Designing American Lenticular Truss Bridges 1878–1900

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The lenticular truss bridge enjoyed a brief period of widespread use in the late-19th century. Although Gustav Lindenthal used this form for his Smithfield Street Bridge in Pittsburgh, the overwhelming majority of these bridges were produced by the Corrugated Metal Company and its successor, the Berlin Iron Bridge Company. Although important points of similarity can be noted between the Smithfield Street Bridge and the Bardwell's Ferry Bridge (a product of the Berlin Iron Bridge Company), a close examination of the two bridges reveals differences in the form of the bridges and in the sizes of the elements of the structure: top and bottom chords, and diagonal web members. By examining the analytical methods available to Lindenthal and to Charles M. Jarvis, the designer of the products of the Berlin Iron Bridge Company, it is found that these differences result from very different approaches to the analysis and design of a lenticular truss bridge. The Berlin Iron Bridge Company bridges were designed as Pauli trusses, a specialization of the lenticular truss in which the top chord is shaped so that its compressive force under full loading is kept constant. While Lindenthal claims that the Smithfield Street Bridge is a Pauli truss, this bridge is actually a parabolic lenticular truss bridge.

Introduction

The lenticular truss is one of the most intriguing structural forms developed in the late-19th century. It was used for a number of well-known, one-of-a-kind bridges, including I. K. Brunel's 1859 Royal Albert Bridge at Saltash, Friedrich Augustus von Pauli's 1857 bridge over the Isar at Grosshesselohe, and Heinrich Gerber's 1860 bridge over the Rhine River at Mainz.[1] The most visible example in the U.S. is Gustav Lindenthal's 1883 (widened in 1891) Smithfield Street Bridge in Pittsburgh, which can be seen in Figure 1.[2] On the other hand, this truss form was also used in the United States for common highway bridges erected by the Corrugated Metal Company and its successor, the Berlin Iron Bridge Company of East Berlin, Connecticut. In this article, the structural components of the Smithfield Street Bridge and several surviving examples of lenticular bridges erected by the Berlin Iron Bridge Company are analyzed, within the context of lenticular-truss design theories of the late-19th century, to ascertain whether different design practices were employed in constructing a one-of-a-kind structure and the common stock of a mass-production bridge company.

American Lenticular Truss Bridges

In a lenticular truss, the top and bottom chords are curved in such a way that the top chord functions as an arch, and the bottom chord functions as a suspension cable.[3] Vertical and diagonal web members are necessary to divide the top and bottom chords into manageable lengths and to transfer deck loads to the top chord and the bottom chord. They also redistribute nonuniform loading, which cannot be resisted by arch and cable action alone. The deck can be placed above the top chord, supported by posts; below the bottom chord, supported by hangers; or in between the two chords.

A U.S. Patent for this truss form was granted to William Douglas in 1878 and formed the basis for the standardized designs of the Corrugated Metal Company and its successor the Berlin Iron Bridge Company. This company enjoyed a period of rapid growth

Figure 1. *Smithfield Street, Pittsburgh, Pennsylvania. The trusses suporting the downstream lanes were completed in 1883. The upstream trusses date to 1891.* Photo by Jack E. Boucher, 1974, HAER photograph PA-2-28.

and sales in lenticular truss bridges between the early 1880s and 1900 when the company was absorbed into the American Bridge Company.[4] The 1882 Bardwell's Ferry Bridge, spanning Deerfield Creek in Shelburne, Massachusetts, is quite typical of the products of this company (Figure 2). The Pine Creek Bridge, on the outskirts of Jersey Shore, Pennsylvania, discussed later and shown in Figure 5, is a further example of a Berlin Iron Bridge Company product. By 1889, at least 664 lenticular truss bridges had been built on this model. A relatively large number of the products of these two companies survive throughout the northeastern U.S. and even as far away as Texas.[5] The contrast between the use of the same unusual bridge form for large-scale, one-off projects by engineers of worldwide reputation and for small-scale, standardized bridges produced by a regional manufacturer has been previously noted, particularly by Victor Darnell.[6]

Comparison of the Smithfield Street and Bardwell's Ferry Bridges

The Smithfield Street Bridge, designed by Lindenthal, and the Bardwell's Ferry Bridge, a representative product of the Corrugated Metal Company, bear strong resemblances to each other. Both have curved top and bottom chords and a horizontal deck suspended slightly below the lowest point of the bottom chord. In both cases, the trusses are subdivided into 13 panels with pins at each panel point. The top chord of both bridges consists of a box section fabricated from plates and angles and stitched together with rivets. The bottom chord of both bridges consists of eyebars. The panels of both bridges are provided with diagonal X-bracing meant to function in tension only—eyebars, provided with turnbuckles, in the Smithfield Street Bridge and rod/turnbuckle combinations in the Bardwell's Ferry Bridge. As a result of these features, both can be concisely characterized as "through lenticular trusses with Pratt webbing." Each Smithfield Street truss spans 360 feet, while the span of the Bardwell's Ferry Bridge is much shorter at 197 feet, 8 inches. The span-to-depth ratio of the two bridges is also different, 7.2 for Smithfield Street and a more conservative 6.4 for Bardwell's Ferry. Both bridges have a portal frame and overhead bracing at mid-height, reducing the overhead clearance to approximately 25 feet for the Smithfield Street and approximately 15 feet for the Bardwell's Ferry, that is, approximately one-half the overall depth of the truss.

Figure 2. *Bardwell's Ferry Bridge, Massachusetts.* Photo by Martin Stupich, 1990, HAER photograph MA-98-4.

Close observation of the elements of each of these bridges, however, reveals some intriguing differences in construction that have implications for the understanding of the underlying design methods.[7] The Smithfield Street Bridge was primarily constructed of steel. Bardwell's Ferry, like all of the Berlin Iron Company bridges built before 1894, is wrought iron. A constant top-chord cross-section is found at Bardwell's Ferry. On the other hand, the cross-section at Smithfield Street changes slightly from 119.35 sq. inches in the panel closest to the end post, to 114.35 sq. inches in the adjacent panel, to 111.35 sq. inches in the remaining panels. Similarly, the bottom chord at Bardwell's Ferry consists of a constant cross-section of four 1-by-3-inch wrought-iron eyebars (12 sq. in. total), while the bottom chord at Smithfield Street varies between 8 and 10 steel eyebars, all of 7 inches depth but of varied thicknesses for a net cross-section area that varies from 101 sq. inches to 98 sq. inches.[8]

Although it could be argued that this shaving of element sizes on the Smithfield Street Bridge results from a need to economize on steel (at the time more expensive than wrought iron), such small savings in materials are generally of more interest to a mass producer like Berlin than to the builder of a single structure. Similar observations can be made concerning the web diagonals.[9] On the Smithfield Street Bridge, the web diagonals are matched within each panel; that is, both legs of the X-bracing are identical, whereas the cross-sections are different at Bardwell's Ferry, with the main diagonals (those pointing towards midspan while sloping downwards) being slightly larger than the counter-diagonals in general. On the Smithfield Street Bridge, the smallest sizes are in the panels closest to the support (5 x 1 in.) while the sizes increase steadily towards midspan to 6 by 1$\frac{3}{16}$ inches, an increase of 42% in cross-sectional area. At Bardwell's Ferry, the sizes of the main diagonals increase from ⅝-inch diameter to 1⅛ inches diameter, a factor of 320%, towards midspan and decrease again in the midspan panel. The counters display a similar trend. Although the changes in dimensions of the chords could possibly result from differing approaches to material economy, the differences regarding the web members can only be the result of differences in analysis and design methods.

