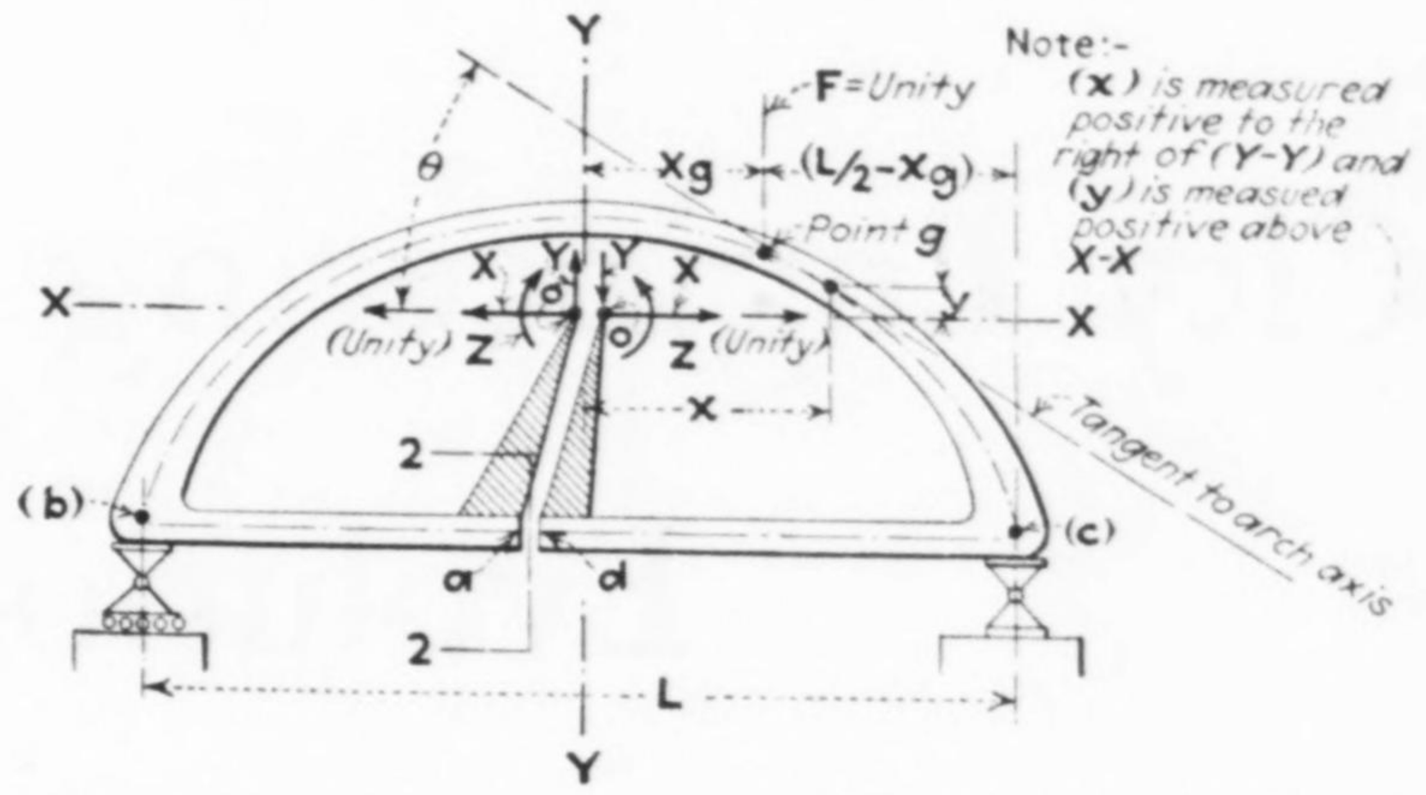


Fig. 2—Diagrammatic bowstring arch for mathematical analysis

neglected (which introduces no material error), the above equations enable the influence lines for the three redundant forces X, Y and Z to be constructed, and from these the stresses in the arch rib and tie may be determined for any condition of loading.

Case II—Temperature and shrinkage effects

In this case $\Delta_{ox} = \Sigma n_x ctds = \Sigma ctdx = 0$ (16) (since the summation is for a closed ring).

Therefore, $X_t = 0$, and in a similar manner it may be shown that $Y_t = 0$ and $Z_t = 0$. The tied arch is, therefore, unaffected by a uniform change in temperature or a shrinkage of the material.

Conclusion—There is much to recommend the bowstring type of arch. It is reasonably economical of material, unaffected by temperature and shrinkage and adaptable to yielding foundations. In addition, it affords full vertical waterway clearance from pier to pier and is not unpleasing in appearance. These properties had considerable bearing on the selection of this type of structure for the Wilson River bridge. The writer ventures the prediction that this structural type will become more popular with American engineers as they investigate its possibilities.

British Standards for Cranes to Insure Greater Degree of Safety

The use of timber is prohibited in any stress-bearing part of new standards for derrick cranes, power-driven and hand-operated, and for traveling jib cranes (contractors' type), just announced by the British Engineering Standards Association, according to the *Commercial Standards Monthly*.

The object of these specifications is to secure the general observance of such fundamental principles as appear desirable to insure reliability and safety without hampering the freedom of the crane maker in his selection of the most appropriate design for the purpose in view.

The derrick-crane specifications provide for the Scotch derrick, the guy derrick and the tower derrick types of cranes, now largely used in building operations. The factors of safety and allowable working stresses specified are applicable to cranes intended for ordinary duty. Higher factors of safety and lower working stresses are recommended for duties of greater severity, or where there is a liability to accidental overloads, as in dockside, quarrying and magnet cranes.