Analysis Methods for Lenticular Trusses

The analysis of a lenticular truss is considerably more complicated than the analysis of other truss forms commonly used in bridge design, such as the parallel chord Pratt truss. This is due to the complex geometry of the chords and the web. It was surely recognized by the designers of these bridges that the lenticular truss with X-braced panels, like many other 19th-century bridge forms, is statically indeterminate; that is, an exact calculation of the forces in the truss members cannot be based on the principles of force equilibrium alone but depends also on the calculation of the deformations of the structure. It was further recognized that the exact calculation of statically indeterminate forces depends on assumptions such as no support settlement and precise sequence of erection that are unrealizable in bridge construction. To bypass the complexities of the calculation of statically indeterminate stresses, simplified procedures were developed that allowed calculation by ordinary means.[10] The assumption that only one diagonal in a panel is active at a time was particularly widespread and reflected the general idea that, under many types of loading conditions, one of the diagonals would be in compression and would buckle harmlessly out of the way.

Two general procedures for the analysis of lenticular trusses are identifiable. The first is described explicitly by Swain and, as will be shown below, is the method employed by Lindenthal in the design of the Smithfield Street Bridge, for which a stress sheet is illustrated in Figure 3.[11] It depends on the assumption that the shape of the top and bottom chords are funicular; that

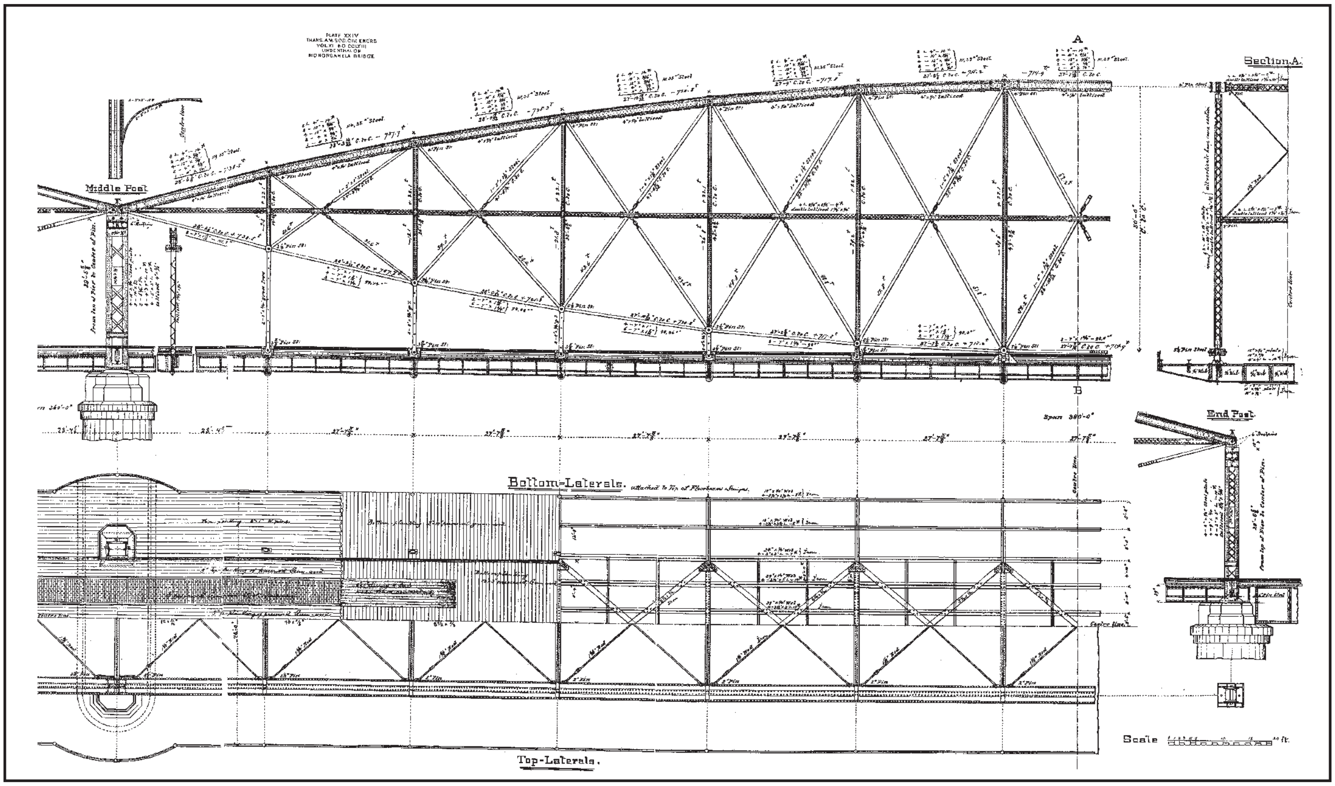


Figure 3. *Stress Sheet, Smithfield Street Bridge, Pittsburgh, Pennsylvania.* Gustav Lindenthal, "Rebuilding of the Monongahela Bridge, at Pittsburgh, Pa." *Transactions of the ASCE* XII, (Sept. 1883): plate XXIV.

is, they are shaped exactly in such a way that the full dead and rolling loads are carried by axial compression in the top chord and axial tension in the bottom chord.[12] It is true that the parabola, on which the form of the top and bottom chords of the Smithfield Street Bridge is based, is a funicular shape for a uniformly distributed load. However, for a series of equal, uniformly spaced, concentrated loads, such as the panel loads on a truss, the chords of a parabola only approximate the true funicular shape. The actual funicular shape has a steeper slope at midspan and flatter slope near the supports. The assumption that the chords are a funicular shape has two important consequences:

1. Under full dead and live load, the diagonals carry no force, and the verticals only distribute half the panel load to the upper chord by tension.
2. The horizontal component of the chord force must be constant, resulting in a larger axial force in the more steeply inclined chord members closer to the support and a lesser axial force in the nearly horizontal chord members closer to midspan.

A review of Lindenthal's stress sheet, Figure 3, shows that the top and bottom chords are designed for a constant horizontal force component of approximately 715 tons. Consequently, Lindenthal was not working with the constant top chord force that is the defining feature of the Pauli truss.

Under the assumption that the diagonal web members only carry forces due to partial loading and are stress free under full uniform loading, the web members need to be designed for partial loading conditions due to live load alone. A mathematical property of a parabolic truss under partial loading, which is used in calculations of maximum and minimum forces in the web members, is that the maximum horizontal component of the force distributed to a diagonal is a constant quantity. A third consequence then emerges of the assumption that the shape of the top and bottom chord is funicular.

3. The maximum force in both diagonals in any given panel is equal, and the horizontal component of the maximum force in each of the diagonals is constant.

Lindenthal's design summary further reflects this approximation of the behavior of the lenticular truss, showing equal maximum forces in both the diagonals in each panel, and a simple calculation reveals that each diagonal is supposed to have a constant horizontal force component of approximately 25 tons.

The strain sheet produced by an engineer of the Corrugated Metal Company for the Bardwell's Ferry Bridge (Figure 4) shows a different and more accurate analysis of a lenticular truss than Lindenthal's analysis of the Smithfield Street Bridge.[13] Under full dead and

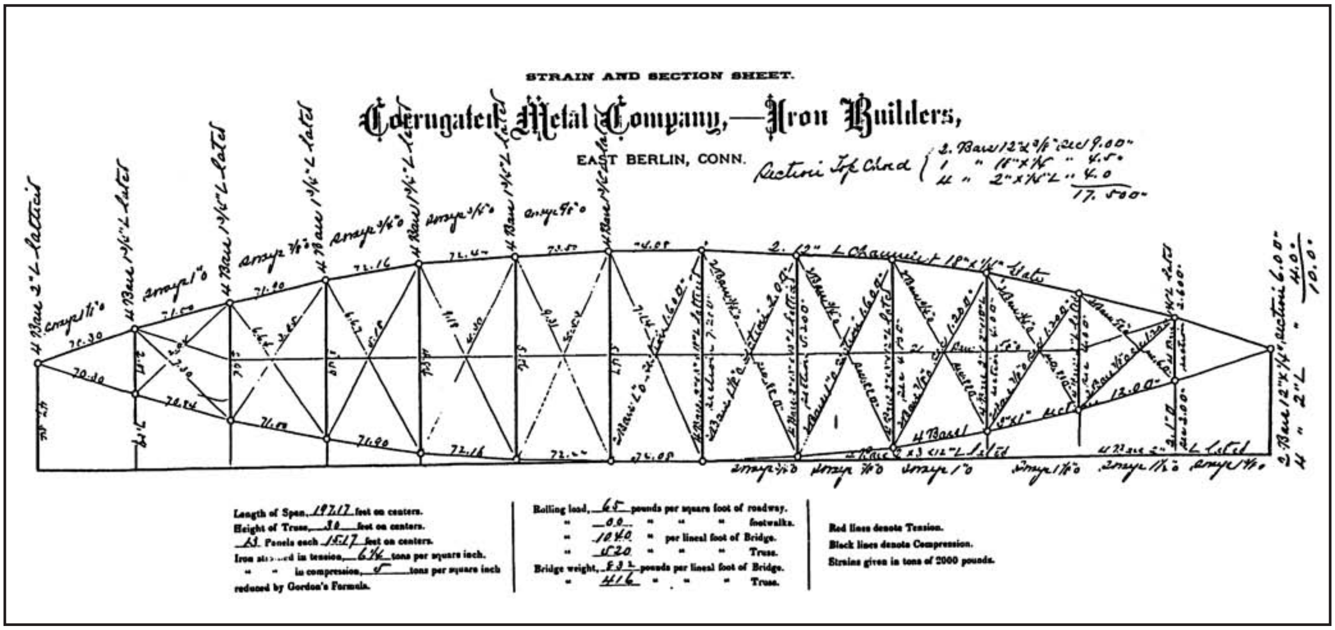


Figure 4. *Strain Sheet, Bardwell's Ferry Bridge.* Reproduced in HAER no. MA-98, "Bardwell's Ferry Bridge," 16.

rolling load, which determines the maximum forces in the top and bottom chords, the truss chord forces are found to increase towards midspan. This requires that the horizontal component of the chord force also increase slightly towards midspan.

Although the strain sheet for the Bardwell's Ferry Bridge is the only known information available on the analysis of this truss, it is possible to infer that the designer was probably employing a variant on the "method of moments" in which the truss is cut at successive intervals, and the forces in each bar are determined by a careful choice of a pole for the calculation of moment equilibrium.[14] This procedure, dubbed Ritter's method, is described by Gerber in an 1865 article on the Pauli system.[15] This method was applied to the analysis of the bar forces in the Bardwell's Ferry Bridge, and agreement was obtained between calculated values within 5% for top and bottom chord bars, within 10% for diagonals, and within 20% for verticals, except for a few bars with very small forces (Table 1). The close agreement, especially for the chords, supports the hypothesis that Ritter's method was employed in the design of this truss. It is further noted that small changes in the height of the panels, which could result from errors in the measurements reported on the HAER drawings, produce relatively large changes in the diagonal and vertical member forces.

The principal design consequence of the use of this refined analysis is that the chords, which generally are of a constant cross-section, must be designed for a greater force than in the simplified analysis procedure. The main diagonals and the verticals are subjected to greater maximum tension and compression, respectively, although the forces in the counter-diagonals are reduced somewhat. Table 2 compares the maximum forces used in the design of the elements of the Bardwell's Ferry Bridge to the forces that result from the simplified analysis method. The table indicates that the Corrugated Metal Company passed up an opportunity to design the chords for a lower force but gained a small advantage in the design of the counters.

Pauli Truss

The Pauli truss is a variant of the lenticular truss, with the top chord carefully shaped so that it has a constant force along the entire length of the truss. Whereas a uniformly loaded parabolic truss, such as the Smithfield Street Bridge, has a constant horizontal component of the chord force, the varying inclination of the chord results in a larger chord force towards the support and the least chord force at midspan. For the loads used in highway bridge design, as will be seen in the following section, the difference in overall truss shape is very slight, with a steeper inclination near the supports and a flatter chord at midspan. This shape has been described as more "cigar-shaped."[16] For railroad bridges, the additional weight of the locomotive results in a different distribution of maximum moments and in greater differences in shape between the parabolic and Pauli truss forms.

Casual use of the term "Pauli truss" has clouded the literature. Authors seem almost equally divided on whether a Pauli truss is a general lenticular truss or the variant described above. Max Becker speaks of the Pauli system but presents the design of a parabolic chord bridge.[17] Augustus J. Du Bois claims, "The form of the Pauli truss is so arranged that the maximum strains in the flanges shall be constant," and presents a fourth-order polynomial for the height of the top chord, based on the assumption that the truss is subjected to a uniformly distributed load.[18] Swain describes the Pauli truss in terms of constant strain in the flanges but does not present design details.[19] Lindenthal in his article on the Smithfield Street Bridge says, "for the channel spans, Pauli trusses were proposed,"[20] whereas the dimensions, the calculated bar forces, and the cross-sections used show that this bridge was not conceived as having a constant maximum chord force. Gerber, a close associate of Pauli's, wrote an authoritative article on the Pauli system that refers in such specific terms to the design of the Mainz Bridge that it becomes quite clear that a Pauli truss is purposely shaped in such a way that the top chord has a constant axial force.[21] Correctly, and contrary to the implication of Du Bois, Gerber notes that only *one* chord can be shaped so that the axial force under maximum loading is constant, while the other (usually the bottom chord) can only approximate this condition. The Pauli truss problem requires the designer to determine the geometry of the chord for a given force instead of determining the forces in members of a given geometry, as in the design of a parabolic bridge truss. Because of the chord geometry of a Pauli truss (a parabolic truss and a funicular truss are different), it should be possible to determine the intended chord profile of each bridge on the basis of physical measurements.

Measurements are available for the Bardwell's Ferry Bridge (in the HAER drawings for this structure) and

Table 1. Comparison of Bar Forces, Bardwell Ferry Bridge

Component	Panel	Bar force (tons) +tension -compression (by Ritter's method)	Strain sheet (tons) +tension -compression	Percentage difference
Top chord	1	68.79	70.30	2.1
	2	69.84	71.50	2.4
	3	70.52	71.90	2.0
	4	71.34	72.16	1.1
	5	72.24	72.47	0.3
	6	73.40	73.50	0.1
	7	73.37	74.08	1.0
Bottom chord	1	-68.79	70.30	2.1
	2	-67.81	-70.84	4.5
	3	-68.95	-71.00	3.0
	4	-69.88	-71.90	2.9
	5	-70.94	-72.16	1.7
	6	-72.03	-72.44	0.6
	7	-73.37	-74.08	1.0
Diagonal	2	6.30	7.3	-13.7
	3	7.22	6.64	8.7
	4	8.23	6.67	23.4
	5	8.74	9.18	-4.8
	6	9.29	9.31	-0.2
	7	7.09	7.14	-0.7
Counter-diagonal	2	2.77	2.04	35.6
	3	3.47	3.85	-9.9
	4	4.14	5.18	-20.2
	5	4.75	4.3	10.5
	6	4.86	4.53	7.3
	7	7.09	7.14	-0.7
Vertical	1	1.10	2.07	-46.8
	2	3.04	2.65	14.8
	3	4.34	3.40	27.7
	4	5.15	4.84	6.4
	5	5.63	5.16	9.0
	6	4.82	5.47	-11.9

Table 2. Comparison of Maximum Forces

Member	Design Force (tons)	Simplified Method
Top chord	74.08 (C)	66.9 (C)
Middle chord	74.08 (T)	66.9 (T)
U1-M2 (main diagonal)	7.3 (T)	4.2 (T)
U5-M6 (main diagonal)	9.3 (T)	6.9 (T)
U1-M2 (short post)	2.1 (C)	1.2 (C)
U5-M5 (long post)	5.2 (C)	4.6 (C)
U2-M1 (counter)	2.0 (T)	4.2 (T)
U6-M5 (counter)	5.5 (T)	6.9 (T)

Note: In most cases, the exact method employed on Bardwell's Ferry produces larger member forces, hence larger member sizes. This applies not only to the chords but also to most of the web diagonals.

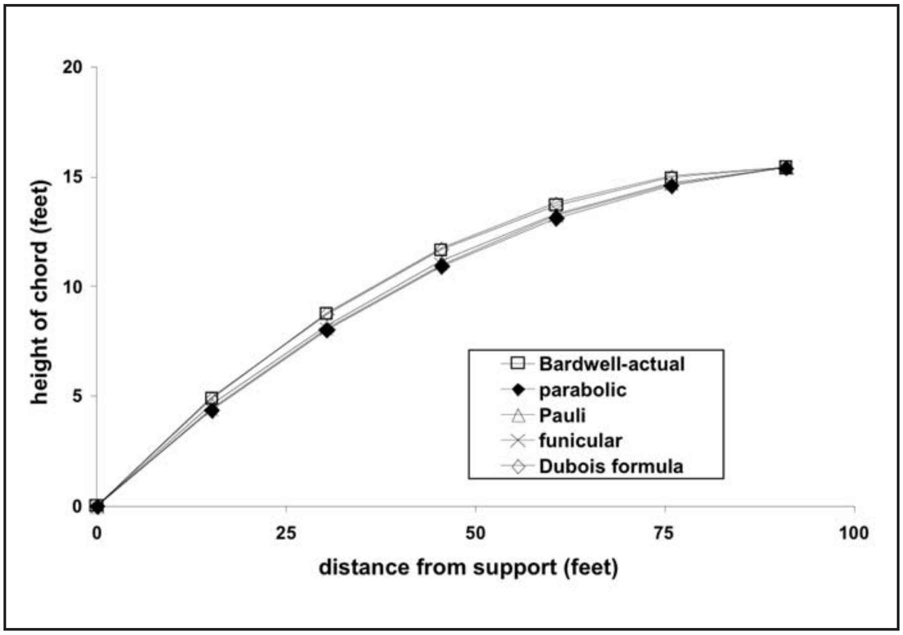
for the 1889 Pine Creek Bridge, shown in Figure 5.[22] The Pine Creek Bridge is a product of the Berlin Iron Bridge Company. It is a variant on the standard design (the web is similar to a Warren truss)—a single diagonal in each panel, alternating in direction, and designed to resist both compression and tension. Although the Douglas patent calls the chord profile of the Corrugated Metal Company bridges “elliptical,” nearly all of the company’s later claims are that this is a “parabolic” truss.[23] Significant deviations from the parabolic shape can be noted however. The Nicholson Township Bridge (1881) deviates substantially from a parabolic shape, probably because of the blacksmith work used to make larger angle changes in the chords at panel points and because the bridge has only three intermediate panel points.[24] It has not been previously noted that the measured drawings of the Bardwell’s Ferry Bridge show that the chords are not parabolic but more “cigar-shaped,” that is, flatter in the center and more steeply inclined near the supports. Figures 6 through 9 compare the actual profiles of three bridges to the profiles of a similar bridge designed exactly in a funicular, Pauli, or parabolic shape. Figure 6 shows a comparison of these profiles for the same span and midspan height as the Bardwell’s Ferry Bridge. It is surprising to see the very small difference between the funicular, parabolic, and Pauli profiles and the larger difference between the actual profile of the bridge and all three of these alternatives. The strain sheet for the Bardwell’s Ferry Bridge makes it clear that this difference was understood because the chord forces increase towards midspan and because the web diagonal descending toward midspan is considered the active diagonal. For the Pine Creek Bridge, figures 7 and 8 show that the actual bottom chord profile is very close to parabolic, while the top chord is closer to a Pauli configuration. While only one chord (usually the top chord) can actually be subjected to an equal force throughout, this raises the strong possibility that the designers of this bridge were intentionally designing a Pauli truss. For comparison, the profile of the Smithfield Street Bridge truss in shown in Figure 9. This bridge is doubtless designed as a parabolic truss. An additional trace on figures 6 through 9, called the Du Bois formula will be explained in the next section.

Analysis and Design of American Lenticular Trusses

The differences previously noted in the cross-sections of the chords along the span of the truss can be explained in terms of the results of the two types of truss analysis described in the two previous sections. The changes in the top and bottom chord sections in the Smithfield Street Bridge result from the analysis method employed by Lindenthal for this structure, in which the horizontal component of the chord force is presumed constant. As a result, the force in the chord is presumed to increase with the inclination of the chord to a maximum in the panel adjacent to the sup-

Figure 5. *Pine Creek Bridge, Jersey Shore, Pennsylvania.* Photo by author.

Figure 6. *Top chord profile of Bardwell's Ferry Bridge compared to parabolic, funicular, and Pauli chord profiles under same loading. Although the parabolic profile is determined on the basis of geometry alone, the funicular and Pauli profiles depend on the loading.* Drawing by author.



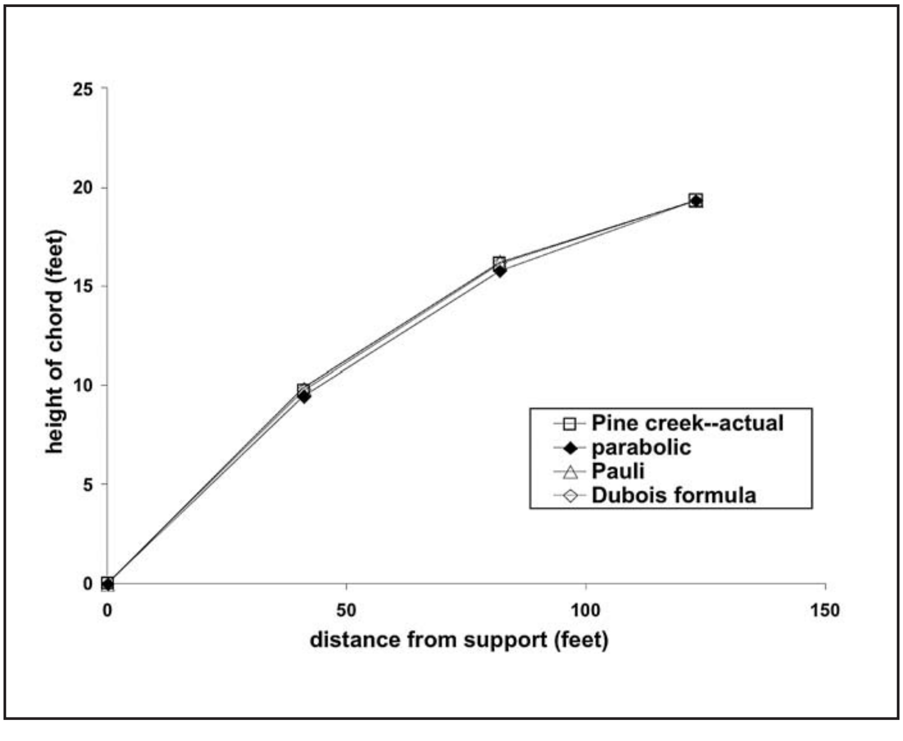


Figure 7. *Pine Creek Bridge, top chord compared to parabolic, funicular, and Pauli chord profiles under same loading.* Drawing by author.

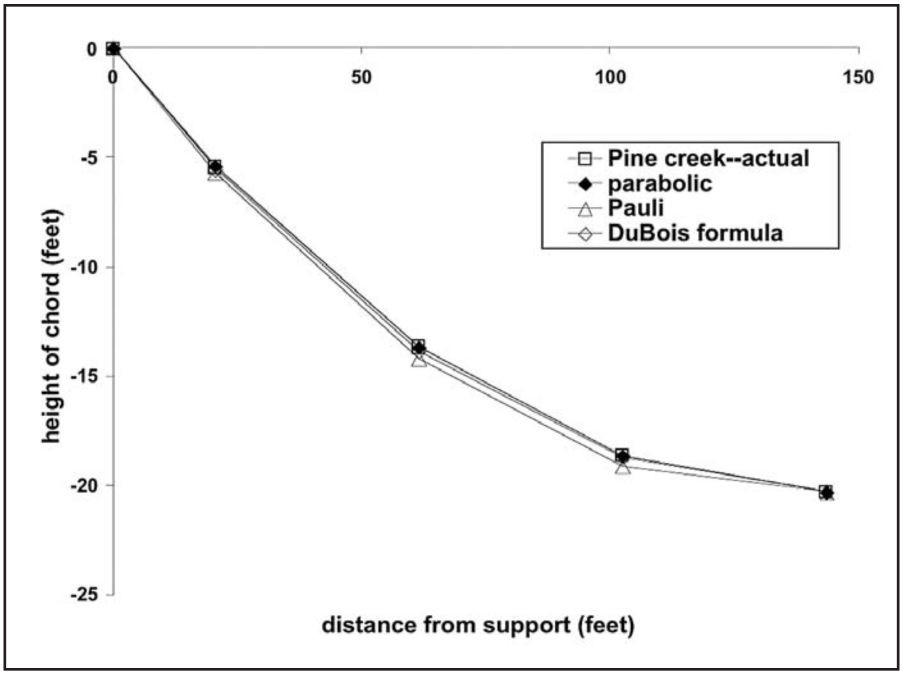


Figure 8. *Bottom chord profile of Pine Creek Bridge compared to parabolic, funicular, and Pauli chord profiles under same loading.* Drawing by author.

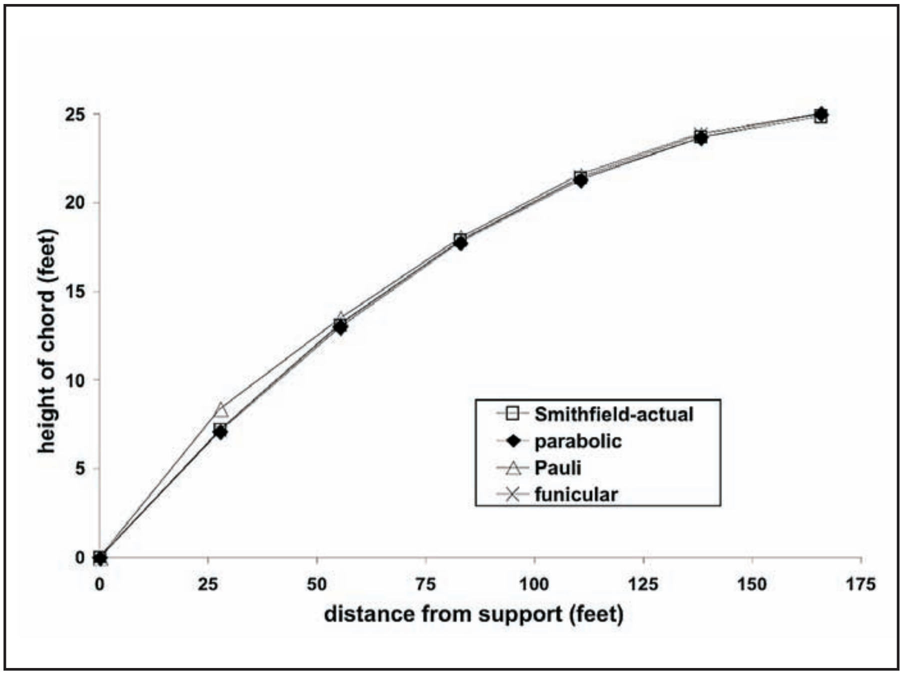


Figure 9. *Chord profile of Smithfield Street Bridge compared to parabolic, funicular, and Pauli chord profiles under same loading.* Drawing by author.

port. The member size also increases slightly from midspan to support. On the other hand, the calculated force in the Bardwell's Ferry Bridge chords is approximately constant, increasing only slightly towards midspan. The cross-section of both the top and bottom chords remains the same over the entire span of the truss. Similarly, the web diagonals and counter-diagonals at Smithfield Street are presumed to have the same maximum force and are of the same cross-section in each panel, whereas the maximum force in the main diagonal is found to be greater than in the counter-diagonal at Bardwell's Ferry. The selections of member sizes reflect these differences in analysis.

William O. Douglas's patent drawings (Figure 10) can be seen as precursors of the later designs of the Corrugated Metal Company. The patent drawings are very unsophisticated and show discernable engineering errors. Although Douglas refers to his truss form as "elliptical," three of the four elevation drawings depict top and bottom chords that form straight lines, referred to in the text of the patent as a "hipped chord," rather than any sort of curve. Douglas apparently did not see the need for counter-diagonals in every panel, as only the two center panels are provided with counters.[25] In the remaining panels, the diagonals are inclined toward midspan from the bottom chord to the top chord. This feature is shown on both the hipped trusses and the "elliptical trusses." For a hipped truss under uniform load, this is the correct inclination of the tension chords. However, it is likely that different loading conditions would produce a reversal of the forces in the diagonal and, unless the diagonals are designed for compression, would require the use of counters. For the elliptical truss, or any chord shape approaching a parabola, the web vertical and diagonal members experience very low forces under full rolling load, and tension diagonals *must* be present in both directions. Moreover, in the drawbridge version in the bottom of Figure 10, Douglas himself notes that the forces will be reversed—tension in the top chord and compression in the bottom chord in the open configuration—but he fails to extend this reasoning to the reversal of forces in the web diagonals: his draw-span configuration is thus unstable when open. All of these features—straight-line chord, reverse inclination of web diagonals, and absence of counter-diagonals—are also apparent in an article published by Douglas in 1877 (Figure 11).[26]

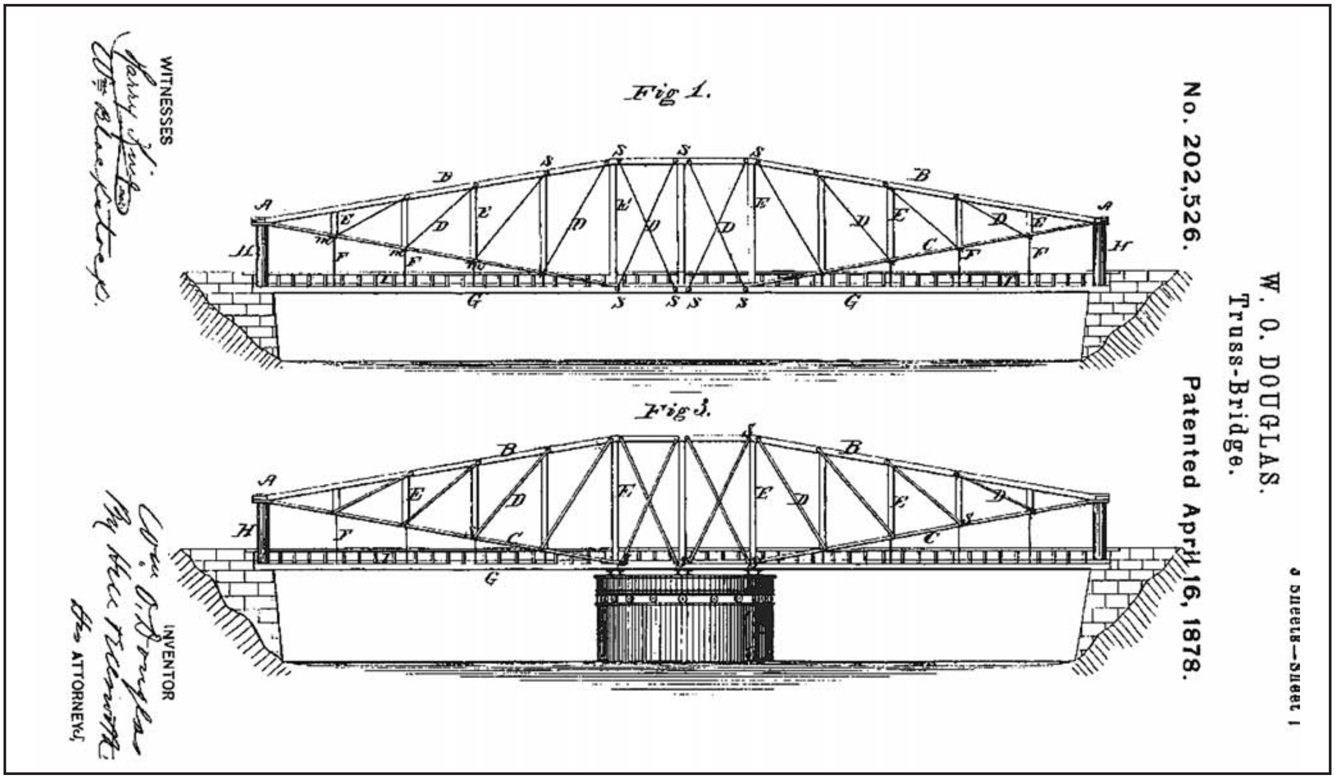


Figure 10. *Elevations for a truss bridge and a swing span.* William O. Douglas, "Improvement in Truss-Bridges," Patent no. 202,526, filed 28 Mar. 1878, issued 16 Apr. 1878.

On the other hand, all of the bridges depicted in later catalogs and all the surviving bridges have curved top chords and counters in every panel. In the design of the Bardwell's Ferry Bridge, for instance, the diagonals that descend towards the midspan of the bridge are clearly considered the main diagonals, by analogy to a parallel chord Pratt Truss, and are considered active under full loading. The most plausible explanation for the significant refinements in engineering introduced from 1878 onwards is the engagement of an experienced engineer.

It is practically certain that this change is attributable to Charles M. Jarvis[27] He was an 1877 graduate of the Sheffield Scientific School at Yale University who was employed by the Corrugated Metal Company as a civil engineer upon graduation. When the company was reorganized as the Berlin Iron Bridge Company in 1882, Jarvis was promoted to chief engineer and vice president.[28] Among Jarvis's instructors at Sheffield Scientific School was Professor Du Bois whose textbook, *The Strains in Framed Structures*, is most likely based on his lectures in civil engineering. The book includes an analytical formula for the shape of the chord of a Pauli Truss (under an implicit assumption of uniformly distributed load) and a description of Ritter's method of calculating strains in trusses.[29] The formula for the shape of the Pauli truss is particularly interesting, as its application to the geometry of the Bardwell's Ferry and Pine Creek bridges yields the as-constructed geometry of these structures (see figures 6–8). The method given by Gerber for the design of a Pauli truss loaded at panel points requires some trial and error to determine the shape of a truss with a given midspan depth.[30] Du Bois's formula, while it represents an approximation to the shape of a Pauli truss loaded at panel points, allows the immediate determination of the shape of the chord. This method was evidently used in the design of the Bardwell's Ferry and the Pine Creek bridges. While the analysis presented here and on the Bardwell's Ferry strain sheet shows that the conditions of a Pauli truss are not strictly satisfied, in that the chord force varies slightly under full loading, it is clear that the intention of the designer was to produce a Pauli truss.

Conclusions

The design of American lenticular trusses was partly based on the more familiar design of the Pratt truss,

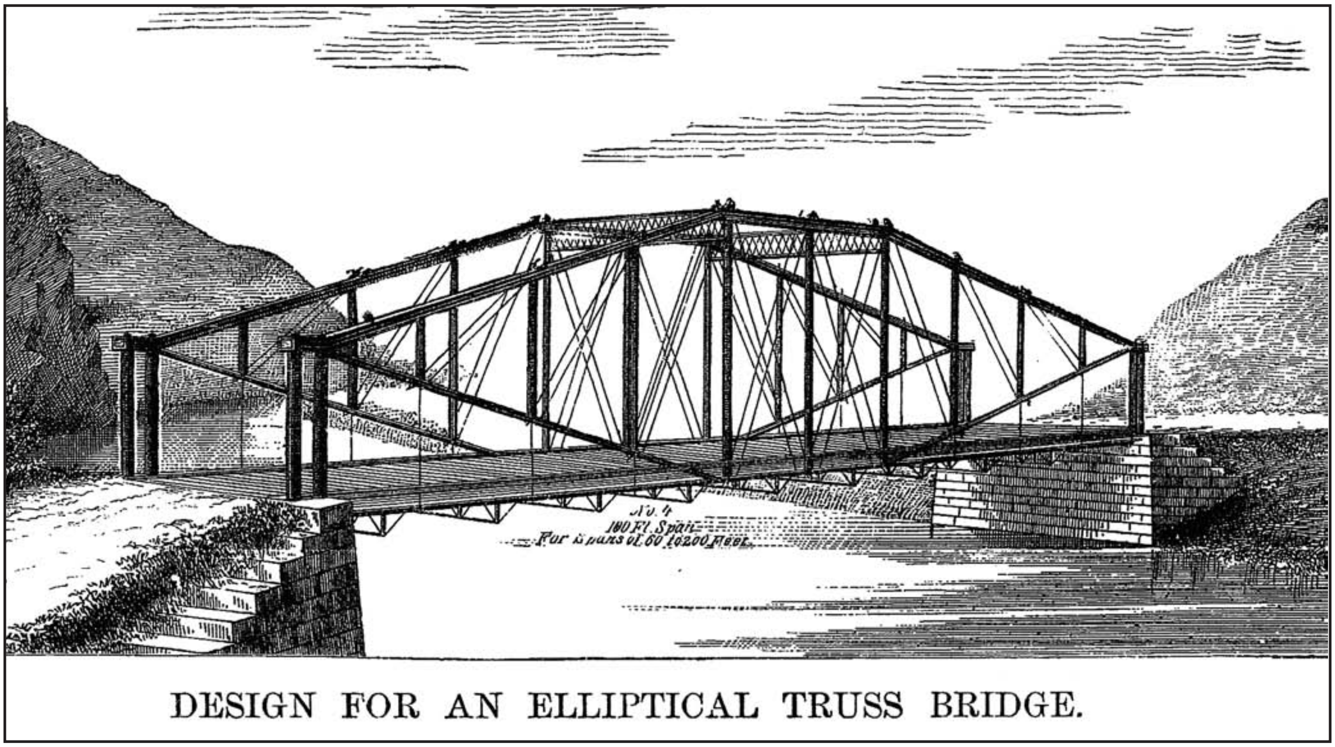


Figure 11. *Douglas's suggestion for an "Elliptical Truss Bridge."* From W. O. Douglas, "Wrought Iron Bridge Designs," *Scientific American Supplement* IV, no. 80 (1877).

with the diagonals descending in the direction from the supports to midspan considered as the main diagonals, and the opposite diagonals in each panel considered as counters.

The term Pauli truss is not interchangeable with the term lenticular truss. The Pauli truss is a specialization of the lenticular truss in which the top chord is designed for a constant maximum force and the bottom chord is allowed to assume approximately the same shape, in order to approximate a constant force along the length of the truss.

The foregoing analysis has determined that the Bardwell's Ferry Bridge and the Pine Creek Bridge were designed as Pauli trusses, based on an idealized formula for the shape of the chord in a Pauli truss. The chord force is very nearly constant, and the form of the chords of these bridges is generally closer to the ideal Pauli form than to a parabolic form. This is an interesting curiosity, in that the Berlin Iron Bridge Company literature referred to their truss type as "parabolic," when, in fact, they were designing and selling Pauli trusses. On the other hand, Lindenthal, who says in his writings that he designed the Smithfield Street Bridge as a Pauli truss, actually produced a parabolic lenticular truss.

Ritter's method, an exact method for the determining of maximum member forces in a truss when only one diagonal in each panel is active for any given loading, is probably the analysis method employed in the design of the Bardwell's Ferry Bridge. For the Smithfield Street Bridge, Lindenthal used a simplified method to determine the maximum forces in a parabolic truss. In designing a monumental one-of-a-kind structure, Lindenthal took simplifying shortcuts, while the engineers of the Corrugated Metal Company, who have a great interest in expedient design, fabrication, and construction, use a more painstaking analysis that generally results in larger forces in the components of the system. The apparently very strong connection of the design of Corrugated Metal Company and Berlin Iron Bridge Company bridges to advanced contemporary ideas in truss-analysis practice can be attributed to the education and to the influence of Jarvis, a point that merits further investigation.

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Notes

1. Martin Trautz, *Eiserne Brücken in Deutschland im 19. Jahrhundert: eine Analyse der ersten eisernen Balkenbrücken in Deutschland unter Berücksichtigung des gesellschaftlichen und politischen Hintergrundes der Zeit am Beispiel der König-Wilhelms-Rhein-Eisenbahnbrücke, der "Hammer Brücke" über den Rhein zwischen Düsseldorf und Neuss* (Düsseldorf: Werner-Verlag, 1991), 101–05, attributes the design of this bridge to Heinrich Gerber, a disciple of Friedrich August von Pauli, and describes the design of the bridge according to the Pauli system.
2. U.S. Dept. of the Interior, "Smithfield Street Bridge," 1974, HAER, no. PA-2., Prints and Photographs Division, Library of Congress, Washington, DC.
3. Berlin Iron Bridge Company, *Iron Bridges, iron roofs, beams, corrugated iron, suspension bridges, iron buildings, girders, columns, corrugated iron fire-proof doors and shutters, turntables: General iron construction* (Hartford, Conn.: Berlin Iron Bridge Co., 1891), 3–4, refers to the top and bottom chord. The HAER documentation drawings cited in this paper refer to the arch and cable of the lenticular truss as the top chord and middle chord, and to the longitudinal beams at deck level as the bottom chord. In this paper, however, the arch and cable of the lenticular truss will be designated the top chord and bottom chord of the truss. This nomenclature is more consistent with the truss action of the main load-carrying members and allows for the full variety of lenticulars, including deck trusses and trusses with the roadway at midheight.
4. "Bardwell's Ferry Bridge," 1990, HAER, no. MA-98, 6 (see n. 2).
5. See Tom Eisenhour, "The Texas Lenticulars: 1 Down, 8 Survive," *Newsletter, Society for Industrial Archeology* 16, no. 3 (Fall 1987):1–3.
6. Victor Darnell, "Lenticular Bridges from East Berlin, Connecticut," *IA: Journal of the Society for Industrial Archeology* 5, no. 1:19–32. A more recent survey of lenticular truss bridges in Massachusetts is provided by Alan Lutenegger and Amy Cerato, "Lenticular Iron Truss Bridges in Massachusetts," *Civil Engineering Practice* 20, no. 1 (Spring-Summer 2005):53–74.
7. The dimensions given below for the elements of the Bardwell's Ferry Bridge are recorded by HAER, while the dimensions for the Smithfield Street Bridge are taken from published design documents.
8. Gustav Lindenthal, "Rebuilding of the Monongahela Bridge, at Pittsburgh, Pa.," *Transactions of the ASCE* XII (September 1883):353–411, plate XXIV.
9. The vertical members in both cases have constant cross-section

and consist of lattice columns, an I-shape at Bardwell's Ferry, and a box shape at Smithfield Street.

10. See, for instance, W[illiam] E. Merrill's calculation of stresses in a post truss in *Iron Truss Bridges for Railroads. Methods of Calculating Strains, with a Comparison of the Most Prominent Truss Bridges, and New Formulas for Bridge Computations; also, the Economical Angles for Struts and Ties* (New York: D. Van Nostrand, 1870), 85–92; John Alexander Low Waddell, *The Designing of Ordinary Iron Highway Bridges,* 4th ed. (New York: J. Wiley, 1889), 44–47, for calculation of stresses in a multiple intersection truss; and the discussion in "Upper Bridge at Slate Run," HAER, no. PA-460, 7 (n. 2).
11. George Fillmore Swain, *Structural Engineering* (New York: J. Wiley, 1924), 50–52; Lindenthal, "Rebuilding," 353–411, (see n. 8). See in particular the stress sheet on plate XXIV.
12. Although the term "rolling loads" implies a series of concentrated loads to modern engineering, in this context rolling loads refer to a uniformly distributed deck load applied over a portion of the deck. The strain sheet for Bardwell's Ferry Bridge in Figure 4, for example, gives the rolling load in pounds per square foot. The portion of the deck chosen for this load application varies from member to member in such a way as to produce the maximum load effect.
13. "Bardwell's Ferry Bridge," HAER no. MA-98, 16 (see n. 4).
14. Lutenegger and Cerato, "Lenticular Iron Truss," 60 (see n. 6) suggest that a graphical approach was used. Although graphical methods of truss analysis were widespread in this time period, the two-decimal-place precision shown on the strain sheet (Figure 4) is impossible to achieve in any graphical method.
15. Hans Gerber, "Ueber Berechnung der Brückenträger nach System Pauli," *Zeitschrift des Vereines deutscher Ingenieure* IX, no. 7 (July 1865):463–86. Gerber refers to the calculation procedure for truss member forces as "Ritter's Methode," referencing August Ritter, *Elementäre Theorie und Berechnung eiserner Dach- und Brücken-Constructionen* (Hanover, Germany: C. Rümpler, 1873), while presenting the method of determining the bar geometry resulting in equal chord forces as the Pauli system.
16. Trautz, *Eiserne Brücken* (see n. 1).
17. Max Becker, *Der Brückenbau in seinen ganzen Umfange und mit besonderer Rücksicht auf die neuesten Constructione*, 4th ed. (Stuttgart, Germany: Mäcken, 1873). Becker's 1853 edition does not mention Pauli trusses.
18. A. Jay Du Bois, *The Strains in Framed Structures,* 7th ed. (New York: John Wiley and Sons, 1891), 60.
19. Swain, *Structural Engineering*, 52 (see n. 11).
20. Lindenthal, "Rebuilding," 355 (see n. 8).
21. Gerber, "Ueber Berechnung," 472–78 (see n. 15).
22. Pennsylvania Bridge Management System Number 41300300100000. This bridge is discussed in "Pennsylvania Historic Bridges Recording Project II," forthcoming, HAER no. PA-614 (see n. 2).
23. William O. Douglas, "Improvement in Truss-Bridges," Patent no. 202,526, Filed 28 Mar. 1878, issued 16 Apr. 1878; also reproduced in "Bardwell's Ferry Bridge," HAER, no. MA-98, 21-2 (see n. 4).
24. "Nicholson Township Lenticular Bridge," HAER, no. PA-468 (see n. 2).
25. William Douglas's statement that "The diagonals … are preferably arranged in pairs, but this is not absolutely essential...." does not refer to counter-diagonals but to the balancing of diagonal forces on each side of the chord. The advertisement, figure 10, shows the main diagonals in pairs, absent counter-diagonals (also see n. 23).
26. William O. Douglas, "Wrought Iron Bridge Designs," *Scientific American Supplement*, IV, no. 80 (1877): 1262–65.
27. According to Robins Fleming of American Bridge Company (the Berlin Iron Bridge Company's successor) in a letter to the editor of *Engineering News-Record* 100, no. 19 (19 May 1928):748–79, "Two or three of these bridges had been built by the Corrugated Metal Company, a small manufacturing firm at East Berlin, Conn., but they were crude affairs, and not until the company acquired the services of Charles M. Jarvis, of Binghamton, as chief engineer, did the company flourish."
28. "Obituary Record of Yale Graduates, 1920–1921," *Bulletin of Yale University*, 19th Series, no. 22 (1 August 1921):198–200.
29. August Ritter's method is described and referenced on p. 28. According to R. H. Chittenden, *History of the Sheffield Scientific School of Yale University* (New Haven: Yale Univ. Press, 1928), 217–19, Jay Du Bois earned a doctorate from Sheffield Scientific School in 1873 and spent two years "in the study of mechanics" in Germany before assuming the post of professor of civil engineering at Yale in 1877. Du Bois is listed as an instructor in the catalog from Charles Jarvis's freshman year 1874–75.
30. See n. 15.